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Lattice QCD solution to the U(1) problem

M. Fukugita*
Yukawa Institute, Kyoto University, Kyoto 606, Japan

Y. Kuramashi and M. Okawa
National Laboratory for High Energy Physics (KEK), Tsukuba, Ibaraki 305, Japan

A. Ukawa
Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305, Japan
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It is shown in quenched lattice QCD that the mass splitting between η' and a pion arises from gauge configurations with a nonzero topological charge Q, its magnitude increasing for larger values of $|Q|$; the contribution from the disconnected quark loop is strongly hindered unless the topological charge is excited. This demonstrates the explicit relation between the large η' meson mass and gauge field topology, which is in the line of argument in the continuum of instantons and the $1/N$ expansion.

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The U(1) problem has been one of the most outstanding problems in hadron physics. Glashow noted the inevitable existence of an isoscalar 0^- meson with a mass of the order of the pion mass in the presence of chiral SU(2)×SU(2) symmetry [1]. Weinberg has since shown that this isoscalar meson mass has to be smaller than $\sqrt{3} \times m_\pi$, which, however, conflicts with experiment [2]. A naive idea to explain a large mass of η [or η' in the case with SU(3)] is that $q\bar{q}$ annihilation into gluons contributes to boosting the flavor-singlet meson mass [3]; i.e., the Okubo-Zweig-Iizuka (OZI) rule is largely violated for the pseudoscalar channels. An objection to this idea was that this process does not violate the conservation of axial U(1) current, and the argument of Weinberg remains intact. An important observation, however, is that the conservation is actually broken due to the presence of an anomaly, when vacuum fluctuations of nonzero topological charge are excited, and this gives rise to a mass of the η' meson [4]. This argument has been developed by Witten [5] and Veneziano [6]: they have derived a Ward identity in the presence of an anomaly within the framework of the $1/N$ expansion, and have predicted the mass of η' which is close to experiment. Whether this picture is valid in our world still remains an issue to be understood within QCD. Furthermore, the dynamical mechanism of topological fluctuations that boosts the η' mass is not clear in their argument.

In a previous publication [7] we calculated the η' meson mass in the quenched approximation of lattice QCD, and showed that η' indeed has a mass much larger than the pseudoscalar octets as the chiral limit is approached. This successful prediction gives us hope that the origin of a large η' mass could be clarified in a further study. A large η'-octet mass splitting arises if the disconnected two-quark-loop amplitude, which contributes only to the η' propagator, substantially cancels the single-quark-loop contribution at large time separations, so that the η' propagator decays faster than the octet propagator. It has been shown in earlier quenched lattice QCD studies that the two-loop amplitude becomes significantly enhanced towards light quarks [8,9] and that the enhancement of the two-loop amplitude is due to gauge configurations of nonzero topological charge [9]. In the present Brief Report, we demonstrate that the η'-octet mass splitting calculated for gauge configurations having a given topological charge increases with the value of the topological charge, thereby establishing a direct connection between the mass of the η' meson and gauge field topology. This appears to be the first case where the role of instantons is compelling to understand the experiment.

The formalism and method of calculation are parallel to those given in Ref. [7]. We extract the splitting between η' and the pseudoscalar octet mass squared $m_0^2 = m_{\eta'}^2 - m_8^2$ from the ratio of the disconnected two quark loop amplitude to the single-quark-loop amplitude, each projected onto the zero momentum state, defined by

$$R(t) = \frac{\langle \eta'(t)\eta'(0)\rangle_{2\text{-loop}}}{\langle \eta'(t)\eta'(0)\rangle_{1\text{-loop}}} . \tag{1}$$

The two-loop contribution is evaluated by the variant wall source technique developed in Ref. [10]. At the same time we calculate the topological charge

$$Q = \frac{1}{32\pi^2} \sum_n \epsilon_{\mu\nu\rho\sigma} \mathrm{Re}\,\mathrm{Tr}\{U_{\mu\nu}(n) U_{\rho\sigma}(n)\} , \tag{2}$$

where $U_{\mu\nu}(n)$ is the plaquette in the $\mu\nu$ plane at site n, by applying the cooling method [11]. We then assemble gauge configurations with the same absolute magnitude of the topological charge $|Q|$ and calculate $R(t;|Q|)$, from which we extract m_0 as a function of $|Q|$.

Our calculations are made at $\beta = 6/g^2 = 5.7$ ($a \approx 0.14$ fm) on a $12^3 \times 20$ lattice in quenched lattice QCD,

*Also at Institute for Advanced Study, Princeton, NJ 08540.

employing the Wilson quark action with $K = 0.1665$ ($m_\pi/m_\rho = 0.59$). We analyzed 240 gauge configurations, each 1000 pseudo-heat-bath sweeps apart. Twenty-five sweeps are made to cool down the configurations in order to calculate topological charge.

