

## Arrow of time in a recollapsing quantum universe

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We argue that the Wheeler-DeWitt equation with a consistent boundary condition is only compatible with an arrow of time that formally reverses in a recollapsing universe. To recover a classically recollapsing universe in terms of wave packets, we impose the usual boundary condition of excluding exponentially increasing wave functions for large scale factors. Consistency of these opposite arrows is then facilitated by quantum effects in the region of the classical turning point. We also discuss in this context the meaning of the time-asymmetric expression used in the definition of “consistent histories.” Since gravitational time dilation diverges at horizons, one has to conclude that collapsing matter must start reexpanding “anticausally” (controlled by the reversed arrow) in this scenario before horizons or singularities can form. There would then also be no mass inflation nor any information loss paradox.

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### I. INTRODUCTION

In conventional statistical physics, the thermodynamical arrow of time is described by assuming the initial entropy to be extremely small compared to its most probable (equilibrium) value, while a final state with low entropy is usually rejected as “improbable.” Since such a special initial state is, of course, equally improbable, this description is not very satisfactory as an *explanation* of the arrow [1, 2]. It is, however, entirely consistent with deterministic dynamical laws (including the Schrödinger equation), since the integration constants can be chosen by fixing the state arbitrarily at any time. The sole use of probability arguments would predict an uninteresting, thermodynamically symmetric history of the state with fluctuations around equilibrium for all times. Quantum cosmology, on the other hand, does not seem to admit even the *formulation* of such a factlike asymmetry in time [3], since the classical time parameter disappears in quantum gravity.

In statistical quantum mechanics, entropy as the usual measure of time asymmetry is calculated by means of the functional

$$S[\rho] = -k_B \text{Tr} \left[ \hat{P} \rho \ln(\hat{P} \rho) \right] \quad (1)$$

from a density matrix  $\rho$  that may correspond to a pure state if  $\hat{P} = \hat{P}^2$  is an appropriate trace-preserving projection operator on the space of density operators. It represents a concept of *relevance* or generalized *coarse graining* [1]. In order to describe the time dependence of the entropy,  $\rho$  must here be given in the Schrödinger picture. The increase of entropy is then described as the transformation of relevant into irrelevant information, with the corresponding Poincaré cycles assumed to be much longer than the age of the Universe. Unless this time dependence of  $\rho$  contains also the “col-

lapse of the wave function” (or, equivalently, Everett’s branching) into *definite*, although dynamically undetermined outcomes of quantum measurements, the formal entropy (1) must include the entropy of lacking information representing this indeterminism. *Physical* entropy is instead defined as a function of “given” classical quantities — and never as a function of quantities which would be represented by any of their superpositions as they may arise in measurementlike situations according to the unitary Schrödinger dynamics. If the collapse represented an asymmetric fundamental dynamical law violating the deterministic Schrödinger equation [4], it would represent an absolute arrow of time and would thus be a candidate for the origin of the thermodynamical arrow. Since there is as yet no experimental hint on such a fundamental irreversible dynamics and its precise nature, this possibility will not be considered here.

As the arrow of time is a cosmic phenomenon, and since the entropy of the Universe seems to be dominated by gravity [5], quantum gravity has to be fundamentally included in the description. It will here be assumed that the Universe is described by a unified canonical quantum theory that possesses a reparametrization invariant classical counterpart such as general relativity. This invariance may be interpreted as a kind of Machian principle with respect to time, that is, as the absence of any *absolute* or preferred time parameter [6]. As there are no trajectories in quantum theory any more (which could at least be parametrized in an arbitrary way), the corresponding Schrödinger equation for gravity or any other reparametrization invariant theory can only be of the stationary form

$$H\Psi = 0, \quad (2)$$

the *quantum Hamiltonian constraint* or generalized *Wheeler-DeWitt equation*. In the absence of any time pa-

parameter, this dynamical law does not allow one to pose an “initial” condition of low entropy at *any* end in time. Although it may be sufficient, and quantum cosmologically even very plausible, that  $\Psi$  depends only on *physical* variables (including all conceivable “clocks”), the general nature of the boundary value problem which is required to determine a solution to the Wheeler-DeWitt equation (2) appears problematic. It may therefore come as a surprise that, at least in the neighborhood of Friedmann-type cosmologies, the Wheeler-DeWitt equation (for constant lapse function  $N$ ) is of the hyperbolic type and thus defines an intrinsic initial value problem with respect to the logarithm  $\alpha = \ln a$  of the expansion parameter (or scale factor)  $a$  [7]. We shall therefore consider this boundary value problem as the appropriate way to impose boundary conditions in quantum cosmology.

Since the potential that appears in the Wheeler-DeWitt equation (2) for these models approaches a simple form in the limit  $a \rightarrow 0$ , cf. Eq. (3) below, it suggests a similar simple structure for its solution [8]. While such a simple structure can then in fact explain the low value of entropy at the big bang (including the absence of initial entanglement and branching), it would do the same for a big crunch since they both correspond to the region of small  $a$  in configuration space. This would be in agreement with a conclusion reached long ago by Gold [9] by means of different, and presumably insufficient, arguments which are based on a classical concept of time. We emphasize that this is a debate of principle which is independent of whether or not the Universe will recontract in reality.

