Renormalization-group equations in the $[SU(6)]^3$ model

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We use the renormalization-group equations to calculate the symmetry-breaking scales in the $[SU(6)]^3$ Pati-Salam model. Using only known parameters, we predict the mass scales consistent with very small values for the light-neutrino masses and rare processes decay such as $K^+ \longrightarrow \pi^+ e^- \mu^+$ for three steps of symmetry breaking. We also find that the proton is stable by gauge and Higgs boson exchange.

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One of the most important advances in particle physics in the last two decades has been the development of the so-called standard model (SM) [1] based on the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The SM agrees with all experimental data, but it does not answer some questions, such as the origin and values of the fermion masses, the number of families, etc. For these reasons physicists believe that the SM is not the most complete of theories.

There have been many attempts to answer the above questions, and new models have been proposed. All of these models imply the existence of new particles, either fermions, gauge bosons, or scalars, additional to those introduced in the SM. Other interesting features are to unify the strong, weak, and electromagnetic interactions embedding the SM in a larger gauge group [2]. However, it is important to consider the unification of quarks and leptons with the three flavor families. Some of these interesting theories that unify the interactions and flavors are the Pati-Salam models [3]. Inspired by these models, we proposed the $G = [SU(6)]^3$ model [4] to unify the nongravitational interactions and the three families. In the Pati-Salam model the quark and lepton numbers are unified into the $SU(4)_C$ gauge group. To unify the three families with the weak interactions, we use the $SU(6)_L \otimes SU(6)_R$ gauge group which has left- and right-handed fermions. In order to have one gauge coupling constant, the $SU(4)_C$ symmetry must be extended to $SU(6)_C$ and a discrete Z_3 symmetry acting upon the three $SU(6)$ groups must be introduced.

In the present work, we use renormalization-group equations (RGE's) to study the breaking of the $[SU(6)]^3$ model. We consider two and three steps to break the G group down to $SU(3)_C \otimes U(1)_Q$. With more steps of symmetry breaking, there are new unknown parameters and it is possible to predict a low-energy parameter as a function of the unknown parameters [5]. One finds that the symmetry-breaking scheme consistent with RGE's is

$$\begin{aligned}[SU(6)]^3 \times Z_3 &\xrightarrow{M_1} Sp(6)_L \otimes Sp(6)_R \otimes SU(3)_C \otimes U(1)_{B-L} \\ &\xrightarrow{M_2} SU(2)_L \otimes U(1)_Y \otimes SU(3)_C \\ &\xrightarrow{M_Z} SU(3)_C \otimes U(1)_Q,\end{aligned}$$

where the energy scales M_1 and M_2 have values of about 10^{10} and 10^5 GeV, respectively. In this model there is no proton decay when mediated by gauge bosons or Higgs bosons, and the grand unification scale can be lower than in other grand unified theories (GUT's) [6]. However, $\Delta B = 2$ processes and neutron-antineutron oscillations are possible.

We give the possible symmetry breaking in agreement with rare processes, such as $K^+ \longrightarrow \pi^+ e^- \mu^+$, and tiny masses for the light neutrinos.

We use $SU(6)_L \otimes SU(6)_C \otimes SU(6)_R \times Z_3$ as the gauge group to unify the nongravitational forces and the three families, where Z_3 is the three-cyclic group acting upon $[SU(6)]^3$. $SU(6)_C$ is the color group, and the fundamental representation consists of three baryonic numbers and three leptonic numbers. This group is broken down to $SU(3)_C \otimes U(1)_{B-L}$, where $U(1)_{B-L}$ is the baryonic number minus leptonic number.

