Induced gravity inflation in the SU(5) GUT

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We investigate the cosmological consequences of a theory of induced gravity in which the scalar field is identified with the Higgs field of the first symmetry breaking of a minimal SU(5) GUT. The mass of the X boson determines a great value for the coupling constant of gravity-particle physics. Because of this fact, a "slow" rollover dynamics for the Higgs field is not possible in a "new" inflation scenario and, moreover, a contraction era for the scale factor in the early Universe exists, after which inflation follows automatically; "chaotic" inflation is performed without problems. Inflation is successfully achieved due to the relationship among the masses of particle physics at that scale: the Higgs-boson, X-boson, and Planck masses. As a result, the particle physics parameter λ is not fine-tuned as usual in order to predict acceptable values of reheating temperature and density and gravitational wave perturbations. Moreover, if the coherent Higgs oscillations did not decay they could explain the missing mass problem of cosmology.

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I. INTRODUCTION

It was more than a decade ago that the inflationary model [1] was proposed in order to solve some problems in cosmology: the horizon and flatness problems, the present isotropy of the Universe, and the possible overabundance of magnetic monopoles. The inflationary scenario was inspired to incorporate theories of particle physics into the early Universe, achieving an interesting jointing of these two different areas of physics. Since that time there has been many alternative models (for a review see Refs. [2–5]) formulated to overcome the problems that inflation suffers, i.e., a smooth ending of the inflationary era (graceful exit), enough e-folds of inflation, sufficient reheat temperature for baryogenesis to take place and the right contrast of density and gravitational perturbations coming from the scalar field fluctuations, among others, to achieve successfully inflation [6]. Although some inflationary scenarios can solve many of the above-mentioned problems, some of their other features (coupling constant's strengths, etc.) are not well understood, due to the lack of a final gravity theory coupled to the other interactions of nature, which are necessary in order to describe the very early Universe. Nevertheless, gravity theories including quadratic terms coming from high-energy theories or Brans-Dicke theory (BDT) plus potentials, induced gravity theories, or others with a particle content are today believed to be the more realistic ones in order to describe the first stage of our Universe and, in accordance with experimental constraints, for low energies these theories should be not much different from Einstein's general relativity (GR) in a four-spacetime manifold.

Many Lagrangians in cosmology inspired from particle physics that should be applicable at the beginning of the Universe have been considered. Some of the scenarios coming from these Lagrangians are not very careful in treating the coupling gravity-particle physics, but they are careful in adjusting the particle physics parameters to the cosmological ones in order to solve the above known problems of cosmology, resulting in a necessary but undesirable fine-tuning. In particular, to obtain the right contrast of density perturbations caused by preinflationary field fluctuations, one has to set λ (from an inflation potential, say, $V = \lambda\phi^4$) to a very small number ($< 10^{-12}$) by hand, which is from the particle physics point of view very unnatural and has no justification.

The cosmological consequences of induced gravity models are well known [7–13], but the particle physics content is still unclear, simply because the Lagrangians used there imply scalar field associated particles with masses greater than the Planck mass ($M_{\rm Pl}$). In our approach gravity is coupled to an SU(5) grand unification theory (GUT), that is, to a lower energy scale. The idea to induce gravity by a Higgs field has already been discussed elsewhere [14–16] and motivation for it came to us, very much as in Ref. [15]; on the one hand, from Einstein's original ideas to incorporate the Mach principle to GR, by which the mass of a particle should be originated from the interaction with all the particles of the Universe, whereby the interaction should be the gravitational one since it couples to all particles, i.e., to their masses or energies. To realize a stronger relationship of the Universe's particles Brans and Dicke [17] introduced their scalar-tensor theory of gravity, letting the active as well as the passive gravitational mass, that is, Newton's gravitational "constant," be a scalar function determinated by the distribution of particles of the Universe.

On the other hand, in modern particle physics the in-

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ertial mass is generated by the interaction with the Higgs field, and it is emphasized that the successful Higgs mechanism also lies precisely in the direction of Einstein's idea of producing mass by a gravitational-like interaction. One can show [18–20] that the Higgs field as a source of the inertial mass of the elementary particles mediates a scalar gravitational interaction, however, of Yukawa type, between those particles which become massive as a consequence of the spontaneous symmetry breaking (SSB): The masses are the source of the scalar Higgs field and the Higgs field acts back by its gradient on the masses in the momentum law.

Then, because of the equivalence principle, it seems natural to identify both approaches. For this reason, a scalar-tensor theory of gravity was proposed [21] where the isospin-valued Higgs field of elementary particles simultaneously plays the role of a variable gravitational constant instead of the scalar field introduced by Brans and Dicke.

In this paper, we discuss some cosmological consequences of this theory of gravity coupled to an isotensorial Higgs field, which breaks down to give rise to both the X- and Y-boson masses and Newton's gravitational constant. Since the symmetry-breaking process of the SU(5) model could be expected to occur in the physical Universe, we are considering inflation there.

Our study carries out, at least, two types of inflationary models: a modified version of "new" and "chaotic" inflation, depending essentially on the initial conditions that the Universe chooses for the Higgs field at the beginning of time. An interesting feature of the models here is that λ fine-tuning is not necessary, just because of the given natural relationship of the different mass scales of particle physics.

II. INDUCED GRAVITY IN THE SU(5) GUT

The scalar-tensor theory with the Higgs mechanism is based on the Lagrange density [20,21] with units $\hbar = c = k_B = 1$ and the signature $(+,-,-,-)$:

$$\mathcal{L} = \left[\frac{\alpha}{16\pi}\mathrm{tr}\Phi^\dagger\Phi \; R + \frac{1}{2}\mathrm{tr}\Phi^\dagger_{||\mu}\Phi^{||\mu} - V(\mathrm{tr}\Phi^\dagger\Phi) + L_M\right] \times\sqrt{-g} \; , \tag{1}$$

where R is the Ricci scalar, and Φ is the SU(5) isotensorial Higgs field. The symbol $||\mu$ means in the following the covariant derivative with respect to all gauged groups and represents in (1) the covariant gauge derivative: $\Phi_{||} = \Phi_{|\mu} + ig[A_\mu, \Phi]$, where $A_\mu = A_\mu{}^a\tau_a$ are the gauge fields of the inner symmetry group, τ_a are its generators, and g is the coupling constant of the gauge group ($|\mu$ means the usual partial derivative); α is a dimensionless parameter to regulate the strength of gravitation and L_M contains the fermionic and massless bosonic fields, which belong to the inner gauge group SU(5); V is the Higgs potential.