Hawking, on the other hand, has repeatedly claimed [10, 11], following objections by Page [12] and Laflamme against his earlier conclusion [13] in support of Gold, that the thermodynamical arrow of time must keep its direction when the Universe has reached its assumed maximal extension and starts recontracting. His arguments are based on the assumptions of a Wheeler-DeWitt equation for closed Friedmann universes together with the Hartle-Hawking (“no boundary”) boundary condition [14]. This boundary condition is based on path integrals (“sums over histories”) as a tool to calculate the wave function. The same dynamical model will be considered here, since it seems indeed to be appropriate for investigating these conceptual issues, although we shall avoid using path integrals in the definition of the boundary condition for the wave function. The no-boundary condition has to be used with caution, since Hawking’s conclusions are essentially based on semiclassical, or even classical, considerations. He has in fact explicitly claimed that the no boundary condition can *only* be used in a semiclassical approximation (see Ref. [10] and the discussion following Ref. [3]). It may thus be worth emphasizing that approximations (such as WKB) cannot give more reliable results than the exact theory, as plausible as they may appear to our classical and time-directed prejudice. If a correct treatment of the Wheeler-DeWitt equation (2) (which does not present insurmountable technical difficulties in a simple minisuperspace model) contradicts the semiclassical results, the latter must be wrong. If, furthermore, it is really true that the no boundary proposal

can only be applied semiclassically, it would then simply be inapplicable to *quantum* cosmology.

Hawking, Laflamme, and Lyons [11], for example (HLL for short), state that “the COBE observations indicate that the perturbations which *lead to the arrow of time arise at a time* during inflation when . . . Einstein gravity should be a good approximation” (our italics). Quantum mechanically this description is not consistent, since the emergence (“arising”) of quasiclassical properties (including spacetime) from a wave function already relies on the most fundamental of all “irreversible” processes (and hence on an arrow of time), namely on decoherence. Decoherence determines which kind of properties emerge in the form of a collapse or branching of the wave function into specific “world components” such as those with definite spacetime geometries and, therefore, with definite proper times along all spacetime orbits. A symmetric treatment would then give classical time an equal opportunity to disappear during the big crunch (if this can still be distinguished from the big bang in quantum gravity) by means of “recoherence” [1, 15–17].

Our paper is organized as follows. In Sec. II we introduce the quantum cosmological model which forms the basis for our discussion. We shall then present in detail our arguments which lead us to the conclusion that the exclusive use of the semiclassical approximation in [11] is not justified. In Sec. III additional degrees of freedom are introduced, which is necessary for a discussion of the arrow of time. We then show that a low entropy condition for the total wave function, consistently posed at small scale factor, must lead to a formal reversal of the arrow of time at the turning point. We also discuss the meaning of consistent histories in this context. Finally, Sec. IV focuses on the consequences of this boundary condition for black holes.

## II. THE QUANTUM FRIEDMANN UNIVERSE

The quantum Friedmann universe is described by a two-dimensional minisuperspace (a strongly restricted configuration space) which is spanned by the expansion parameter  $a$  or its logarithm  $\alpha$  and the amplitude  $\phi$  of a homogeneous scalar field representing matter in this model. These variables may then be supplemented by the amplitudes  $x_n$  of all higher multipoles of geometry and matter on the Friedmann sphere (“perturbations”) [18], which for the conceptual part of the discussion need not be assumed to be small, and therefore should be able to describe a realistic quantum universe. The Wheeler-DeWitt equation in minisuperspace is given by

$$\left( \frac{\partial^2}{\partial \alpha^2} - \frac{\partial^2}{\partial \phi^2} - e^{4\alpha}(1 - e^{2\alpha}m^2\phi^2) \right) \psi(\alpha, \phi) = 0. \quad (3)$$

This is equivalent to Eq. (2.6) of HLL, except for a different factor ordering that is irrelevant for our discussion.

Any boundary condition which is imposed to determine a solution to (3) clearly has to be understood as a condition to fix the (stationary) *wave function*, even if it is technically expressed by means of (parametrizable) Feynman *paths*. These paths have no *a priori physical* meaning in quantum theory. Their superposition in

a path integral, in particular, always represents a wave function and never merely an ensemble of paths or of (consistent or inconsistent) histories. The central point of our argument is that the mathematical boundary value problem for the wave function  $\psi$  (and its generalization to full superspace) “knows” neither of its intended physical interpretation, nor of any external or derivable concept of time that could possibly *distinguish* initial from final boundaries. Physical conclusions should then only be drawn from the solution of this equation, which must *depend* on its exact boundary condition, and may turn out to be *approximately* compatible with parametrizable trajectories in *certain regions* of configuration space where concepts of geometrical optics are applicable, i.e., where narrow wave packets which form local *components* of the solution propagate without dispersion. (Such wave packets may decohere from one another when further degrees of freedom are taken into account.) Obviously, this seems to be the case in that region which represents the present era of our observed Universe, but may not necessarily hold everywhere in superspace, including that part which corresponds to our Universe’s early past or late future (as far as they are defined at all in timeless quantum gravity). The concept of trajectories with their own “time” parameters (perhaps to be justified by a saddle point approximation in the path integral representation) must, in particular, not be assumed to be applicable for  $a \rightarrow 0$ , where the cosmological boundary conditions of low entropy are to be imposed.

