Grand unification scale effects in supersymmetric unification

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We write a Monte Carlo program allowing arbitrary splitting among GUT scale particles consistent with experimental constraints for both the minimal supersymmetric model and the missing doublet model. The resulting correlations among the low energy parameters are discussed and several advantages of the missing doublet model are pointed out.

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Since the observation [1] of the superiority of supersymmetric over nonsupersymmetric grand unification in predicting the values of the standard model couplings at the Z scale, supersymmetry (SUSY) has received a greatly increased amount of attention. For a recent review see [2]. It is now customary to require unification of b and τ Yukawa couplings at the grand-unified theory (GUT) scale in addition to gauge coupling unification. Then the observed b/τ mass ratio yields a prediction for the top quark mass,

$$145 < m_t < 218 \text{ GeV} , \tag{1}$$

which seems to be in good agreement with both indirect evidence from measurements at the Z and with preliminary direct evidence from Fermilab [3]. In the simplest unification scheme which we study here one considers the GUT scale particles degenerate at some mass M_X and SUSY partners degenerate at some mass M_S and writes a system of five coupled differential equations for the three gauge couplings, the top Yukawa coupling and the ratio of b/τ Yukawa couplings. In actuality one expects the SUSY partners to be spread above and below M_S. In the standard supergravity-inspired model for SUSY splittings with $M_{1/2} = 0$ (light gluino case), the squark and slepton squared masses separately average to a common value M_0^2 which can then be identified with M_S^2. In the general case it is also possible to define M_S in such a way that the one-loop corrections to $\alpha_3(M_Z)$ due to this splitting cancel exactly. The percentage corrections to the other (much larger) inverse couplings are then also expected to be small. If $\tan(\beta)$ is large (compared to 5 say) it is necessary to enlarge this system to six equations and consider the effects of the b and τ Yukawas on the running of the other couplings. In this work we will restrict our attention to $\tan(\beta) < 5$ which is in any case theoretically preferred from proton decay and other considerations. The system of differential equations as given, for example, in [4] then relates the following 10 parameters: (1) the unification scale M_X; (2) the unified gauge coupling $\alpha_0(M_X)$; (3) the top Yukawa at the unification scale $\alpha_t(M_X)$; (4) the SUSY scale M_S; (5) the ratio $\tan(\beta)$ of the Higgs vacuum expectation values; (6) the weak angle $\sin^2(\theta_W)$; (7) the fine-structure constant at the Z scale $\alpha(M_Z)$; (8) the strong-coupling constant at the Z scale $\alpha_3(M_Z)$; (9) the value of the top quark mass M_t; (10) the value of the b/τ mass ratio at the b quark scale. In the "top-down" approach that we follow, random values are chosen for the first five parameters, the system is evolved to lower energy and solutions are stored in a file that can be queried for correlations. A "solution" is defined as a set of values for these ten parameters that is consistent with the renormalization-group system of differential equations and with the experimental constraints

$$\alpha(M_Z)^{-1} = 127.9 \pm 0.2 , \tag{2a}$$

$$\sin^2[\theta_W(M_Z)] = 0.2324 - 0.002[M_t^2/(138 \text{ GeV})^2 - 1] \pm 0.0003 , \tag{2b}$$

$$m_b/m_\tau = 2.39 \pm 0.10 . \tag{2c}$$

In the light gluino variant, the gluino slows the running of the strong-coupling constant between M_S and m_b, and if, in addition, the standard supergravity-inspired model is chosen for soft SUSY breaking, the charginos and two of the neutralinos are at the Z scale [5]. In either the light or heavy gluino case, the solution space is a compact region in the ten-dimensional space defined above which can be projected onto any of its 45 planes to explore correlations between the various parameters. For example, in Fig. 1 we show the correlation between M_S and M_X with the values of $\alpha_3(M_Z)$ indicated by shape coding. The great current interest in SUSY is due to the fact that this compact region overlaps with current measurements of $\sin^2(\theta_W)$, $\alpha_3(M_Z)$, and m_t. In addition it contains sufficiently high values of M_X to be consistent with the current nonobservation of proton decay and sufficiently low values of M_S to be consistent with theoretical arguments relating this scale to that of electroweak symmetry breaking. It is interesting to note that if the top quark is in the region between 159 and 189 GeV as suggested by the Fermilab events, the solutions give $\tan(\beta) < 2.3$ very close to the value $\tan(\beta) = 1.88$ required [6] in the light gluino scenario with radiative breaking. However there are some problems. First of all radiative breaking in the light gluino scenario requires a much lower physical top quark mass (124 GeV) [5,6] than Eq. (1) found in the numerical solutions. This problem also exists in the no-scale version of the heavy gluino case [7] and to a lesser extent in the full minimal SUSY [minimal su-

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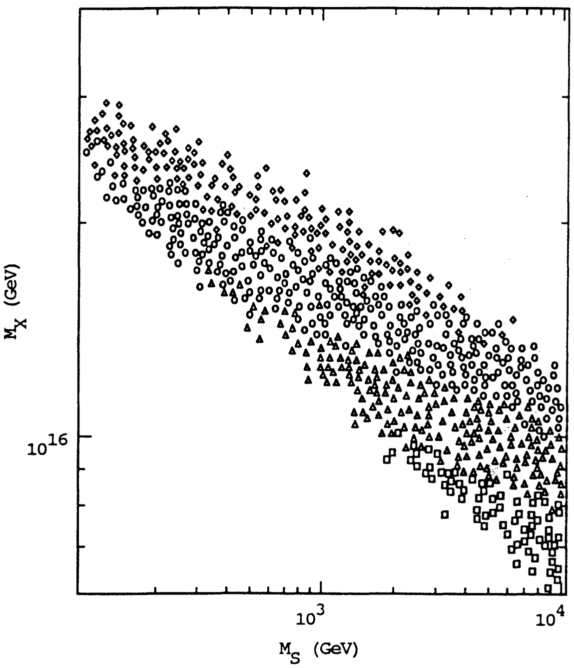

FIG. 1. Correlation between M_S and M_X in the solution space of the minimal supersymmetric model. Solutions are printed as squares, triangles, circles, and diamonds if the value of $\alpha_3(M_Z)$ is in the first, second, third, and fourth quadrants, respectively, of the full allowed range $0.11 < \alpha_3(M) < 0.136$.

persymmetric standard model (MSSM)] model imposing proton decay constraints [8,9] where the solution space rapidly shrinks to zero if the top quark moves above 150 GeV.