Naturally from the first term of Eq. (1) it follows that $\alpha\ \mathrm{tr}\Phi^\dagger\Phi$ plays the role of a variable reciprocal gravitational "constant." The aim of our theory is to obtain GR as a final effect of a symmetry-breaking process and in that way to have Newton's gravitational constant G induced by the Higgs field; similar theories have been considered to explain Newton's gravitational constant in the context of a spontaneous symmetry-breaking process to unify gravity with other fields involved in matter interactions; see Refs. [14–16] .

In the minimal SU(5) GUT the Higgs field which breaks the SU(5) symmetry to $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_{HC}$, is in the adjoint representation a 5×5 traceless matrix taking the form, in the unitary gauge,

$$\Phi = \phi\mathbf{N}, \quad \mathbf{N} \equiv \sqrt{2/15}\ \mathrm{diag}(1,1,1,-3/2,-3/2) \tag{2}$$

(ϕ a real valued function). In this paper we analyze the cosmological consequences when this symmetry breaking is responsible for the generation of gravitational constant as well as the SU(5) standard particle content, i.e., the X- and Y-boson masses.

The Higgs potential takes the form, using Eq. (2),

$$\begin{aligned} V(\mathrm{tr}\Phi^\dagger\Phi) &= \frac{\mu^2}{2}\mathrm{tr}\Phi^\dagger\Phi + \frac{\lambda}{4!}(\mathrm{tr}\Phi^\dagger\Phi)^2 + \frac{3}{2}\frac{\mu^4}{\lambda} \\ &= V(\phi) = \frac{\lambda}{24}\left(\phi^2 + 6\frac{\mu^2}{\lambda}\right)^2 , \end{aligned} \tag{3}$$

where we added a constant term to prevent a negative cosmological constant after the breaking. The Higgs ground state v is given by

$$v^2 = -\frac{6\mu^2}{\lambda} \tag{4}$$

with $V(v) = 0$, where λ is a dimensionless real constant, whereas μ^2 (< 0) is so far the only dimensional real constant of the Lagrangian.

In such a theory, the potential $V(\phi)$ will play the role of a cosmological "function" (instead of a constant) during the period in which Φ goes from its initial value Φ_0 to its ground state $v\mathbf{N}$, where furthermore

$$G = \frac{1}{\alpha v^2} \tag{5}$$

is the gravitational constant to realize from (1) the theory of GR [21]. In this way, Newton's gravitational constant is related in a natural form to the mass of the gauge bosons, that for the case of the SU(5) GUT is

$$M_X = M_Y = \sqrt{\frac{5\pi}{3}}gv \; . \tag{6}$$

As a consequence of (5) and (6) one has that the strength parameter for gravity, α , is determined by

$$\alpha = \frac{10\pi}{3}\left(g\frac{M_{\mathrm{Pl}}}{M_X}\right)^2 , \tag{7}$$

where $M_{\mathrm{Pl}} = 1/\sqrt{2G}$ is the Planck mass and $g^2 \approx 0.02$. In order to be in accordance with proton decay experiments, $\alpha \lesssim 10^7$ must be valid, since the X-boson mass cannot be smaller than approximately 10^{15} GeV. For the detailed calculations in Secs. III and IV we take for α the upper limit ($\alpha = 10^7$). In this way, the coupling between the Higgs field and gravitation is very strong: the fact that $\alpha \gg 1$ is the price paid in recovering Newton's gravitational constant at that energy scale. On the other hand, for the BDT the value of its corresponding α ($= 2\pi/\omega$) must be $\alpha < 10^{-2}$ to fit well the theory with the experimental data [22]. Therefore, in this respect there are important differences between our presentation here and the Brans-Dicke one and also with regard to most of the induced gravity approaches, where to achieve successful inflation typically $\alpha \ll 1$ [8,10], and in that way, the existence of a very massive particle ($> M_{\rm Pl}$) is necessary, which after inflation should decay into gravitons, making an acceptable nucleosynthesis scenario difficult later [23,24].

From (1) one calculates immediately the gravity equations of the theory:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{8\pi V(\mathrm{tr}\Phi^\dagger\Phi)}{\alpha\ \mathrm{tr}\Phi^\dagger\Phi}g_{\mu\nu} = -\frac{8\pi}{\alpha\ \mathrm{tr}\Phi^\dagger\Phi}T_{\mu\nu} - \frac{8\pi}{\alpha\ \mathrm{tr}\Phi^\dagger\Phi}\left[\mathrm{tr}\Phi^\dagger_{(||\mu}\Phi_{||\nu)} - \mathrm{tr}\frac{1}{2}\Phi^\dagger_{||\lambda}\Phi^{||\lambda}\ g_{\mu\nu}\right] - \frac{1}{\mathrm{tr}\Phi^\dagger\Phi}\left[\mathrm{tr}(\Phi^\dagger\Phi)_{|\mu||\nu} - tr(\Phi^\dagger\Phi)^{|\lambda}{}_{||\lambda}\ g_{\mu\nu}\right] , \quad (8)$$

where $T_{\mu\nu}$ is the energy-momentum tensor belonging to $L_M\sqrt{g}$ in (1) alone, and the Higgs field equations

$$\Phi^{||\lambda}{}_{||\lambda} + \frac{\delta V}{\delta\Phi^\dagger} - \frac{\alpha}{8\pi}R\Phi = 2\frac{\delta L_M}{\delta\Phi^\dagger} . \quad (9)$$

Now we introduce the new real valued scalar variable

$$\chi \equiv \frac{1}{2}\left(\frac{\mathrm{tr}\Phi^\dagger\Phi}{v^2} - 1\right) , \quad (10)$$

which describes the excited Higgs field around its ground state; for instance $\Phi = 0$ implies $\chi = -1/2$ and $\Phi = v\mathbf{N}$ implies $\chi = 0$. With this new Higgs variable Eqs. (8) and (9) are now

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \left[\frac{8\pi}{\alpha v^2}\frac{V(\chi)}{(1+2\chi)}\right]g_{\mu\nu} = -\frac{8\pi}{\alpha v^2}\frac{1}{(1+2\chi)}\hat{T}_{\mu\nu} - \frac{8\pi}{\alpha}\frac{1}{(1+2\chi)^2}\left[\chi_{|\mu}\chi_{|\nu} - \frac{1}{2}\chi_{|\lambda}\chi^{|\lambda}g_{\mu\nu}\right] - \frac{2}{1+2\chi}\left[\chi_{|\mu||\nu} - \chi^{|\lambda}{}_{||\lambda}g_{\mu\nu}\right] \quad (11)$$

and

$$\chi^{|\mu}{}_{||\mu} + \frac{1}{(1+\frac{4\pi}{3\alpha})}\frac{4\pi}{3\alpha v^2}\frac{\delta V}{\delta\chi} = \frac{1}{(1+\frac{4\pi}{3\alpha})}\frac{4\pi}{3\alpha v^2}\hat{T} , \quad (12)$$

where $\hat{T}_{\mu\nu}$ is the *effective* energy-momentum tensor given by

$$\hat{T}_{\mu\nu} = T_{\mu\nu} + \frac{(1+2\chi)}{4\pi}M^2_{ab}\left(A^a{}_\mu A^b_\nu - \frac{1}{2}g_{\mu\nu}A^a{}_\lambda A^{b\lambda}\right), \quad (13)$$

where M^2_{ab} is the gauge boson mass square matrix.