A conventional way of solving (3) would consist in specifying  $\psi$  and its derivative  $\partial\psi/\partial\alpha$  on a hypersurface of fixed  $\alpha = \alpha_0$ , and then integrating with respect to  $\alpha$ . This procedure (which in particular is indicated by the hyperbolic nature of this equation) could be used to impose an initial condition at small  $a$ , for example, but it would then in general *not* lead to a reflection of the wave from the repulsive curvature potential  $e^{4\alpha}$  at very large values of  $\alpha$ . One would instead obtain an exponentially increasing wave function in the “classically forbidden” region. Such an exponentially increasing wave function was, in fact, claimed to have been found from the no-boundary proposal [14, 19].<sup>1</sup> A reflection from the “repulsive” curvature potential at large  $\alpha$ , however, seems to be needed if one wants to describe a recollapsing universe in quantum cosmology. Although it is far from clear what kind of “norm” one has to use for a wave function in quantum cosmology,<sup>2</sup> we regard it as appropriate to

proceed in accordance with standard quantum mechanics and to *exclude* exponentially *increasing* solutions in classically forbidden regions. (Note that we do not impose the stronger assumption of  $\mathcal{L}^2$  integrability in  $\alpha$ .) Although there are no strictly classical forbidden regions due to the indefinite nature of the kinetic energy, without this condition one would not be able at all to represent the trajectory of a recollapsing universe in quantum mechanical terms, that is, by a propagating wave packet (or stationary “wave tube”). This boundary condition of excluding exponentially increasing wave functions for large  $\alpha$  restricts the freedom of choice for the initial conditions of this second-order equation (again in terms of the variable  $\alpha$ ) to only *one* remaining function of  $\phi$  (and the amplitudes  $x_n$  in full superspace). It thereby constrains the solution to contain the reflected partial waves “from the beginning” [16, 21], precisely as the boundary condition at  $r = 0$  in conventional scattering problems requires outgoing waves to be present at  $r \rightarrow \infty$ . Wave mechanically these two parts of the wave function must interfere, and cannot, in general, be conceptually separated from one another, since there is no dependence on an external time. Even decoherence can only occur as a process with respect to an “intrinsic time”  $\alpha$ . In contrast to the stationary treatment of conventional scattering problems, which is justified by the construction of time-dependent wave packets, the stationary form of the Wheeler-DeWitt equation is exact.

Note that the terms “initial” and “final” have here quite different meanings with respect to classical and quantum dynamics. With respect to the wave equation they refer to increasing  $\alpha$ , while the classical language refers to parameters of (possibly returning) orbits, which do not exist in quantum theory. Since the Wheeler-DeWitt equation is fundamentally stationary and real, there is no absolute time variable which could give rise to a reference phase  $e^{i\omega t}$  that might enable one to distinguish between “incoming” and “outgoing” parts  $e^{\pm ik\alpha}$  of the wave function. The signs of all momenta can then only have relative meaning.

The no-boundary condition cannot easily be transformed into the wave mechanical form appropriate for directly solving the Wheeler-DeWitt equation. HLL claim that it corresponds to a “boundary condition at *one* end of the four geometry,” that is, to a *classical* initial or final condition. In minisuperspace this condition is claimed to read [see their Eq. (2.8)]

$$a = 0, \quad \frac{da}{d\tau} = 1, \quad \phi = \phi_0, \quad \frac{d\phi}{d\tau} = 0, \quad (4)$$

where  $\tau = it$  is an imaginary time parameter. Because of the fourth condition, a real time parameter  $t$  would, in spite of the generally indefinite sign of the kinetic energy of gravity, lead to a sign which is incompatible with the potential  $V(a, \phi) := -e^{4\alpha} + e^{6\alpha} m^2 \phi^2 < 0$  in the Planck era. Quantum mechanically this is known to require a wave function depending exponentially on  $\alpha$  if  $p_\phi^2$  can be neglected in correspondence to the fourth condition in (4). While the first condition defines the boundary, the third one merely labels the starting points of the paths on it. Since all paths are furthermore assumed to con-

<sup>1</sup>Technically, this arises from a definite prescription for the rotation of the conformal factor in the Euclidean path integral [14].

<sup>2</sup>A proposal to use the Klein-Gordon norm in this context has recently been made in [20]. In its present formulation, however, it can only describe expanding universes. If a Klein-Gordon current (the same holds for a Schrödinger current) hits a potential barrier without a condition of exponentially decreasing wave functions being imposed in the “forbidden” region, it will continue undamped into the region inside the barrier. No “corrections” would then be required in the allowed region.

tribute with equal initial amplitude (at  $\alpha \rightarrow -\infty$ ), they form a boundary condition  $\psi(-\infty, \phi) = \text{const}$ , which is compatible with the fourth condition in (4). Other boundary conditions at  $\alpha \rightarrow -\infty$  may correspond to initially “superluminal” trajectories with  $|d\phi/dt| > |d\alpha/dt|$ , parametrized by real values of  $t$ . As mentioned above, there are no absolutely forbidden regions, but *certain* trajectories will be reflected, while others are bent to superluminal angles. Analogous results are obtained for propagating wave packets in the WKB approximation if the exponentially increasing solutions are excluded.

