Stationarity of inflation and predictions of quantum cosmology

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We describe several different regimes which are possible in inflationary cosmology. The simplest one is inflation without self-reproduction of the Universe. In this scenario the universe is not stationary. The second regime, which exists in a broad class of inflationary models, is eternal inflation with the self-reproduction of inflationary domains. In this regime local properties of domains with a given density and given values of fields do not depend on the time when these domains were produced. The probability distribution to find a domain with given properties in a self-reproducing universe may or may not be stationary, depending on the choice of an inflationary model. We give examples of models where each of these possibilities can be realized, and discuss some implications of our results for quantum cosmology. In particular, we propose a new mechanism which may help solve the cosmological constant problem.

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I. INTRODUCTION

According to the first versions of inflationary theory, inflation was an important but extremely short intermediate stage of the evolution of the Universe. Later it was discovered that in many versions of this theory inflation never ends because of the process of self-reproduction of inflationary domains. This realization dramatically changed our point of view on the fate of the Universe and on its global structure [1].

Self-reproduction of the inflationary universe is possible both in the old inflationary scenario [2] and in the new inflationary scenario [3, 4]. However, the significance of the existence of this regime was not fully realized until it was shown to occur in the chaotic inflation scenario [5]. In the simplest versions of this scenario inflationary domains may jump for an indefinitely long time at densities close to the Planck density [6]. Quantum fluctuations of all physical fields and metric are extremely large in such domains. As a result, the Universe becomes divided into exponentially large domains filled with matter with all possible types of symmetry breaking [1], and maybe even with different types of compactification of space-time [7]. Variations in the laws of low-energy physics in different domains are typically discrete, such as the change of symmetry breaking from SU(5) to SU(3)×SU(2)× U(1). However, continuous changes are also possible, such as the change of an effective gravitational constant in the inflationary Brans-Dicke cosmology [8, 9].

This gave us a possibility to justify the weak anthropic principle in the context of inflationary cosmology, and even to speculate about the Darwinian approach to particle physics and cosmology [10] and about some kind of natural selection of the "constants" of particle physics which lead to a greater physical volume of those domains which can be occupied by observers of our type [1, 6, 8].

The next step toward a justification of the anthropic principle in quantum cosmology was made in [11], after the appearance of the baby Universe theory [12]. This theory was based on two basic assumptions. The first assumption is that the coupling constants may take different values in different quantum states of the universe. This idea is very intriguing and it may have a good chance of being correct. The second (and quite independent) assumption was a particular choice of measure on the space of all quantum states of the universe. The choice advocated in [12] effectively was based on the exponentiation of the square of the Hartle-Hawking wave function of the Universe $\Psi \sim \exp\Big(\frac{3M_P^4}{16V}\Big)$, where V is the vacuum energy (or the effective potential of the scalar field ϕ) [13, 14]. This gave the probability to live in the Universe with the non-negative cosmological constant $\Lambda = \frac{8\pi}{M_P^2}V$: $P(\Lambda) \sim \exp\Big(\exp\frac{3\pi M_P^2}{\Lambda}\Big)$. This probability distribution is peaked at $\Lambda = 0$. However, this result is a consequence of the "wrong" (negative) sign of the gravitational action, which makes calculations unreliable. Moreover, an extended version of this approach suggests that the probability to obtain negative cosmological constant is even higher [15].

Another possibility would be to exponentiate the square of the wave function $\Psi \sim \exp\Big(-\frac{3M_P^4}{16V}\Big)$, which was first suggested by one of the present authors [16], see also [17]. However, as was argued in [5], both wave functions are related to the probability P_c for some events to happen at a given point [18], without taking into account that different parts of the Universe with different values of V grow at a different rate. It may be natural therefore to use the measure of probability P_p introduced in [5], which is proportional to the physical volume of those parts of the Universe where such events may happen. This, combined with the first assumption of the baby Universe theory, may give us a possibility to justify

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not only the weak anthropic principle, but the strong anthropic principle as well [1, 11]. Moreover, the use of the probability distribution P_p can make anthropic considerations much more precise and quantitative.

Various properties of the probability distribution $P_p(\phi, t)$ to find a given field ϕ in a unit physical volume have been investigated in [5, 6, 19]. It was found that this distribution has an important advantage over other probability distributions. In a sufficiently big universe the normalized probability distribution $P_p(\phi, t)$ in many realistic theories very rapidly approaches a stationary regime $P_p(\phi)$, which does not depend at all on the unsettled issues related to the probability of quantum creation of the universe and on the choice between the Hartle-Hawking wave function $\Psi \sim \exp\left(\frac{3M_P^4}{16V}\right)$ and the tunneling wave function $\Psi \sim \exp\left(-\frac{3M_P^4}{16V}\right)$ [6, 20].

This encouraging result indicates that the stationary distribution $P_p(\phi, t)$ plays a very important role in quantum cosmology. However, this approach has its own problems. For example, as was shown in [6, 8], the shape of the probability distribution P_p may depend on the choice of the time parametrization. The reason can be understood as follows. Let us consider two *infinite* boxes, one with apples, another with oranges. One can pick up one apple and one orange, then again one apple and one orange, etc. This may give an idea that the number of apples is equal to the number of oranges. But one can equally well take each time one apple and two oranges, and conclude that the number of oranges is twice as large as the number of apples. The main problem here is that we are making an attempt to compare two infinities, and this gives an ambiguous result. Similarly, the total volume of a self-reproducing inflationary universe diverges in the future. When we make slices of the Universe by hypersurfaces of constant time t, we are choosing one particular way of sorting out this infinite volume. If one makes the slicing in a different way, the results will be different. This forces us to be very cautious when using various probability distributions in quantum cosmology [8].

Nevertheless, it is very tempting to consider various cosmological problems using the probability distribution P_p as a guide. In many cases one can get simple and unambiguous results. In some other cases, especially when one investigates speculative possibilities related to the baby Universe theory and the choice between different coupling constants, one should not take the corresponding results too seriously. This being said, it would be most interesting to see whether this approach is capable, at least in principle, to give us any new insight into such profound problems as the cosmological constant problem.

The first attempts to study this question were not very enlightening. The best result which one could obtain was to reduce the interval of possible values of the vacuum energy V_0 from $-10^{94}\ \mathrm{g\,cm^{-3}} \le V_0 \le 10^{94}\ \mathrm{g\,cm^{-3}}$ to $-10^{-29}\ \mathrm{g\,cm^{-3}} \le V_0 \le 10^{-27}\ \mathrm{g\,cm^{-3}}$ [11]. This was achieved by justifying anthropic bounds in the context of inflationary cosmology. Note that the constraint $-10^{-29}\ \mathrm{g\,cm^{-3}} \le V_0$ follows from the fact that the Universe with the negative vacuum energy $V_0 < -10^{-29}\ \mathrm{g\,cm^{-3}}$ would collapse within 10^{10} yr. This constraint does not differ very much from the observational constraints on the vacuum energy $|V_0| \le 10^{-29}\ \mathrm{g\,cm^{-3}}$. The anthropic constraint $V_0 \le 10^{-27}\ \mathrm{g\,cm^{-3}}$ on the *positive* cosmological constant follows from the theory of galaxy formation [21]. Unfortunately, it allows the vacuum energy to be about 2 orders of magnitude greater than $10^{-29}\ \mathrm{g\,cm^{-3}}$. This disagreement remains the most difficult part of the cosmological constant problem. We will argue in this paper that under certain conditions this part of the problem can be resolved.

Another problem to be discussed is related to the recent argument of Refs. [22, 23] that in the context of inflationary quantum cosmology it is most probable that the density perturbations are produced by topological defects. This argument is related to the baby Universe theory, the possibility to choose between the theories with different coupling constants, and the probability distribution P_p. We will try to formulate this argument in a more exact form and examine its validity, taking into account the results of Ref. [6].

In order to compare theories with different coupling constants (and different values of the cosmological constant) in the context of quantum cosmology, one should know first of all how inflation can be realized in each of them. In particular, self-reproduction of the universe and stationarity of the probability distribution P_p are not generic properties of all inflationary models. There are some inflationary models where self-reproduction does not occur, while there are other in which it does, but the probability distribution P_p is not stationary: it constitutes what we called a runaway solution [8, 9]. In the main part of this paper we will consider a large class of inflationary models where each of these regimes can be realized.

In Sec. II we will discuss the main features of the chaotic inflation scenario in the simple theory of a scalar field ϕ minimally coupled to gravity, with the effective potential $\frac{\lambda}{4}\phi^4$ [24]. In this discussion we will follow refs. [5, 6]. In Sec. III we will consider the same model, but with the scalar field ϕ nonminimally coupled to gravity due to the term $\frac{1}{2}\xi\phi^2 R$ in the Lagrangian. Models of this type have been extensively studied by many authors, see, e.g., [25, 26]. However, the theory of self-reproduction of the universe and the behavior of the distribution P_p in these models have not been addressed so far. Meanwhile, as we will see, this behavior can be quite nontrivial. Depending on the value of the coupling constant ξ, each of the regimes mentioned in the previous paragraph can be realized in these models. In Sec. IV we will generalize this model by including one-loop quantum gravity corrections (conformal anomaly). The model we will consider is a hybrid of the standard chaotic inflation scenario (with an arbitrary coupling $\frac{1}{2}\xi\phi^2 R$) and the Starobinsky model [27]. We will show, in particular, that one of the inflationary branches in Starobinsky model, which had been considered unphysical for the reason that the Hubble constant on this branch was growing rather than decreasing, may have a very interesting interpretation when taking into account the self-reproduction of the inflationary universe.

