Electroweak oblique corrections from quark-lepton substructure

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We consider the electroweak oblique corrections from quark-lepton substructure through its form factor. We derive general formulas for the oblique correction parameters T, U, and S for a general class of the form factors. Then we precisely calculate them for the case of the single-pole form factor. It turns out that the effects on T become sizable when the new physics scales break global isospin symmetry at the order of the weak interaction scale.

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I. INTRODUCTION

Recent precision experiments at high energies have confirmed the standard-model predictions of the strong and electroweak interactions [1]. This seems to suggest that the field theory based on the gauge principle precisely holds, at least in the presently accessible energy region. Many theorists, however, do not take that the standard model is the ultimate theory of the nature, because it involves too many semiempirical parameters, it requires unnatural fine-tuning of the bare parameters, etc.

They expect that there exists some new physics such as compositeness [2], supersymmetry [3], technicolor [4], etc., and that the standard model is an effective theory which holds in the far less energy region than the new physics scale. Within the presently accessible energy region, the effects of new physics are expected to be revealed through small deviations from the standard model in precision experiments. Recently many people have studied the contributions to the electroweak oblique corrections from the new physics models [5], but only a few from the composite-model viewpoint [6]. In this paper, we investigate the contribution to the electroweak oblique corrections from quark-lepton substructure through its form factor at the vertices with the weak bosons. At low energies, the effects of the form factor through the tree diagrams [7] are very small, but they can become sizable in the loop-diagram effects because the momentum of the internal lines can be very large.

In the composite models of quarks, leptons, gauge bosons, and/or Higgs scalars, the interactions among the composite particles may lose their meaning at the energy scale of subconstituent physics. Naively, we can describe this situation by multiplying the interaction vertices by form factors which suppress the vertex above the compositeness scale Λ. In general, the form factor $\mathcal{F}(q^2)$ is a function of the relevant four-momentum squared q^2, such that $\mathcal{F}(0) = 1$, and $\mathcal{F}(q^2) = 0$ for $q^2 \gg \Lambda^2$. Unfortunately, there exists no such function $\mathcal{F}(q^2)$ which is analytic in the whole complex q^2 plane. In this sense, it is merely an approximation which is useful much below Λ^2, and only the gross features make sense around Λ^2 and beyond. We assume that the form factor $\mathcal{F}(q^2)$ is analytic except for a number of points in the q^2 plane. Then it is written as a sum of functions which is analytic except for a point and vanishes at $q^2 = \infty$. Each function can be expanded with positive power terms in $\Lambda^2/(\Lambda^2 - q^2)$ with its singular point Λ^2. Thus the most general form factor is a finite or infinite linear combination of the terms with the form

$$\left(\frac{\Lambda^2}{\Lambda^2 - q^2}\right)^n \quad (n = \text{positive integer}). \tag{1}$$

The plan of this paper is as follows. In Sec. II, we consider the general functional form of the form factors, and derive general formulas of the oblique correction parameters T, U, and S. In Sec. III, we perform the calculation of them for the case of the single-pole form factor (see Appendix for the detailed derivation), and investigate their dependence on the compositeness scale Λ. In Sec. IV, we show the S-T and the S-U plane trajectories of the form factor effects and compare them with the phenomenological bounds.

II. GENERAL FORM FACTOR

Now we incorporate the form factor $\mathcal{F}(q^2)$ to the electroweak-interaction Lagrangian of the isodoublet fermion ψ (quark or lepton) in the form

$$\mathcal{L} = \overline{\psi}(i\not{\mathcal{D}} - m)\psi - \frac{1}{2}g\overline{\psi}\mathcal{F}(-\mathcal{D}^2)\tau_l \not{W}_l\gamma_L\mathcal{F}(-\mathcal{D}^2)\psi, \tag{2}$$

where $\mathcal{D}^\mu \equiv \partial^\mu + ieQA^\mu$ is the covariant derivative with respect to the electromagnetic gauge symmetry $\mathrm{U}(1)_{\mathrm{em}}$, A^μ and W_l^μ (l=1,2,3) are the photon and the weak boson fields (to be diagonalized), respectively, e and g are the electromagnetic and the weak coupling constants, respectively, m, Q, and τ are the mass, the electric charge, and the Pauli matrices, respectively, and $\gamma_L = (1-\gamma_5)/2$. In