Instead of solving the wave function, however, HLL follow *trajectories* which start at  $\alpha = -\infty$  and are parametrized by *imaginary* values of time until they reach the border line represented by  $V(a, \phi) = 0$ .<sup>3</sup> They then “continue” these trajectories by means of a real time parameter (corresponding to an oscillating WKB wave function) through the whole history of a universe and obtain in this way quite different final conditions at the other end of these classical paths. Such a classical condition should, however, already be part of the wave mechanical initial condition which has to fix the wave function completely. These final conditions, derived by classical methods, are interpreted by HLL as defining corrections to the wave function for small  $a$ , although they must possess the same measure as the original component in an appropriate sense, for example in terms of the conserved Klein-Gordon current.

This resulting asymmetry of the *trajectories* (albeit not defined with respect to any *absolute* sense of time) is the basis of their conclusion that the arrow of time must continue beyond the turning point. Although classically consistent, it is here a consequence of the asymmetric treatment of both ends (motivated by the usual interpretation of the path integral as a propagator in time), and not just of the use of trajectories, which especially in the Planck era is a doubtful concept. A symmetric treatment of paths or trajectories would select those (much fewer) ones which obey equivalent conditions at both ends, although they would not have to be individually symmetric (as had originally been expected by Hawking [13]). The direction of calculation cannot be based on the direction of any “causality” still to be derived. This is true even for the construction of a WKB wave function that includes a second sheet describing the reflected wave.

If the asymmetric *procedure of calculation* did in fact

<sup>3</sup>The only consequence of the imaginary time parameter in this forbidden region of configuration space is that the path integral leads to an exponential WKB wave function (which is here *chosen* to increase with  $a$  along these pseudotrajectories, in precise analogy to what we require in the forbidden region for large  $a$ ). In the limit  $\hbar \rightarrow 0$  the penetration depth for the wave function into this region would vanish. It therefore appears misleading to interpret this situation as describing “Euclidean spacetimes.” When conventional quantum theory is applied to gravity, the wave function describes nothing but a stationary probability amplitude for three-geometries.

lead to corrections to the wave function at small  $a$ , this would simply demonstrate that the original boundary condition for small  $a$  is incompatible under the assumed wave equation with that required at large  $a$  for describing the reflection. There would, however, be no justification for this final condition affecting only *one* end of the quasitrajectories, which quantum mechanically have to be represented by propagating wave packets. The exact Wheeler-DeWitt equation does not even provide conceptual means to apply a boundary condition of the form  $\psi(-\infty, \phi) = \text{const}$  to only one “sheet” of the wave function, since the concept of separate sheets is facilitated only by the nonlinearity of the Hamilton-Jacobi equation. Any insistence on semiclassical concepts as being essential for the interpretation of the theory would demolish its claim as representing *quantum* gravity. In the Planck era, for example, trajectories are no better justified than in a hydrogen atom.

How may one consistently interpret the path integral used in the no-boundary proposal? If it were to represent wave propagation according to the “Klein-Gordon dynamics” (3) from some “initial time”  $\alpha_0$  to, say,  $\alpha_1$ , the paths would have to be parametrized by  $\alpha_0 \leq \alpha \leq \alpha_1$ . This propagation would then be defined regardless of any reflection from the repulsive potential at large  $\alpha$  (although an arbitrary initial condition would in general be inconsistent with the corresponding final condition if the wave function were propagated that far). Such an interpretation of the path integral is subtle and even questionable, since a composition law, as it must of course be valid for the wave equation (3), no longer seems to hold for the quantum cosmological path integral [22]. The path integral quantization has, therefore, occasionally been suggested to be more general than the canonical (wave function) quantization [23]. Such a “generalized quantum theory” would, however, be speculative, and transcend the realm of the empirically confirmed theory. The usual evaluation of path integrals by means of a WKB approximation, on the other hand, does not even correspond to the Heisenberg-Schrödinger quantization, since it represents the Bohr-Sommerfeld level of quantum theory.

One may thus rather interpret the path integral in a first step as representing a propagation with respect to a *formal* parameter  $t$  according to  $\Psi(t) = e^{-iHt}\Psi(0)$ , where  $H$  is the Wheeler-DeWitt Hamiltonian, followed by a projection onto the corresponding zero frequency mode, that is, onto a solution of the Wheeler-DeWitt equation  $H\Psi = 0$  by integrating over  $t$  from  $-\infty$  to  $+\infty$  [24, 25]. In minisuperspace this construction would read explicitly

$$\psi(a, \phi) = \int_{-\infty}^{+\infty} dt \int da' d\phi' G(a, \phi; a', \phi'; t) \psi^{(0)}(a', \phi'), \quad (5)$$

with the formal propagator  $G(a, \phi; a', \phi'; t)$  which would propagate a formal initial wave function  $\psi^{(0)}$  that has to be given on the full configuration space (including *all* values of  $a$ ), not only on a boundary. Although this initial function may be *chosen artificially* to contain a factor  $\delta(a)$ , perhaps multiplied by a constant in  $\phi$ , such

an assumption would not represent a natural choice of a boundary condition for this formal “dynamics.” Any information about a direction of propagation in formal time  $t$  would be lost by the integration. The no boundary proposal thus can only yield wave functions which are given “at once” on the full configuration space. We also note that the construction (5) with a  $\delta$  function as mentioned above, if calculated exactly in simple models, does not allow the construction of narrow wave packets which follow classical trajectories [25].