The results obtained in Secs. II–IV are of some interest independently of the speculative discussion contained in the last part of our paper, Sec. V, where we compare various theories in the context of quantum cosmology. To avoid possible misunderstandings we should emphasize from the very beginning that in this paper the discussion is carried out in the context of the baby Universe theory. We will compare different quantum states of the Universe (we will call them different "Universes"), which are described by theories with different coupling constants. This approach differs considerably from the more conventional approach developed in [6, 8], where different exponentially large causally disconnected parts of the same Universe, which may have different laws of low-energy physics inside each of them, have been compared to each other. The reason for this difference is that the total volume of different Universes may grow at a different rate, depending on the coupling constants, vacuum energy, etc. On the other hand, as it was shown in [6, 8], the total volume of all parts of the Universe described by a stationary probability distribution P_p grows at the same rate. Therefore when comparing different Universes the main effect may arise from comparing the rates of expansion for various values of coupling constants. Meanwhile, when considering different parts of the same Universe, the relative fraction of the volume of a stationary self-reproducing Universe in a given state (i.e., in a state with given fields, given effective coupling constants, etc.) is controlled by the normalized probability distribution P_p. Comparing different parts of the same Universe has a much simpler interpretation than the speculative possibility of comparing different quantum states of the Universe. However, we believe that some "theoretical experiments" with the baby Universe theory may be useful, since they allow us to look at many problems of quantum cosmology from a new perspective. In particular, in Sec. V we will consider inflation in the Starobinsky model. We will argue that if the cosmological constant in this model is non-negative, then it is most probably zero.

II. ELEMENTARY CHAOTIC INFLATION MODEL

In this section we will describe the classical evolution of the inflaton field with the effective potential $V(\phi) = \frac{\lambda}{4}\phi^4$. The equations of motion during inflation in this theory can be written as

$$H = \left(\frac{8\pi V(\phi)}{3M_P^2}\right)^{1/2} = \left(\frac{2\pi\lambda}{3}\right)^{1/2} \frac{\phi^2}{M_P} ,$$
$$\dot\phi = -\frac{V'(\phi)}{3H} = -\left(\frac{\lambda}{6\pi}\right)^{1/2} M_P\,\phi . \qquad (1)$$

According to these equations, the inflationary regime ($|\dot H| < H^2$) occurs at $\phi > \phi_e$, where $\phi_e = M_P/\sqrt{\pi}$. However, these equations are valid only at densities smaller than the Planck density, $V(\phi) < M_P^4$, or $\phi < \phi_P = (\lambda/4)^{-1/4} M_P$. We will call ϕ_e the end of inflation boundary and ϕ_P the Planck boundary.

The inflaton field fluctuates in de Sitter space during a time interval $\Delta t = H^{-1}$ with an amplitude approximately equal to the Gibbons-Hawking temperature:

$$\delta\phi = \frac{H}{2\pi} . \qquad (2)$$

Quantum fluctuations then act on the coarse-grained background field as stochastic forces, producing a Brownian motion on the value of the inflaton field. In average, the value of the scalar field at any given point will follow the classical evolution toward the end of inflation. However, those rare domains in which the inflaton field grows due to quantum fluctuations will inflate more, since the rate of expansion H is proportional to ϕ^2. Beyond a certain value of the inflaton field $\phi = \phi_s$, for which the amplitude of quantum fluctuations becomes larger than its change due to classical motion in the same time interval, $\delta\phi > \Delta\phi = \dot\phi H^{-1}$, we enter the regime of self-reproduction of the Universe. In the case of a simple quartic potential, such a value is given by $\phi_s = (2\pi\lambda/3)^{-1/6} M_P$.

The Brownian motion of the inflaton during the self-reproduction of the Universe can be described in the physical frame, which takes into account the growth of the proper volume of the inflationary domain, with an ordinary diffusion equation [19, 6]:

$$\frac{\partial\mathcal{V}}{\partial t} = \frac{\partial}{\partial\phi}\left(\frac{H^{3/2}}{8\pi^2}\frac{\partial}{\partial\phi}\left(H^{3/2}\mathcal{V}\right) + \frac{V'}{3H}\,\mathcal{V}\right) + 3H\mathcal{V} , \qquad (3)$$

where $\mathcal{V}(\phi,t)$ is the total volume of all domains containing scalar field ϕ. In the terminology of Ref. [6], this is the non-normalized probability distribution P_p.

It is possible to find solutions to this equation subject to the appropriate boundary conditions at the Planck boundary and the end of inflation. Such solutions are generically of the form

$$\mathcal{V}(\phi,t) = e^{\alpha t}\, P_p(\phi) , \qquad (4)$$

where α is some constant and P_p is a time-independent normalized probability distribution to find a field ϕ in a unit physical volume. In case that the dependence of $\mathcal{V}(\phi,t)$ on ϕ and t can be factorized as in Eq. (4) we will speak about stationary solutions for $\mathcal{V}(\phi,t)$ and for $P_p(\phi)$. Dependence of these solutions on the conditions at the boundary where inflation ends is exponentially weak [6]. However, in general, solutions strongly depend on the boundary conditions at large ϕ. The simplest boundary condition one can impose is $\mathcal{V}(\phi_P) = 0$. One can argue that inflation ceases to exist at $V(\phi) > M_P^4$ because of large quantum fluctuations [6]. An advanced version of this argument was recently given in [22]; we will consider it in Sec. IV. One can show that the stationary solution for $\mathcal{V}$ with the boundary condition $\mathcal{V}(\phi_{\max}) = 0$ (whatever is the value of the $\phi_{\max} \gg \phi_e$) is given by [6]

$$\mathcal{V}(\phi,t) \sim e^{d(\lambda)H_{\max}(\lambda)t}\, P_p(\phi) . \qquad (5)$$

The coefficient $d(\lambda)$ in the chaotic inflation scenario can be interpreted as a fractal dimension of inflationary domains at the upper boundary $\phi = \phi_{\rm max}$ [6]. (For a discussion of the fractal dimension in the context of the new inflationary scenario see [28].)

If the upper boundary $\phi_{\rm max}$ coincides with the Planck boundary ϕ_P defined by the condition $V(\phi_P) = M_P^4$, then the distribution $\mathcal{V}$ grows as

$$\mathcal{V}(\phi, t) \sim e^{(3-f(\lambda))\sqrt{\frac{8\pi}{3}}M_P t}\ P_p(\phi)\ , \tag{6}$$

where $f(\lambda) = 3 - d(\lambda)$. Another useful form of Eq. (6) is

$$\mathcal{V}(\rho, t) \sim e^{[3-f(\lambda)]\sqrt{\frac{8\pi}{3}}M_P t}\ P_p(\rho)\ , \tag{7}$$

where $\mathcal{V}(\rho, t)$ is the total volume of all domains with a given density and $P_p(\rho)$ is the probability distribution to find a domain of a unit volume containing matter with density ρ. If one divides these equations by the ϕ-independent factor $e^{[3-f(\lambda)]\sqrt{\frac{8\pi}{3}}M_P t}$ one obtains the normalized stationary probability distribution P_p discussed in [6]. In this paper, however, we will often talk about the unnormalized distributions (5), since they show the growth of the total volume of all domains filled with a given field ϕ [29].

It is important to study how the fractal dimension $d(\lambda)$ depends on the coupling constant λ [6]:

λ	1	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}
d	0.9719	1.526	1.915	2.213	2.438	2.604	2.724

As we see, $d(\lambda)$ grows with decreasing λ toward the usual space dimension 3. This means that $f(\lambda)$ decreases with λ; it can be shown that $f(\lambda)$ vanishes in the limit $\lambda \to 0$.

Note that the distributions $\mathcal{V}(\phi, t)$ and P_p depend on the choice of time parametrization. For example, instead of usual time t measured by observers by their clock, one can use "time" $\tau = \ln \frac{a(x,t)}{a(x,0)} = \int H(\phi(x,t), t)\ dt$. Here $a\,(x,t)$ is a local value of the scale factor in the inflationary universe. Obviously, the time τ measures the logarithm of the local expansion of the Universe. Solution of the diffusion equation for $\mathcal{V}(\phi, \tau)$ is also stationary, but it looks slightly different [6]:

$$\mathcal{V}(\phi, \tau) \sim e^{(3-1.1\sqrt{\lambda})\tau}\ \tilde{P}_p(\phi)\ , \tag{8}$$

which corresponds to a fractal dimension $3 - 1.1\sqrt{\lambda}$. In what follows it will be important for us, that even though the distributions $\mathcal{V}(\phi, t)$ and $\mathcal{V}(\phi, \tau)$ differ from each other, both share the same property: they grow at large t (at large τ) with a rate which increases as λ goes to zero.

One should emphasize [6] that the factor $e^{\alpha t} \sim e^{d(\lambda)H_{\rm max}(\lambda)t}$ in (5) [as well as the factor $e^{(3-1.1\sqrt{\lambda})\tau}$ in (8)] gives the rate of growth of the combined volume of all domains with a given field ϕ (or of all domains containing matter with a given density) *not only at very large ϕ, where quantum fluctuations are large, but at small ϕ as well, and even after inflation* [30]. This result may seem absolutely unexpected, since the volume of each particular inflationary domain grows like $e^{3H(\phi)t}$, and after inflation the law of expansion becomes completely different. One should distinguish, however, between the growth of each particular domain, accompanied by a decrease of density inside it, and the growth of the total volume of all domains containing matter with a given (constant) density. In the standard big bang theory the second possibility did not exist, since the energy density was assumed to be the same in all parts of the Universe ("cosmological principle"), and it was not constant in time.

