Cosmological models with a time-dependent Λ term

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(Received 20 May 1994)

Cosmological models with a time-dependent Λ term of the form $\Lambda_1 + \Lambda_2 R^{-m}$ are considered for both dust and noncoherent radiation source terms. A general expression for the age of the Universe in the matter-dominated case is presented and some of its consequences are analyzed. The relevance of constraints imposed by the observed γ-ray flux, the extragalactic background light, and the gravitational lensing phenomena are briefly considered. The Landau-Lifshitz fluctuation theory, as presented by Pavon, is applied to obtain constraints on the allowable set of parameters in the radiation-dominated model.

PACS number(s): 98.80.Hw

A lesson given by the history of cosmology is that the concept of the cosmological term revives in the days of crisis. Over 60 years ago Eddington wrote [1] "...Λ-term is the strongest pillar of the theory of relativity and I would as soon think to reverting to Newtonian theory as of dropping the cosmical constant...." And though the motives for which Eddington was so strong a supporter of retaining the cosmological constant are not so persuasive now as they were in his times (at least for him), today we have more reasons than ever to belive that the cosmological term is the necessary ingredient of any cosmological model.

The possibility of adding a cosmological vacuum energy density to the Einstein field equations raises the question of empirical justification of such a step. A positive cosmological constant helps overcome the age problem, connected on the one side with the high estimates of the Hubble parameter and with the age of the globular clusters on the other. Further, it seems that in order to retain the cold dark matter theory in a spatially flat Universe most of the critical density should be provided by a positive cosmological constant [2,3].

Observational data indicate that the cosmological constant, if nonzero, is smaller than 10^{-55} cm^{-2}. However, since everything that contributes to the vacuum energy acts as the cosmological constant it cannot just be dropped without serious considerations. Moreover, particle physics expectations for Λ exceed its present value by the factor of order 10^{120} that is in a sharp contrast to observations.

To explain this apparent discrepancy the point of view has been adopted which allows the Λ term to vary in time [4–12]. The idea is that during the evolution of the Universe the energy density of the vacuum decays into the particles thus leading to the decrease of the cosmological constant. As a result one has the creation of particles although the typical rate of the creation is very small.

In the models proposed so far the variability of Λ is generally of the form

$$\Lambda(t) = \alpha R^{-m} + \beta \left(\frac{\dot{R}}{R}\right)^2, \quad (1)$$

where R is a scale factor; α, β, m are constants and an overdot denotes differentiation with respect to the cosmic time coordinate. Though different in assumptions Ozer and Taha [4] and Chen and Wu [7] concluded that the time dependence of Λ should have $\beta = 0$ and $m = 2$. The law of Gasperini [6] involves $\beta = 0$ and $\frac{18}{5} < m < 4$, whereas Freese *et al.* [5] advocated the law of variation with $\beta = 0$ and $m = 4(\kappa - 1)$, where κ is a phenomenological constant parameter. We remark here that Berman [11] proposed explicit time dependence $\Lambda \propto t^{-2}$.

In the other approach to the problem Peebles and Ratra [14] considered a spatially flat cosmological model with a scalar field with the power law potential. Such a model produces a Λ-like term decreasing in time. It should be noted however that the resulting equation of state approaches the Λ equation of state in the limit of vanishing power. In the Peebles-Ratra model there is no creation of particles; all potential energy is converted into the kinetic energy of the scalar field.

Since the (modified) Einstein field equations do not specify models completely additional assumptions have to be made. Pavon [13] proposed to analyze the models from the point of view of their thermodynamic correctness by studying the fluctuations of fluxes around their mean value within the framework of the Landau-Lifshitz fluctuation hydrodynamic theory. Unfortunately the applications of this theory without the detailed knowledge of the particle content of the Universe and their mutual interactions are limited to the radiation-dominated era. Pavon's analysis extended later by Salim and Waga [10] to the $\beta \neq 0$ case provides definite constraints on the admissible features of such models.

Cosmological models with a time-dependent Λ term or its analogue have been subjected to numerous tests. Observed γ-ray flux [5], extragalactic background light [15], the number count of faint galaxies [16], the cosmic background radiation anisotropies [17], and the gravitational lensing phenomena [18] have been successfully applied to constraint the allowable set of parameters of the models.