Another potential problem in the MSSM may be emerging with regard to the prediction for $\alpha_3(M_Z)$. In the supergravity (SUGRA) inspired MSSM with light gluinos, the solution space is restricted [5] to the narrow region

$$0.122 < \alpha_3(M_Z) < 0.132 \ . \tag{3}$$

(Lower values can be obtained if one in effect gives up the idea of a universal gaugino mass and puts the charginos and neutralinos other than the photino at M_S [10].) This $\pm 4\%$ prediction is consistent with measurements at the Z [11,12] which however have large errors. It may also be consistent with deep-inelastic results analyzed in the light gluino case [13]. However, the vast body of quarkonium and other low-energy measurements analyzed in the light gluino case [14,15] seem to require

$$\text{light gluino}: \quad \alpha_3(M_Z) \approx 0.115 \pm 0.003 \ . \tag{4a}$$

There is a similar problem in the heavy gluino scenario. In Fig. 1, solutions are found in the range $0.11 < \alpha_3(M_Z) < 0.136$. This is consistent with the values at the Z [11,12] but is marginally inconsistent with the values found from the more accurate low-energy data [15] in the heavy gluino case: namely,

$$\text{heavy gluino}: \quad \alpha_3(M_Z) = 0.097 \pm 0.003 \ . \tag{4b}$$

Other analyses [13,16] of low-energy data arrive at values about 10% higher than this but, in any case, there is widespread agreement that the low-energy data is inconsistent with a value of $\alpha_3(M_Z)$ above 0.117 [15]. On the other hand, if one analyzes the solution space more closely one finds that no solutions exist with $\alpha_3(M_Z) < 0.117$ unless $M_X < 10^{16}$ and $M_S > 1$ TeV. Both of the latter are undesirable. One expects M_S to be of order of the electroweak scale $\approx$200 GeV because of the hierarchy problem. In addition the lightest supersymmetric particle should have a mass no greater than this for cosmological reasons. Finally, the radiative breaking constraint runs into an extreme fine-tuning problem if M_S is much larger than this value. Similarly $M_X < 10^{16}$ runs into problems with proton decay. Although the leptoquark bosons could be as low as 10^{15} GeV without causing too rapid proton decay, the triplet Higgs bosons must be above 10^{16} GeV. Since M_X, the energy above which the theory becomes SU(5) symmetric, corresponds to the maximum mass of the GUT scale particles, the consistency of the theory is in danger unless $M_X > 10^{16}$.

The purpose of this paper is to investigate whether these emerging problems might be alleviated by GUT scale effects, i.e., by mass splitting among the GUT scale particles. We consider both the MSSM and the missing doublet model (MDM) [17,18] which differ in the GUT scale Higgs content of the theory. We neglect the effect of GUT scale splitting on the running of the Yukawa couplings due to the quasi-fixed-point behavior of the top Yukawa coupling. This causes the top Yukawa coupling to be very insensitive to perturbations at the GUT scale. GUT scale effects have been investigated recently prior to our work [19] and those results can be compared with ours below. The effect on the Yukawa couplings has been treated by Wright [19] in the MSSM and found to be very small in the low $\tan(\beta)$ region that we investigate in this paper. Some non-negligible effect is observed in the b/τ ratio but that does not seem to feed back significantly into the other parameters discussed here. Langacker and Polonsky [19] have treated the GUT scale effects on the gauge couplings in the MSSM with inclusion of SUSY splitting effects. Polansky and Pomarol [19] have studied the effect of GUT scale splitting on the SUSY-breaking parameters. Although significant effects are seen on the masses of individual SUSY particles, their results do not seem inconsistent with those presented here. Some of our results for the heavy gluino case of the MDM can be seen in broad outline in the quasianalytic work of Hagiwara and Yamada and Yamada [19]. However we find significant departures from, for example, their quoted prediction for the $\alpha_3(M_Z)$ range due to their neglect of the effect of the Yukawa couplings on the running of the gauge couplings. In the MSSM the Higgs particles fall into $\mathbf{24}$, $\mathbf{5}$, and $\overline{\mathbf{5}}$ representations while in the MDM they occur in $\mathbf{75}$, $\mathbf{50}$, $\overline{\mathbf{50}}$, $\mathbf{5}$, and $\overline{\mathbf{5}}$. The MDM provides a natural explanation for the color Higgs triplet obtaining a GUT scale mass while the electroweak Higgs doublet has a mass at the electroweak scale. The running of the gauge couplings in the high-energy region is, at the one-loop level, determined by the equations

$$\frac{2\pi}{\alpha_i^2(Q)} \frac{d\alpha}{dt} i = b_i + b_i^V + b_i^H \ , \qquad (5)$$

where $t = \ln(Q)$, b_i represents the contribution of particles at the SUSY scale and below, b_i^V represents the contribution of the GUT scale leptoquark supermultiplet at mass M, and b_i^H represents the contribution of the GUT scale Higgs supermultiplet. The low-energy and leptoquark contributions are

$$b_i = (\tfrac{33}{5}, 1, -3) \ , \qquad (6)$$

$$b_i^V = (-10, -6, -4) \ , \qquad (7)$$

for $i = 1, 2, 3$ in that order. The high-energy contributions from Higgs supermultiplets are tabulated in Tables I and II adopted from [19] for the MSSM and MDM cases, respectively. Since the Higgs singlet in Table I decouples from the running of the gauge couplings, in the MSSM one can entertain the notion that the contributing GUT scale particles are (at least approximately) degenerate. In the MDM, however, as can be seen from Table II there are unavoidable mass splittings which have important consequences for the solution space. In both the MSSM and the MDM, proton decay will be too rapid if the color Higgs triplet with hypercharge $\pm\frac{1}{3}$ is below 10^{16} GeV. We therefore require that M_D and, in the MDM, M_Σ and M_Φ are above this value while allowing unconstrained values of the other masses. Since the D_1 and D_2 in the MDM have relatively minor effect on the gauge coupling running, we equate their masses for simplicity. The solution space of the MSSM is enlarged to 12 dimensions with the replacement of the single M_X by the three parameters M_V, M_D, and M_Σ. In the MDM the solution space becomes 13-dimensional with M_X replaced by M_V, M_D, and M_Σ, and M_Φ. We equate the (dimensional reduction) gauge couplings and the b and τ Yukawa couplings at M_X defined to be the maximum mass of the GUT scale particles. For each choice of M_S, $\tan(\beta)$, and the GUT scale parameters we integrate to low-energy decoupling each particle at energy scales below its mass. The contributions of the GUT scale particles are treated only in one-loop approximation. We also require

$$100 \text{ GeV} < M_S < 1 \text{ TeV} \ , \qquad (8)$$

TABLE I. GUT scale Higgs particles in the MSSM and their contributions to the running of the gauge couplings.