The continuity equation (energy-momentum conservation law) reads

$$\hat{T}_\mu{}^\nu{}_{||\nu} = 0 , \quad (14)$$

which has no source since in the present theory, SU(5) GUT, all the fermions remain massless after the first symmetry breaking and no baryonic matter is originated in this way.

Another important difference with the BDT is due to the existence of the potential term, which shall play the role of a positive cosmological function [see the square brackets on the left hand side of Eq. (11)]; it takes, in terms of χ the simple form,

$$V(\chi) = \frac{\lambda v^4}{6}\chi^2 \quad (15)$$

which at the ground state vanish, $V(\chi = 0) = 0$. From Eqs. (11) and (5) one recovers GR for the ground state

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi G\ \hat{T}_{\mu\nu} \quad (16)$$

with the effective energy-momentum tensor (13). Newton's gravitational "function" is $G(\chi) = \frac{1}{\alpha v^2}\frac{1}{1+2\chi}$ and Newton's gravitational constant $G(\chi = 0) = G$.

From (12) one can read directly, if the explicit form of $V(\chi)$ is introduced, the mass of the Higgs boson M_H , and therefore its Compton range l_H,

$$M_H = \sqrt{\frac{\frac{4\pi}{9\alpha}\lambda v^2}{(1+\frac{4\pi}{3\alpha})}} , \quad l_H = \frac{1}{M_H} , \quad (17)$$

that is, the χ field possesses a finite range, a characteristic which accounts for the difference between this theory and the BDT. From Eq. (17) one sees that the mass of the Higgs particle is a factor $\sqrt{\frac{4\pi}{3\alpha}} \approx 10^{-4}$ smaller than the one derived from the SU(5) GUT without gravitation. This is a very interesting property since the Higgs boson mass determines the scale of the symmetry breaking, or equivalently, $\sqrt{\lambda}/\alpha$ will be in a natural way a very small value, avoiding a fine tuning of λ in order to accomplish a successful Universe (see later discussion).

Next, we proceed to investigate the cosmological consequences of such a theory.

III. FRW MODELS

Let us consider a Friedmann-Robertson-Walker (FRW) metric. One has with the use of (5) that Eqs. (11) are reduced to

$$\frac{\dot{a}^2+\epsilon}{a^2} = \frac{1}{1+2\chi}\left(\frac{8\pi G}{3}[\rho+V(\chi)] - 2\frac{\dot{a}}{a}\dot{\chi} + \frac{4\pi}{3\alpha}\frac{\dot{\chi}^2}{1+2\chi}\right) \tag{18}$$

and

$$\frac{\ddot{a}}{a} = \frac{1}{1+2\chi}\left(\frac{4\pi G}{3}[-\rho-3p+2V(\chi)] - \ddot{\chi} - \frac{\dot{a}}{a}\dot{\chi} - \frac{8\pi}{3\alpha}\frac{\dot{\chi}^2}{1+2\chi}\right) , \tag{19}$$

where $a = a(t)$ is the scale factor, ϵ the curvature constant ($\epsilon = 0$, $+1$, or -1 for a flat, closed, or open space, respectively), ρ and p are the matter density and pressure, assuming that the effective energy momentum tensor (13) has in the classical limit the structure of that of a perfect fluid. An overdot stands for a time derivative.

In the same way Eq. (12) results in

$$\ddot{\chi} + 3\frac{\dot{a}}{a}\dot{\chi} + M_H^2\chi = \frac{4\pi G}{3}\frac{(\rho-3p)}{(1+\frac{4\pi}{3\alpha})} , \tag{20}$$

where the Higgs potential is already inserted: i.e.,

$$V(\chi) = \frac{\lambda v^4}{6}\chi^2 = (1+\tfrac{4\pi}{3\alpha})\frac{3}{8\pi G}M_H^2\chi^2 . \tag{21}$$

The Higgs boson mass demarcates the time epoch for the rolling over of the potential, and therefore for inflation. Note that $V(\chi) \sim M_{Pl}^2 M_H^2 \chi^2$; this fact is due to the relationship (5) to obtain GR once the symmetry breaking takes place.

The continuity equation (14) takes the simple form

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho+p) = 0 , \tag{22}$$

which is sourceless, meaning that the Higgs mechanism produces no entropy processes, although the Higgs field is coupled to the perfect fluid through Eq. (20). If one takes the equation of state of a barotropic fluid, i.e., $p = \nu\rho$ with the dimensionless constant ν, Eq. (22) can be easily integrated $\rho = \frac{M}{a^{3(1+\nu)}}$, where M is the integration constant.

At this point we would like to make several remarks concerning the scale factor equations (18) and (19). First of all, the equations allow static solutions ($\dot{a} = \ddot{a} = \dot{\chi} = \ddot{\chi} = \dot{\rho} \equiv 0$) for dust particles ($\nu = 0$):

$$\chi = 1 , \quad a^2 = \frac{1}{(1+\frac{4\pi}{3\alpha})}\frac{1}{M_H^2}$$

and

$$\rho = \frac{3}{2\pi}(1+\tfrac{4\pi}{3\alpha})M_{\rm Pl}^2 M_H^2 . \tag{23}$$

For the radiation case, $\nu = 1/3$, there are no static solutions.

For the dynamical behavior one notes that the Higgs potential is indeed a positive cosmological function, which corresponds to a positive mass density and a negative pressure [see Eqs. (18) and (19)], and represents an ideal ingredient to have inflation. But, on the other hand, there is a negative contribution to the acceleration equation (19) due to the Higgs-kinematic terms, i.e., terms involving $\dot{\chi}$ and $\ddot{\chi}$; terms involving the factor $1/\alpha \sim 10^{-7}$ are simply too small compared to the others and can be neglected.