The purpose of this work is to analyze general features of the solutions of the Einstein field equations with a different time dependence of the "cosmological constant:" namely,

$$\Lambda(t) = \Lambda_1 + \Lambda_2 R^{-m}, \quad (2)$$

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where Λ_1, Λ_2, and m are constants. Special emphasis is given to the problem of the age of the model Universe. We consider also the relevance of the constraints imposed on the model from the observed γ-ray flux, the extragalactic background light, and the gravitational lensing phenomena.

To avoid great negative values of the Λ term in the past, we shall assume that $\Lambda_2 > 0$, placing no additional requirements on Λ_1. Such a cosmological term allows changes of a sign of the effective cosmological constant.

For the Friedmann-Lemaitre-Robertson-Walker cosmologies the Einstein field equations with the variable cosmological constant and a source term given by a stress-energy tensor of a perfect fluid read

$$3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} = 8\pi(\Lambda + \rho), \tag{3}$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = 8\pi(\Lambda - p), \tag{4}$$

where ρ is the fluid energy density and p is its pressure.

The equation of state is taken in the form

$$p = \alpha\rho, \tag{5}$$

where α is a constant satisfying $0 \leq \alpha \leq 1/3$. From Eqs. (3) and (4) one obtains

$$\frac{d\rho R^3}{dt} + p\frac{dR^3}{dt} + R^3\frac{d\Lambda}{dt} = 0. \tag{6}$$

Eliminating the source terms and putting $\gamma = 1 + 3\alpha$, $\delta = 1 + \alpha$, and $a_i = 8\pi\Lambda_i$, the first integral may be easily obtained:

$$\dot{R}^2 = a_1\delta\frac{R^2}{\gamma+2} + a_2\delta\frac{R^{2-m}}{\gamma - m + 2} + b_1R^{-\gamma} - k \tag{7}$$

and

$$\dot{R}^2 = \frac{\delta a_1}{\gamma+2}R^2 + \delta a_2 R^{-\gamma}\ln R + b_2R^{-\gamma} - k, \tag{8}$$

for $\gamma \neq m+2$ and $\gamma = m+2$, respectively,

Assuming that all considered models evolve from a singular state it could be easily shown that the above equations place a general limit on the value of m, namely,

$$0 < m < \gamma + 2, \tag{9}$$

provided $b_1 \geq 0$. If $\alpha = \frac{1}{3}$ one has $m < 4$. Similar restrictions follow, as we shall see, from the thermodynamic considerations.

Equation (7) may be easily integrated in terms of exponential, trigonometric, or hyperbolic function for the special choices of parameters. As such solutions are self-evident we shall not treat them separately.

Let $a_1 > 0$, $a_2 > 0$, and $b_1 > 0$, then for $k \leq 0$ the model expands forever. The particular case $k = 1$ is more complicated. Let $r = V(R_{\max})$ be a maximal value of the effective potential

$$V(R) = -a_1\delta\frac{R^2}{\gamma+2} - a_2\delta\frac{R^{2-m}}{\gamma - m + 2} - b_1R^{-\gamma}. \tag{10}$$

If $r < -1$ the model expands forever. If $r = -1$ there is a singular solution corresponding to the Einstein static universe. Moreover, there are solutions either asymptotically tending to or asymptotically starting from the Einstein universe. The condition $r > -1$ leads to the oscillatory solutions.

Let $a_1 < 0$, $a_2 > 0$, and $b_1 > 0$. Since for great values of R the first term is always dominant there exists a maximal value of R regardless of the sign of k. However, whether the model has an oscillatory character or there exist singular and asymptotic solutions depends on the exact values of the parameters. For $m \leq 2$ one has only the oscillatory solutions, whereas for $2 < m < 2 + \gamma$ the effective potential has a local maximum, and hence if $r = -1$, the singular and asymptotic solutions are admissible.

To this end let us consider the special case of $b_1 = 0$. If $a_1 > 0$, $a_2 > 0$, and $0 \leq m \leq 2$, only oscillatory models with $k \leq 0$ are admissible. In the case with $2 < m \leq m + \gamma$ the basic features of solutions are similar to the case with nonzero b_1. If $a_1 < 0$ only the oscillatory solutions may be constructed.

It should be noted that though the condition $a_1 \leq 0$ is allowed it does not mean that the "observed" effective cosmological constant is necessarily negative. In fact it may be positive and smaller than 10^{-55} cm^{-2}, or even be zero now due to a fine cancellation. Moreover, since the model with the negative a_1 either recollapses or has singular or asymptotical solutions the effective cosmological term may be always non-negative. It might be, of course, negative as well.