j	Rep.	Mass	$b_1(j)$	$b_2(j)$	$b_3(j)$
$H_{(8,1)}, \overline{H}_{(8,1)}$	(8,1,0)	M_Σ	0	0	3
$H_{(1,3)}, \overline{H}_{(1,3)}$	(1,3,0)	$M_\Sigma/5$	0	2	0
$H_{(1,1)}, \overline{H}_{(1,1)}$	(1,1,0)	M_Σ	0	0	0
$D, \overline{D}$	(3,1,$\pm\frac{1}{3}$)	M_D	$\frac{2}{5}$	0	1
Sum			$\frac{2}{5}$	2	4

TABLE II. GUT scale Higgs particles in the MDM and their contributions to the running of the gauge couplings.

j	Rep.	Mass	$b_1(j)$	$b_2(j)$	$b_3(j)$
$H_{(8,3)}, \overline{H}_{(8,3)}$	(8,3,0)	M_Σ	0	16	9
$H_{(3,1)}, \overline{H}_{(3,1)}$	(3,1,$\pm\frac{5}{3}$)	$0.8M_\Sigma$	10	0	1
$H_{(6,2)}, \overline{H}_{(6,2)}$	(6,2,$\pm\frac{5}{6}$)	$0.4M_\Sigma$	10	6	10
$H_{(1,1)}, \overline{H}_{(1,1)}$	(1,1,0)	$0.4M_\Sigma$	0	0	0
$H_{(8,1)}, \overline{H}_{(8,1)}$	(8,1,0)	$0.2M_\Sigma$	0	0	3
$D_1, \overline{D}_1$	(3,1,$\pm\frac{1}{3}$)	M_{D_1}	$\frac{2}{5}$	0	1
$D_2, \overline{D}_2$	(3,1,$\pm\frac{1}{3}$)	M_{D_2}	$\frac{2}{5}$	0	1
$H_{50}, \overline{H}_{50}$		M_Φ	$\frac{173}{5}$	35	34
Sum			$\frac{277}{5}$	$\frac{285}{5}$	$\frac{295}{5}$

in accord with the theoretical prejudices discussed above and $M_t > 131$ GeV in the heavy gluino case as required by the Fermilab experiments. In the light gluino case nonstandard decay modes could allow a lighter top quark. The solution space is characterized by minimum and maximum values of various parameters which are tabulated for the various models in Table III.

In the MSSM we find that the inclusion of GUT scale effects does not appreciably lower the range of $\alpha_3(M_Z)$ which lies between 0.117 and 0.133. For the heavy gluino case with arbitrary GUT scale splitting subject to the proton decay constraint, we show in Fig. 2(a) the correlation between M_S and $\alpha_3(M_Z)$ with values of M_t indicated in shape coding. The solutions are plotted as squares, triangles, circles, or diamonds if the top quark is in the first, second, third, or fourth quadrant, respectively, of the full range from 144 to 207 GeV. In Fig. 2(b), the same correlation is shown for the light gluino case ($M_{\tilde{G}}$ <5 GeV). Our result for the $\alpha_3(M_Z)$ range in the heavy gluino case of the MSSM is in good agreement with the findings of [19] namely $\alpha_3(M_Z) > 0.115$ (Langacker and Polonsky) and $\alpha_3(M_Z) > 0.118$ (Wright). Figure 3 shows, in the light gluino case, the correlation between M_V and M_D with values of M_Σ indicated in shape coding. The solution is plotted in Fig. 3 as a square, triangle, circle, or diamond if the M_Σ value falls into the first, second, third, or fourth quadrant, respectively, of the full range given in Table III. In Fig. 4, we show the correlation between $\tan(\beta)$ and M_t in the light gluino case. The quadrant values of the top Yukawa coupling at M_X are shown in shape coding. The solution is plotted as a square, triangle, circle, or diamond, respectively, if the $\alpha_t(M_X)$ value falls in the first, second, third, or fourth quadrant of the full solution space range which is 0.198< $\alpha_t(M_X) < 1.0$ in the light gluino case (see Table III). We do not show the corresponding correlations in the heavy gluino case of the MSSM since these have been discussed recently by other authors [19] and we find general agreement with their results in spite of slightly differing approximations made. For example, the minimum value of M_V given in Table III agrees exactly with that of Wright [19].

In the MDM the situation is quite different. The intrinsic mass splittings at the GUT scale have the effect of

TABLE III. Minimum and maximum values for the (at least partially) unconstrained parameters in the solution spaces of the missing doublet model and the MSSM with and without GUT scale splittings. Imposed lower or upper limits are underlined. All masses in the table are in GeV. The SUSY scale is restricted to 100 GeV< M_S < 1 TeV.

		MSSM (degenerate)	MSSM	MDM
α_0^{-1}	$M_{\tilde{G}} = M_S$	23.6,25.4	22.7,27.1	3.8,25.9
	$M_{\tilde{G}} < M_b$	23.4,24.5	22.6,26.3	4.1,25.0
M_V	$M_{\tilde{G}} = M_S$	$(1.2,3.0)10^{16}$	$(0.7,8.2)10^{16}$	$(2.3,21)10^{15}$
	$M_{\tilde{G}} < M_b$	$(2.3,4.0)10^{16}$	$(1.2,8.7)10^{16}$	$(3.6,19)10^{15}$
M_D	$M_{\tilde{G}} = M_S$		$(\underline{1.0}, 24)10^{16}$	$(\underline{1.0}, 18)10^{16}$
	$M_{\tilde{G}} < M_b$		$(\underline{1.0}, 33)10^{16}$	$(\underline{1.0}, 16)10^{16}$
M_Σ	$M_{\tilde{G}} = M_S$		$(0.32,15)10^{16}$	$(\underline{1.2}, 6.7)10^{16}$
	$M_{\tilde{G}} < M_b$		$(0.62,19)10^{16}$	$(\underline{1.2}, 6.4)10^{16}$
M_Φ	$M_{\tilde{G}} = M_S$			$(\underline{1.0}, 7.8)10^{16}$
	$M_{\tilde{G}} < M_b$			$(\underline{1.0}, 6.7)10^{16}$
$\alpha_t(M_X)$	$M_{\tilde{G}} = M_S$	0.214,$\underline{1.00}$	0.195,$\underline{1.00}$	0.013,0.387
	$M_{\tilde{G}} < M_b$	0.299,$\underline{1.00}$	0.198,$\underline{1.00}$	0.010,0.354
$\alpha_3(M_Z)$	$M_{\tilde{G}} = M_S$	0.117,0.127	0.117,0.133	0.095,0.114
	$M_{\tilde{G}} < M_b$	0.122,0.132	0.119,0.137	0.095,0.113
M_t	$M_{\tilde{G}} = M_S$	145,207	144,207	$\underline{131}$,193
	$M_{\tilde{G}} < M_b$	146,203	146,197	108,184
$\sin^2\theta_W$	$M_{\tilde{G}} = M_S$	0.2291,0.2325	0.2296,0.2324	0.2302,0.2329
	$M_{\tilde{G}} < M_b$	0.2304,0.2324	0.2301,0.2324	0.2306,0.2332

