

QCD radiative enhancement of the decay $b \rightarrow c\bar{c}s$

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A substantial enhancement of the $b \rightarrow c\bar{c}s$ decay rate is found due to the QCD interaction within the $\bar{c}s$ pair which may amount to 30% or more. Some general features of the calculation of the QCD radiative corrections in the two first orders are discussed.

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The present theoretical understanding of B decays conspicuously runs into a problem with explaining the experimental observation of a rather low semileptonic branching ratio [1], which requires [2] an enhancement of the nonleptonic decay rate by as much as 20%–30% over the existing theoretical predictions. Therefore a more thorough theoretical study of the B decays is imperative. In this Brief Report the decay $b \rightarrow c\bar{c}s$ is discussed, whose rate is suppressed by the presence of two massive charmed quarks as compared to the dominant nonleptonic decay $b \rightarrow c\bar{u}d$. An enhancement of the decay $b \rightarrow c\bar{c}s$ would somewhat relax the problem of a low semileptonic branching ratio. However, such an enhancement would also worsen another possible problem, which is perhaps hinted at by an experiment [3], that is, the problem of a low average charm yield per B decay.¹ Nevertheless, within the present uncertainty of the measured average charm yield in B decays, the data can still accommodate a substantial enhancement of the decay $b \rightarrow c\bar{c}s$. In any case, it is important to have the potentially essential effects calculated. Here it is found that the first-order QCD correction to the total rate of the decay $b \rightarrow c\bar{c}s$ due to the interaction within the quark pair $\bar{c}s$ has a relative magnitude $\delta_{\bar{c}s}\alpha_s/\pi$ with an unusually large coefficient $\delta_{\bar{c}s} = 4.46$ for $m_c/m_b = 0.3$, unlike the case of the decay $b \rightarrow c\bar{u}d$, where a similar factor due to the $\bar{u}d$ pair is equal to 1.

QCD radiative effects in the decay rate of the b quark are usually analyzed within the leading logarithmic approximation in $\ln(m_W/m_b)$ or in the next-to-leading logarithmic approximation (see, e.g., [5,6]). However, since $\ln(m_W/m_b)$ is not really a sufficiently large parameter, the nonlogarithmic terms may be quite essential, and it might be more reasonable to rely instead on complete calculation of the QCD radiative corrections at first and second orders in α_s . The error in the logarithmic terms, induced by such a truncation of the series, is then not more than about 5% (in the total rate), which is not larger than other uncertainties in the calculation, in particular not larger than the nonlogarithmic terms.

Both perturbative and nonperturbative effects in the inclusive decay rates of B hadrons are conveniently cal-

culated by representing [7] the decay rate through the absorptive part of the B self-energy, arising at second order in the weak interaction. For first-order QCD corrections to the rate of the decay $b \rightarrow c\bar{q}_2q_1$, one thus should consider the absorptive part arising from all possible unitary cuts of the graphs of three types shown in Fig. 1. (These graphs ignore the “penguin” contribution, arising for $q_1 = q_2$, which is to be discussed separately.) In fact, however, it is clear that graphs of the type in Fig. 1(c), i.e., with gluon exchange between the bc line and the \bar{q}_2q_1 loop, are vanishing because of the color trace over the loop (a gluon cannot interact through the loop with two colorless W bosons²). The graphs of the type in Fig. 1(a) do not involve the interaction of the quarks q_1 and q_2 in the loop with gluons. Therefore the correction arising from these graphs can be adapted [8] from the old calculations [9] of QED corrections to muon decay (see also [10,11]). As to the effects of gluon exchange within the colorless \bar{q}_2q_1 loop, these were discussed thus far for both quarks being massless, in which case the correction reduces to the familiar factor α_s/π . This limit is justified for the case of the decay $b \rightarrow c\bar{u}d$, but, as will be shown, is misleading for the decay $b \rightarrow c\bar{c}s$, where the charmed quark in the loop has mass, which is not small in comparison with m_b .

Thus, at first order in α_s , the rate of the decay $b \rightarrow c\bar{c}s$ can be written as

$$\Gamma_{c\bar{c}s} = \Gamma_{c\bar{c}s}^{(\text{bare})} \left(1 + \frac{\alpha_s}{\pi} [\delta_{bc} + \delta_{\bar{c}s} + \delta_{\text{penguin}}] \right), \quad (1)$$

where $\Gamma_{c\bar{c}s}^{(\text{bare})}$ is the rate without any QCD corrections, δ_{bc} arises from gluon exchange on the bc line, $\delta_{\bar{c}s}$ is due to gluon interactions within the $\bar{c}s$ loop, and δ_{penguin} is due to effects of the penguin type. It is the goal of the present Brief Report to calculate the correction factor $\delta_{\bar{c}s}$.