For the matter-dominated universe, i.e., when $p = 0$, Eqs. (7) and (8) could be rewritten in terms of the observed quantities, such as the present value of the Hubble constant H_0, the deceleration parameter q_0, and the cosmological density parameter Ω_0, to yield the age of the Universe:

$$t_0H_0 = \int_0^1 \frac{dx}{\sqrt{\frac{1}{3}\tilde{a}_1(x^2 - \frac{1}{x}) + \frac{\beta_1}{3-m}(x^{2-m} - \frac{1}{x}) + \beta_2(\frac{1}{x} - 1) + \frac{1}{x}}}, \tag{11}$$

for $m \neq 3$ and

$$t_0H_0 = \int_0^1 \frac{dx}{\sqrt{\frac{1}{3}\tilde{a}_1(x^2 - \frac{1}{x}) + x^3\ln(x)\beta_1 + \beta_2(\frac{1}{x} - 1) + \frac{1}{x}}}, \tag{12}$$

for $m = 3$, where

$$\frac{\tilde{a}_2}{R_0^m} = \frac{3}{2}\Omega_0 - \tilde{a}_1 - 3q_0 = \beta_1, \tag{13}$$

$$\frac{k}{R_0^2H_0^2} = \frac{3}{2}\Omega_0 - q_0 - 1 = \beta_2, \tag{14}$$

and $\tilde{a}_i = a_iH_0^{-2}$. Note that our definition of the effective cosmological constant is three times Ω_Λ.

It should be noted that when $m = 2$, Eq. (11) does not explicitly depend on Ω_0. On the other hand, (11) does not depend on m, if $\beta_1 = 0$.

A straight consequence of Eq. (14) is the possibility of expressing the present value of the scale factor by observable parameters provided k is nonzero. We have

$$R_0 = T_H \left(\frac{k}{\beta_2}\right)^{\frac{1}{2}}, \qquad (15)$$

where $T_H = H_0^{-1}$. However, if $k = 0$, the scale factor cannot be determined without knowledge of the exact values of the normalized cosmological constants and the power m. In this case we have instead of standard $\Omega_0 = 2q_0$ a little more complicated $\Omega_0 = \frac{2}{3}q_0 + \frac{2}{3}$. Note that for the critical density both models give $q_0 = \frac{1}{2}$.

Before preceding further we shall evaluate the rate of particle creation (annihilation) n, that is defined as

$$n = \frac{1}{R_0^3}\left.\frac{d\rho R^3}{dt}\right|_0. \qquad (16)$$

Making use of (6), (13), and (15) yields

$$n = \frac{mH_0^2\beta_1}{8\pi R_0}. \qquad (17)$$

Since we have chosen $a_2 > 0$ we have particle creation only.

It is believed that luminous matter contributes only a small fraction of Ω_0, and the luminous matter plus the dark matter give $\Omega_0 = 1$. However, recent estimates from observations of galaxy clustering and their dynamics indicate that the mean mass density is about one-third of the critical value [19]. Madsen *et al.* [20] indicate that the critical value of the cosmological density parameter favored by numerous workers is not generically connected with inflation, thus allowing low energy cosmological models. All that is known for certain about the present value of Ω_0 could be exhibited by means of the inequality [22]

$$0.03 \leq \Omega_0 \leq 2. \qquad (18)$$

Observational evidence does not rule out the negative deceleration parameter and the tightened limits on the present value of q_0 are [21]

$$-1.27 \leq q_0 \leq 2. \qquad (19)$$

Various methods place the age of the Universe between 10 and 20 Gyr [22]. Though it seems unlikely that T_0 is less than 12 Gyr. Analyses of the age of the oldest stars in our galaxy and adding realistic incubation time yield $T_0 = 15$ Gyr.

The Hubble parameter is usually taken to be $h\,100$ (km sec^{-1} Mpc^{-1}), where $0.4 \leq h \leq 1$. Since the Hubble time $T_H = 9.778\,h^{-1}$ (Gyr) is related to the Hubble parameter it follows then that it would be necessary to introduce modifications in the standard cosmology to satisfy requirements that follow from observations if the exact value of H_0 exceeds the Sandage-Tammann [23] value and is equal to these advocated by Jacoby *et al.* [24], or van der Bergh [25], i.e., $h = 0.8$ and $h = 0.76$, respectively.