significantly reducing the solution values of $\alpha_3(M_Z)$. Figure 5(a) shows the correlation between M_S and $\alpha_3(M_Z)$ with M_t indicated by shape coding. Figure 5(b) shows th same correlation except here the gluino is put below the b mass. The solution spaces overlap well with Eqs. (4a) and (4b). Figure 6(a) and 6(b) show the correlation between M_V and M_Φ in the heavy and light gluino cases, respectively. The solutions are plotted as squares, triangles, circles, and diamonds if the M_D value lies in the first, second, third, or fourth quadrant of its full range given in Table III. One sees many solutions with high values of M_D and low values of M_V. Such solutions would have the interesting consequence that proton decay would procede through the leptoquark gauge bosons leading to the "standard model" proton decay mode $e^+\pi^0$ unlike the typical supersymmetry signature of a νK^+ dominant decay. Figures 7(a) and 7(b) show the correlation between $\tan(\beta)$ and M_t in the heavy and light gluino scenarios, respectively. The quadrant value of the GUT scale Yukawa coupling $\alpha_t(M_X)$ is indicated in shape coding with squares, triangles, circles, and diamonds being used if $\alpha_t(M_X)$ lies in the first, second, third or fourth quadrant of its full range given in Table III. The data show solutions for very low values of M_t unlike the situation in the MSSM. This is only of academic interest if the events observed at Fermilab are indeed due to top quark production. Even if the Fermilab events are not due to top quark production it is now experimentally ruled out that a top quark with a standard model decay chain could lie below 131 GeV. However, in the light gluino scenario, if M_S is also relatively low a light top quark could have evaded the Fermilab search since it would have the nonstandard dominant decay chain

$$t \to \tilde{t}\tilde{G}$$
$$\qquad \hookrightarrow \tilde{W} b$$
$$\qquad\qquad \hookrightarrow q\bar{q}\tilde{G}\ .$$

Such a decay chain would not lead to energetic leptons and hence would not trigger the Fermilab top quark detector. Thus in the light gluino case, it is possible that the top quark lies significantly lower than the 174 GeV suggested by the Fermilab events which in that case would have to be attributed to standard model backgrounds or, perhaps, to other SUSY sources. Such a light top quark might be in agreement with that required by radiative breaking in the light gluino case [6,5]. It might also resolve the anomalously large top quark production cross section implied if the observed events are due to a 174-GeV top quark. A light top quark is not, however, a prediction of the light gluino case however since solutions with higher top mass are also seen in Fig. 7(b).

Another new feature of the missing doublet model is that low values of the top Yukawa coupling are allowed unlike the case in the MSSM where most of the solutions involve top Yukawa couplings at M_X close to the "perturbative limit," $\alpha_t = 1$.

In Table III we summarize the solution space of the three models under consideration. We require $M_S < 1$ TeV and plot the minimum and maximum values of each of the parameters which are at least partially unconstrained by experimental or theoretical bounds. The results in the light and heavy gluino cases are shown separately. The predictions for the top mass and $\sin^2\theta_W$ have comparable ranges to the experimental indications and are in good agreement with them. This is especially impressive in the case of $\sin^2\theta_W$ where the ranges are at the 1% level. One should note, however, that there are in some cases strong correlations between the various pa-

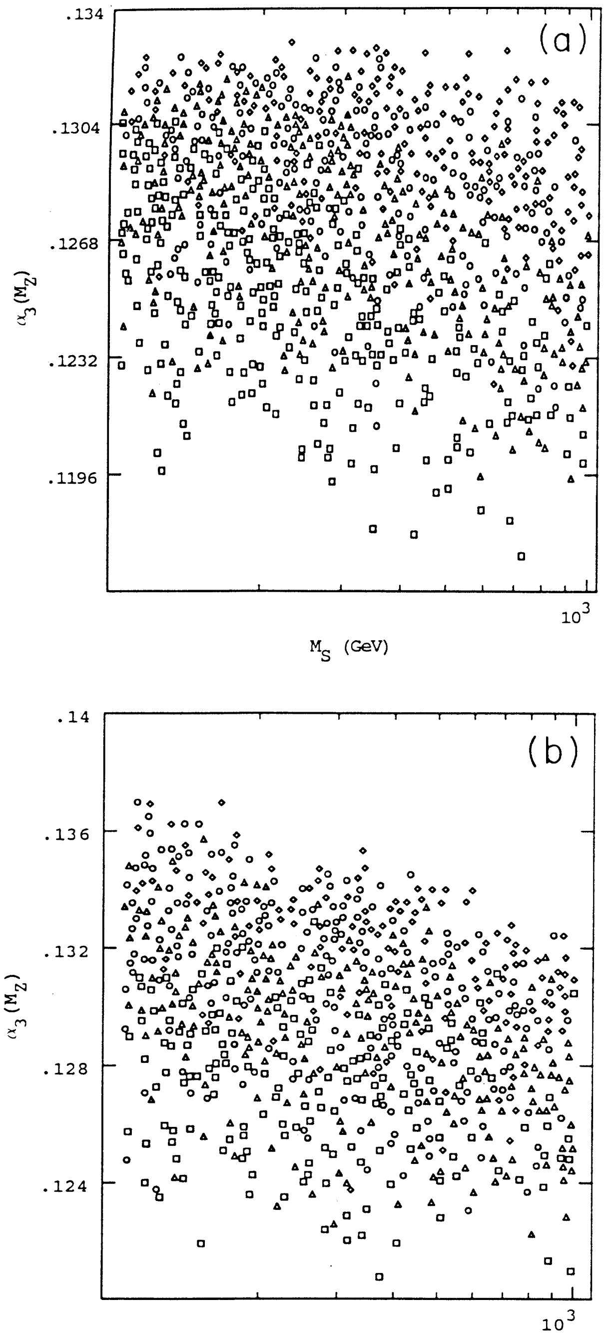

FIG. 2. (a) Correlation between M_S and $\alpha_3(M_Z)$ in the heavy gluino case of the MSSM with GUT scale corrections. The shape coding indicates the quadrant values of the top quark mass (see text). (b) As in (a) except here for the light gluino case.