It should be noted that as far as the final result for the correction $\delta_{\bar{c}s}$ is concerned, it is not new: A complete calculation of the $O(\alpha_s)$ correction to nonleptonic weak decay with arbitrary masses of final quarks has been done

²In particular, this explains why there is no first-order correction proportional to $\alpha_s \ln(m_W/m_b)$: The graphs of the types in Fig. 1(a) (gluon exchange on the bc line) and in Fig. 1(b) (gluon exchange within the loop) contain renormalization of the $V-A$ currents, which is not logarithmic.

¹Clearly, it is impossible to solve simultaneously both these problems by an enhancement of the decay $b \rightarrow c\bar{c}s$ [4].

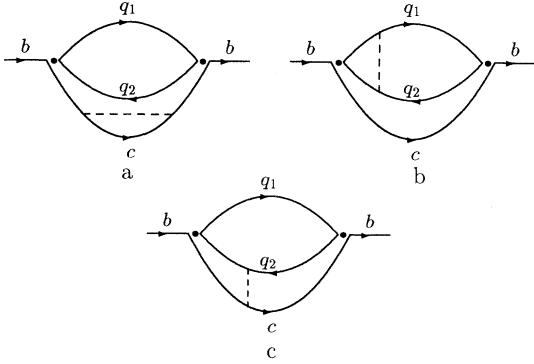


FIG. 1. Three types of graphs, whose unitary cuts describe the first QCD radiative corrections to the inclusive decay rate $b \rightarrow c\bar{q}_2q_1$. The small solid circles represent the W boson propagators and the dashed lines correspond to gluons. The gluon vertices can be anywhere on the (a) bc line, (b) quark lines in the loop, or (c) one vertex anywhere on the bc line and the other vertex on either line in the loop.

to Hokim and Pham [12], who in particular have found a strong dependence of the correction on masses of the final quarks.³ The calculation described here, to some extent, explains the origin of the large correction to the decay $b \rightarrow c\bar{c}s$ as due to enhancement of the contribution of low invariant masses of the $\bar{c}s$ pair, which also indicates a possible additional enhancement due to a larger appropriate value of α_s .

Starting with a relevant term in the weak Lagrangian of the form

$$L_{\text{int}} = 2\sqrt{2}G_F V_{cb}V_{q_2q_1} (\bar{c}_L\gamma_\mu b_L)(\bar{q}_1L\gamma_\mu q_2L) \quad (2)$$

and parametrizing the spectral density of the current $j_\mu = (\bar{q}_1L\gamma_\mu q_2L)$ as

$$\sum_n \langle 0 | j_\mu^\dagger(-q) | n \rangle \langle n | j_\mu(q) | 0 \rangle = -\frac{3}{8\pi} A(q^2) (q^2 g_{\mu\nu} - q_\mu q_\nu) + \frac{3}{8\pi} B(q^2) q_\mu q_\nu, \quad (3)$$

one can write the total decay rate of $b \rightarrow c\bar{q}_2q_1$ as

$$\Gamma_{c\bar{q}_2q_1} = 6\Gamma_0 m_b^{-8} \int_{(m_1+m_2)^2}^{(m_b-m_c)^2} \{ A(q^2) q^2 (m_b^2 + m_c^2 - q^2) + \frac{1}{2} [A(q^2) + B(q^2)] [(m_b^2 - m_c^2)^2 - q^2 (m_b^2 + m_c^2)] \} \times \sqrt{\lambda(m_b^2, q^2, m_c^2)} dq^2, \quad (4)$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the standard kinematical function, and

$$\Gamma_0 = \frac{3G_F^2 |V_{cb}V_{q_1q_2}|^2 m_b^5}{192\pi^3} \quad (5)$$

is the lowest-order parton decay rate with massless quarks in the final state.

It should be emphasized that Eq. (4) is applicable to calculation of only the effects associated with the gluon exchanges within the \bar{q}_2q_1 loop, i.e., the ones which are discussed in this Brief Report and does not include the effects of gluon exchange on the bc line [like the one shown in Fig. 1(a)] or the effects of gluon exchange between the loop and the bc line, which arise starting from order α_s^2 . In the absence of QCD radiative effects, the form factors A and B are readily calculable. In the case when $q_2 = c$ and $q_1 = s$, so that $m_1 = 0$ and $m_2 = m_c$, one finds

$$A_0 = \frac{2}{3} \left(\frac{q^2 - m_c^2}{q^2} \right)^2 \left(1 + \frac{m_c^2}{2q^2} \right),$$