We now turn to the discussion of constraints imposed on the model from observational data. Provided the baryon number is conserved, decay of the cosmological term into baryons may be constrained by the observed γ-ray flux. Integration of Eq. (6) and comparison with the observations [26] yields the condition

$$\frac{m}{3-m}\beta_1 h^2(1 - x_{\rm eq}^{3-m}) \lesssim 2.6\times 10^{-4}, \qquad (20)$$

where $x_{\rm eq} = R(t_{\rm eq})/R_0$ is a normalized scale factor when matter and radiation were equal. Since $x_{\rm eq}$ is expected to be small and since it seems unlikely that $m \ll 1$, it follows that the adiabatic relation of the matter-dominated era is satisfied and consequently the density of matter scales as R^{-3}.

Therefore one has

$$x_{eq} \cong \left(3\Omega_0^r - \frac{m\beta_1}{4-m}\right)\frac{1}{3\Omega_0^m}, \qquad (21)$$

and Ω_0^r and Ω_0^m are the energy density of radiation and matter, respectively. From (21) it follows that β_1 cannot exceed $3\,(4-m)\,\Omega_0^r m^{-1}$. Numerically for $m = 2$ that is expected to be a best candidate condition (20) yields $\beta_1 h^2 \lesssim 1.3\times 10^{-4}$.

Let us assume that photons created during the vacuum decay have Planckian spectrum and are in equilibrium with those already present. Using the method of Valle, Wesson, and Stabell [27] in the present context the intensity of the extragalactic light $I(\lambda)$, where λ is the wavelength, may be computed and compared with recent data provided by the Cosmic Background Explorer (COBE) [28]. As in [15] we assume that matter decoupled from radiation not much earlier $t_{\rm eq}$. Unfortunately the method invented by Overduin *et al.* and applied to the model proposed by Freese *et al.* cannot be applied here. In our model the fraction of the radiation energy density to the vacuum energy density is necessarily a function of the scale factor. Therefore we have calculated the $I(\lambda)$ for observed parameters, starting from $x_{\rm eq}$ to the present epoch. In the calculations three massless neutrino species have been assumed. From

$$\rho^\gamma(x) = \frac{m\beta_1 H_0^2}{8\pi(4-m)}\left(x^{-m} - x^{-4}\right) + \frac{3H_0^2\Omega_0^\gamma}{8\pi}x^{-4}, \qquad (22)$$

where ρ^γ is photon energy density and $\Omega_0^\gamma = \frac{8\pi\rho_0^\gamma}{3H_0^2}$, it follows that $\beta_1 < \frac{3(4-m)\Omega_0^\gamma}{m}$. Note that neutrinos after decoupling redshifts as x^{-4}. The calculations of the extragalactic background light for $\Omega_0 = 0.2$, $m = 2$, and $h = 0.5$ are presented in Fig. 1, and indicate that if the above condition holds calculated $I(\lambda)$ does not exceed

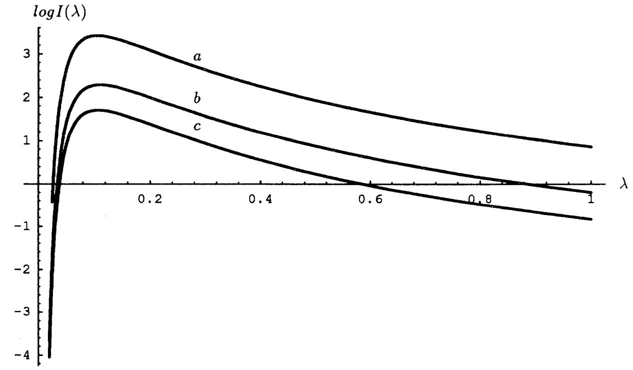

FIG. 1. Comparison of intensity of the extragalactic background light $I(\lambda)$ in units 10^{-64} cm^{-3} calculated for $\Omega_0 = 0.2$, $h = 0.5$, and $m = 2$ to the actual intensity of the cosmic microwave background as measured by COBE. (a) COBE; (b) $\beta_1 = 5\times10^{-5}$; (c) $\beta_1 = 2\times10^{-4}$.

the observed flux as expected.