For inflation it is usually taken that $\ddot{\chi} \approx 0$, but in fact the dynamics should show up this behaviour or at least certain consistency. For instance, in GR with the *ad hoc* inclusion of a scalar field ϕ as a source for the inflation, one has that at the "slow rollover" epoch $\ddot{\phi} \approx 0$ and therefore $\dot{\phi} = -V'/3H$, which implies that

$$\frac{\ddot{\phi}}{3H\dot{\phi}} = -\frac{V''}{9H^2} + \frac{1}{48\pi G}\left(\frac{V'}{V}\right)^2 \ll 1 , \tag{24}$$

where $H = \dot{a}/a$ is the Hubble expansion rate (a prime denotes the derivative with respect to the corresponding scalar field; see Ref. [6]). In the present theory, if one considers the Higgs potential term in Eq. (18) as the dominant one[1] and Eq. (20) without source, i.e., $p = \frac{1}{3}\rho$, one has indeed an extra term due to the variation of Newton's "function" $G(\chi)$, that is,

$$\frac{\ddot{\chi}}{3H\dot{\chi}} = \left(\frac{1}{1+\frac{4\pi}{3\alpha}}\right)\frac{4\pi G}{3} \times\left[-\frac{V''}{9H^2} + \frac{1}{48\pi G}\left(\frac{V'}{V}\right)^2(1+2\chi) - \frac{1}{24\pi G}\frac{V'}{V}\right] . \tag{25}$$

[1]From now on, we shall always consider the dynamics to be dominated by the Higgs terms, in Eqs. (18)–(20), instead of the matter density term, from which it is not possible to drive inflation.

If $\chi < 0$ the last term does not approach to zero during the rolling down process for $\alpha \gg 1$. Thus, instead of a "slow," one has rather a "fast" rollover dynamics of the Higgs field along its potential down hill. On the other side, for $\chi \gg 1$ there is indeed a "slow" rollover dynamics.

With this in mind one has to look carefully at the contribution of $\ddot{\chi}$: if one brings $\ddot{\chi}$ from (20) into (19) one has that $M_H^2\chi$ competes with the potential term $M_H^2\chi^2$, and during the rolling down of the potential, when χ goes from $-1/2$ to 0, $M_H^2\chi < 0$ dominates the dynamics, and therefore instead of inflation one ends with deflation or at least with a contraction era for the scale factor.

Resuming, if one starts the Universe evolution with an ordinary new inflation scenario ($\chi_0 < 0$),[2] it implies in this theory a "short" deflation instead of a "long" inflation period, since the Higgs field goes relatively fast to its minimum. This feature should be present in theories of induced gravity with $\alpha > 1$ and also for the BDT with this type of potential (see for example the field equations in Ref. [25]). Considering the opposite limit, $\alpha < 1$, induced gravity models [8] have proved to be successful for inflation, also if one includes other fields [12]; induced gravity theories with a Coleman-Weinberg potential are also shown to be treatable for a very small coupling constant λ with $\chi_0 < 0$ [10,11], or with $\chi_0 > 0$ [9] and $\alpha < 1$ as well as $\alpha > 1$ [7]. For extended or hyperextended inflation models [26,27] this problem does not arise due to the presence of vacuum energy during the rollover stage of evolution, which is supposed to be greater than the normal scalar field contribution.

With this concern one has to prepare a convenient scenario for the Universe to begin with. In the next section we analyze the initial conditions of our models.

IV. INITIAL CONDITIONS AND INFLATION

The initial conditions that we have chosen are simply $\dot{a}_0 = \dot{\chi}_0 = 0$. Equations (18) to (20) must satisfy the following relations

The size of the initial Universe is, if $\epsilon = 1$,

$$a_0^2 = \frac{1 + 2\chi_0}{\frac{8\pi G}{3}\rho_0 + (1 + \frac{4\pi}{3\alpha})M_H^2\chi_0^2} \; ; \tag{26}$$

its acceleration has the value

$$\frac{\ddot{a}_0}{a_0} = \frac{1}{1+2\chi_0}\left\{ -\frac{4\pi G}{3}\left[1 + \frac{1}{(1+\frac{4\pi}{3\alpha})} + \left(1 - \frac{1}{(1+\frac{4\pi}{3\alpha})}\right)3\nu\right]\rho_0 + (1 + \tfrac{4\pi}{3\alpha})M_H^2\chi_0^2 + M_H^2\chi_0\right\} \tag{27}$$

and, for the Higgs field,

$$\ddot{\chi}_0 + M_H^2\chi_0 = \frac{4\pi G}{3}\frac{(1-3\nu)}{(1+\frac{4\pi}{3\alpha})}\rho_0 \, . \tag{28}$$

The initial values ρ_0 and χ_0 as well as M_H are the cosmological parameters to determine the initial conditions of the Universe. The value of M_H fixes the time scale for which the Higgs field breaks down into its ground state. In order to consider the Higgs terms as the dominant ones (see footnote 1), one must choose the initial matter density $\rho_0 < \frac{3\chi_0^2}{4\pi}(1 + \frac{4\pi}{3\alpha})M_{\rm Pl}^2 M_H^2$. Let us say for a Higgs boson mass $M_H \sim 10^{-1}M_X = 10^{14}$ GeV, one has a typical time of $M_H^{-1} \sim 10^{-38}$ sec, and therefore $\rho_0 < \frac{3\chi_0^2}{4\pi}10^{66}\mathrm{GeV}^4$.

The question of the choice of the initial value χ_0 is open: for example, for "new inflation" $\chi_0 < 0$ [28–30], whereas for "chaotic inflation" [31] $\chi_0 > 0$. Therefore, we are considering both cases, which imply two different cosmological scenarios.

Scenario (a) ($\chi_0 < 0$). From Eq. (26) it follows that if the initial value of the Higgs field is strictly $\chi_0 = -1/2$, the Universe possesses a singularity. If the Higgs field sits near its metastable equilibrium point at the beginning ($\chi_0 \gtrsim -1/2$, $\Phi_0 \gtrsim 0$), then χ grows up since $\ddot{\chi}_0 > 0$, and from Eq. (27) one gets $\ddot{a}_0 < 0$, i.e., a maximum point for a_0; thus at the beginning one has a contraction instead of an expansion. Let us call this *rollover contraction*; see Fig. 1(a).