The reason why there is a universal expansion rate (5) can be understood as follows. Because of the self-reproduction of the Universe there always exist many domains with $\phi \sim \phi_{\rm max}$, and their combined volume grows almost as fast as $e^{3H_{\rm max}t}$. Then the field ϕ inside some of these domains decreases. The total volume of domains containing some small field ϕ grows not only due to expansion $\sim e^{3H_{\rm max}t}$, but mainly due to the unceasing process of expansion of domains with large ϕ and their subsequent rolling (or diffusion) toward small ϕ. The above-mentioned universality of the expansion law will play a crucial role in our discussion of quantum cosmology in Secs. V and VI. In what follows we will turn to other models, where this law may or may not hold.

III. NONMINIMAL COUPLING TO GRAVITY

In this section we will describe the classical evolution of the inflaton field with a generic chaotic potential, coupled to the curvature scalar with a small nonminimal coupling ξ:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{16\pi}R - \frac{1}{2}\xi\phi^2 R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]\ . \tag{9}$$

In this theory the effective Planck mass takes the form $M_P^2(\phi) = M_P^2 - 8\pi\xi\phi^2$. We can write the equations of motion for the homogeneous field ϕ during inflation, in the slow roll-over approximation, as

$$\begin{aligned} R \simeq 12H^2 &= \frac{32\pi V(\phi)}{M_P^2 - 8\pi\xi\phi^2}\ , \\ (1-6\xi)\ 3H\dot{\phi}\phi &= 4V(\phi) - \phi V'(\phi) - \frac{M_P^2}{8\pi}\ R\ . \end{aligned} \tag{10}$$

We will consider two different cases: $\xi > 0$ and $\xi < 0$. For simplicity we will assume that $|\xi| \ll 1/6$.

1. Case $\xi > 0$. In this case there is a clear bound on the inflaton field,

$$\phi < \phi_c \equiv \frac{M_P}{\sqrt{8\pi\xi}}\ , \tag{11}$$

in order that gravity be attractive, i.e., $M_P^2(\phi) = M_P^2 - 8\pi\xi\phi^2 > 0$. We will consider a typical potential of chaotic inflation, $V(\phi) = \lambda\phi^4/4$. The equations of motion (10) are then

$$3H^2 = \frac{2\pi\lambda\phi^4}{M_P^2 - 8\pi\xi\phi^2}\ , \qquad 3H\dot{\phi}\phi = -\frac{\lambda\phi^4 M_P^2}{M_P^2 - 8\pi\xi\phi^2}\ . \tag{12}$$

The Planck boundary is given by

$$\phi_P^2 = \frac{2M_P^2}{\sqrt{\lambda} + 16\pi\xi} \ . \tag{13}$$

Note that $\phi_P < \phi_c$; in particular, $\phi_P = \phi_c\left(1 - \frac{\sqrt{\lambda}}{32\pi\xi}\right)$ for $\sqrt{\lambda} \ll 16\pi\xi$.

Therefore, if $\sqrt{\lambda} < 16\pi\xi \ll 1$, the inflaton field will stop just before ϕ_c. On the other hand, the range of inflation corresponds to $|\dot{H}| < H^2$, which gives

$$\frac{M_P^2}{\pi} < \phi^2 < \phi_i^2 \ , \tag{14}$$

where the upper boundary of the inflationary regime is given by $\phi_i = \phi_c\,(1 - 2\xi)$. Therefore, in order that inflation ends before reaching Planck boundary (13), we require $\sqrt{\lambda} < 64\pi\xi^2$. The classical motion of the coarse-grained inflaton field is affected by its quantum fluctuations $\delta\phi = \frac{H}{2\pi}$.

An inflationary domain of the universe will enter the self-reproduction regime when the amplitude of quantum fluctuations of the inflaton field in that domain $\delta\phi$ is larger than the corresponding classical motion $\Delta\phi = \dot{\phi}H^{-1}$ in the interval $\Delta t = H^{-1}$. This happens for $\phi > \phi_s$, where

$$\phi_s^2 = \phi_c^2\left(1 - \frac{\lambda}{768\pi^2\xi^3}\right) \ . \tag{15}$$

Domains with $\phi > \phi_s$ will inflate and produce more domains with even higher values of the inflaton, until they reach the upper boundary of inflation ϕ_i.

To investigate various regimes which are possible in the model (9) with $\xi > 0$ one should compare ϕ_e, ϕ_i, ϕ_s, and ϕ_P for various relations between λ and ξ. One can show that inflation does not exist at all for $\xi \gtrsim 1/6$. In the case $\xi \ll 1/6$, which we are considering in this paper, inflation occurs at $\phi > \phi_e = M_P/\sqrt{\pi}$. There exist three different regimes.

(1) $\sqrt{\lambda} < 64\pi\xi^2$. In this case $\phi_e < \phi_i < \phi_s < \phi_P < \phi_c$. Therefore there is inflation but no self-reproduction because $\phi_i < \phi_s$.

(2) $64\pi\xi^2 < \sqrt{\lambda} < 16\pi\xi$. In this case $\phi_e < \phi_s < \phi_P < \phi_i < \phi_c$. There is inflation and self-reproduction.

(3) $\sqrt{\lambda} > 16\pi\xi$. In this regime $\phi_i > \phi_c$ and we recover the usual results of chaotic inflation with a quartic potential.

To be more accurate, when $\sqrt{\lambda}$ grows near $64\pi\xi^2$, it has three different critical values when ϕ_i, ϕ_s, and ϕ_P change their mutual positions from $\phi_i < \phi_s < \phi_P$ to $\phi_s < \phi_P < \phi_i$. However, these critical values do not differ much from $64\pi\xi^2$.

In this theory there are no runaway solutions since the quantum diffusion of the inflaton stops just before the critical value (11). For arbitrary potentials we may or may not have inflation, but there will never be runaway solutions, even if the parameters allow for a Planck boundary, because of the bound (11).

2. Case $\xi < 0$. The Planck mass in this case is given by $M_P^2(\phi) = M_P^2 + 8\pi|\xi|\phi^2$. Thus, it is always positive, and there is no bound on ϕ. In fact, for large values of the inflaton field, $M_P(\phi)$ will become dominated by its evolution.

Let us consider again $V(\phi) = \lambda\phi^4/4$, and later comment on alternatives. The equations of motion are

$$\begin{aligned} 3H^2 &= \frac{2\pi\lambda\phi^4}{M_P^2 + 8\pi|\xi|\phi^2} \ , \\ 3H\dot{\phi}\phi &= -\frac{\lambda\phi^4 M_P^2}{M_P^2 + 8\pi|\xi|\phi^2} \ . \end{aligned} \tag{16}$$

The Planck boundary is given by

$$\phi^2(\sqrt{\lambda} - 16\pi|\xi|) < 2M_P^2 \ . \tag{17}$$

Therefore, if $\sqrt{\lambda} < 16\pi|\xi|$, then the inflaton will never reach Planck density. On the other hand, the end of inflation for small $|\xi|$, is given by $\phi_e = M_P/\sqrt{\pi}$.

The amplitude of quantum fluctuations of the inflaton field at small $|\xi|$ is given by (2). Self-reproduction in this case will occur, for small $|\xi|$, at

$$\frac{\delta\phi}{\Delta\phi} = \frac{H\phi}{M_P^2} \simeq \left(\frac{2\pi\lambda}{3}\right)^{1/2}\frac{\phi^3}{M_P^3} > 1 \ . \tag{18}$$

There are runaway solutions in this case, since the quantum diffusion of the inflaton has no boundary for large ϕ, see Eq. (17), and the probability distribution will move toward the maximum expansion rate. For large ϕ, this becomes

$$H \simeq \sqrt{\frac{\lambda}{12|\xi|}}\,\phi \ , \tag{19}$$

while the value of Planck mass also grows with ϕ, as $M_P = (8\pi|\xi|)^{1/2}\,\phi$.

Runaway solutions are probability distributions satisfying a diffusion equation similar to (3) that move forever toward large values of the field ϕ [8, 9]. The way it moves will depend on the type of potential. For $\lambda\phi^4$ it is an explosive behavior, as we will see. The probability distribution gives a statistical description of the quantum diffusion process toward large ϕ, but it proves useful to analyze the particular behavior of those relatively rare domains in which the field ϕ increases in every quantum jump of amplitude (2). We can compute the speed at which those domains move toward large values of the field ϕ from

$$\dot{\phi} = \frac{\delta\phi}{\Delta t} = \frac{H^2}{2\pi} = \frac{\lambda}{24\pi|\xi|}\,\phi^2 \ , \tag{20}$$

where H is given by (19). We find an explosive solution

$$\phi(t) = \phi_0\,\left(1 - \frac{\lambda}{24\pi|\xi|}\,\phi_0\,t\right)^{-1} \ . \tag{21}$$

We see that those first domains of the diffusion process reach infinity in finite time. Note that the total volume of such domains at that time will be finite, and then they will start growing at an infinitely large rate. This behavior is explosive, it corresponds to probability distributions that are nonstationary and singular at $\phi \to \infty$.