The gauge fields of $SU(6)_C$ are given by

$$\lambda_\alpha A_\alpha = \begin{bmatrix} D^1 & G_2^1 & G_3^1 & \bar{X}_1 & \bar{Y}_1 & \bar{Z}_1 \\ G_1^2 & D^2 & G_3^2 & \bar{X}_2 & \bar{Y}_2 & \bar{Z}_2 \\ G_1^3 & G_2^3 & D^3 & \bar{X}_3 & \bar{Y}_3 & \bar{Z}_3 \\ X_1 & X_2 & X_3 & D^4 & P_1^- & P^0 \\ Y_1 & Y_2 & Y_3 & P_1^+ & D^5 & P_2^+ \\ Z_1 & Z_2 & Z_3 & \bar{P}^0 & P_2^- & D^6 \end{bmatrix}, \tag{1}$$

where G_j^i for $i,j = 1,2,3$ are the $SU(3)_C$ nondiagonal gauge bosons; X_i, Y_i, and Z_i are leptoquark gauge bosons with electrical charges $-\frac{2}{3}$, $\frac{1}{3}$, and $-\frac{1}{3}$, respectively; $P_a^\pm$, $a = 1,2$, and P^0 are dilepton gauge bosons; and D_i for $i = 1, \ldots, 6$ are the neutral gauge bosons associated with the diagonal generators of $SU(6)_C$.

$SU(6)_{L(R)}$ contains $SU(2)_{L(R)} \otimes SU(3)_{HL(HR)}$ where $SU(3)_{HL(HR)}$ is the horizontal gauge symmetry which unifies the three families. The representation matrix for $SU(6)_{L(R)}$ is given so that $SU(2)_{L(R)}$ is universal with respect to the three families; i.e., the generators may be written in an $SU(2)_{L(R)} \otimes SU(3)_{HL(HR)}$ basis as

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$$\sigma_i \otimes I_3, \quad I_2 \otimes \lambda_\alpha, \quad \sigma_i \otimes \lambda_\alpha, \tag{2}$$

where σ_i are the 2×2 Pauli matrices, λ_α are the 3×3 Gell-Mann matrices and I_2 (I_3) is the 2×2 (3×3) identity matrix.

The fermions of the model are in

$$Z_3[\psi(6,1,\bar{6})] = \psi(6,1,\bar{6}) + \psi(\bar{6},6,1) + \psi(1,\bar{6},6), \tag{3}$$

where

$$\psi(\bar{6},6,1) = \begin{pmatrix} d_1 & d_2 & d_3 & E_1^- & L_1^0 & T_1^- \\ u_1 & u_2 & u_3 & E_1^0 & L_1^+ & T_1^0 \\ s_1 & s_2 & s_3 & E_2^- & L_2^0 & T_2^- \\ c_1 & c_2 & c_3 & E_2^0 & L_2^+ & T_2^0 \\ b_1 & b_2 & b_3 & E_3^- & L_3^0 & T_3^- \\ t_1 & t_2 & t_3 & E_3^0 & L_3^+ & T_3^0 \end{pmatrix}_L \tag{4}$$

contains the ordinary left-handed fermions; $\psi(1,\bar{6},6)$ has the charged conjugate fields for the electrically charged fields in $\psi(\bar{6},6,1)$, but not for the neutral ones, because we are using Majorana fields, and $\psi(6,1,\bar{6})$ represents exotic leptons with positive and negative electric charge and neutral fields. In this model there are no exotic quarks. The ordinary leptons and quarks are linear combinations of the leptons and quarks in $Z_3[\psi(\bar{6},6,1)]$.

To achieve the desired symmetry breaking (SB) in three different steps, we introduce appropriate Higgs scalars. Using the branching rules for $SU(6)\to SU(2)_{L(R)} \otimes SU(3)_{HL(HR)}$, we have

$$\begin{aligned} \mathbf{6} &\to (2,3), \\ \mathbf{15} &\to (1,6) + (3,\bar{3}), \\ \mathbf{21} &\to (1,\bar{3}) + (3,6), \\ \mathbf{35} &\to (3,1) + (1,8) + (3,8). \end{aligned} \tag{5}$$

Note that the embedding for $SU(6)_{L,R} \to SU(2)_{L(R)} \otimes SU(3)_{HL(HR)}$ is such that $SU(2)_{L,R}$ is universal—this is family independent—while, for $SU(6)_C \to SU(3)_C \otimes U(1)_{B-L}$,

$$\mathbf{6} \longrightarrow (3,1) + (1,3). \tag{6}$$

We can see from Eq. (5) that the vacuum expectation values (VEV's) of the Higgs scalar associated with irrep **6** necessarily breaks $SU(2)_{L,R}$, but irreps **15**, **21**, and **35** can respect such symmetry. So irrep **6** should be used at the last step of SB when the breaking of $SU(2)_L$ should take place. The breaking of $SU(2)_R$ should produce Majorana masses for three right-handed neutrinos. So the Higgs fields carrying this task should couple to $\psi(1,\bar{6},6) \otimes \psi(1,\bar{6},6)$, and thus they should be either of the form $\phi(1,15,\overline{15})$ or of the form $\phi(1,21,\overline{21})$.