Normally it is argued that in BDT with a constant (or slowly varying) potential producing a finite vacuum energy density, the vacuum energy is dominant and is used to both to expand the Universe and to increase the value of the scalar field. This "shearing" of the vacuum energy to both pursuits is the cause of a moderate power law inflation instead of an exponential one [32]. In this scenario the Universe begins with a contraction, and therefore the same shearing mechanism, moreover here due to the Higgs field, drives a "friction" process for the contraction, due to the varying of $G(\chi)$, making the deflation era always weaker. Furthermore, one can see from Eq. (27) that the cause of the deacceleration in scenario (a) is the negative value of χ; then if $\dot{a} < 0$ from Eq. (20) it follows $\ddot{\chi} \sim -\frac{\dot{a}}{a}\,\dot{\chi} > 0$, which implies an "antifriction" for χ that tends to reduce the contraction; see Ref. [33].

One may wonder if the rollover contraction can be stopped. As long as χ is negative the contraction will not end, but if χ goes to positive values, impulsed by special initial conditions, one could eventually have that the dynamics dominating term, $M_H^2(\chi^2 + \chi)$, be positive enough to drive an expansion. But due to the nature of Eq. (20), if χ grows, the term $M_H^2\chi$ will bring it back to

[2] The subindex zero stands for the initial values (at $t = 0$) of the corresponding variables.

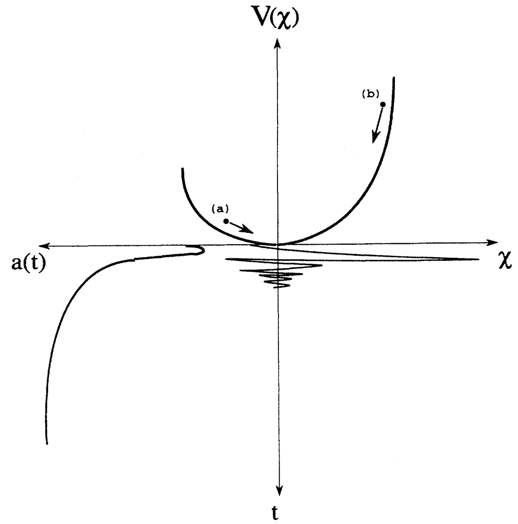

FIG. 1. (a) The Higgs field comes down the hill from $\chi_0 < 0$. During this time the Universe contracts, in a "new deflation" scenario with "fast" rollover evolution of χ. It is possible, however, for very special initial values that the field in (a) evolves to (b) resulting a normal exponential expansion. (b) The Higgs fields come down the hill, but now from $\chi_0 > 0$. During that time the Universe expands exponentially in a type of "chaotic" inflationary scenario.

negative values and cause an oscillating behavior around zero, its equilibrium state, with an amplitude which is damped with time due to the redshift factor $3H\dot{\chi}$. Therefore, one has to seek special values of χ_0, which will bring χ dynamically from negative values to great enough positive values to end up with sufficient e-folds of inflation. This feature makes clear that this scenario is not generic for inflation, but depends strongly on special initial conditions; in this sense, this is another type of fine-tuning, which is however always present by choosing the initial value of the inflaton field. For instance, in the SU(5) GUT energy scale with $M_H = 10^{14}$ GeV, one finds by numerical integration that obtaining the required inflation implies $\chi_0 \approx -0.15\,509$ [see Figs. 2(a) and 3] but not a very different number from this, otherwise the deflation era does not stop and the Universe evolves to an Einstein Universe with a singularity; one could consider whether GR singularities are an inevitable consequence of particle physics. It was also assumed, of course, that during the deflation phase the stress energy of other fields, e.g., radiation or nonrelativistic matter, are smaller than the Higgs one, otherwise an expansion follows.

It is interesting to note that in the SU(5) GUT with a finite-temperature effective potential coupled conventionally to GR, inflation takes place only after the temperature $T \lesssim 0.05\sigma$ ($\sigma = 2 \times 10^{15}$ GeV) and $|\phi - \sigma| > 0.03\sigma$, since there the "inflation pressure" $\rho(\phi, T) + 3p(\phi, T)$ is negative. Then if these requirements are not satisfied, there can appear a deflation era in the Universe; see [33]. In our presentation, temperature corrections are not considered, but the only possible initial Higgs values for a successful scenario (a) are ($\chi_0 \sim -0.1$ or $\phi \sim 0.8v$) those which correspond to the beginning of inflation in GR with temperature corrections. In both cases inflation begins when the Higgs initial value is not very far from its equilibrium state v, that means, ϕ is not located very close to zero as usual. Furthermore, if we were to consider temperature corrections, a negative contribution to the potential comes in (see Ref. [33]), making the deflation stronger, since it should be added to the already considered coming from $\ddot{\chi} > 0$ in Eq. (19). However, for the above-mentioned special values of the Higgs field, because of the temperature corrections a positive contribution is now expected to the acceleration of the scale factor, which should compete with $\ddot{\chi}$ to determine the scale factor evolution. Finally, only for some special values of χ_0, a short deflation is followed by a successful inflation period.

Scenario (b) ($\chi_0 > 0$). One could consider initial conditions whereby the Higgs terms $\chi_0^2 + \chi_0 > 0$ dominate the dynamics to have a minimum for a_0, i.e., $\ddot{a}_0 > 0$, and to begin on "a right way" with expansion i.e., inflation. That means one should start with a value $\chi_0 > 0$ (far from its minimum) positive enough to render sufficient e-folds of inflation. Thus, the "effective" inflation potential part is similar to the one proposed in the "chaotic" inflationary model [31], due to the form of the potential

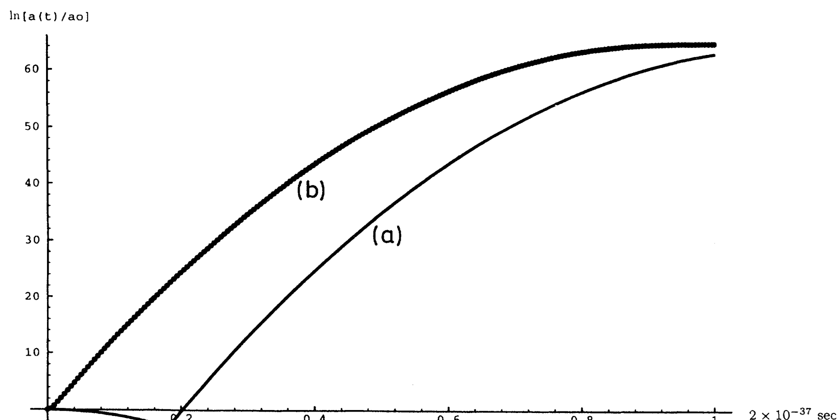

FIG. 2. The scale factor $a(t)$ is shown for both inflationary models (a) and (b) in a logarithmic scale. The model (a) begins with a "fast" contraction followed automatically by an inflation if $\chi_0 \approx -0.15\,509$. The upper curve [scenario (b)] shows the behavior of inflation if $\chi_0 \approx 130/3$ (chaotic exponential expansion).