It is clear that classical *trajectories* in minisuperspace (derived from a boundary condition or not) are generically asymmetric. However, they would merely represent a situation well known from classical mechanics, where solutions from a symmetric Lagrangian are also *not* symmetric, without in general offering any thermodynamical insights. The solutions considered by Laflamme and Shellard [26] for Kantowski-Sachs universes, for example, were chosen to *start* at or near a disklike singularity, and must then evolve into a cigarlike one. In contrast to the recollapsing Friedmann minisuperspace trajectory, these initial and final singularities are distinct in configuration space. (In the vacuum case, the disklike singularity would merely represent a topological one, caused by the chosen foliation of the upper Kruskal wedge according to the Schwarzschild coordinate  $r$ . Classical solutions with matter may similarly “bounce” from the diskshape for earlier times.)

In special situations, a Klein-Gordon type equation (here with variable “mass term”) may nonetheless be consistent with the concept of initial conditions for *reflected trajectories* (which would then allow imposing very different boundary conditions for *additional* degrees of freedom, see Sect. III). This would require, first, that geometrical optics is applicable with sufficient precision along the *whole* trajectory, and second, that there is a region on a spacelike initial hypersurface in minisuperspace to which reflected quasitrajectories, which started there, never return. A simple example is a plane timelike potential barrier in Minkowski space, hit nonorthogonally by a spatially bounded wave packet (for this purpose equivalent to an ensemble of trajectories). It may easily be constructed from plane waves which fulfill the boundary condition of vanishing at the barrier. This would allow completely free initial conditions on a *partial* initial (or “source”) region, and thereby determine the wave function in the resulting disjoint final (or “image”) region on the same Cauchy surface (such as  $\alpha = \text{const}$ ). The initial region would itself have to be selected by that half of the complete initial conditions which remain free after the barrier condition at large  $\alpha$  has been imposed, while the final region would then be determined in this way.

Even this *ad hoc* distinction between initial and final regions according to this special ensemble of classical solutions or “light rays” fails wave mechanically if the wave packets show sufficient dispersion that prevents them from remaining disjoint. This turns out to be the case precisely in the minisuperspace characterizing closed Friedmann universes, where reflected wave packets are found to be scattered over their whole configuration space as a consequence of the specific form

of the repulsive curvature potential [3, 21]. This dispersion demonstrates that the semiclassical approximation for (3) *cannot* be valid all around the region of a classically expanding *and* recollapsing trajectory. Quantum effects are thus essential not only in the “Planck era.”

### III. DECOHERENCE AND “CONSISTENT HISTORIES”

If the two partial wave packets which formally represent the “expanding” and the “collapsing” universe intersect or overlap in minisuperspace (as they do repeatedly in the two-dimensional quantum Friedmann model even without any dispersion), they must interfere unless they decohere from one another. If the environmental degrees of freedom that contribute to this decoherence can themselves be described by a WKB approximation, this decoherence simply means that the partial wave packets, which are then dispersion free, travel in disjoint slices of the complete configuration space, and hence do not overlap any more. In general, however, decoherence results from quantum scattering processes which follow an arrow of time determined by a Sommerfeld radiation condition of negligible *initial* correlations. It would therefore represent circular reasoning to continue the statistically interpreted collapse of the wave function along a trajectory describing a reversal of the expansion of the Universe in order to *derive* a continuing thermodynamical arrow of time. Instead, one has to expect recoherence (derived from an inverse Sommerfeld condition) to occur there.

The arrow of time requires statistical considerations, and therefore additional degrees of freedom (such as the “environmental” ones which are responsible for decoherence). In the Friedmann model, the entropy may be defined by means of the functional (1) from the wave functions  $\varphi(\alpha, \phi; \{x_n\})$  for the higher multipoles  $x_n$  defined by the ansatz [18, 27]

$$\Psi(\alpha, \phi, \{x_n\}) = \text{Re} [\psi(\alpha, \phi)\varphi(\alpha, \phi; \{x_n\})]. \quad (6)$$

$\varphi$  is here assumed to depend only slowly on  $\alpha$  and  $\phi$ , while  $\psi$  may, in certain regions, be approximated by a WKB solution of the form  $e^{iS(\alpha, \phi)}$  with a Hamilton-Jacobi function  $S$ . The ansatz (6) is more general than a product of individually real factors [28]. In particular, one may derive from it a “time-dependent Schrödinger equation” for the multipoles,

$$i \frac{\partial \varphi}{\partial t} := i \nabla S \cdot \nabla \varphi \approx H_{\text{eff}} \varphi, \quad (7)$$

that is approximately valid along the parametrized trajectories  $\phi(t), \alpha(t)$  in minisuperspace defined by means of the gradient  $\nabla S$ . The orbit parameters  $t$  (or “WKB times”) assume the role of time as a “controller of motion” for this effective dynamics. This “complexification” of the time-dependent Schrödinger equation, which is derived from the *real* Wheeler-DeWitt wave function, represents a strong spontaneous symmetry breaking as it is typically described by means of a nonlinear approximation in quantum theory [29, 30].