For the irrep **15** of $SU(6)_{L,R}$, we have explicitly the components

$$\begin{aligned} (1,6)_{T_{3L(R)}=0} &\longrightarrow \phi^{[1,2]}, \phi^{[3,4]}, \phi^{[5,6]}, \phi^{[1,4]-[2,3]}, \phi^{[1,6]-[2,5]}, \phi^{[3,6]-[4,5]}, \\ (3,\bar{3})_{T_{3L(R)}=1} &\longrightarrow \phi^{[1,3]}, \phi^{[1,5]}, \phi^{[3,5]}, \\ (3,\bar{3})_{T_{3L(R)}=0} &\longrightarrow \phi^{[1,4]+[2,3]}, \phi^{[1,6]+[2,5]}, \phi^{[3,6]+[4,5]}, \\ (3,\bar{3})_{T_{3L(R)}=-1} &\longrightarrow \phi^{[2,4]}, \phi^{[2,6]}, \phi^{[4,6]}. \end{aligned} \tag{7}$$

For the symmetry breaking at M_1 scale, we use

$$\phi_1 = Z_3\phi_1(\overline{15},1,15) = \phi_{1[a,b]}^{[A,B]} + \phi_{1[A,B]}^{[\alpha,\beta]} + \phi_{1[\alpha,\beta]}^{[a,b]}, \tag{8}$$

where $a,b,c,\ldots,A,B,C,\ldots,\alpha,\beta,\gamma,\ldots$ refer to $SU(6)_L$, $SU(6)_R$, and $SU(6)_C$ tensor indices, respectively. The VEV's of ϕ_1 may be chosen in the directions $[a,b] = [1,6] = -[2,5] = [3,4]$, $[A,B]$ similar to $[a,b]$, and $[\alpha,\beta] = [5,6] = [4,5]$.

For the second step of symmetry breaking at M_2 scale, we use one more $Z_3[\phi_2(15,\overline{15},1)]$ Higgs representation:

$$\phi_2 = Z_3\phi_2(15,\overline{15},1) = \phi_{2[\alpha,\beta]}^{[a,b]} + \phi_{2[a,b]}^{[A,B]} + \phi_{2[A,B]}^{[\alpha,\beta]}, \tag{9}$$

with the VEV's in the following directions. For the first two terms of ϕ_2, $[a,b] = [1,2] = [3,6] = -[4,5]$, $[A,B]$ similar to $[a,b]$, and $[\alpha,\beta] = [4,5]$, while for the third term, $[A,B] = [2,4] = [2,6] = [4,6]$ and $[\alpha,\beta] = [4,6]$.

The final stage of the breaking is achieved by using a $Z_3[(\bar{6},6,1)]$ representation with the VEV's displayed in Ref. [4].

In this model the only gauge boson interactions that could produce baryon number violation is given by

$$\begin{aligned} Z_3[\bar{\psi}(\bar{6},6,1)iD_\mu\gamma^\mu\psi(\bar{6},6,1)] &\approx \sum_{i=1}^{3}\{\bar{E}_i^-\mathbf{X}d_i + E_i^0\mathbf{X}u_i + \bar{L}_i^-\mathbf{Y}d_i + L_i^+\mathbf{Y}u_i \\ &\quad + T_i^-\mathbf{Z}d_i + T_i^0\mathbf{Z}u_i\} + \text{H.c.}, \end{aligned} \tag{10}$$

where i refers to the family index. The gauge bosons always change a quark for a lepton, and they have a well-defined baryon number; it is then not possible to write down an effective Hamiltonian that changes the baryon number obtained from the gauge boson exchange. Therefore proton decay is forbidden; i.e., $\Delta B = 0$ by gauge bosons exchange.