Figure 1 shows the distribution of topological charge. The values actually measured on the lattice are not necessarily integers, since we are not working at a coupling sufficiently close to the continuum limit and hence Q defined in (2) receives a finite lattice correction of $O(a^2)$. Defining $\bar{Q}$ to be the value of Q rounded off to the nearest integer, we find that $(Q-\bar{Q})/\bar{Q}$ shows a Gaussian-like distribution with a full width at half maximum of 0.11 (see Fig. 2; we have excluded configurations with $\bar{Q}=0$ for this figure).

In Fig. 3 we present $R(t;|\bar{Q}|)$ for typical values of $|\bar{Q}|$: $|\bar{Q}| = 0, 2, 5$. For the one-quark-loop amplitude in the denominator we used the average over the whole ensemble since it shows little variation depending on $|\bar{Q}|$. We see that the function $R(t;|\bar{Q}|)$ is consistent with zero for $\bar{Q}=0$, and it shows a clear increase as $|\bar{Q}|$ increases. This demonstrates that the effect of instantons generates the two-loop contribution; the contribution from the two-loop diagram is strongly suppressed unless the topological charge is excited. A large mass of η' does not arise merely from $q\bar{q}$ annihilation into gluons; this suggests that the prototype OZI rule holds well even for the 0^- channel if the topological effect can be switched off. We note in this context that the validity of the OZI rule has been demonstrated for π-π scattering where the quark annihilation amplitude in the $I=0$ channel with purely gluonic intermediate states has been seen to be very small compared to other contributions [10].

The dependence of the η'-octet mass splitting m_0 on $|\bar{Q}|$ is presented in Fig. 4 in lattice units [12]. We have extracted m_0 by fitting the data to the form

$$R(t;|\bar{Q}|) \approx \frac{m_0(|\bar{Q}|)^2}{2m_8} t + \text{const} , \tag{3}$$

where the value of m_8 is taken from a single exponential fitting of the pion propagator calculated for the entire ensemble. The fitting range is chosen to be $4 \le t \le 8$, where the cutoff at a small t is made in order to avoid a possible contamination from higher excited states, in agreement with the analysis for the pion propagator that indeed exhibits such a contribution. We set the constant term to be zero following Ref. [7]. As Fig. 4 shows, m_0 increases with $|\bar{Q}|$. The mass splitting m_0 calculated from the entire ensemble is 0.379(13), or, using the physical scale, 550 MeV at $m_\pi/m_\rho = 0.59$ for this figure.

We have repeated the analysis at two other values of the hopping parameter $K = 0.164$ ($m_\pi/m_\rho = 0.74$) and 0.165 (0.70) on the $12^3 \times 20$ lattice, and also at $K = 0.168$ ($m_\pi/m_\rho = 0.42$) on a $16^3 \times 20$ lattice with 40 configurations. The connection between topological charge and the η'-octet mass splitting as observed in Figs. 3 and 4 is also seen at those values of K. An increasingly rapid increase of $m_0(|\bar{Q}|)$ for nonzero $|\bar{Q}|$ toward light quark masses, expected from the existence of the associated fermion zero modes, was not clearly observed, however, probably due to still heavy quarks ($m_\pi/m_\rho \ge 0.42$) employed in the analysis.

Let us comment on the relation between our results

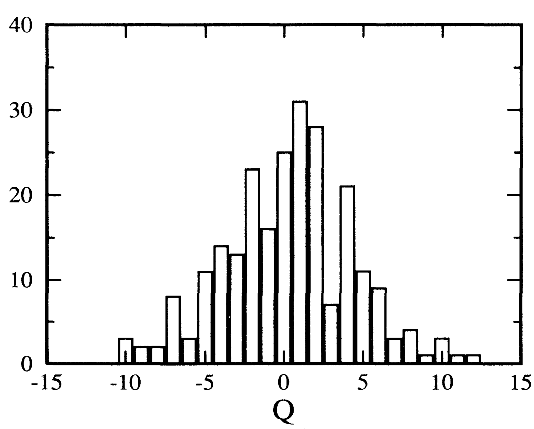

FIG. 1. Distribution of topological charge Q in quenched lattice QCD at $\beta = 5.7$ on a $12^3 \times 20$ lattice. Bins are centered at integer values with unit width.