In order to be compatible with the conventional quan-

tum mechanical description of the observed world, Eq. (7) must describe measurement processes according to von Neumann's unitary ("second") dynamics in a time direction of growing entropy and entanglement. In addition to physical entropy, the statistical entropy calculated from  $\varphi(t, \{x_n\})$  by means of (1) must therefore contain the entropy that measures the missing information about the outcome of all measurementlike processes which occurred in the respective "past" of  $t$ . The time-dependent Schrödinger equation must hence *not* be used to determine the wave function describing the state in our past by calculating backwards in time, starting only with a "branch" wave function that represents the present state of the "observed world" (with definite classical properties). Such a calculation would miss the deterministic predecessors of those "nonobserved Everett components" that are physically meaningful and important in forming superpositions which may define *observed* past states. For precisely the same reason, the Schrödinger equation must then also not be used to calculate the formal "future" of  $\varphi$  beyond the turning point of the cosmic expansion *if* the arrow of time reverses at this point. On returning quasitrajectories in minisuperspace, one would expect anticausal and nonunitary contributions to (7) to occur for the same reason (the boundary condition) which leads to causality and branching during expansion. In this case the *physical* direction of time is reversed with respect to the formal parameter of the trajectory. Equation (7) can neither be always meaningful nor exact if  $\varphi$  is defined by Eq. (6).

If (7) is instead used to *define*  $\varphi$  along the complete trajectory [31], starting with an initial state of low entropy [for example at the same end as used to apply (4)], an ever increasing entropy will result for all times smaller than the corresponding Poincaré times (which would greatly exceed any conceivable duration of the Universe). In contrast, solutions of the Wheeler-DeWitt equation of the form (6) with relative states  $\varphi(\alpha, \phi; \{x_n\})$  of low entropy everywhere for small  $a$  would necessarily describe a reversing arrow along turning quasitrajectories. Close to the turning point there would be a region of indefinite de- or recoherence, that is, a region that cannot be interpreted in classical terms (similar to the Planck era).

Wave packets constructed from that half of the solutions of the Wheeler-DeWitt equation which obey the boundary condition at large  $a$  must automatically render initial and final conditions (in the sense of trajectories) quantum dynamically compatible with one another. They cannot, in general, form complete quasiclassical histories, though, since they must represent *superpositions* of many very different classical worlds at least on one leg of their histories because of the quantum scattering that occurs at the turning point [27]. These superpositions must decohere into very *different* branches on both legs, i.e., there does not exist any classical connection between different legs across some "turning point."

Can such wave packets then at least consistently describe quasiclassical worlds during *one* of their half-cycles? Low entropy conditions at both ends of quasiclassical time would be compatible with the observed time asymmetry if we happened to live close to one end, and

if the world were "informationally opaque" somewhere during its complete history (for example, by closely approaching thermodynamical equilibrium in the middle), so that no information could survive the turning point of the expansion [32]. The second condition has been questioned to be realistic [33], since our Universe seems to remain transparent to light all the way until it approaches the big crunch (thereby preventing the electromagnetic field from becoming thermalized in between). Electromagnetic radiation into "empty space" would then have to be inhibited by the existence of a "visible dark future sky" (a time-reversed Olbers paradox), that is, by a reduced emission power of antennae which are pointed into empty space. This was found *not* to be the case [34]. The argument is not completely convincing, though, because of the defocusing effects that must affect retarded waves through the whole unknown lifetime of our Universe. Any "conspirative" correlations which would be required for their focusing onto reversed sources in the contraction era will hardly be locally detectable now. The consistency problem of opposite arrows in one universe may appear more severe with respect to gravitation because of the irreversible formation of black holes, but it is in both examples based on the unrealistic *classical* field equations and does not take into account the essential "quantum scattering" of the whole Universe at its turning point. Since the Universe must become informationally opaque due to these quantum effects, its initial conditions (at small  $a$ ) for all *relevant* (information-carrying) degrees of freedom appear practically free and therefore admit a condition of low entropy at both ends of a quasitrajectory.

We emphasize that the presumed time dependence of the density matrix used in (1) may completely describe an arrow of time, regardless of any interpretation in terms of probabilistic "quantum events" (or their time ordering). The occurrence of "events" is in fact described in the form of *decoherence* by means of the smooth Schrödinger dynamics [35], while the (very general [1]) Zwanzig type coarse graining  $\hat{P}$  used in the definition of the entropy functional (1) has simply to be chosen compatible with it. A monotonic increase of the corresponding entropy is then described by a "master equation" of the form

$$\left(\frac{d(\hat{P}\rho)}{dt}\right) \approx \left(\frac{d(\hat{P}\rho)}{dt}\right)_{\text{master}} := \frac{\hat{P}e^{-i\hat{L}\Delta t}\hat{P}\rho - \hat{P}\rho}{\Delta t} \approx -i\hat{P}\hat{L}\hat{P}\rho - \hat{G}_{\text{ret}}\hat{P}\rho, \quad (8)$$

with the Liouville operator  $\hat{L} := [H, \dots]$ , a positive time scale  $\Delta t$  which is larger than some "relaxation time" [36, 37], and a positive operator  $\hat{G}_{\text{ret}}$ . It can be derived as an approximation from the Schrödinger (von Neumann) equation for  $\rho$  by assuming an appropriate initial condition  $\rho_i = \rho(t_i)$  (precisely as in the derivation of Boltzmann's equation from Newton's). Using square roots in the diagonal form of  $\hat{G}_{\text{ret}}$ , this master equation can also be written as a Lindblad equation.