The possible Higgs contributions to proton decay arise from the interactions between the quarks and the scalar multiplets through a gauge-invariant effective operator with at least four fermionic fields and at least one Higgs boson which develops a VEV with $\Delta B = 1$ [11].

The Higgs sectors we are using to produce the spontaneous SB (SSB) and to give masses to the fermions are $Z_3[\phi(1,\bar{6},6)]$ and $Z_3[\phi_i(1,\overline{15},15)]$. With the $Z_3[\phi(1,\bar{6},6)]$ we can generate the couplings

$$
\begin{aligned}
Z_3[\psi(\bar{6},6,1)_L\psi(1,\bar{6},6)_L]\phi(6,1,\bar{6}) \\
= [\psi(\bar{6},6,1)_L\psi(1,\bar{6},6)_L]\phi(6,1,\bar{6}) \\
+[\psi(6,1,\bar{6})_L\psi(\bar{6},6,1)_L]\phi(1,\bar{6},6) \\
+[\psi(1,\bar{6},6)_L\psi(6,1,\bar{6})_L]\phi(\bar{6},6,1), \qquad (11)
\end{aligned}
$$

where the last two terms produce the mixing of the exotic fermions with the ordinary fermions, to lead the seesaw mechanisms for neutral and charged leptons [7], and the other term produces mass matrices for ordinary fermions [4]. From the above Lagrangian we see that the interactions between ordinary fermions and Higgs bosons do not lead to $\Delta B = 1$ because the Higgs fields $\phi(6,1,\bar{6})$ have $\Delta B = 0$ quantum numbers.

Because of the fact that quarks have one-third of the baryon number, we need at least three quark operators to obtain a $\Delta B = 1$ effective operator, which implies the necessity of having the existence of a Higgs multiplet with three indices of $\mathrm{SU}(6)_C$. In this model we are using only Higgs representations with one or two $\mathrm{SU}(6)_C$ indices, and therefore, we do not have any $\Delta B = 1$ effective operator after the SSB. The lowest Higgs representation that could produce proton decay in this model is the $Z_3[\phi(20,\overline{20},1)] = Z_3[\phi^{abc}_{\alpha\beta\gamma}]$, where the 20-dimensional representation is the totally antisymmetry third-rank tensor, with the $\mathrm{SU}(6)^3$-invariant effective operator

$$\psi^\alpha_a\psi^\beta_b\psi^\gamma_c\psi^\delta_d\phi^{abc}_{\alpha\beta\gamma}\psi^{\xi\rho\sigma}_{lmn}\phi^{dl}_{i,\delta\xi}\phi^{mn}_{j,\rho\sigma}. \qquad (12)$$

However, there are effective operators that give rise to neutron-antineutron oscillations, i.e., $\Delta B = 2$. The Yukawa Lagrangian that gives mass to the exotic fermions at M_1 and M_2 energies is

$$Z_3\{\psi(1,\bar{6},6)\psi(1,\bar{6},6)[\phi_1(1,15,\overline{15}) + \phi_2(1,15,\overline{15})]\}; \qquad (13)$$

there also exists a self-coupling of the four scalar multiplet $\phi(1,15,\overline{15})$ that can produce with the above Yukawa Lagrangian an effective operator of six-quark fields that lead to $\Delta B = 2$. The scales predicted by RGE's give tiny $\Delta B = 2$ processes consistent with the experimental bounds, such as the Pati-Salam model [8].

The coupling constants g_i obey the renormalization-group equations which have the form [9]

$$\mu\frac{\partial}{\partial\mu}g_i = \beta(g_i), \qquad (14)$$

and in the one-loop approximation the coupling constants evolve as

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M) - (B_i/6\pi)\ln(M/\mu), \qquad (15)$$

where $\alpha_i = g_i^2/4\pi$ and the coefficient B_i is defined as

$$B_i = 11C_2 - b_i. \qquad (16)$$

C_2 is the quadratic Casimir operator of the adjoint representation which gives the contribution of the gauge bosons that do not get mass at the M scale [10]. The second term b_i gives the contribution of the fermions with a mass smaller than this scale.