The behavior of T_0 as a function of parameters $m, \tilde{a}_1$, q_0, and Ω_0 may be easily inferred from Eq. (11). The general rule is that decreasing the deceleration parameter the present age of the Universe becomes greater. Recall that if $\beta_1 = 0$ then T_0 does not explicitly depend on m and therefore any (p, m) section, where p stands for Ω_0, q_0, or $\tilde{a}_1$ is divided by $p = p_c$, where parameters with superscript c satisfy the condition

$$\frac{3}{2}\Omega_{0c} = 3q_{0c} + \tilde{a}_{1c}, \tag{23}$$

into two regions in which exclusively particle creation or annihilation takes place.

Consider at first the (Ω_0, m) sections; by (23) for each q_0 and $\tilde{a}_1$ there exists a critical value Ω_{0c} such that for $\Omega_0 < \Omega_{0c}$, T_0 is a decreasing function of m, and the particle annihilation takes place, for $\Omega_0 = \Omega_{0c}$ has its minimum, whereas for $\Omega_{0c} < \Omega_0$ it is an increasing function of m and the creation occurs. The general tendency is that Ω_{0c} increases with q_0 and $\tilde{a}_1$. Since, as we have remarked before, T_0 is independent of Ω_0 when $m = 2$, one has the following qualitative picture: for $m < 2$, T_0 decreases with Ω_0, while for $m > 2$, T_0 is an increasing function.

The (q_0, m) sections show that T_0 is always a decreasing function of q_0. By (23) there exists a critical value q_{0c}, such that for $q_0 < q_{0c}$, T_0 increases with m, while for $q_0 > q_{0c}$, T_0 is the decreasing function. The general tendency is that q_c increases with Ω_0 and decreases with $\tilde{a}_1$.

To this end we remark that by (23) the qualitative behavior of $(m, \tilde{a}_1)$ sections are similar to the cases treated above.

Since a primary goal of introducing a cosmological term is, in addition to its presence in the particle physics considerations, to increase T_0, it is interesting to evaluate expected age of the model universe as a function of Ω_0 and $\tilde{a}_1$ for chosen values of β_1 and m. The results of such calculations for $\beta_1 = 10^{-4}$ and $m = 2$ are presented in Figs. 2 and 3.

It has been pointed out that in the Λ-constant models the gravitational lensing optical depth, τ, i.e., the probability that quasar is multiply imagined by a gravitational lens along the line of sight of an observer is a sensitive indicator of the value of the cosmological constant. The general tendency is that τ increases with Λ in spite of uncertainity with the notion of a distance [29]. It is therefore natural to examine our model from the point of view of this test. The normalized optical depth is given by the expression [30]

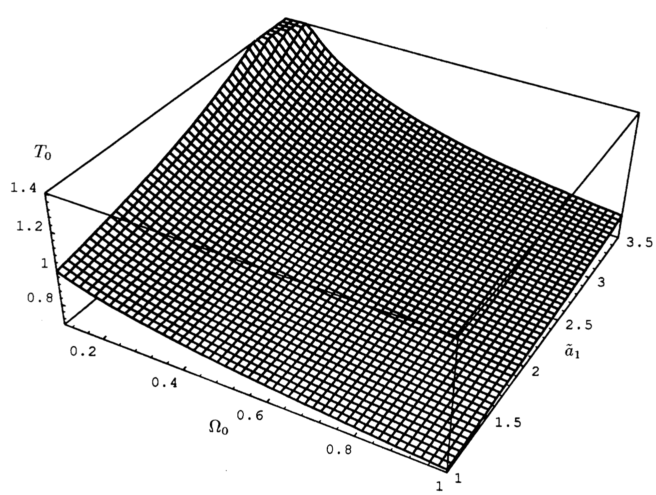

FIG. 2. The age of the Universe T_0 as a function of Ω_0 and $\tilde{a}_1$ for $m = 2$ and $\beta_1 = 10^{-4}$.