If the master equation holds for a coarse graining of the specific form

$$\hat{P}\rho := \sum_n P_{\alpha_n} \rho P_{\alpha_n}, \quad (9)$$

where the  $P_{\alpha_n}$  form a complete set of orthogonal projection operators on the Hilbert space of the quantum

$$C_{\alpha_n \dots \alpha_1} := e^{-iH(t_f - t_n)} P_{\alpha_n} e^{-iH(t_n - t_{n-1})} \dots P_{\alpha_2} e^{-iH(t_2 - t_1)} P_{\alpha_1} e^{-iH(t_1 - t_i)}, \quad (11)$$

defines probabilities for consistent histories in the sense of Griffiths [38]. Relaxation times correspond to the (extremely short) decoherence times in this case, while the projectors  $P_{\alpha_i}$  may in general be moderately time-dependent. They may be chosen to project onto the stable pointer basis for the coarse-grained degrees of freedom [37]. Such consistent histories result from a successive application of Fermi's probabilistic golden rule [as used in steps of  $\Delta t$  in (8)], which simply neglects interference of the probability amplitudes after the occurrence of assumed "quantum events." This formal "consistency" of *probabilities for histories* must not, however, mislead to circumventing the quantum measurement problem by interpreting the Feynman path integral as representing an *ensemble* of paths or histories from which an element can be "picked out" by a mere increase of knowledge. This would be as mistaken as simply replacing the wave function of an electron by the corresponding probability distribution of particle positions.

If the master equation (8) holds for  $\hat{P}\rho$ , while  $\rho$  itself obeys the von Neumann equation  $id\rho/dt = \hat{L}\rho$  with an appropriate initial condition, the probabilities (10) are not changed by inserting in addition the final density matrix  $\rho_f = e^{-iH(t_f - t_i)} \rho_i e^{iH(t_f - t_i)}$  in order to obtain a symmetric form,

$$p_{\alpha_1 \dots \alpha_n} = (\text{Tr} [\rho_f C_{\alpha_n \dots \alpha_1} \rho_i (C_{\alpha_n \dots \alpha_1})^\dagger])^{1/2}. \quad (12)$$

This can be seen as follows. Using (9) and (10), the cyclic property of the trace, and  $P_{\alpha_k} P_{\alpha_l} = \delta_{kl} P_{\alpha_k}$ , one has

$$\begin{aligned} & \text{Tr} (\rho_f C_{\alpha_n \dots \alpha_1} \rho_i (C_{\alpha_n \dots \alpha_1})^\dagger) \\ &= \text{Tr} \left( e^{-iH(t_n - t_i)} \rho_i e^{iH(t_n - t_i)} P_{\alpha_n} \dots \rho_i \dots P_{\alpha_n} \right) \\ &= \text{Tr} (P_{\alpha_n} \rho(t_n) P_{\alpha_n} P_{\alpha_n} \dots \rho_i \dots P_{\alpha_n}) \\ &= \text{Tr} (\hat{P}\rho(t_n) P_{\alpha_n} \dots \rho_i \dots P_{\alpha_n}). \end{aligned}$$

Taking into account the "successive probabilities"  $p_{\alpha_n} := \text{Tr}[P_{\alpha_n} \rho(t_n)] = p_{\alpha_1 \dots \alpha_n}$  which arise from the master dynamics, this expression is equal to  $(p_{\alpha_1 \dots \alpha_n})^2$ , in accordance with (12). The symmetric form is thus based on the factual asymmetry which is represented by the master equation. It is caused by the special *initial* condition. The probabilities would in general be changed drastically, however, by inserting  $\rho_f$  in this or a similar symmetric expression [38–40] if the master equation did *not* hold as an approximation through all times from  $t_i$  to  $t_f$ . It would, in particular, not hold if the entropy (1) with

states, the familiar expression

$$p_{\alpha_1 \dots \alpha_n} = \text{Tr} [C_{\alpha_n \dots \alpha_1} \rho_i (C_{\alpha_n \dots \alpha_1})^\dagger], \quad (10)$$

for probabilities of time-ordered series of events  $\alpha_1(t_1), \alpha_2(t_2) \dots \alpha_n(t_n)$ , with

respect to a fixed relevance concept  $\hat{P}$  were low both at  $t_i$  and  $t_f$ , thus forming a thermodynamically time-symmetric (though possibly unitarily evolving) universe (cf. also Page [41]).

In the Heisenberg picture (and without any collapse)  $\rho_i$  and  $\rho_f$  are identical, while the introduction of two *independent* density matrices  $\rho_i$  and  $\rho_f$  in order to define a symmetric "transition probability"  $\text{Tr}(\rho_f \rho_i)$  (with or without considering "histories"), would interpret the whole Universe as *one* probabilistic "scattering event." This is, of course, particularly dubious in the absence of external observers.