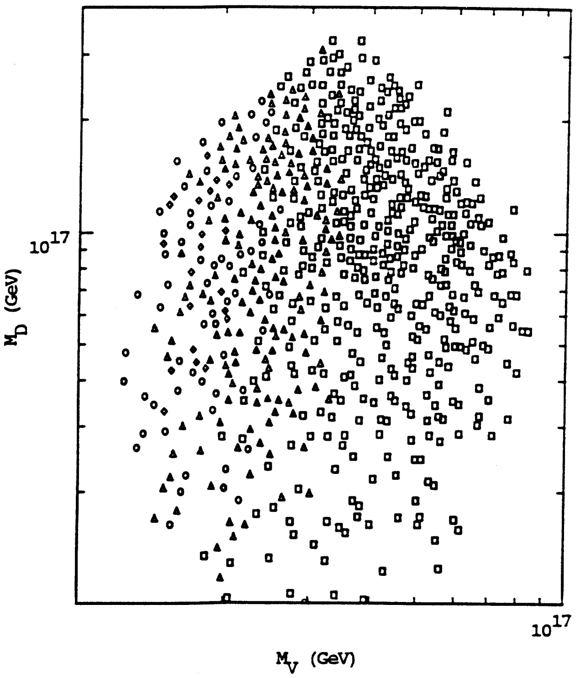

FIG. 3. Correlation between M_V and M_D in the light gluino case of the MSSM with GUT scale corrections. The shape coding indicates the quadrant values of M_Σ (see text).

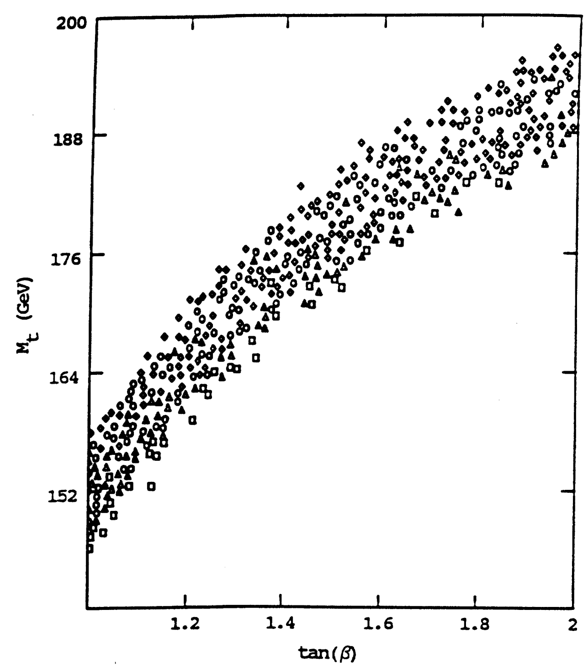

FIG. 4. Correlation between $\tan(\beta)$ and M_t in the light gluino case of the MSSM with GUT scale corrections. The quadrant value of the GUT scale Yukawa coupling $\alpha_t(M_X)$ is indicated by the shape coding. (See text.)

rameters that cannot be seen in Table III. Some of these are displayed in the figures.

Our results may be summarized as follows.

(1) The minimal supersymmetric model with a SUSY threshold below 1 TeV and a GUT scale spectrum consistent with proton decay is inconsistent with low-energy measurements of the strong-coupling constant that suggest a value $\alpha_3(M_Z) < 0.117$. Allowing splitting of the GUT scale degeneracy does not improve agreement.

(2) The missing doublet model predicts, to within

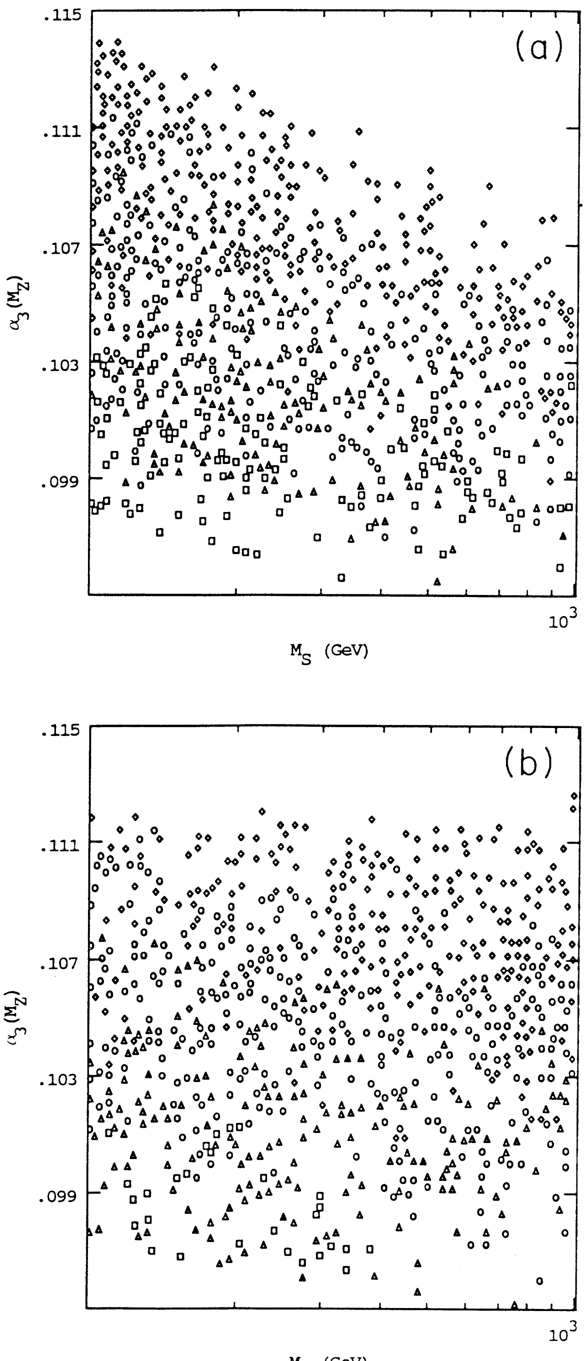

FIG. 5. (a) Correlation between M_S and $\alpha_3(M_Z)$ in the heavy gluino case of the MDM with GUT scale corrections. The shape coding indicates the quadrant values of the top quark mass (see text). (b) As in (a) except here for the light gluino case.