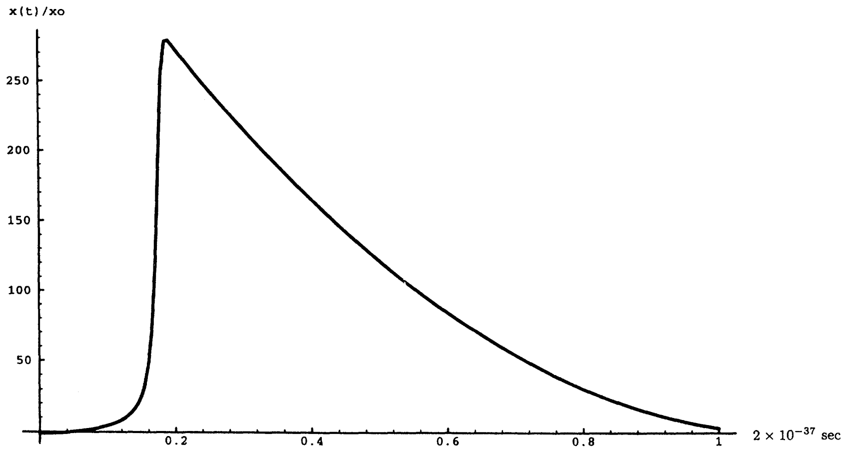

FIG. 3. The Higgs field of scenario (a) as a function of time. The Higgs field goes first very fast until it reaches $\chi \gtrsim 130/3 (\approx 280|\chi_0|)$; at that point H evolves faster than χ, to proceed with an inflationary phase.

and to the value of $\chi_0 > 0$ to have the desired inflation. In other words, both this and chaotic inflation scenarios are generic [5]; see Fig. 1(b).

Now let us see how the dynamics of both models works: The curvature term ϵ/a^2 in (18) can be neglected only after inflation began; during the rollover contraction (a) it plays an important role. The terms $\frac{\dot{a}}{a}\dot{\chi}$ will be comparable to $8\pi GV(\chi)/3$ until the high oscillation period ($H < M_H$) starts. For instance, in the chaotic scenario (b), the slow rollover condition[3] $\ddot{\chi} \approx 0$ is valid, which implies $\dot{\chi}/\chi = -M_H^2/3H$, and then from

$$H^2 \approx \frac{1}{1+2\chi}\left[M_H^2\chi^2 - 2H\dot{\chi}\right] \tag{29}$$

(with $\dot{\chi} < 0$) it follows that for $\chi > 2/3$ the Hubble parameter will be dominated by the potential term to have

$$H \approx M_H \frac{\chi}{\sqrt{1+2\chi}} \ , \tag{30}$$

which for $\chi \gg 1$ goes over into $H/M_H \sim \sqrt{\chi/2} \gg 1$, giving cause for the slow rollover ("chaotic") dynamics. Indeed, the rollover time is $\tau_{\rm roll} \sim 3H/M_H^2$, i.e.,

$$H\tau_{\rm roll} \approx 3H^2/M_H^2 \approx 3\frac{\chi^2}{1+2\chi} \gtrsim 65 \ , \tag{31}$$

yielding enough inflation; for $\chi_0 \gg 1$ it follows $H\tau_{\rm roll} \sim 3\chi_0/2 \gtrsim 65$, which implies $\chi_0 \gtrsim 130/3$ ($\phi_0 \gtrsim 9.3v$); this value can be checked by numerical integration; see Figs. 2(b) and 4. Note that H/M_H does not depend on the energy scale of inflation, but on the initial value χ_0. In other words, enough inflation is performed automatically and independently of α as was pointed out in Ref. [13]. Then if one considers inflation at a lower energy scale, χ_0 can be smaller than 130/3, since at that smaller energy scale the e-folds required are less.

On the other hand, if χ_0 is negative, the rollover contraction phase in scenario (a) happens, but in this case Eq. (29) indicates $H/M_H \approx |\chi| < 1$, that is, the scale factor evolves slower than the Higgs field; and with $\chi_0 \approx -0.155\,09$, χ evolves to values greater or equal than 130/3 to gain conditions very similar to scenario (b); see Figs. 3 and 4.

Summarizing, for the two possibilities of Universe models, one has the following: In the chaotic scenario (b), the initial value should be $\chi_0 > 130/3$ in order to achieve sufficient e-folds of inflation. And in the scenario (a), only for special initial values of the Higgs field ($\chi_0 \approx -0.15\,509$), the Universe undergoes a small contraction which goes over automatically into a sufficiently long inflation period; otherwise, for other initial negatives values of χ_0, the Universe contracts to a singularity.

At the end of inflation the Higgs field begins to oscillate with a frequency $M_H > H$ and the Universe is now dominated by the oscillations, which drive a normal Friedmann regime [34]. This can be seen as follows: First when $H \gtrsim M_H$ with $H \approx$ const, $\chi \approx e^{-3H/2\ t}\cos M_H t$ is valid; later on when $H \ll M_H$, $H \sim 1/t$ and $\chi \sim 1/t \cos M_H t$ give rise to $a \sim t^{2/3}$, i.e., a matter-dominated Universe with coherent oscillations, which will hold on if the Higgs bosons do not decay. In Figs. 5 and 6 the behavior of the scale factor and the Higgs field is shown until the time $100 M_H^{-1}$; the numerics fit very well the "dark" matter dominated solutions. Let us consider this possibility more in detail: then, the average over one oscillation of the absolute value of the effective energy density of these oscillations, $\rho_\chi \sim V(\chi) = \frac{3}{4\pi}M_{\rm Pl}^2 M_H^2 \chi^2$, is such that

$$\frac{\rho_\chi}{\rho_{\chi_*}} = \left(\frac{t_*}{t}\right)^2 \ , \tag{32}$$

where t_* is the time when the rapid oscillation regime begins. If $M_H = 10^{14}$ GeV, $t_* \approx 10^2 M_H^{-1} = 10^{-36}$ sec for which $\chi_* \approx 10^{-3}$, then it follows that $\rho_{\chi_*} = 10^{60}$ GeV^4. Thus nowadays when $t_n \sim 10^{17}$ sec, one should have

[3]This condition is equivalent to both known prerequisites $3H \gg \ddot{\phi}/\dot{\phi}$, $3\dot{\phi}/\phi$ of induced gravity.