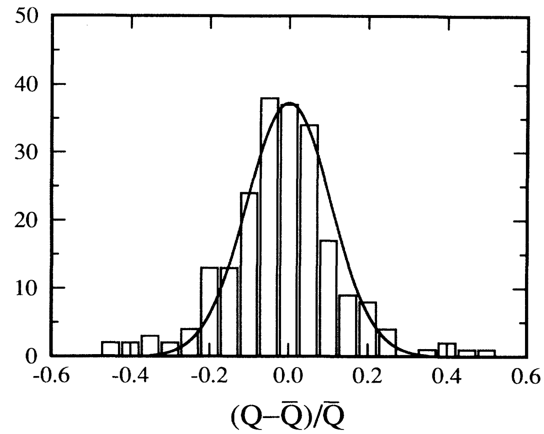

FIG. 2. Distribution of $(Q-\bar{Q})/\bar{Q}$ with $\bar{Q}$ the nearest integer value of Q. Configurations with $\bar{Q}=0$ are excluded.

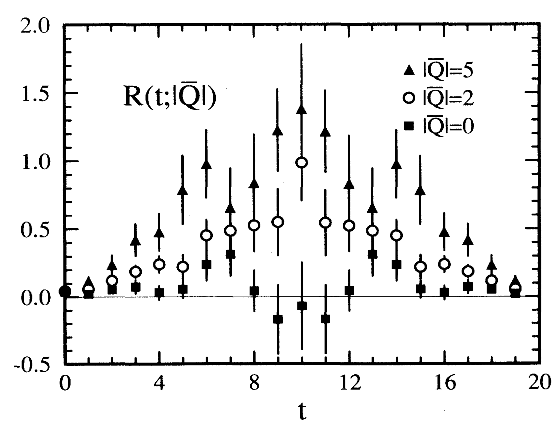

FIG. 3. Representative values of the ratio $R(t;|\bar{Q}|)$ of two- and single-quark-loop contributions to the η' propagator for several values of $|\bar{Q}|$ at $\beta = 5.7$ and $K = 0.1665$ on a $12^3 \times 20$ lattice.

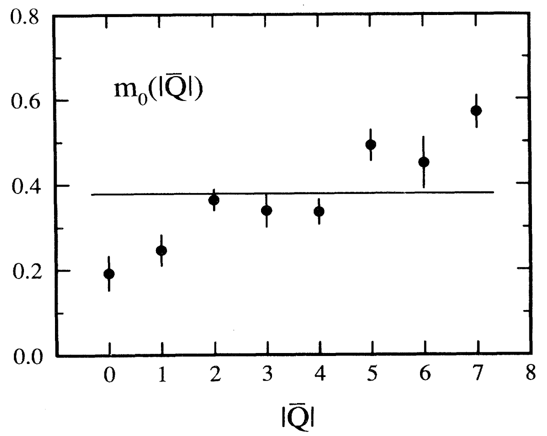

FIG. 4. η'-octet mass splitting m_0 in lattice units as a function of $|\bar{Q}|$ at $\beta = 5.7$ and $K = 0.1665$ on a $12^3 \times 20$ lattice in quenched QCD. The horizontal line represents the value for the entire ensemble.

and the fermionic definition of topological charge on the lattice [13]. In the continuum the quark propagator $G(x,y)$ on a background gluon field with topological charge Q satisfies $\int d^4x\, \mathrm{Tr}[\gamma_5 G(x,x)] = Q/m_q$. Based on this relation it has been suggested [13] that topological charge on the lattice may be defined as $Q_f = Z_P m_q \sum_n \mathrm{Tr}[\gamma_5 G(n,n)]$, where Z_P is a renormalization factor. This provides an interesting view to understand the correlation we have observed between m_0 and topological charge, since the two-loop contribution to the η' propagator $\langle \eta'(t)\eta'(0)\rangle$ summed over t is proportional to $\langle Q_f^2 \rangle$. However, the choice of Z_P for ensuring a value of Q_f close to an integer and independent of the quark mass used for the quark propagator is quite subtle [13]. This problem is avoided in our calculation employing the gluon field definition of topological charge given in (2) and the cooling method.

In sum, we clarified the issue concerning the U(1) problem: it is the dynamical role of instantons that boosts the η' mass, which leads to an apparent large violation of the OZI rule in the pseudoscalar channel. We remark that our results are obtained in the quenched approximation. In the presence of dynamical sea quarks, topologically nontrivial gauge configurations are suppressed towards a vanishing quark mass due to fermionic zero modes. It remains an intriguing problem to investigate how the η' mass survives the suppression in full QCD. The mechanism may be subtle, involving a balance between the suppression and an increase of η' mass for larger values of topological charge as was shown in the present work. A numerical elucidation of this effect, however, will require very long runs, since full QCD simulations carried out to date exhibit long-range correlations in the fluctuation of topological charge [14,15].

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