It should be noted that the existence of runaway solutions or even inflation is not generic. In fact, for arbitrary potentials of the type $V(\phi) = \frac{\lambda}{2n}\phi^{2n}$, $n > 2$, the equations of motion (10) become, for large ϕ, $8\pi|\xi|\phi^2 \gg M_P^2$,

$$\begin{aligned} \dot{\phi} &= -2|\xi|H\phi\,(n-2)\ , \\ H &= \left(\frac{\lambda}{6n|\xi|}\right)^{1/2} \phi^{n-1}\ . \end{aligned} \tag{22}$$

It can easily be seen that there is inflation ($|\dot{H}| < H^2$) only for

$$(n-1)(n-2) < \frac{1}{2|\xi|}\ . \tag{23}$$

In particular, for an exponential potential in the limit of very large ϕ one effectively has $n = \infty$. Thus the nonminimally coupled term prevents inflation in the theory with an exponential potential at very large ϕ. Furthermore, one can also compute the self-reproduction regime for an arbitrary chaotic potential like above. For those values of n for which there is inflation, the condition $\dot{\phi}H^{-1} < \delta\phi$ reads

$$4\pi\sqrt{6n|\xi|}\,(n-2)\,\frac{|\xi|}{\sqrt{\lambda}} < \left(\frac{\phi}{M_P}\right)^{n-2} < 8\pi\sqrt{2n}\,\frac{|\xi|}{\sqrt{\lambda}}\ , \tag{24}$$

where the last inequality comes from the Planck boundary in the large inflaton limit. It is clear that in order to have self-reproduction we need

$$(n-2)^2 < \frac{4}{3|\xi|}\ . \tag{25}$$

In conclusion, as we increase $|\xi|$, first self-reproduction disappears, and then even inflation itself is no longer sustained for a given n. Therefore, runaway solutions are very special and appear only for theories satisfying both (23) and (25).

IV. CHAOTIC INFLATION AND CONFORMAL ANOMALY

A. Starobinsky model with inflaton field

In this section we will describe inflation in the combined model, including scalar fields and the R^2 terms, which may appear in the theory because of the one-loop quantum gravity effects. Note however, that these terms by themselves can lead to the existence of inflationary regime, as was first realized by Starobinsky [27]. Therefore in this subsection we will remember some basic features of the Starobinsky model. After that we will add a nonminimal coupling to the inflaton field.

The equations of motion associated with the Starobinsky model in the presence of an inflaton field with potential $V(\phi)$ can be written as

$$\begin{aligned} \nabla^2\phi &= -\,V'(\phi)\ , \\ -\frac{M_P^2}{8\pi}&\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) \\ &= g_{\mu\nu}V(\phi) + \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2 \\ &\quad + \frac{M_P^2}{8\pi}\left(\frac{1}{6M^2}\,{}^{(1)}H_{\mu\nu} + \frac{1}{H_0^2}\,{}^{(3)}H_{\mu\nu}\right)\ , \end{aligned} \tag{26}$$

where

$$\begin{aligned} {}^{(1)}H_{\mu\nu} &= 2\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^2\right)R + 2RR_{\mu\nu} \\ &\quad -\tfrac{1}{2}g_{\mu\nu}R^2\ , \\ {}^{(3)}H_{\mu\nu} &= R_\mu{}^\lambda R_{\lambda\nu} - \tfrac{2}{3}RR_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R^{\rho\sigma}R_{\rho\sigma} \\ &\quad +\tfrac{1}{4}g_{\mu\nu}R^2\ . \end{aligned} \tag{27}$$

The parameters H_0 and M in the original version of the Starobinsky model were related to the conformal anomaly, but in a later version the term $\frac{R^2}{6M^2}$ was simply added to the Lagrangian by hand [31, 32]. The value of H_0 is of the same order as M_P, but it can be somewhat smaller if there are many matter fields (of spin 0, 1/2, and 1) contributing to the conformal anomaly. In fact, one loop gravitational corrections in our theory are somewhat more complicated, especially because the theory of the inflaton field ϕ is not conformally invariant. Nevertheless when the number of other fields contributing to conformal anomaly is sufficiently large, i.e., when the masses M^2 and H_0^2 are sufficiently small, our approximation may be reasonable.

During inflation one can write the equations of motion for the homogeneous fields ϕ and H, in the slow rolling approximation, as

$$\dot{H} = -\frac{M^2}{6}\left(1 - \frac{H^2}{H_0^2} - \frac{8\pi V(\phi)}{3M_P^2H^2}\right)\ , \tag{28}$$

$$\dot{\phi} = -\frac{V'(\phi)}{3H}\ . \tag{29}$$

Let us neglect the scalar field first, i.e., consider the original Starobinsky model first. All possible inflationary and noninflationary regimes in this model have been described in a particularly detailed way in [33]. However, as we will see, with an account taken of the self-reproduction of the inflationary universe we can get some additional (and rather unexpected) information about this model.

First of all, let us remember that there exist three different inflationary regimes in this model, and only two of them are usually considered in the literature. Namely, the first stage of inflation occurs for $H_0^2 \gg M^2$ in the regime with $|\dot{H}| < M^2/6 \ll H_0^2$. In the limiting case $\dot{H} = 0$ inflation occurs with the Hubble constant $H = H_0$. Just as in the new inflation model, this regime is unstable, and inflation enters the second regime with $\dot{H}$ asymptotically approaching $-M^2/6$. During this process the Hubble constant decreases, until it reaches the value $H \sim M$, which corresponds to the end of inflation.

However, there also exists another, rather unusual inflationary branch, namely, Eq. (28) at $V(\phi) = 0$, $H \gg H_0$, reads

$$\dot{H} = \frac{M^2}{6H_0^2} H^2 . \tag{30}$$

Obviously this is an inflationary regime with $|\dot{H}| \ll H^2$ for $M^2 \ll 6H_0^2$. In this regime the Hubble constant indefinitely grows, and approaches infinitely large values within a finite time $\Delta t \sim 6H_0/M^2$. For this and some other reasons this branch was believed to be unphysical and not very interesting [33, 22]. Neglecting the possibility of inflation with positive $\dot{H}$ can lead to important constraints on the rate of inflation in the Starobinsky model.

Indeed, Eq. (28) in the limit $\dot{H} \to 0$ has two possible solutions [34]:

$$H^2 = \frac{H_0^2}{2} \left(1 \pm \sqrt{1 - \frac{32\pi V(\phi)}{3H_0^2 M_P^2}}\right) . \tag{31}$$

In the limit of small $V(\phi)$, these two branches obviously correspond to the Starobinsky inflation with $H^2 = H_0^2$, and to the scalar field driven inflation with $H^2 = \frac{8\pi V(\phi)}{3M_P^2}$. One can easily see, that for $\dot{H} < 0$ the Hubble constant H on the upper branch becomes smaller than H_0. Thus, one can consider H_0 as an upper bound on the rate of inflation in a rather general class of models. Typically this bound is very close to the Planck bound $H_{\rm max} \sim M_P$, but in certain cases H_0 can be somewhat smaller than M_P [22]. This is a very interesting observation, since it provides a natural upper boundary which is necessary for finding the probability distribution P_p [6]. For $H_0 \ll M$ the upper boundary for inflation becomes even lower. In this case there is no inflation close to the upper branch of Eq. (31), and the lower branch cannot go higher than $H_{\rm max} \sim H_0/\sqrt{2}$, which corresponds to $V(\phi) \sim \frac{3H_0^2 M_P^2}{32\pi}$.

However, if inflation near the upper branch of (31) is possible, then one cannot always neglect the possibility of inflation with $\dot{H} > 0$. Indeed, in the usual chaotic inflation models classical motion of the scalar field shifts it to smaller values of $V(\phi)$. However, quantum jumps of the field ϕ in the regime of self-reproduction can move it against the classical flow, toward the highest possible values of $V(\phi)$. Similarly, the classical motion shifts H toward the singularity at the inflationary trajectory with $\dot{H} > 0$. However, if this trajectory allows self-reproduction, the Hubble constant H may drift toward its very large values, and then in some of inflationary domains it may jump back toward $H < H_0$. Since the rate of expansion of the Universe at the branch with $H > H_0$ is very large, the volume of the corresponding parts of the Universe grows at a very high rate, and the existence of the "pathological" inflationary branch with $H > H_0$ will give a dominant contribution to the overall rate of the Universe expansion $e^{dH_{\rm max}t}$. In this case $H_{\rm max}$ will be greater than H_0. One can still argue that $H_{\rm max}$ should not be much greater than M_P, since in this case the notion of classical space-time ceases to exist, and the usual derivation of the stochastic equations for P_p becomes invalid [6].

In order to study the process of the universe self-reproduction in the Starobinsky model, we should find the amplitude of quantum fluctuations of the scalar curvature $R \sim 12H^2$, which are related in the following way to the fluctuations of the canonically normalized scalaron field $\delta\varphi$:

$$\delta R = (48\pi C)^{1/2} \frac{M^2}{M_P} \delta\varphi , \tag{32}$$

where $C = 1 + R/3M^2 - R/6H_0^2$ [34]. Both the scalaron $\delta\varphi$ and the inflaton fluctuations $\delta\phi$ satisfy approximate massless equations in de Sitter space, whose solution is well known, and whose amplitude is approximately given by Gibbons-Hawking temperature, $H/2\pi$. (Scalaron has a tachyonic mass squared $-M^2$, where $M^2 \ll H^2$ [34].) Using (31) we find $C^{1/2} \simeq 2H/M$, and therefore

$$\delta\varphi = \delta\phi = \frac{H}{2\pi} ,$$
$$\delta H = \frac{H}{2\pi}\sqrt{\frac{\pi}{3}}\frac{M}{M_P} . \tag{33}$$

Let us now consider inflation at $H \simeq H_0$, with $V(\phi) \simeq 0$. Self-reproduction will occur in this case if the classical shift of the Hubble constant within the time H^{-1} is smaller than the amplitude of fluctuations of H. This condition implies $|\frac{\dot{H}}{H}| < \delta H$. Together with Eqs. (28) and (33), this condition gives

$$\left|1 - \frac{H_0^2}{H^2}\right| < \sqrt{\frac{3}{\pi}}\frac{H_0^2}{MM_P} . \tag{34}$$

Some care should be taken when applying this criterion at $H \simeq H_0$: it should be satisfied for all H within the interval δH from H_0. This gives the following criterion of the self-reproduction at $H \simeq H_0$, which essentially coincides with the criterion for inflation there: $M \leq \sqrt{3}H_0$. A similar criterion can be obtained from the condition that the fractal dimension for inflation at $H = H_0$ is positive: $M \leq 3H_0$, see Eq. (40) in the next subsection.