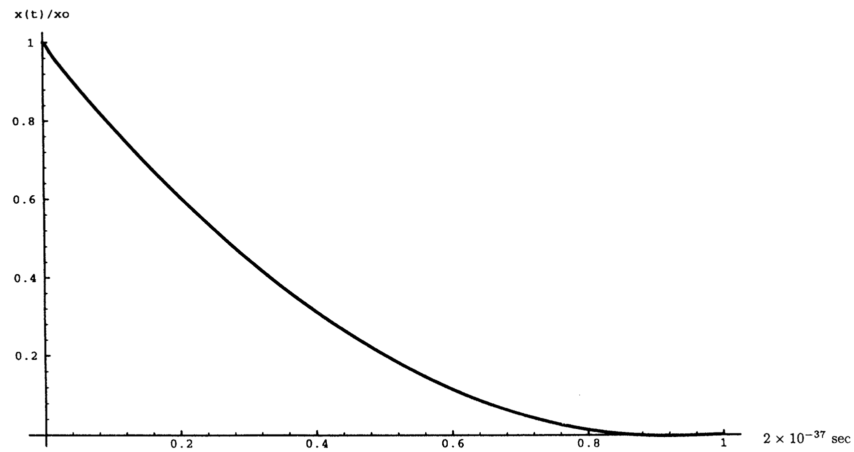

FIG. 4. The same as in Fig. 3 but here with initially $\chi_0 \gtrsim 130/3$. The exponential expansion takes place directly.

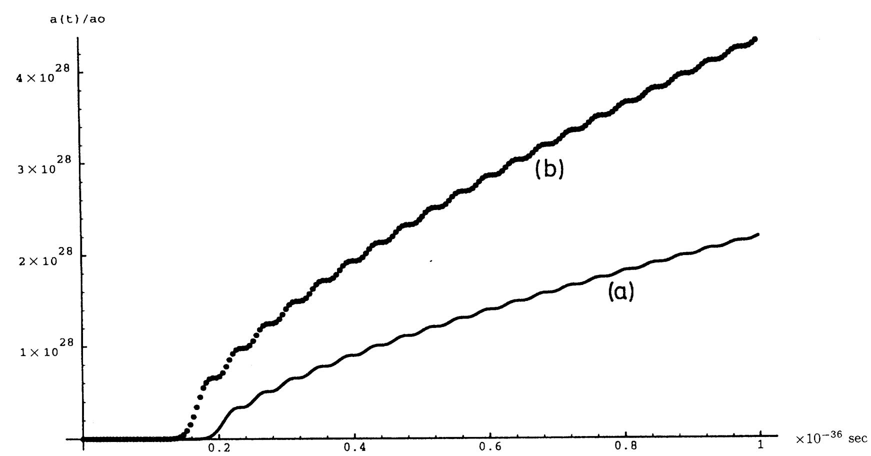

FIG. 5. Again the scale factor evolution as in Fig. 2, but now until $t = 10^2 M_H^{-1}$. One notes that the inflation time is approximately $t = 2 \times 10^{-37}$ s, later on, the Universe is "dark" matter dominated, perhaps until today, if reheating did not take away the coherent Higgs oscillations. It can be seen the track imprinted the Higgs coherent oscillations in the scale factor evolution at that time scale; later on, this influence will be imperceptible.

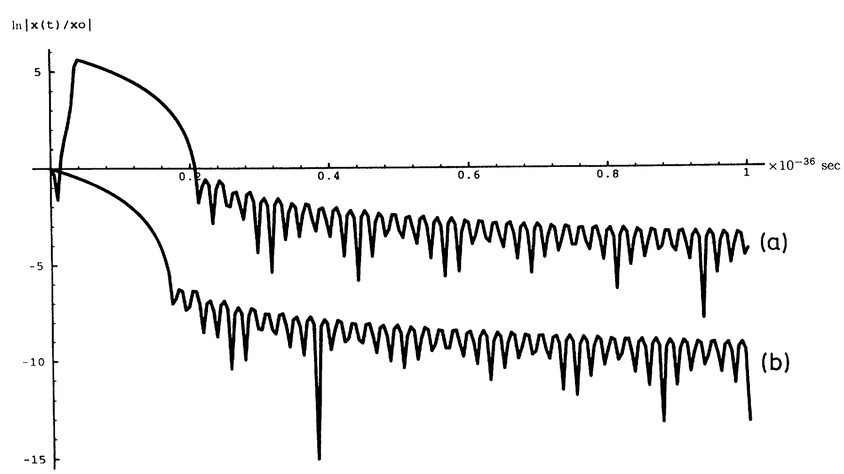

FIG. 6. The evolution of the Higgs oscillations is shown in logarithmic scale during and after inflation. In scenario (a) the Higgs field jumps from very small values to 130/3 to achieve inflation, later it begins to oscillate. In scenario (b), the Higgs field diminishes until it begins to oscillate.

$$\rho_{\chi_n} = 10^{60}\left(\frac{10^{-36}}{10^{17}}\right)^2 \text{ GeV}^4$$
$$= 10^{-46} \text{ GeV}^4 \approx 10^{-29} \text{ g/cm}^3 ; \quad (33)$$

i.e., the Higgs oscillations could solve the missing mass problem of cosmology, implying the existence of cold dark matter, since after some time as the Universe expands the Higgs particles will have a very slow momentum owing to their big mass. But in going so far we have neglected other meaningful facts in the evolution of our physical Universe, as could be the influence of such an oscillation for baryogenesis and/or for nucleosynthesis; their repercussion must still be investigated; however, they are not in scope of this paper.