$$B_0 = \frac{m_c^2}{q^2} \left(\frac{q^2 - m_c^2}{q^2} \right)^2. \quad (6)$$

Using these expressions in Eq. (4) and integrating over q^2 gives the well-known result for the “bare” rate of the decay $b \rightarrow c\bar{c}s$ in Eq. (1):

$$\Gamma_{c\bar{c}s}^{(\text{bare})} = \Gamma_0 I \left(\frac{m_c}{m_b}, \frac{m_c}{m_b} \right), \quad (7)$$

where

$$I(x, x) = \sqrt{1 - 4x^2} (1 - 14x^2 - 2x^4 - 12x^6) + 24x^4 (1 - x^4) \ln \left(\frac{1 + \sqrt{1 - 4x^2}}{1 - \sqrt{1 - 4x^2}} \right). \quad (8)$$

The QCD radiative corrections due to interactions within the $\bar{c}s$ loop are expressed through radiative corrections to the form factors A and B in the spectral density in Eq. (3). The calculation of the $O(\alpha_s)$ corrections to the spectral density with unequal masses of quarks has been done in connection with the QCD sum rules both for the longitudinal form factor B [13–15] and for the transversal one A [16]. For the case relevant here, where one of the quarks is massless, the result reads as

$$A(q^2) = A_0(q^2) \left\{ 1 + \frac{4}{3} \frac{\alpha_s}{\pi} \left[f_1 \left(\frac{q^2}{m_c^2} \right) + \frac{2q^2}{2q^2 + m_c^2} f_2 \left(\frac{q^2}{m_c^2} \right) \right] \right\},$$

$$B(q^2) = B_0(q^2) \left\{ 1 + \frac{4}{3} \frac{\alpha_s}{\pi} \left[f_1 \left(\frac{q^2}{m_c^2} \right) - 1 \right] \right\}, \quad (9)$$

with

$$f_1(z) = \frac{13}{4} + 2 \text{Li} \left(\frac{1}{z} \right) + \ln z \ln \frac{z}{z-1} - \frac{3}{2} \ln(z-1) + \ln \frac{z}{z-1} + \frac{1}{z} \ln(z-1) + \frac{1}{z-1} \ln z \quad (10)$$

and

³I am thankful to Xuan-Yem Pham for sending me a copy of Ref. [12].

$$\begin{aligned}
 f_2(z) = & -\frac{5}{2} - \frac{1}{z} - \frac{1}{z-1} \\
 & + \left(\frac{z-1}{z}\right) \left(\frac{3}{2} + \frac{1}{2z}\right) \ln(z-1) \\
 & + \frac{z}{(z-1)^2} \ln z, \quad (11)
 \end{aligned}$$

where $\text{Li}(x) = -\int_0^x \ln(1-t)dt/t$ is the standard dilogarithm function.⁴

The integral in Eq. (4) with the radiatively corrected values of the form factors can be easily done numerically, thus giving the value of the correction factor $\delta_{\bar{c}s}$ in Eq. (1). The results of such calculation are shown in Fig. 2 in terms of the behavior of $\delta_{\bar{c}s}$ as a function of m_c/m_b . In particular, at a “reference” point $m_c/m_b = 0.3$ one finds $\delta_{\bar{c}s} = 4.46$, which is significantly larger than the analogous correction factor for the $b \rightarrow c\bar{u}d$: $\delta_{\bar{u}d} = 1$. If one uses the value $\alpha_s = 0.2$, then the discussed correction is about 30%. However, a closer inspection of the integral for the correction in Eq. (4) shows that the integrand has a maximum at $q^2 \simeq 2m_c^2 \simeq 0.2m_b^2$ for the discussed here range of values of m_c/m_b . Therefore the appropriate value of α_s can in fact be larger. Naturally, a quantitative clarification of this point requires a higher-order calculation.⁵ It can be also noted that the enhancement of the contribution of relatively low values of q^2 is due to the logarithmic growth of the function $f_1(z)$ in Eq. (10) in the threshold region $z \rightarrow 1$, which is a consequence of the “hybrid” anomalous dimension [18] of the current ($\bar{c}\Gamma s$).

We therefore conclude that the decay $b \rightarrow c\bar{c}s$ is enhanced by about 30% or more by the correction proportional to $\delta_{\bar{c}s}$. To assess the resulting fraction of this decay in the total decay rate, one should also take into account the corrections with δ_{bc} and δ_{penguin} and measure the result against, say, the semileptonic mode $b \rightarrow cl\nu$ with $l = e$ or $l = \mu$, whose rate contains only the QCD correction associated with the bc line, i.e., with $\delta_{bc}^{(l)}$. The penguin effect is negative and is about 3%–5% in magnitude [5]. The term δ_{bc} is also negative, but due to the charmed quark mass its magnitude is somewhat smaller than that of the negative $\delta_{bc}^{(l)}$. In effect, the term with δ_{penguin} approximately cancels against the difference $\delta_{bc} - \delta_{bc}^{(l)}$ in the ratio $\Gamma_{c\bar{c}s}/\Gamma_{cl\nu}$ and the net $O(\alpha_s)$ correction is dominated by the large $\delta_{\bar{c}s}$. Therefore it is quite possible that a sizable part of the existing theoretical deficiency of nonleptonic decays of B can be eliminated by a 30%