In the present work, we do not use the conventional normalization. We will normalize all gauge group generators by T_{3L}, which is given by $T_{3L} = \mathrm{diag}(1,-1,1,-1,1,-1)/2$ [4] such that

$$\mathrm{Tr}(T_aT_b) = \tfrac{3}{2}\delta_{ab}. \qquad (17)$$

We will use this new normalization to compute the Casimir operator. For example, for an $\mathrm{SU}(n)$ subgroup of $\mathrm{SU}(6)_C$, in the Gell-Mann matrix representation, the Casimir operator for the adjoint representation is given by

$$C_2(\mathrm{SU}(n)) = 3n, \qquad (18)$$

where the factor 3 arises from the new normalization.

The Casimir operators of the subgroups of $\mathrm{SU}(6)_{L(R)}$ are different because we are using a special embedding where $\mathrm{SU}(2)_{L(R)}$ is universal with respect to the three families. The fundamental matrix representation T^j is given by Eq. (2) with the appropriate coefficient to get $\frac{3}{2}$ as the normalization factor, and the Casimir operators can be calculated by the expression

$$C_2(G) = f^{1jk}f^{1jk} = -\tfrac{2}{3}\mathrm{Tr}([T^1,T^j][T^1,T^j]), \qquad (19)$$

where f^{ijk} are the structure constants of the G group.

With the above normalization the electromagnetic charge is defined as

$$Q = T_{3L} + T_{3R} + \sqrt{\tfrac{5}{9}}Y_{B-L}, \qquad (20)$$

where $Y_{B-L} = \sqrt{\frac{9}{20}}\ \mathrm{diag}(\frac{1}{3},\frac{1}{3},\frac{1}{3},-1,1,-1)$.

If we break down the model in three steps, we can consider some possibilities depending on how to break the $\mathrm{SU}(6)_C$ color group or the $\mathrm{SU}(6)_L\otimes\mathrm{SU}(6)_R$ group. One of this possibilities is to break $\mathrm{SU}(6)_C$ down to $\mathrm{SU}(3)_C\otimes\mathrm{U}(1)_{B-L}$, and the other is to break it down to $\mathrm{SU}(4)_C$ at the M_1 mass scale. The second possibility does not give an appropriate solution to the RGE, and this is ruled out in this class of models; however, in other kind of models such as SO(10) [11] this chain is viable because the Casimir invariants are different. So a realistic phenomenological breakdown that can give solutions to the RGE is

$$
\begin{aligned}
[\mathrm{SU}(6)]^3\times Z_3 &\xrightarrow{M_1} G_L\otimes G_R\otimes\mathrm{SU}(3)_C\otimes\mathrm{U}(1)_{B-L} \\
&\xrightarrow{M_2} \mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y\otimes\mathrm{SU}(3)_C \\
&\xrightarrow{M_Z} \mathrm{SU}(3)_C\otimes\mathrm{U}(1)_Q,
\end{aligned}
$$

and the RGE's in the one-loop approximation for the SM couplings are given by

$$
\begin{aligned}
\alpha_L^{-1}(M_Z) &= \alpha_G^{-1} - \frac{11a-b}{6\pi}\ln\frac{M_1}{M_2} - \frac{22-b'}{6\pi}\ln\frac{M_2}{M_Z}, \\
\alpha_C^{-1}(M_Z) &= \alpha_G^{-1} - \frac{9a-b}{6\pi}\ln\frac{M_1}{M_2} - \frac{99-b'}{6\pi}\ln\frac{M_2}{M_Z}, \qquad (21) \\
\alpha_Y^{-1}(M_Z) &= \alpha_G^{-1} - \left[\frac{9}{14}\frac{11a-b}{6\pi} - \frac{5}{14}\frac{-b}{6\pi}\right] \\
&\quad\times\ln\frac{M_1}{M_2} - \frac{-b'}{6\pi}\ln\frac{M_2}{M_Z},
\end{aligned}
$$

where b and b' are the fermionic contributions at M_1 and M_2 scales, respectively, a is the Casimir operator for the group $G_{L(R)}$ which can be equal to $\mathrm{Sp}(6)_{L(R)}$ or $\mathrm{SU}(2)_{L(R)}$. It is important to note that we are considering unbroken parity symmetry at the gauge group and it is spontaneously broken at the M_2 scale. The group

$U(1)_Y$ comes from two different gauge groups $U(1)_{B-L}$ and G_R, which contain $SU(2)_R$.