However, this condition is not strong enough to ensure self-reproduction for $|H - H_0| \geq H_0$. The corresponding condition follows from (34). At small H this condition reads $M < \frac{H^2}{M_P}\sqrt{\frac{3}{\pi}}$; at large H this condition is $M < \frac{H_0^2}{M_P}\sqrt{\frac{3}{\pi}}$.

Therefore one should consider several different regimes in our model.

(1) $M \geq 3H_0$. There is no inflation in the Starobinsky model, but inflation may exist due to the scalar field ϕ, for $V(\phi) < \frac{3H_0^2 M_P^2}{32\pi}$.

(2) $\frac{H_0^2}{M_P}\sqrt{\frac{3}{\pi}} < M < 3H_0$. There is inflation and self-reproduction at $H \simeq H_0$. However, self-reproduction occurs only in a narrow band $H_0 \pm \Delta H$ (34), with

$$\Delta H \sim \frac{\sqrt{3}}{2\sqrt{\pi}}\,\frac{H_0^3}{MM_P}\ . \tag{35}$$

This means, in particular, that only a small part of the branch with $H > H_0$, $\dot{H} > 0$ is of any interest for us; the points which go beyond $H_0+\Delta H$ never return, and move toward a singularity. Therefore in this case the effective maximal value of the Hubble constant will be of the order of $H_0 + \Delta H$ (35). After inflation at the upper branch, the Hubble constant becomes smaller, and inflation continues at the lower branch, which will be responsible for the density perturbations in the observable part of the Universe.

(3) $M < \frac{H_0^2}{M_P}\sqrt{\frac{3}{\pi}}$. In this case self-reproduction occurs at the whole branch with $H > H_0$, $\dot{H} > 0$, at least until inflationary domains enter the area where the curvature becomes higher than the Planck one. In this case there is no upper bound for the Hubble constant near $H = H_0$; the upper bound for H is expected to be of the same order as M_P.

In what follows we will consider the stationary probability distribution in case (2), assuming that $H_0 \ll M_P$, $\frac{H_0^2}{M_P}\sqrt{\frac{3}{\pi}} \ll M < 3H_0$. In this case self-reproduction of the Universe occurs in a very narrow region ($\Delta H \ll H_0$) near H_0, see Eq. (35). The results of our investigation will be useful for us when we will discuss the cosmological constant problem.

The corresponding diffusion equation will be written in terms of the canonically normalized fields ϕ and φ, where φ is the scalaron field introduced in [34], see Eq. (32). According to [34], this field satisfies the same equation as a tachyon field with the mass squared $-M^2$ in de Sitter background. We will assume also that $V(\phi) = \frac{m^2}{2}\phi^2$. Assuming $H \simeq H_0$, one can show that self-reproduction of the Universe occurs in the intervals

$$\begin{aligned} \phi &< \frac{3H_0^3}{2\pi\, m^2}\ , \\ \varphi &< \frac{3H_0^3}{2\pi M^2}\ . \end{aligned} \tag{36}$$

The diffusion equation for $\mathcal{V}(\phi,\varphi,t)$ for $H \simeq H_0$ can be written in the form

$$\begin{aligned} \frac{\partial \mathcal{V}}{\partial t} &= \frac{H_0^3}{8\pi^2}\frac{\partial^2\mathcal{V}}{\partial\varphi^2} - \frac{M^2}{3H_0}\frac{\partial}{\partial\varphi}(\varphi\mathcal{V}) \\ &\quad +3H_0\left(1-\frac{2\pi\phi^2 m^2}{3M_P^2H_0^2}\right)\mathcal{V} \\ &\quad + \frac{H_0^3}{8\pi^2}\frac{\partial^2\mathcal{V}}{\partial\phi^2} + \frac{m^2}{3H_0}\frac{\partial}{\partial\phi}(\phi\mathcal{V})\ . \end{aligned} \tag{37}$$

There is a solution in the limit of large time t:

$$\mathcal{V}(\phi,t) \sim \exp(dH_{\rm max}t)\ \exp\left(-2\pi^2\, a\,\frac{\phi^2}{H_0^2}\right)\ . \tag{38}$$

In our case, the maximum rate of inflation which provides the largest relative volume is given by $H_{\rm max} \simeq H_0 + \Delta H \simeq H_0$. In the limit of the quantum diffusion dominating classical drift, $H_0^2 \gg mM_P$, we find

$$a \simeq \frac{m}{\sqrt{\pi}M_P}\ , \tag{39}$$

while the fractal dimension is given by

$$d = 3 - \frac{M^2}{3H_0^2} - \frac{m}{2\sqrt{\pi}M_P}\ . \tag{40}$$

The stationary solution (38) is a Gaussian centered at $\phi = 0$. It does not depend on φ in the domain of self-reproduction (36).

B. Starobinsky model with nonminimally coupled scalar field

In this section we will briefly describe the classical evolution of the inflaton field with a generic chaotic potential, coupled to the curvature scalar with a small coupling $\xi < 0$, $|\xi| \ll 1/6$, in the context of the Starobinsky model. In this case our equations of motion are somewhat modified:

$$\begin{aligned} \nabla^2\phi &= -\,V'(\phi) + |\xi|\phi R\ , \\ -\frac{M_P^2(\phi)}{8\pi}\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right) &= \left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^2\right)|\xi|\phi^2 + \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2 \\ &\quad + g_{\mu\nu}V(\phi) + \frac{M_P^2(\phi)}{8\pi}\left(\frac{1}{6M^2(\phi)}\ {}^{(1)}H_{\mu\nu} + \frac{1}{H_0^2(\phi)}\ {}^{(3)}H_{\mu\nu}\right)\ . \end{aligned} \tag{41}$$

Here ${}^{(1)}H_{\mu\nu}$ and ${}^{(3)}H_{\mu\nu}$ are given by Eq. (27) and

$$\begin{aligned} M_P^2(\phi) &= M_P^2 + 8\pi|\xi|\phi^2\ , \\ M(\phi) &= \frac{M_P(\phi)}{M_P}\ M\ , \\ H_0(\phi) &= \frac{M_P(\phi)}{M_P}\ H_0\ . \end{aligned} \tag{42}$$

During inflation ($|\dot{H}| < H^2$), the equation of motion for H can be written as

$$H^2 = \frac{8\pi V(\phi)}{3M_P^2(\phi)} + \frac{H^4}{H_0^2(\phi)} - \frac{6H^2\dot{H}}{M^2(\phi)}\ . \tag{43}$$

In the limit of $\dot{H} \to 0$, the equation of motion for the scalar field reads

$$3H\dot{\phi} = 4V(\phi) - V'(\phi) - \frac{3M_P^2}{2\pi} H^2 \left(1 - \frac{H^2}{H_0^2}\right) , \quad (44)$$

On the other hand, Eq. (43) in the limit $\dot{H} \to 0$ has two possible solutions:

$$H^2(\phi) = \frac{H_0^2(\phi)}{2} \left(1 \pm \sqrt{1 - \frac{32\pi V(\phi)}{3H_0^2(\phi) M_P^2(\phi)}}\right) . \quad (45)$$

In the limit of small ϕ and $V(\phi)$, the upper branch corresponds to the Starobinsky inflation with $H^2 = H_0^2$. Inflation at large ϕ is strongly model dependent. The main qualitative difference with the regime studied in the previous subsection is the following. If $V(\phi)$ grows at large ϕ more slowly than $3H_0^2(\phi)M_P^2(\phi)/32\pi$, then there may be no upper bound for the Hubble constant $H(\phi)$.

For example, there is no upper bound for $H(\phi)$ in the theory $V(\phi) = \frac{\lambda}{4}\phi^4$ for $\lambda < 24\pi\xi^2 H_0^2/M_P^2$. In this case, the value of the inflaton field can increase indefinitely in the regime of self-reproduction,

$$\left|1 - \frac{H^2}{H_0^2}\right| < \frac{H\phi}{2\pi} , \quad (46)$$

and we find what we have called runaway solutions, that is, non stationary probability distributions that move forever toward large values of the fields [8, 9]. The rate of inflation (45) in the limit $8\pi|\xi|\phi \gg M_P^2$ at the upper branch becomes

$$H \simeq H_0(\phi) \simeq \frac{H_0}{M_P} (8\pi|\xi|)^{1/2} \phi , \quad (47)$$

which increases indefinitely with ϕ. In this case, self-reproduction occurs for all values of ϕ if $8\pi|\xi| < H_0^2/M_P^2$, see Eq. (46). The rate of inflation at the lower branch also grows without limit:

$$H \simeq \sqrt{\frac{\lambda}{12|\xi|}}\, \phi . \quad (48)$$

This regime coincides with the one that we have found at the end of Sec. III, see Eq. (19). The reason is very simple: If the effective Planck mass grows very rapidly with the growth of ϕ, then inflation never approaches the Planck boundary, and the effects associated with the conformal anomaly always remain small.