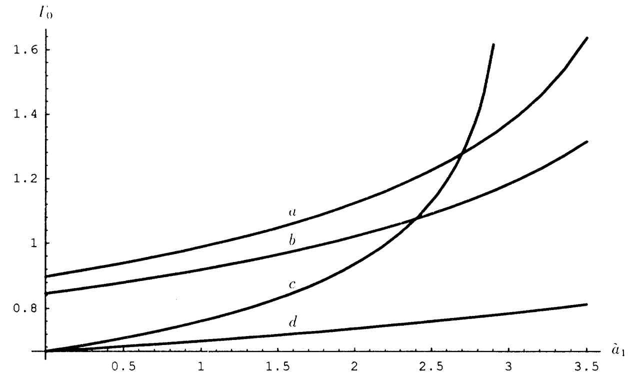

FIG. 3. T_0 as a function of $\tilde{a}_1$ for $m = 2$ and $\beta_1 = 10^{-4}$. T_0 is presented for (a) $\Omega_0 = 0.1$; (b) $\Omega_0 = 0.2$; (c) spatially flat model; (d) $\Omega_0 = 1$.

$$f(z) = \int_0^{D_S} \frac{D_{OL}^2 D_{LS}\, dD_{OL}}{(1 - \beta_2 D_{OL}^2)\, D_{OS}^2}, \tag{24}$$

where D is a proper motion distance, and the subscripts O, L, and S represent the observer, lens, and source, respectively. Since the probability that a quasar is lensed is the product of its optical depth, its magnification bias, the selection effect and presumably other factors, it is evident that the large τ are strongly disfavored.

As we have remarked before, the main difference between the present and the Peebles-Ratra model stems from the fact that in the latter model the ordinary Λ equation of state is approached in the limit of vanishing power and that only spatially flat cosmologies are considered. Nevertheless, it is interesting to compare its predictions. Ratra and Quillen have shown that for $\Omega_0 = 0.2$ and reasonable choice of the power, say, 4 in the Peebles-Ratra model, $f(z)$ at $z = 2.5$ is approximately factor 1.8 smaller than in the flat, constant Λ-dominated model with the same value of Ω_0. Numerical calculations of $f(z)$ in the model at hand have been carried out for $\beta_1 = 10^{-4}$. In our model, for a given value of the baryonic density parameter, reduction of the optical depth is necessarily connected with $k = -1$ models, i.e., with decreasing of $\tilde{a}_1$. Inspection of Fig. 4 shows that $f(z)$ strongly increases with a_1. Numerically, the ratio $\tau(z)/\tau_{\rm EdS}$, where $\tau_{\rm EdS}$ denotes the optical depth in the Einstein–de Sitter model, for $\Omega_0 = 0.2$ at $z = 2.5$ is approximately 4.3 for a flat model (a), 3 for the case (b), 2.2 for (c), and 1.7 for (d). Since the high estimates of the Hubble constant require the greater a_1 one concludes therefore that there would be a problem with a matching the age of the model with the reasonable optical depth. One possible way to cure the situation is to assume the dust obscuration, as it was proposed by Fukugita and Peebles [31].

Let us consider more closely the special case of the

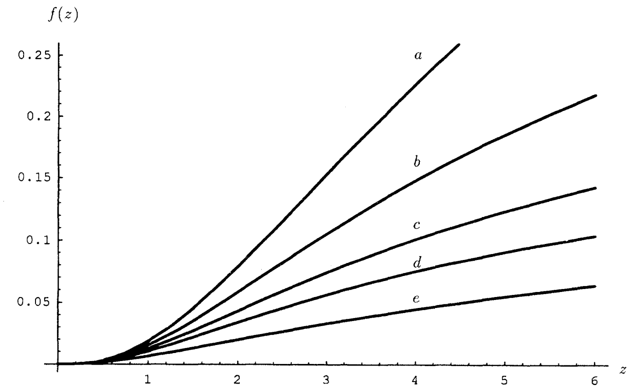

FIG. 4. The normalized optical depths $f(z)$ as a function of a redshift of a source evaluated for $\beta_1 = 10^{-4}$, $m = 2$, and $\Omega_0 = 0.2$. The optical depths are shown for the five cases: (a) $\tilde{a}_1 = 2.3999\,(T_0 = 1.08)$ (spatially flat model); (b) $\tilde{a}_1 = 2$ $(T_0 = 1.02)$; (c) $\tilde{a}_1 = 1.5$ $(T_0 = 0.96)$; (d) $\tilde{a}_1 = 1$ $(T_0 = 0.92)$; (e) Einstein–de Sitter-type $(T_0 = 2/3)$.