There is, however, another problem: if one tries to explain the today observed baryonic mass of the Universe, then owing to inflation this mass is given by $M(t) \approx M/a^{3\nu}$ and is too small. For solving this problem one has to assume that some amount of the Higgs oscillations decay into baryons and leptons. At the time around t_* the Higgs field should decay into other particles with a decay width Γ_H to give place to a normal matter or radiation Universe expansion, producing the reheating of the Universe [35–37]. If reheating takes place, the still remaining energy of the scalar field at t_* is converted into its decay products. This would mean that the cosmological "function" disappears to give rise then to the known matter of the Universe. But if coherent oscillations still stand, they are the remnants of that cosmological "function," which is at the present, however, invisible to us in the form of cold dark matter. Now suppose that they really did decay. Mathematically, the way of stopping the oscillations or to force the decay is to introduce a term $\Gamma_H\dot{\chi}$ in Eq. (20). The Universe should then reheat up to the temperature $T_{\rm RH} \approx \sqrt{M_{\rm Pl}\Gamma_H}$, where Γ_H depends, of course, on the decay products. For example, if the coherent oscillations decay into two light fermions [8] it is valid:

$$\Gamma_H \approx g^2 M_H \approx g^2\frac{2\sqrt{2\pi}}{3}\frac{\sqrt{\lambda}}{\alpha}M_{\rm Pl} , \quad (34)$$

where we have used Eq. (17) with $(1+\frac{4\pi}{3\alpha}) \approx 1$. For the reheating this would mean that

$$T_{\rm RH} \approx \sqrt{M_{\rm Pl}\Gamma_H} = g\sqrt{\frac{2\sqrt{2\pi}}{3}\frac{\sqrt{\lambda}}{\alpha}M_{\rm Pl}} . \quad (35)$$

Now consider again Eq. (17), from which it follows that

$$\lambda = \frac{9\alpha}{4\pi}\left(\frac{M_H}{v}\right)^2 = \frac{25\pi}{2}g^4\left(\frac{M_{\rm Pl}}{M_X}\frac{M_H}{M_X}\right)^2 . \quad (36)$$

If one chooses, $\frac{M_{\rm Pl}}{M_X} = 10^4$, $\frac{M_X}{M_H} = 10^1$, one has

$$\lambda = \frac{25\pi}{2}10^6 g^4, \quad (37)$$

for which one does not have the usual λ fine-tuning, $\lambda < 10^{-12}$; see Ref. [8]. Coming back to the reheat temperature, one obtains

$$T_{\rm RH} \approx g^2 10^2\frac{M_{\rm Pl}}{\sqrt{\alpha}} = 10^{15} \text{ GeV} , \quad (38)$$

which should be enough for baryogenesis to occur. The right baryon asymmetry could be generated in this model if the masses of the Higgs triplets (as decays products) are between 10^{11} and 10^{14} GeV; see Ref. [37]. Also one should be aware of the production of gravitational radiation as a decay output of the oscillations [23,24]; however, it could also be possible that other decay channels are important, since the symmetry breaking takes here place at a much more smaller energy scale than the Planck one.

The contrast of density perturbations $\delta\rho/\rho$ can be considered in scenario (b), or in scenario (a) when χ has evolved to its positive values to have very similar slow rollover conditions as in (b). Then one has [13,38]

$$\left.\frac{\delta\rho}{\rho}\right|_{t_1} \approx \frac{1}{\sqrt{1+4\pi/3\alpha}}H\left.\frac{\delta\chi}{\dot{\chi}}\right|_{t_1} \approx \sqrt{\frac{3}{\pi\alpha}}\frac{M_H}{v}\left.\frac{\chi^2}{1+2\chi}\right|_{t_1} , \quad (39)$$

where t_1 is the time when the fluctuations of the scalar field leave H^{-1} during inflation. At that time, one finds that

$$\left.\frac{\delta\rho}{\rho}\right|_{t_1} \approx \frac{2}{\sqrt{3}}\frac{\sqrt{\lambda}}{\alpha}\left.\frac{\chi^2}{1+2\chi}\right|_{t_1}$$
$$\approx \frac{1}{2}\sqrt{\frac{3}{2\pi}}\frac{M_H}{M_{\rm Pl}}\chi\bigg|_{t_1} \approx 10\frac{M_H}{M_{\rm Pl}} = 10^{-4} , \quad (40)$$

which corresponds exactly to the ratio of the masses already chosen and fixes an upper limit to the density perturbations that give rise to the observed astronomic structures, for which $\chi(t_1)\gtrsim 30$, corresponding to more than 50 e-folds before inflation ends. The natural smallness of $\sqrt{\lambda}/\alpha$ avoids the usual fine-tuning of λ necessary to keep $\delta\rho/\rho$ sufficient small. The perturbations on the microwave background temperature are for the same reason approximately well fitted. The gravitational wave perturbations considered normally should also be small [8]:

$$h_{\rm GW} \approx \frac{H}{M_{\rm Pl}} \approx \frac{M_H}{M_{\rm Pl}}\sqrt{\frac{\chi}{2}} \approx 10^{-5} , \quad (41)$$

again to be evaluated when the scale in question crossed outside H^{-1} during inflation.

V. CONCLUSIONS

The scalar-tensor theory with Higgs mechanism applied to the SU(5) GUT can drive successful inflation without forcing the parameter λ of particle physics to be very small. This is performed by means of the natural relationship among the fundamental masses of physics at that energy scale: the Higgs-boson, X-boson, and Planck masses, on the strength of Eqs. (5), (6), and (17), achieving an interesting bridge in particle physics. Especially interesting is the Higgs boson mass, coming from a Yukawa-type equation (12), which is smaller by a factor $\sqrt{\frac{4\pi}{3\alpha}}$ than the one derived from a SU(5) GUT without gravitation. Because of the smallness of $\sqrt{\frac{4\pi}{3\alpha}}$, a contraction period could arise in the early Universe if $\chi_0 < 0$; otherwise, if $\chi_0 > 0$, one has a normal chaotic kind of inflationary scenario. On the one hand, it would seem that "chaotic" initial conditions are more appealing, because of their generic character. But, on the other hand, it could be also true that some unknown, quantum or classical, initial conditions put χ_0 on a very special value, a consequence of which is that one has at first a deflation and then automatically the desired inflation as follows from scenario (a).

After inflation, the Universe is oscillation dominated, and without its decay one could explain the missing mass problem of cosmology given today in the form of cold dark matter.

Our presentation is, however, not free of difficulties: The model, as a whole, cannot explain immediately the baryon mass of the Universe, today observed for which one is forced to look for a reheating scenario after inflation. Perhaps, this must take place, but the question whether too much gravitational radiation is generated to eventually spoil a normal nucleosynthesis procedure remains open at this energy scale. Nevertheless, it is not necessary to adjust the parameter λ to achieve a density perturbation spectrum required for galaxy formation: the fact that $\alpha = 10^7$ in itself makes $\delta\rho/\rho \approx \sqrt{\lambda}/\alpha$ reasonably small, and on the other hand, without attaching the reheat temperature too low. We also obtain sufficiently small gravity perturbations to be in accordance with the measured anisotropy of the microwave background spectrum.

We think that the "natural" relationship among the fundamental masses achieved by the theory sheds some light on the understanding of the present known problems of cosmology.

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