V. INFLATION, QUANTUM COSMOLOGY, AND THE COSMOLOGICAL CONSTANT PROBLEM

A. Quantum cosmology predictions for the minimal model

Now we will turn to the discussion of predictions of quantum cosmology, assuming that one can live in different quantum states of the Universe with different coupling constants. One should emphasize that this assumption in its most radical form (all values of coupling constants are possible) is extremely speculative, being based on some particular interpretation of the baby Universe theory. We do not really know which constants can be considered adjustable, and which ones are "true constants"; in what follows we will consider several different possibilities.

As a working hypothesis we will assume that the most probable quantum state of the Universe is the state where the total number of observers of our type can be greater. This condition can be rather ambiguous [8], but we can use it as a starting point of our investigation of inflationary quantum cosmology.

The main idea can be illustrated by considering the simplest model of the scalar field ϕ minimally coupled to gravity ($\xi = 0$) with the effective potential $V(\phi) = \frac{\lambda}{4}\phi^4$. As we already mentioned, the total volume of different parts of the Universe with a given value of the scalar field ϕ (or with a given density) in this model grows in time as $\mathcal{V}(\phi, t) \sim e^{(3-f(\lambda))H_{\max} t}\, P_p(\phi)$, where $f(\lambda) \to 0$ decreases in the limit $\lambda \to 0$ [6]. It is clear then that the greatest rate of expansion can be reached in the limit $\lambda \to 0$ [6, 22, 23].

This is a rather general conclusion. The overall rate of expansion of the Universe grows when the effective potential becomes more and more flat. A similar result was known in chaotic inflation even without taking into account the Universe self-reproduction: The total degree of inflation there was proportional to $\exp(c/\sqrt{\lambda})$, where $c \sim 1$ [1]. Thus, the size the Universe after inflation becomes exponentially large for small λ. Moreover, it was known that if one has several scalar fields ϕ_i with coupling constants λ_i, the last stages of inflation are typically driven by the field ϕ_i with the smallest λ_i. This helped, to some extent, to understand why coupling constants of the inflaton field are so small: They may be large, but the structure of the part of the observable part of the Universe was formed at the last stage of inflation, which was driven by the field with the smallest coupling constant λ_i [1]. However, the results obtained in [6] are much stronger, and, being interpreted in a certain way, they can be even dangerous.

Indeed, let us take the baby Universe theory seriously and assume that we can actually compare different universes with different coupling constants. The total number of observers of our type which may appear for a given set of coupling constants λ_i at a given time t is presumably given by the symbolic equation

$$\mathcal{N}(\lambda_i, t) \sim \int d\rho_0\, d\rho\, P_{\rm creat}(\rho_0, \lambda_i)\, P_{\rm life}(\rho, \lambda_i) \times \int_0^t \mathcal{V}(\rho, t', \rho_0, \lambda_i)\, dt' . \quad (49)$$

Here $P_{\rm creat}(\rho, \lambda_i)$ is the probability of creation of an (inflationary) universe with initial density $\rho_0 \sim V(\phi_0)$ in the theory with coupling constants λ_i, $P_{\rm life}(\rho, \lambda_i)$ is the probability for life of our type to appear in a unit volume of such a universe at density ρ. This equation can be written more accurately, but the main point we are going to make will not depend on many details.

Note that in the standard big bang cosmology there would be no need in integrating over time in addition to integrating over density ρ, since ρ would be a definite function of time t all over the Universe. In our case domains of space with given ρ appear at all sufficiently large values of time t. In particular, suppose that the probability distribution $\mathcal{V}(\rho, t, \rho_0, \lambda_i)$ is stationary, $\mathcal{V}(\rho, t, \rho_0, \lambda_i) \sim e^{\alpha(\lambda_i)\,t}\, P_p(\rho)$, as in Eqs. (4) and (7) [29]. It is important that this solution *does not* depend on the initial condition ρ_0. Therefore one can take an integral over ρ_0, absorbing all information about the probability distribution $P_{\rm creat}(\rho_0, \lambda_i)$ into some function $P_{\rm creat}(\lambda_i)$. This yields

$$\mathcal{N}(\lambda_i, t) \sim e^{\alpha(\lambda_i)\,t}\, P_{\rm creat}(\lambda_i)\, \alpha^{-1}(\lambda_i) \times \int d\rho\, P_{\rm life}(\rho, \lambda_i)\, P_p(\rho)\ . \tag{50}$$

Typically $P_{\rm creat}(\lambda_i)$ is a smooth function of the coupling constants λ_i. For example, one can take for the probability of quantum creation either the square of the tunneling wave function $\exp\left(-\frac{3M_P^4}{8\,\rho_0}\right)$ [16, 17], or the square of the Hartle-Hawking wave function $\exp\left(\frac{3M_P^4}{8\,\rho_0}\right)$ [14] (which, in our opinion, does not describe quantum creation of the Universe [1]). Independently of this choice, after the integration over ρ_0 one obtains an irrelevant normalization constant $P_{\rm creat}$, which does not depend on λ_i. The functions $\alpha(\lambda_i)$ and $P_{\rm life}(\rho, \lambda_i)$ entering Eq. (50) are also not expected to exhibit any singular behavior with respect to λ_i. The only function which strongly depends on λ_i in this equation is $e^{\alpha(\lambda_i)\,t}$.

For example, if the upper boundary for inflation in the theory $\frac{\lambda}{4}\phi^4$ coincides with $V(\phi) = M_P^4$, one has $e^{\alpha(\lambda_i)\,t} \sim e^{[3-f(\lambda)]H_{\max}t} \sim e^{[3-f(\lambda)]\sqrt{\frac{8\pi}{3}}M_P t}$. It is clear from Eq. (50) that most of the observers of our type should live at an indefinitely large time interval from the big bang, i.e., at $t \to \infty$ [6]. (The conditions for the appearance of life in each particular domain of the Universe at a given density ρ does not depend on time in the limit $t \to \infty$, whereas the number of such domains grows as $e^{\alpha(\lambda_i)\,t}$.) But in this limit the relative number of observers living in the Universes with a nonvanishing λ becomes suppressed by a factor $e^{-f(\lambda)\sqrt{\frac{8\pi}{3}}M_P\,t}$ as compared with the number of observers living in the Universes with $\lambda \to 0$.

Thus the factor $e^{-f(\lambda)\sqrt{\frac{8\pi}{3}}M_P\,t}$ always wins over all anthropic considerations [8]. (A similar argument holds in more traditional versions of the baby Universe theory [35].) Meanwhile, the probability of existence of life of our type becomes strongly suppressed in a Universe with small λ. For example, the typical amplitude of density perturbations produced during inflation is proportional to $10^2\sqrt{\lambda}$. Therefore, one could argue that in the limit $\lambda \to 0$ there will be no galaxies, and no people to live there. The conclusion that quantum cosmology picks up flat potentials was interpreted in [22, 23] as suggesting that inflation does not produce density perturbations (these perturbations decrease as $10^2\sqrt{\lambda}$ in the limit $\lambda \to 0$), and one should use topological defects in order to account for galaxy formation. However, in the limit of absolutely flat potentials there will be neither inflationary density perturbations nor topological defects. Typically, superheavy topological defects are produced after inflation only if the potential $V(\phi)$ is very curved; in fact, it is very difficult to produce such defects in the context of inflationary cosmology. In certain cases these difficulties can be somewhat weakened, and strings can be produced even at low energy density in some theories with very flat potentials [36]–[38]. On the other hand, in the same class of theories it is also possible to obtain $\frac{\delta\rho}{\rho} \sim 5\times10^{-5}$ [the result following from the Cosmic Background Explorer (COBE) data in the normalization of $\frac{\delta\rho}{\rho}$ used in [1]] even for extremely flat potentials at a very small energy density $V(\phi)$ [37, 38]. Thus, the use of topological defects produced after inflation may not have any obvious advantages over the standard inflationary mechanism of generation of density perturbations. The only constraint on the applicability of each of these mechanisms is the reheating constraint: If the energy density at the end of inflation is too low, then particle production will be exponentially suppressed, and there will be no observers to enjoy life in such a Universe.

This is a typical anthropic constraint and, as we have already mentioned, it is not strong enough to win over the factor $e^{-f(\lambda)\sqrt{\frac{8\pi}{3}}M_P\,t}$. For example, the expression $\frac{\delta\rho}{\rho} \sim 10^2\sqrt{\lambda}$ is only statistically correct, which means that in the eternally self-reproducing inflationary universe there will always appear exceptional domains where $\frac{\delta\rho}{\rho} \sim 5\times10^{-5}$ even for $\lambda \ll 10^{-13}$. Such parts of the Universe may be extremely rare, but eventually (because of the factor $e^{-f(\lambda)\sqrt{\frac{8\pi}{3}}M_P\,t}$) their total volume will become much greater that the volume of more regular domains with $\lambda \sim 10^{-13}$ and $\frac{\delta\rho}{\rho} \sim 10^2\sqrt{\lambda}$. The same arguments may apply to the possibility of large fluctuations with an unusually high baryon density. This suggests that according to our scenario most of the observers should live in those parts of the Universe where conditions for existence of life appear as a result of an extremely improbable fluctuation. The total number of observers living in such domains should be large only because of the indefinitely large time of existence and exponential growth of a self-reproducing inflationary Universe. One could expect that most observers of our type should live in terrible and irregular conditions, perhaps on the verge of being extinct. Many would argue that this conclusion contradicts observational data, even though some pessimists would agree with this conclusion and use it as an advanced version of the doomsday prediction.