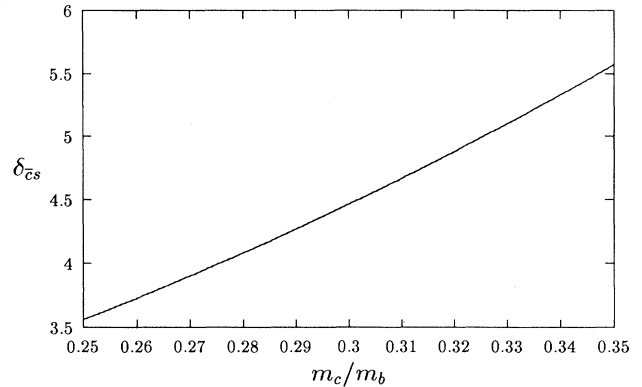


FIG. 2. Correction factor $\delta_{\bar{c}s}$ in Eq. (1), arising from gluon exchange within the $\bar{c}s$ loop, vs the mass ratio m_c/m_b . The range of m_c/m_b shown covers well the ratio of the actual quark masses with the existing uncertainty.

or larger enhancement of the decay $b \rightarrow c\bar{c}s$.

In order to completely quantify the issue of QCD radiative effects in the B decay rates and to possibly achieve an accuracy of about 5% in theoretical predictions for the rate of each inclusive mode, a complete calculation at second order in α_s is needed. Although no attempt at such a calculation is done in this Brief Report, I would like to conclude with a simple general remark concerning a calculation of the $O(\alpha_s^2)$ corrections to the inclusive decay rates of the b quark by the unitary cuts of graphs similar to those in Fig. 1, albeit at the next order of QCD perturbation theory. Also, in that order, one can split the graphs into a few classes. One class is where the gluons are attached only to quarks on the bc line. For the dominant decay $b \rightarrow c\bar{u}d$, this correction cancels in the ratio $\Gamma_{c\bar{u}d}/\Gamma_{cl\nu}$. Another is where the gluon corrections are fully contained within the \bar{q}_2q_1 loop, which are reduced to the corrections to the form factors A and B in the spectral density (3). For the pair of massless quarks $\bar{u}d$, this can be read off the corresponding calculation [19] for e^+e^- annihilation into light hadrons. Third class is where one gluon is exchanged on the bc line and the other within the \bar{q}_2q_1 loop and is thus a product of the first-order corrections. Finally, because of the color trace over the loop, the gluon exchange between the bc line and the \bar{q}_2q_1 loop gives a nonvanishing result only when there are two gluons exchanged, each starting on the bc line and ending on the \bar{q}_2q_1 loop. An example of such graph is shown in Fig. 3. It is with the latter graphs

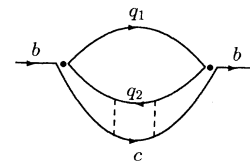


FIG. 3. Only type of graphs at second order in α_s with gluon exchange between the bc line and the \bar{q}_2q_1 loop, which gives a contribution to the inclusive decay rate. Each of the two gluons should start anywhere on the bc line and end on either of the quark lines in the loop.

⁴The correction to the longitudinal form factor B coincides with the correction [13] for a scalar or pseudoscalar density up to an additive constant, corresponding to the normalization of the (pseudo)scalar operator in order α_s . This constant is fixed [14,15] unambiguously for the longitudinal part of the vector or axial current. I am thankful to P. Ball for pointing out to me the papers [14,15], where this point is clarified.

⁵It has been recently argued [17] that the natural normalization scale for α_s in δ_{bc} is also quite low.

that the terms proportional to $[\alpha_s \ln(m_W/m_b)]^2$ and to $\alpha_s^2 \ln(m_W/m_b)$ are associated and which may also contain large nonlogarithmic terms.

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part, by DOE Grant No. DE-AC02-83ER40105. When this paper was finished, a revised version of Ref. 6 appeared, where a similar estimate of 30% enhancement of $\Gamma_{c\bar{c}s}$ was found. I thank P. Ball and V. M. Braun for pointing out to me their revised estimate. As discussed in this Brief Report, the actual enhancement can in fact be larger due to a larger value of α_s at the relatively low relevant invariant mass of the $\bar{c}s$ pair.

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