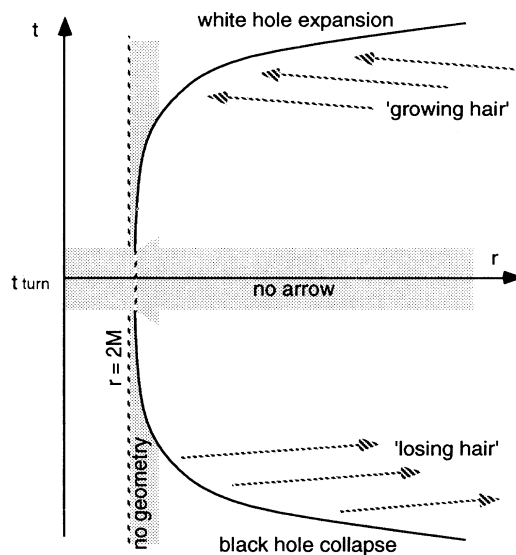


FIG. 1. Symbolic (quasi) trajectory of the radius of a dust shell (or the surface of a star) in a thermodynamically symmetric universe, represented in compressed Schwarzschild coordinates. As the gravitational collapse is assumed to occur long before the maximum expansion of the universe is reached at  $t = t_{\text{turn}}$ , light rays appear almost horizontally due to this scale compression. Advanced radiation must cause the matter to expand again and grow hair in this scenario, although observers would experience the "future" side of the diagram in the reversed time direction. Because of drastic quantum effects near the classical turning point [3, 21], there is in fact only a "probabilistic" connection between quasiclassical expansion and recontraction of the universe.

#### IV. CONSEQUENCES FOR BLACK HOLES

The dominating aspect characterizing the low entropy of the initial Universe seems to be its homogeneity. This is a consequence of the negative heat capacity of gravitating systems. Gravitational contraction is thus controlled by the radiation arrow of time, for conventional stars described by the virial theorem, and during gravitational collapse by the no-hair theorem. The initial homogeneity has been expressed by means of the Weyl tensor hypothesis [5] which excludes inhomogeneous past singularities from spacetime. In a thermodynamically time-symmetric Universe such a condition would also have to apply to formal “future” singularities, which must thus be constrained by the same boundary condition. In timeless quantum gravity these two boundaries become *identical*. We emphasize that the singularity theorems of classical general relativity do not apply here since in quantum gravity there is no classical spacetime which could obey the Einstein equations.

How this could be achieved even in the presence of massive spherical black holes (for which Hawking radiation can be neglected, and singularities might thus be expected to survive until the big crunch) has been described elsewhere [1, 3]. Their external Schwarzschild metric is static (invariant under translations of the Schwarzschild time  $t$ ). Because of the diverging gravitational time dilation of collapsing matter, the spacelike hypersurfaces characterized by this time coordinate will approach the turning point of the cosmic expansion at spatial infinity before any collapsing spherical dust shell (or a spherical matter shell that is part of a collapsing star) has reached the horizon that is expected to form. Classically it would then very soon (as measured in proper time) have to pass it, and to collapse further into a singularity, thus demonstrating the incompatibility of exactly spherical black holes with a thermodynamically time symmetric *classical* universe. Spherical black holes are, however, compatible with a time symmetric *quantum* universe (see Fig. 1).

Less symmetric matter concentrations could in principle, even according to the classical theory, enter a time-symmetric and “informationally opaque” state at extremely high density, from which they would have to “grow hair” again by means of the advanced radiation that must become relevant in the collapse era of the Universe. Since such a state of matter appears classically not very realistic, genuine quantum effects representing “inconsistent histories” (the invalidity of probabilistic master equations for measurement processes) appear indis-

pensable close to the Schwarzschild time which corresponds to the turning point of a time symmetric closed universe. Although hard to interpret in a classical picture, they are readily described by reflected (and thereby scattered) wave packets which (1) solve the Wheeler-DeWitt equation and (2) are compatible with the wave mechanical low entropy (initial) boundary condition everywhere at  $a \rightarrow 0$  [21]. Classically one would be confronted with the severe consistency problem of having two opposite boundary conditions, while wave mechanically there is only one low entropy boundary. Consistency between different quasitrajectories is then warranted by the coherent quantum scattering at the turning point.

We emphasize that quantum cosmology with a boundary condition of low entropy for  $a \rightarrow 0$  would immediately solve many of the problems of the classical gravitational theory. The first concerns the “information loss paradox” for black holes, which does not occur because of the absence of horizons. Hawking radiation would always remain in a pure, although highly correlated, quantum state. Furthermore, since no singularities would form (except for the cosmological one) the principle of cosmic censorship would be implemented. Finally, a time-symmetric quantum universe would prevent the occurrence of mass inflation inside a rotating black hole, since no Cauchy horizon could ever form. The cosmological scenario from mass inflation [42] would then become obsolete. We emphasize that, in spite of their speculative nature, these considerations are solely based on symmetry arguments.

The continuation of the classical concept of time beyond the turning point can thus only be formal. If the “psychological arrow of time” is determined by the thermodynamical one, the Universe can only be observed expanding. In particular, “information-gaining systems” (observers) cannot *continue* to exist from the expansion into the collapse era. The different quasiclassical branches of the wave function which are connected by quantum scattering at the turning point should rather be interpreted as all representing different *expanding* universes, which disappear at the turning point by means of destructive interference (similar to their coming into existence as separate Everett branches from a symmetric initial state at the big bang).

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