Since this conclusion certainly looks unpleasant, let us see whether we have real reasons to worry.

(1) There is a simple formal reason to have doubts about the results of the approach developed above. If the Universe enters the stage of eternal self-reproduction, then after a sufficiently large time there is no difference between the Universes formed at two different moments. That is why we introduced the integration over time t in Eq. (49). However, we also introduced a cutoff in this integral at some time t. The only reason of doing so is that otherwise the integral diverges and we get infinitely large number of observers for any given λ. This

not well motivated introduction of the upper bound in the integral in (49) is the main reason why the probability distribution P_p does depend on the choice of time parametrization [6,8]. Therefore, one may argue that the versions of the theory which allow self-reproduction of the Universe and, consequently, an infinitely large number of observers, are preferable as compared with the versions without self-reproduction, where the number of observers may be finite. However, if there are many branches of the Universe which allow self-reproduction, their comparison is ambiguous since it involves comparison of infinities, see Introduction.

Unfortunately, this simple argument does not sound entirely convincing in our case. Indeed, as we already mentioned in Sec. II, even the choice of a radically different time parametrization τ, where time is measured not by clocks but by rulers (τ is the logarithm of the local expansion of the Universe) does not change our conclusion: The total volume of all inflationary domains with a given ϕ (or with a given ρ) grows at a much greater rate in the limit $\lambda \to 0$. Nevertheless, it remains not quite clear whether one has any physical reason to compare *different Universes at the same time t.*

(2) Our investigation was based on the assumption that different universes with different values of the coupling constant λ may actually exist. Moreover, we assumed that the coupling constant λ may take all possible values, including zero. In the context of the baby Universe theory these assumptions look reasonable, but the baby Universe paradigm may be wrong, or it may have limited validity, being applicable to the vacuum energy of the Universe (cosmological constant), but not to other parameters. Also, the problem disappears if λ can take only a discrete number of values, not including zero.

(3) Our estimate gives us the total number of observers of our type for different values of λ. But is it correct to say that we consider observers *of our type* if they belong to the Universe with different coupling constants? Shall we perhaps compare sheep to sheep and wolves to wolves? Does it make any sense to say that it was improbable for the authors to be born in Spain and in Russia, because they could have been born in China or India where the total population is much larger?

(4) In our discussion we assumed that the number of observers of our type is directly proportional to the volume of the Universe. But one cannot get any crop even from a very large field without having seeds first. The idea that life appears automatically once there is enough space to be populated may be too primitive. More generally, in this paper we are making an attempt to describe emergence of life solely in terms of physics. It is certainly a most economical approach, but this approach may not be correct, especially if consciousness has its own degrees of freedom [1].

Unfortunately, we are not in a position to discuss these issues in this paper. Quantum cosmology is developed by trial and error, and we are not pretending to have final answers to all these questions. Instead of that we will try to develop our scheme somewhat further, in an attempt to see whether or not the conclusions we have reached in this section are general.

B. Quantum cosmology in more complicated models of inflation

The model which we considered in the previous section has only one dimensionless parameter, λ. Meanwhile, in the model (9) there appears another parameter, ξ. We do not really know whether or not one is allowed to vary each of these two parameters in the context of the baby Universe theory. Let us assume for a moment that the parameter ξ is fixed and positive, as in the first model considered is Sec. III. We will also assume that $\xi \ll 1/6$, since otherwise we do not have inflation in this model, which makes the total volume small and finite. In the theories with $\xi \ll 1/6$ there are several different possibilities discussed in Sec. III. In particular, for $\sqrt{\lambda} < 64\pi\xi^2$ there is inflation but no self-reproduction. In this case stationarity of P_p is impossible, and the growth of volume of inflationary domains is finite. Thus, it is unfavorable [from the point of view of increasing the number of observers of our type $\mathcal{N}(\lambda, t)$] to have $\sqrt{\lambda} < 64\pi\xi^2$.

On the other hand, for $\sqrt{\lambda} > 16\pi\xi$ the number $\mathcal{N}(\lambda, t)$ grows with the decrease of λ, as we have shown in the previous subsection. This pushes the coupling constant λ into the region

$$64\pi\xi^2 < \sqrt{\lambda} < 16\pi\xi \; . \tag{51}$$

In other words, if the coupling constant ξ is fixed, the maximum of $\mathcal{N}(\lambda, t)$ appears not at $\lambda = 0$, but somewhere in the interval (51). This corresponds to density perturbations in the interval $2 \times 10^4\xi^2 < \frac{\delta\rho}{\rho} < 5 \times 10^3\xi$. Thus, instead of the problem of vanishing $\frac{\delta\rho}{\rho}$ we have the usual problem of requiring the coupling constants to be very small, in this case related to the coupling constant ξ.

On the other hand, if one can vary both λ and ξ, the results appear to be quite different. In this case the greatest number of observers will live in the universes with $\xi < 0$, $\sqrt{\lambda} < 16\pi|\xi|$. Indeed, as follows from Eq. (21), there exist some domains with a finite volume, in which the scalar field ϕ reaches infinitely large values within finite time $t = \frac{24\pi|\xi|}{\lambda\phi_0}$. After that time the rate of growth of the total volume of the Universe containing indefinitely large field ϕ becomes infinite, for any value of λ. It takes some more time (because of quantum jumps and classical rolling) before the rate of growth of volume of the Universe with small ϕ (and ρ) also becomes infinite. It is important, however, that the rate of expansion of domains containing a finite field ϕ (or finite density ρ) becomes infinitely large within some finite time. It hardly makes any sense to compare the number of observers in different Universes if this number is infinite in each Universe even at a finite time t. This is a much stronger manifestation of the same ambiguity which we encountered before, when the integral in (49) became infinite in the limit $t \to \infty$.

Our conclusions will not change much if we add the terms R^2 to our model. As follows from Eq. (40), the overall rate of expansion of the whole Universe grows when the curvature of $V(\phi)$ (i.e., the mass m) decreases.

On the other hand, if one can vary both λ and ξ in the theory $\frac{\lambda}{4}\phi^4$, then for negative ξ, and $\lambda < 24\pi\xi^2 H_0^2/M_P^2$ one obtains runaway solutions, and the rate of expansion becomes infinitely large within a finite time.

Let us summarize our results. The theory $\frac{\lambda}{4}\phi^4$ with $\xi = 0$ is a particular version of the more general class of models with arbitrary ξ. In fact, even if one starts with the model with $\xi = 0$ for $\phi = 0$ and $R = 0$, one almost inevitably obtains an effective coupling constant $\xi \neq 0$, which depends logarithmically on ϕ and R, as a result of quantum corrections [39]. If one varies λ for a given $\xi > 0$, one does not obtain the dangerous result that $\mathcal{N}(\lambda, t)$ has a maximum at $\lambda = 0$. Instead one can find a maximum of $\mathcal{N}(\lambda, t)$ somewhere in the interval $64\pi\xi^2 < \sqrt{\lambda} < 16\pi\xi$. On the other hand, if one can vary both λ and ξ, then the only conditions which one can get are $\xi < 0$ and $\sqrt{\lambda} < 16\pi|\xi|$. Under these conditions the total volume of the Universe (and the total number of observers which will occupy this volume later) becomes infinite within a finite time t, which makes any further analysis ambiguous. Therefore at present we do not think that one should worry too much about the conclusion that quantum cosmology prefers vanishing effective potentials $V(\phi)$, even though this issue deserves further consideration.

In addition, we should emphasize again that all results obtained in this section have been derived under the very speculative assumption that one can compare *different Universes* at the same time. If one would compare different exponentially large *parts of the Universe* with different laws of low-energy physics, there would be no difference in the rate of growth $e^{\alpha(\lambda_i)t}$ of these parts [6, 8], and our conclusions would be quite different, see Discussion.

C. Extended Starobinsky model and the cosmological constant problem

Even though there are many potential problems associated with the approach developed in this paper, it certainly allows us to look at the old problems of quantum cosmology from a different perspective. Let us try to apply our methods to the problem of the cosmological constant.

The main lesson we have learned from our previous investigation is that the total number of observers is almost entirely controlled by the factor $e^{dH_{\max}t}$. If one considers the usual inflationary models where inflation is driven by the scalar field ϕ, then one can expect that adding a vacuum energy V_0 to the effective potential will only increase $dH_{\max}$. For example, if the upper boundary for inflation is determined by the condition $V(\phi) = M_P^4$, then $H_{\max} = \sqrt{\frac{8\pi}{3}}M_P$, independently of V_0. A similar result is valid if there is no inflation in Starobinsky model (if, e.g., $H_0 \ll M$), and the upper bound for the scalar-field-driven inflation is given by $H_0/\sqrt{2}$, independently of V_0. Then the only factor which depends on V_0 is the fractal dimension d. One can easily understand, for example, that with an increase of V_0 the potential $\frac{m^2}{2}\phi^2+V_0$ near its upper bound becomes more flat, which typically increases d. We have verified this conjecture by changing V_0 and finding d numerically. Thus one may guess that quantum cosmology pushes the cosmological constant $\Lambda \equiv \frac{8\pi V_0}{M_P^2}$ toward its largest possible values. This would be a very undesirable conclusion, especially since the anthropic principle allows a positive vacuum energy V_0 to be as large as $10^{-27}\mathrm{g}\cdot\mathrm{cm}^{-3}$, which is 2 orders of magnitude greater than the present observational constraints on V_0.