radiation-dominated universe, i.e., when $p = \frac{1}{3}\rho$. It is expected that this very phase of the evolution occur in the early Universe, and therefore we shall consider the expansion only. As in the matter-dominated case we have now two expressions for $\dot{R}^2$ with $m \neq 4$ and $m = 4$. However, contrary to the case of the matter-dominated era it is expected that this period is rather short and practically adds nothing to the total age of the Universe. Therefore we shall not present the analogues of Eqs. (11) and (12) here. Setting the chemical potential to zero—that is justifiable during the radiation era—the entropy rate in a comoving volume V is given by

$$\dot{S} = -\frac{V}{T}\dot{\Lambda}, \tag{25}$$

where T_0 is the temperature. The second law of thermodynamics requires $\dot{\Lambda} < 0$. From Eq. (2) we have

$$\dot{\Lambda} = -m\Lambda_2 R^{-m} H, \tag{26}$$

and hence m and the "second cosmological constant" Λ_2 must be of the same sign. Now in order to apply the Landau-Lifshitz fluctuation theory as presented by Pavon one has to decide if $\dot{\Lambda}$ is steady for large values of the scale factor.

It is useful to rewrite the fluid energy density in terms of the scale factor. For $m \neq 4$ one has

$$8\pi\rho = \frac{m\, a_2}{4-m} R^{-m} + 3b_1 R^{-4}. \tag{27}$$

Now, from the Landau-Lifshitz theory we have

$$\left.\frac{\langle \delta\dot{\Lambda}(t_1)\delta\dot{\Lambda}(t_2)\rangle}{\langle \dot{\Lambda}(t_1)\dot{\Lambda}(t_2)\rangle}\right|_{t_1=t_2} \propto \frac{TR^{m-3}}{m\Lambda_2 H}. \tag{28}$$

From (7) it follows that expression (28) is decreasing in cosmic time and thus the expansion of the Universe is governed by classical equations if $0 < m < 4$, that is conformed with our earlier statement.

Let us now summarize and add a few remarks to the results presented in this work. We have obtained a general formula that describes the age of the Universe as a function of observable parameters by solving gravitational field equations with a time-dependent cosmological term. Assuming $h = 0.5$ the age of the Universe as expressed in the units of the Hubble time is approximately confined between 0.56 and 1.12. On the other hand, if $h = 0.8$ then $0.81 \leq T_0 \leq 1.62$. Regardless of the exact value of the cosmological density it is a simple task to adopt the parameters in the model at hand to satisfy the expected age of the Universe.

Since Ω_0, m, q_0, and the present value of the effective cosmological constant are interrelated, (18) and (19) yield

$$-5.955 < \tilde{a}_1 + \frac{\tilde{a}_2}{R_0^m} < 6.81. \tag{29}$$

Taking $\Omega_0 = 0.2$ and $m = 2$ one has

$$-5.7 < \tilde{a}_1 + \frac{\tilde{a}_2}{R_0^m} < 5.11, \tag{30}$$

that in view of the foregoing discussion is practically condition on $\tilde{a}_1$.

The γ-ray test for $m = 2$ shows that $\beta_1 h^2 \lesssim 1.26 \times 10^{-4}$, whereas the extragalactic background light tests show that for the same value of the parameter m one has the following constraint $\beta_1 h^2 \lesssim 7.5 \times 10^{-5}$. The large cosmological constant predicts too many lensing events and since the differences in the optical depths associated with substantial variations of Λ are greater than uncertainties of the definitions of τ alone, one concludes that the analyses of the gravitational lensing provide a powerful tool in attempts to determine the world model. In order to keep balanced the expected age of the Universe, say, 15 Gyr with high estimates of the Hubble constant and reasonable gravitational lensing frequency one is forced to introduce some external agent such as the dust obscuration or even more radical $z = 0.5$ cutoff.

It should be noted that observational evidence is consistent also with a low density $\Lambda = 0$ model [32] although there may be problems if the Universe is older than 12 Gyr and the Hubble constant is substantially greater than 50 km Mpc^{-1} sec^{-1}.

Historically, the Λ term has periodically appeared and disappeared in cosmology, and the "...most recent cycle of interest derives from a mutually supportive combination of aggressive theoretical prejudice and new, suggestive, observations..." [33]. It is expected that new observations that are going to be made provide the definitive answer regarding the existence and the very nature of this important parameter.

The author wishes to thank Andrzej Baran for his help with numerical calculations.

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