It is possible to predict from the above equations and the data from the CERN e^+e^- collider LEP the mass scales. For $G_{L(R)} = \mathrm{Sp}(6)_{L(R)}$ this yields

$$\begin{aligned} M_1 &= M_Z \exp\left[\frac{2\pi}{144 \times 11}(81\alpha^{-1} - 180\alpha^{-1}x - 27\alpha_C^{-1})\right] \\ &\approx 1.42 \times 10^{10}\ \mathrm{GeV}, \\ M_2 &= M_Z \exp\left[\frac{2\pi}{144 \times 11}(18\alpha^{-1} + 9\alpha^{-1}x - 55\alpha_C^{-1})\right] \\ &\approx 3.53 \times 10^{5}\ \mathrm{GeV}, \end{aligned} \tag{22}$$

where $C_2(\mathrm{Sp}(6)_{L(R)}) = 11$, and for the parameters at low energies we are using $\alpha^{-1}(M_Z) = 127.9$, $\alpha_C = 0.113$, and $x = \sin^2\theta_W = 0.233$ [12]. For $G_{L(R)} = SU(2)_{L(R)}$ the RGE's lead to inconsistent values for M_1 and M_2. From the above result, we conclude that this Pati-Salam model can be broken down to $SP(6)_{L(R)}$, which has horizontal symmetry, and in the second step to the SM. This second step lies in an experimentally attractive regime for the future colliders.

Another interesting but no realistic scheme is to break down the model by two steps. This case corresponds in the above equations to $M_1 = M_2$ with mass scale

$$M \approx 2.12 \times 10^4\ \mathrm{GeV}. \tag{23}$$

In this model the unification scale is much lower than in the other models, E_6, SO(10), SU(5), etc., but this does not give very small values for the light-neutrino masses.

Another interesting feature is the intermediate scale M_2 when the symmetry is broken down by three steps. At this scale the horizontal symmetry is broken and the corresponding neutral gauge boson that produce flavor-changing neutral currents at the tree level get mass. They give rise to the decay $K^+ \to \pi^+e^-\mu^+$ and this can be written naively [14]:

$$\Gamma(K^+ \longrightarrow \pi^+e^-\mu^+) \approx \frac{G_{\mathrm{eff}}^2 10^{-5}\ \mathrm{GeV}^8}{256\pi^3 m_K^3}, \tag{24}$$

where $G_{\mathrm{eff}}/\sqrt{2} \approx 4\pi\alpha/8M_{Z'}^2 \sin^2\theta_W$. From the experimental data $\Gamma_i^{\mathrm{expt}} \leq 11.173 \times 10^{-27}$ [13], we find for the mass of the neutral gauge bosons the value

$$M_{Z'} \geq 10^5\ \mathrm{GeV},$$

where we take the values of α and $\sin^2\theta_W$ at the M_Z scale. The result obtained above is in agreement with the renormalization-group equations with three steps breaking the scheme when $[SU(6)]^3$ is broken down to $G_{L(R)} = \mathrm{Sp}(6)_{L(R)}$.

In conclusion, the $[SU(6)]^3$ Pati-Salam model [15] with the above Higgs set does not produce proton decay by gauge or Higgs bosons exchange. However, this predicts $\Delta B = 2$. Proton stability in this kind of model depends on the Higgs bosons [3]. Using the RGE with three steps of symmetry breaking, it is possible to find a lower grand unification scale than other models that give very small values for the light-neutrino masses. The intermediate scale gives mass to the gauge bosons that produce rare processes, and it agrees with present data. It would be interesting to consider other steps of horizontal symmetry breaking, but these introduce new unknown parameters. The mass scale predicted by two steps of SB is lower than the scale predicted by $K^+ \longrightarrow \pi^+e^-\mu^+$ decay. This suggests that it is necessary to consider at least three steps of SB to have a realistic model.

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