However, this conclusion is not general. Indeed, let us consider Starobinsky model together with the scalar field with the effective potential $V(\phi) + V_0$, where V_0 is some constant, and $V(\phi)$, as before, vanishes in its minimum, e.g., $V(\phi) = \frac{m^2}{2}\phi^2$. We will assume that $H_0 \ll M_P$, and $\frac{H_0^2}{M_P}\sqrt{\frac{3}{\pi}} \ll M < 3H_0$. As we have shown in the Sec. IV, the maximal rate of expansion $H_{\max} \simeq H_0 + \Delta H \simeq H_0$ is achieved at top of the Starobinsky branch, for the smallest value of the effective potential $V(\phi) = 0$, and the self-reproduction of the Universe occurs in a very narrow region near H_0, $\Delta H \ll H_0$ (35). However, in the present case the analogue of Eq. (31) at small ϕ yields

$$H^2 = \frac{H_0^2}{2}\left(1 \pm \sqrt{1 - \frac{4\Lambda}{3H_0^2}}\right) , \tag{52}$$

where $\Lambda = \frac{8\pi}{M_P^2}V_0$ is the cosmological constant. Thus, adding the cosmological constant $\Lambda > 0$ diminishes the maximal value of the Hubble constant H. In the limit $\Lambda \ll H_0^2$ one obtains

$$H_{\max} \simeq H_0\left(1 - \frac{\Lambda}{6H_0^2}\right) . \tag{53}$$

Adding a cosmological constant changes the fractal dimension d as well. The corresponding changes can be approximately described by substituting $H_{\max}$ for H_0 in (40):

$$d(\Lambda) = 3 - \frac{M^2}{3H_0^2} - \frac{M^2\Lambda}{9H_0^4} - \frac{m}{2\sqrt{\pi}M_P} . \tag{54}$$

This gives

$$e^{dH_{\max}t} \approx \exp\left[d(\Lambda)\, H_0\left(1 - \frac{\Lambda}{6H_0^2}\right)\, t\right] . \tag{55}$$

Note that both factors in the exponent decrease with a growth of Λ. In the τ-parametrization of time the last factor disappears, being absorbed into the definition of τ [6], and the resulting exponential factor acquires the form

$$e^{d(\Lambda)\,\tau} = \exp\left[\left(3 - \frac{M^2}{3H_0^2} - \frac{M^2\Lambda}{9H_0^4} - \frac{m}{2\sqrt{\pi}M_p}\right)\tau\right] . \tag{56}$$

Therefore, the exponential factor decreases with increasing Λ for either choice of the time parametrization. This suggests that out of all possible Universes with $\Lambda \geq 0$ it is most probable to live in those Universes with $\Lambda \simeq 0$.

Perhaps this is the reason why we live in a Universe (or in a part of the Universe) with a vanishingly small value of the cosmological constant.

Note that both in this case and in the case of the coupling constant λ we are speaking about probability distributions that become infinitely sharp in the limit $t \to \infty$. It strongly resembles the infinite sharpness of the distribution of probability to find a Universe with a given cosmological constant in the context of the baby Universe theory, $P(\Lambda) \sim \exp\left(\exp \frac{3\pi M_P^2}{\Lambda}\right)$ [12]. The existence of such a peak in the baby Universe theory was extremely counterintuitive. Indeed, if our Universe had lived for only 10^{10} yr, it could not "know" the value of its energy density with infinitely high precision. In our case the explanation of the infinite sharpness of the probability distribution is very simple: The Universe lives for infinitely long time, and even a very small deviation from $\Lambda = 0$ eventually makes a lot of difference.

A note of caution is needed here before one gets too excited. First of all, we have obtained our results in the context of a particular inflationary theory, which may or may not be correct. Our conclusions would be different if one is allowed to vary not only the cosmological constant but other coupling constants as well. Even more immediate problem arises if one considers the possibility to have a negative cosmological constant, since in this case our exponential factor becomes even greater. This is similar to the problem of the negative cosmological constant in the baby Universe theory [15].

An obvious way out of this difficulty is to note that the universe with $V_0 \ll -10^{-29}$ g cm^{-3} would collapse within the time smaller than 10^{10} yr, and nobody would discuss the cosmological constant problem in such a Universe. Unfortunately, as we already mentioned, anthropic considerations typically are not strong enough to fight against indefinitely growing or decreasing exponents. However, anthropic considerations could be quite sufficient if we were able to find some natural cutoff in our integrals (49) at very large t. Taking into account all our doubts concerning the measure of integration, this possibility is not inconceivable.

Another possibility is that a negative cosmological constant is forbidden by some law of nature. This is known to be the case in globally supersymmetric theories, where the cosmological constant can only be positive or zero. In locally supersymmetric theories this property can be violated. Still there is a chance that in future theories the problem of a negative cosmological constant will be less urgent.

Finally, it is quite possible that the problem of a negative cosmological constant should be addressed at a somewhat more advanced level. If there is any analogy between our approach and the baby Universe theory, this analogy suggests that perhaps we still did not make "exponentiation of the exponent," we still did not take into account nonlocal interactions of exponentially expanding Universes with each other. This interaction may be less efficient if the Universes must disappear soon after being created, which is the case if the cosmological constant is negative. A possible solution of the problem of negative cosmological constant in the context of the inflationary baby Universe theory was envisaged in Ref. [40].

VI. DISCUSSION

This paper consists of two main parts. In the first part of the paper, Secs. II–IV, we have studied several different regimes which are possible in inflationary cosmology with an account taken of the process of self-reproduction of inflationary domains. It appears that by changing the coupling constants in a simple class of inflationary models one can go from the models where inflation is possible, but there is no self-reproduction of the Universe, to the models where the Universe is self-reproducing, and it can be described by a stationary probability distribution $P_p(\phi)$. Some additional modifications lead to models where self-reproduction of the Universe is so active that the corresponding probability distribution within finite time moves toward infinitely large values of the inflaton field ϕ. This classification of possible inflationary regimes, as well as the investigation of self-reproduction in the models of a scalar field nonminimally coupled to gravity and in Starobinsky model, can be of some interest independently of a more speculative discussion contained in Sec. V.

The investigation performed in Sec. V is based on the assumption that coupling constants can take different values in different quantum states of the Universe (which we call different "Universes"). The basic assumption which we are making is that we are typical observers, and therefore we live in those Universes where most other observers live. Thus, by finding out the values of coupling constants in the Universes occupied by most of the observers, we may find an "explanation" of the values of the coupling constants in our own Universe.

This approach is very ambiguous, even though we understand it much better than the Euclidean approach to the baby Universe theory. It is amazing that in some cases these two approaches give similar results. One of these results seems especially interesting. If one considers self-reproduction of the Universe in the context of the Starobinsky model and assumes that the cosmological constant Λ may take different non-negative values in different parts of the Universe, then our results imply that a typical observer should live in a state with $\Lambda = 0$. Of course, one should not consider this result as a solution of the cosmological constant problem until we understand why the cosmological constant cannot be negative. We clearly realize how far we are from any final and rigorously proven conclusions.

In the beginning of our paper we have mentioned that the possibility of having different coupling constants appears even without any recourse to the baby Universe theory. Indeed, during the process of self-reproduction of inflationary domains the Universe becomes divided into exponentially large regions where all possible laws of the low-energy physics compatible with inflation can be realized [1]. Since these parts of the Universe are exponentially large and causally disconnected, for all practical purposes one may consider them as separate universes. Thus one could expect that all results of the investigation performed in Sec. V should be valid for the distribution of probability to live in a part of the Universe with given values of coupling constants. This would make our discussion much less speculative.

We have used this approach in [6, 8]. However, the results obtained in [6, 8] differ considerably from the results of our investigation of the baby Universe theory. The reason is that in all parts of the Universe which can be produced from a single inflationary domain by the process of classical motion and quantum diffusion (or tunneling), the exponential factor $e^{\alpha t}$ in the stationary distribution (4) is universal. It does not depend on the value of the effective cosmological constant (vacuum energy) in each particular minimum of the effective potential, on the curvature of the effective potential near each of its minima, etc. All phases which can exist in the theory and which can transform to each other due to classical motion and quantum jumps appear to be in a kind of "thermal equilibrium" with the same "temperature" α. Only those parts of the phase space of the theory which evolve absolutely independently of each other can have different values of α [41]. Therefore in the theories where the probability distribution is stationary, the most important tool for comparison of different branches of inflationary Universe is not the overall factor $e^{\alpha t}$, which we have studied in this paper, but the normalized probability distribution P_p. This distribution does not have any singularities encountered in our treatment of the baby universe model. Consequently, this approach is not expected to lead to any troubles with too flat effective potentials. Since the probability distribution P_p is not singular, one can use it in combination with the anthropic principle in order to explain the values of effective coupling constants in our part of the Universe [6, 8].

On the other hand, the theory of inflationary baby Universes can be more powerful in solving the cosmological constant problem. That is why we have mentioned both possibilities in this paper. The main reason why we decided to discuss here the results obtained by both methods despite all uncertainties involved can be explained as follows. For many years the general attitude toward quantum cosmology was rather sceptical. Even some authors of quantum cosmology believed that this theory, being very important for investigation of creation of the Universe, does not have any testable observational consequences. However, our investigation suggests that within the context of quantum cosmology there may exist a rather strong relation between the values of coupling constants, the structure of the Universe at ultimately large distances, and the properties of interactions at nearly Planckian scales. By testing one part of the picture we may get some nontrivial information about its other parts. This may give us a possibility to test the basic principles of quantum cosmology as well.

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