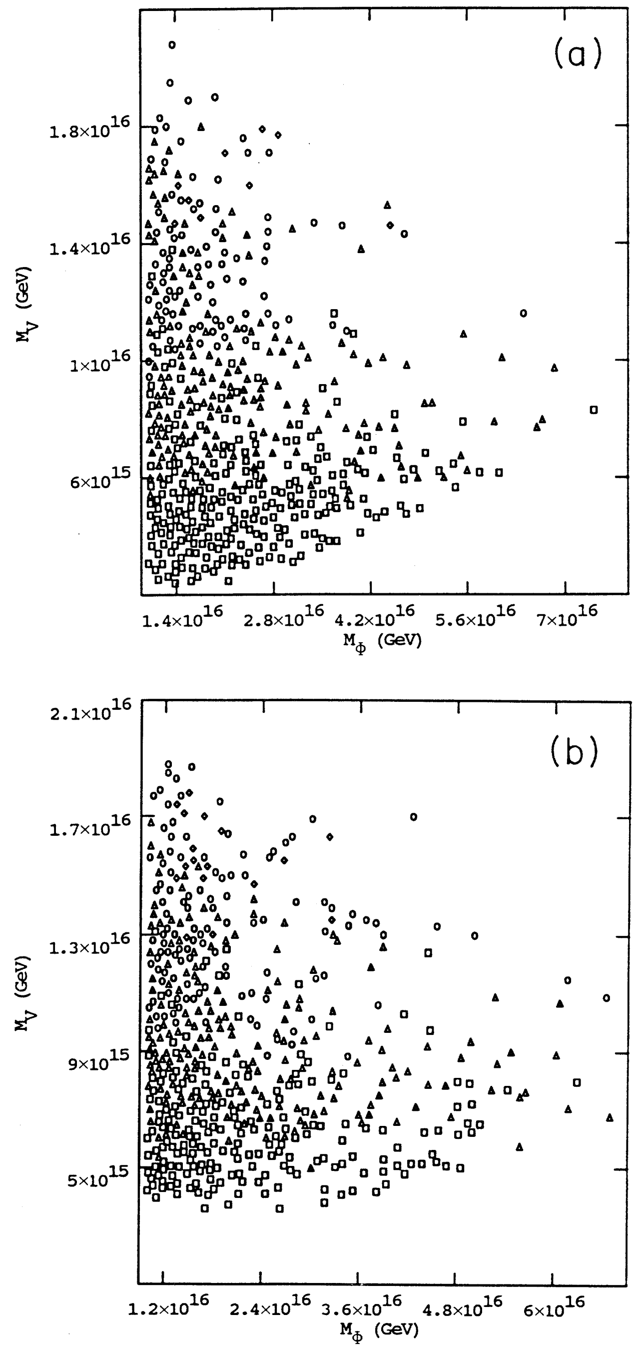

FIG. 6. (a) Correlation between M_V and M_Φ in the heavy gluino case of the MDM with GUT scale correlations. The shape coding indicates the quadrant values of M_D (see text). (b) As in (a) except here for the light gluino case.

$\pm 9\%$, a low value of the strong-coupling constant in agreement with low-energy data. The unification predictions might however be expected to rise somewhat if one allows smooth thresholds [20].

(3) In the MDM the GUT scale, defined as the mass above which the theory is SU(5) symmetric, reaches above 10^{17} GeV, improving agreement with string theory expectations. In many solutions the leptoquark gauge multiplet lies an order of magnitude below the Higgs multiplet allowing standard-model-like $e\pi$ proton decay modes to dominant over the νK modes usually expected in supersymmetry.

(4) In the MDM the top quark mass predictions extend to lower masses. In the light gluino scenario such a light top could evade the Fermilab bounds due to nonstandard decay modes and could provide agreement with radiative breaking ideas. Higher top mass values are, however, also found among the unification solutions with both light and heavy gluinos.

(5) In the MDM the top Yukawa coupling remains comfortably in the perturbative region from low energies to the GUT scale as can be seen from the comparisons in Table III.

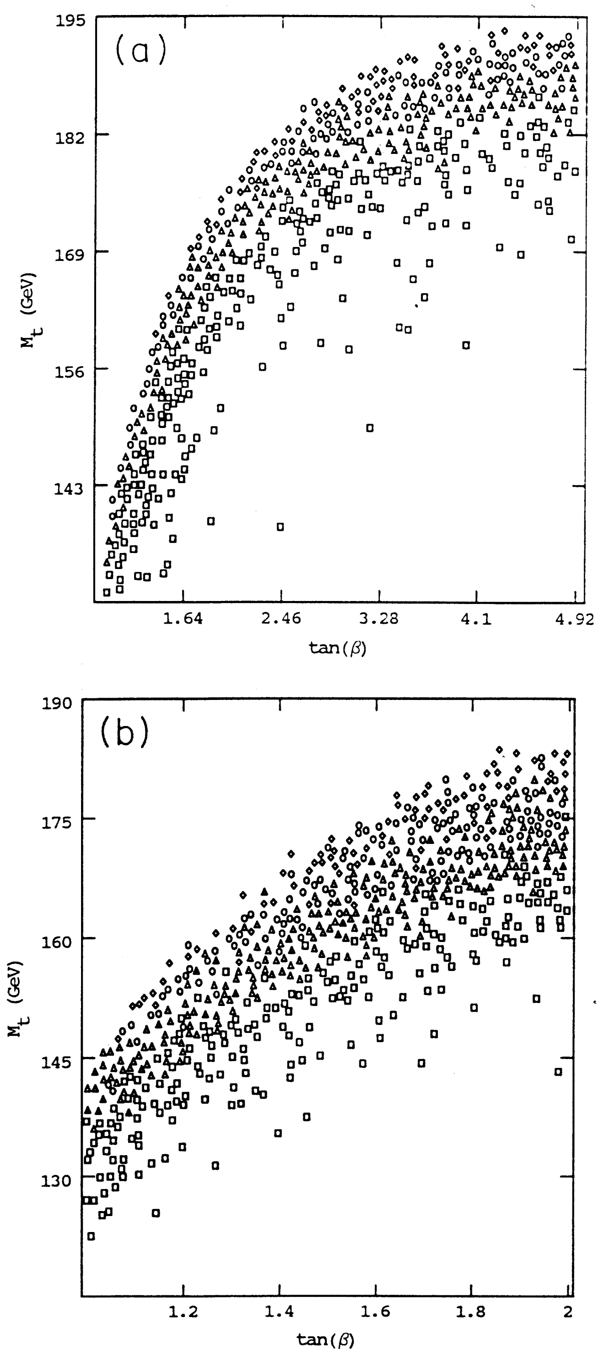

FIG. 7. (a) Correlation between $\tan(\beta)$ and M_t in the heavy gluino case of the MDM with GUT scale corrections. The quadrant value of the GUT scale Yukawa coupling $\alpha_t(M_X)$ is indicated by the shape coding. (See text.) (b) As in (a) except for the light gluino case.

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APPENDIX A

In this appendix we give some details of our numerical techniques used to efficiently demarcate the unification solution spaces. We wish to solve a set of five coupled differential equations for the three gauge couplings $\alpha_i(Q)$, the top Yukawa coupling $\alpha_t(Q)$, and the ratio $r \equiv \sqrt{(\alpha_b/\alpha_\tau)}$. These equations are of the form

$$4\pi \frac{d}{dt}\alpha_i^{-1} = -2[b_i + b_i^V + b_i^H + b_{ij}\alpha_j/(4\pi) \\ -a_{it}\alpha_t/(4\pi)] \ , \tag{A1}$$

$$4\pi \frac{d}{dt}\ln(\alpha_t) = 2(d_{tt}\alpha_t - c_{ti}\alpha_i) \ , \tag{A2}$$

$$4\pi \frac{d}{dt}\ln(r) = (d_{bt} - d_{\tau t})\alpha_t - (c_{bi} - c_{\tau i})\alpha_i \ . \tag{A3}$$

Here $t \equiv \ln(Q)$ and the normalization is such that the coefficients coincide with those listed in [4]. In the numerical analysis, we add to these the effects of the b and τ Yukawa couplings (treated as nonrunning) and of the two-loop contributions to the Yukawa couplings. These however give negligible contributions in the case of small $\tan(\beta)$ to which we restrict our attention. The boundary conditions are defined as

$$\alpha_0^{-1}(M_X) = \alpha_1^{-1}(M_X) = \alpha_2^{-1}(M_X) - 1/(6\pi)$$
$$= \alpha_3^{-1}(M_X) - 1/(4\pi) \ ,$$

$$r(M_X) = 1 \ .$$

In addition to the GUT scale b_i^X and b_i^H tabulated in Tables I and II, the b_i are given in terms of the individual contributions by

$$b_1 = [2.2 + N_\tau/2 + 17(N_c + N_t)/30 + N_b/6](1 + N_S/2) + (N_h + 4N_{\tilde{h}} + N_S)/10 \ , \tag{A4}$$

$$b_2 = [\tfrac{7}{3} + N_\tau/6 + (N_c + N_b + N_t)/2](1 + N_S/2) - 22N_W/3 + 4N_{\tilde{W}}/3 + (N_h + 4N_{\tilde{h}} + N_S)/6 \ , \tag{A5}$$

$$b_3 = -11 + 2N_{\tilde{G}} + 2N_f(1 + N_S/2)/3 \ , \tag{A6}$$

with a quark number

$$N_f = 3 + N_c + N_b + N_t \ . \tag{A7}$$

The contributions from the GUT scale gauge supermultiplet are

$$b_i^V = (-10N_V, -6N_V, -4N_V) \ , \quad i = 1,2,3 \ .$$

The contributions from the GUT scale Higgs supermultiplets can be read from Table I or II in the MSSM or MDM, respectively. The N's are zero far below the corresponding particle mass and unity far above. In this paper we work in the θ function approximation where the N_i's are taken to be $\theta(Q - M_i)$. For example, $N_{\tilde{G}}$ is taken to be a θ function at the gluino mass and N_S is taken to be a θ function at the SUSY threshold treated as degenerate apart from the particles explicitly separated out in Eqs. (A4)–(A6). It is clear from these equations how to separate N_S into the contributions from nondegenerate squarks and sleptons if desired. The effects of the light quarks and leptons are included as constants. The θ-function approximation could be made exact if the mass dependence of the threshold effects were correctly included elsewhere in extracting the coupling constants from experiment. Since this cannot be done without prior knowledge of the SUSY spectrum, it would at present be more accurate to use a smooth function for the N_i as has been done elsewhere [14,20] (but not with total consistency). We leave this, however, for future study. For definiteness we put the light Higgs boson mass M_h at 60 GeV. In the heavy gluino case we take a degenerate SUSY spectrum ($N_{\tilde{G}} = N_{\tilde{W}} = N_{\tilde{h}} = N_S$). In the light gluino case, we take $M_{\tilde{G}} < M_b$, $M_{\tilde{W}} = M_{\tilde{h}} = 49$ GeV. The heavy Higgs boson and the charged Higgs boson are put at M_S. This approximate spectrum is suggested in the minimal SUGRA model with a universal gaugino mass put to zero at the GUT scale. Reasonable splittings among the squarks and sleptons will not significantly affect our conclusions.

The two-loop contributions to the gauge running are defined by the coefficients

$$B_{11} = 19N_f(1 + N_S)/30 + 9(N_h + N_{\tilde{h}}/2 + N_S/2)/50 \ , \tag{A8}$$

$$B_{12} = 3N_f(1 + N_S)/10 + 9(N_h + N_{\tilde{h}}/2 + N_S/2)/10 \ , \tag{A9}$$

$$B_{13} = 44N_f(1 + N_S)/30 \ , \tag{A10}$$

$$B_{21} = N_f(1 + N_S)/10 + 3(N_h + N_{\tilde{h}}/2 + N_S/2)/10 \ , \tag{A11}$$

$$B_{22} = -136N_W/3 + 64N_{\tilde{W}}/3 + 49N_f(1 - N_S/7)/6 + 13N_h/6 + 29(N_{\tilde{h}} + N_S)/12 \ , \tag{A12}$$

$$B_{23} = 2N_f(1 + N_S) \ , \tag{A13}$$

$$B_{31} = 11N_f(1 + N_S)/60 \ , \tag{A14}$$

$$B_{32} = 3N_f(1 + N_S)/4 \ , \tag{A15}$$

$$B_{33} = -102 + 38N_f(1 - 2N_S/19)/3 + 48N_{\tilde{G}} \ . \tag{A16}$$

The top Yukawa contributions to the gauge running are defined by the coefficients

$$a_{1t} = 17N_t(1 + 35N_S/17)/10 \ , \tag{A17}$$

$$a_{2t} = 3N_t(1 + 3N_S)/2 \ , \tag{A18}$$

$$a_{3t} = 2N_t(1 + N_S) \ . \tag{A19}$$

The running of the Yukawa couplings is defined by the coefficients

$$d_{Jt} = 9N_t(1 + N_S/3)/2, \ 3N_t(1 - N_S/3)/2, \ 3N_t(1 - N_S) \ , \quad J = t, b, \tau \ , \tag{A20}$$

$$c_{ti} = 17N_t(1 + N_S/51)/20, \ 9N_t(1 + N_S/3)/4, \ 8N_t(1 - N_S/3) \ , \quad i = 1,2,3 \ , \tag{A21}$$

$$c_{bi} = N_b(1 + 13N_S/15)/4, \ 9N_b(1 + N_S/3)/4, \ 8N_b(1 - N_s/3) \ , \quad i = 1,2,3 \ , \tag{A22}$$

$$c_{\tau i} = 9N_\tau(1 - N_S/5)/4, \ 9N_\tau(1 + N_S/3)/4, \ 0 \ , \quad i = 1,2,3 \ . \tag{A23}$$

The first step is to choose random values for M_S and the GUT scale parameters between some minimum and maximum values. The minimum and maximum values are chosen to satisfy Eq. (8) and the perturbativity requirement for the top Yukawa coupling, $\alpha_t(M_X) < 1$, as well as the proton decay constraints discussed in the text but are otherwise unconstrained. The parameters are controlled by a set of random numbers ranging from zero to one. We ensure that no solutions are missed outside of the resulting envelope by monitoring the distributions of random numbers and requiring that no solutions are found in the outer 5%; otherwise the boundaries are moved out until this is true. This procedure is based on the assumption that the solution space is simply connected.

From these input values we must integrate down to low energies and record the "solutions" that are consistent with experimental constraints of Eqs. (2). It is inefficient to numerically integrate every random choice of the input parameters down to the b quark scale. Instead we proceed as follows. In step (2) we use the analytic approximation techniques discussed in [23] to estimate, from M_S and the GUT scale parameters, the Z scale values of $\alpha_1(M_Z)$, $\alpha^{-1}(M_Z)$, $\sin^2\theta_W$, and $\alpha_3(M_Z)$. In general these predictions are good to within a few percent but it would be dangerous to rely on this. Each prediction $p_i(Z)$ is required to satisfy

$$p_i(Z) = p_{i0}(Z) \pm e_i \tag{A24}$$

before a full numerical integration to low energies is performed. Until the first solution is found $p_{i0}(Z)$ is taken within the experimental range and a generous error (much larer than the experimental error) is taken for the e_i. As long as e_i is large enough the exact values of p_i and e_i are not critical. After the first solution is found $p_{i0}(Z)$ is repeatedly readjusted to the average of the successful predictions. This average need not overlap the experimental values since the analytic results have some systematic error and only the results of successful numerical runs are accepted as solutions. After the second solution is found e_i is repeatedly readjusted to four times the rms deviation of the successful predictions. In this way the program rapidly "learns" which random choices of the input parameters are worth numerically integrating. As a check, with a probability of 10%, a numerical integration is performed on input choices which lead to predictions within 8 standard deviations of the successful predictions. The number of successful predictions from these runs is monitored and found to be negligible (usually zero). We are thus confident that no valid solutions are being lost by this procedure which can significantly speed up the generation of solutions. A few low statistics runs have been made without use of the prediction subroutine as a final check of the results.

Although $\tan(\beta)$ is technically an input parameter it is inefficient to choose $\tan(\beta)$ before integrating to low energies. For each value of the first four input parameters M_X, $\alpha_0(M_X)$, $\alpha_t(M_X)$, and M_S, a range of values of $\tan(\beta)$ will lead to a solution. Most solutions will be lost if a predetermined value of $\tan(\beta)$ is insisted upon. Apart from a small effect to be discussed below, nothing depends on $\tan(\beta)$ until one reaches the top quark energy scale. Therefore, without choosing $\tan(\beta)$ we first integrate down to an intermediate scale randomly chosen between 230 and 690 GeV in 125 steps in $\ln(Q)$. We then integrate in a further 125 steps from this intermediate scale down to M_Z. At each step below 220 GeV we assume that the energy is a possible value for M_t. Based on the couplings at that point we analytically estimate the value of $\sin^2\theta_W(M_Z)$. This can be done quite accurately since the extrapolation in $\ln(Q)$ is small. If the assumed value of M_t and the estimated value of $\sin^2(\theta_W)$ are in agreement with the experimental correlation Eq. (2b) we define a value of $\sin(\beta)$ from the relation

$$\sin(\beta) = M_t[173\text{ GeV }\sqrt{4\pi\alpha_t(M_t)}(1+4\alpha_3(M_t)/(3\pi)+11\alpha_3(M_t)^2/\pi^2)]^{-1} . \tag{A25}$$

We then repeat this procedure at the following steps until no further consistent values of M_t and $\sin^2\theta_W$ are found. Then from among the acceptable pairs of M_t and $\sin(\beta)$ we choose one solution at random. We decouple the top and proceed down to the Z, where the exact numerical value of $\sin^2\theta_W$ is then found. The solution is kept if the experimental constraints of Eqs. (2a) and (2b) are satisfied. In the usual procedure of fixing $\tan(\beta)$ in advance, Eq. (A25) becomes a nonlinear equation to be solved for M_t and $\alpha_t(M_t)$ which introduces some error in addition to the inefficiency described above. However, our procedure also involves some small errors. The first is that the top quark is decoupled at the lowest acceptable value of M_t instead of at the actual solution chosen. These values however differ typically by less than 10 GeV so that the effect on the gauge couplings at M_Z is very small. In addition there is another small error alluded to above. The usual procedure in extrapolating from M_X down to M_Z is to define an effective standard model field theory below M_S. The effective standard Higgs coupling is

$$\alpha_t^{\text{std}}(M_S) = \alpha_t(M_S)\sin^2\beta . \tag{A26}$$

The coefficients are defined to govern the running of this "effective" standard model Higgs coupling below M_S. When one reaches the top scale the top mass relation is

$$M_t = 173\text{ GeV }\sqrt{4\pi\alpha_t^{\text{std}}(M_t)} \times \left[1+\frac{4\alpha}{3\pi}3(M_t)+11\alpha^2(M_t)/\pi^2\right] . \tag{A27}$$

The difference between this and Eq. (A25) is extremely small for M_S less than a TeV since the top Yukawa coupling is running very slowly in this region. To further minimize the discrepancy while preserving the advantages of not choosing $\tan(\beta)$ in advance, we perform the scaling of Eq. (A26) with an "average" value of $\sin(\beta)$ corresponding to $\tan(\beta_0) = 1.8$.

If after reaching the Z scale the solution is still viable we extrapolate down to the b quark scale in a further 125 steps including in the strong-coupling running the effect of the three-loop coefficient:

$$b_{333} = -2857/2 + 5033N_f/18 - 325N_f^2/5 . \tag{A28}$$

Since the electromagnetic effects on the running of α_3 and r are very small in this region and the effects of the running of the electromagnetic coupling on α_3 and r are even more negligible we neglect to switch to an effective U(1) gauge theory below M_W. Instead we run the full standard model couplings (decoupling the W at 82 GeV). This slight approximation and that discussed above at the top quark scale could be avoided if, instead of using effective field theories in the various regions, the full supersymmetric theory was adhered to with a natural smooth decoupling of particle contributions below their mass. Equation 2(c) corresponds to a running b quark mass of 4.25±0.17 GeV. QCD corrections bring this up to a physical b quark mass of 4.95±0.20. We require therefore that the b/τ mass ratio, r, reach the value of Eq. (2c) at the scale 4.95 GeV. We neglect the running of the τ mass from the b quark scale down to the τ mass scale since this is extremely small compared to the uncertainty in the b quark mass. If the couplings pass the final test of Eq. (2c) the 10-, 12-, or 13-dimensional solution in the case of the MSSM, the MSSM with GUT scale splitting, and the MDM, respectively, is recorded.

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