(2), m and Q should be taken as matrices operating on the (global) isodoublet space of the fermion ψ: i.e.,

$$\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}, \; Q = \begin{pmatrix} Q_u & 0 \\ 0 & Q_d \end{pmatrix}, \; m = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}.$$

The subscripts u and d denote the up-type and the down-type fermions, respectively. We have no reason to exclude the possibility that Λ violates isospin symmetry at the order of weak interaction scale Λ_W. Then Λ and accordingly $\mathcal{F}(-\mathcal{D}^2)$ are also matrices in the isodoublet space of the fermion, i.e., $\Lambda = \begin{pmatrix} \Lambda_u & 0 \\ 0 & \Lambda_d \end{pmatrix}$ with $\Lambda_u - \Lambda_d = O(\Lambda_W)$. In the low energy region where $q^2 \ll \Lambda^2$, i.e., $\mathcal{F}(-\mathcal{D}^2) \approx 1$, the Lagrangian (2) is reduced to the Hung-Sakurai form [8] which is equivalent to the standard model as far as the non-Higgs sector is concerned.

In (2), we consider only the form factor operating on the fermion field ψ, but not that operating on the weak boson W_l^μ, because the latter does not contribute to the oblique corrections where the invariant mass of the weak boson is much smaller than the compositeness scale. The Lagrangian (2) preserves the electromagnetic gauge symmetry, but not the full electroweak symmetry $SU(2)_L \otimes U(1)_Y$. There is no nontrivial way to incorporate a form factor operating on the matter field so that it preserves the full gauge symmetry. Such a violation of the gauge symmetry can be taken as a physical consequence of compositeness of the fermions and/or the weak bosons.

To derive the Feynman rule for the Lagrangian (2), we expand the form factor $\mathcal{F}(-\mathcal{D}^2)$ in terms of the field A^μ. The term of the zeroth order in A^μ contributes the factor $\mathcal{F}(-\partial^2)$ to the vertex. The term linear in A^μ gives a vertex attached with an additional photon line with the following factor in the momentum space:

$$\mathcal{V}_\mu(k,q) = -2\mathcal{F}'(k^2)k_\mu - \mathcal{F}''(k^2)\left[q^2 k_\mu + (k\cdot q)q_\mu\right] - \frac{4}{3}\mathcal{F}'''(k^2)(k\cdot q)^2 k_\mu + O(q^3)\ , \tag{3}$$

where the primes denote the derivatives of $\mathcal{F}(k^2)$ with respect to k^2, and k_μ and q_μ are the momenta of the photon and the initial fermion, respectively. Equation (3) is derived as follows. Since the most general form factor is a finite or an infinite linear combination of (1), it is sufficient to prove (3) for the form factor of the form (1). Then

$$\mathcal{F}(-\mathcal{D}^2) = \left(\frac{\Lambda^2}{\Lambda^2+\partial^2}\right)^n - \sum_{j=1}^{n}\left(\frac{\Lambda^2}{\Lambda^2+\partial^2}\right)^j \times \frac{ieQ(\partial_\mu A^\mu + A^\mu\partial_\mu)}{\Lambda^2}\left(\frac{\Lambda^2}{\Lambda^2+\partial^2}\right)^{n+1-j} . \tag{4}$$

Therefore,

$$\mathcal{V}_\mu(k,q) = -\sum_{j=1}^{n}\left(\frac{\Lambda^2}{\Lambda^2-(k+q)^2}\right)^j \times \frac{eQ(2k_\mu+q_\mu)}{\Lambda^2}\left(\frac{\Lambda^2}{\Lambda^2-k^2}\right)^{n+1-j} . \tag{5}$$

Then the first two coefficients in the expansion of $\mathcal{V}_\mu(k,q)$ in q are calculated to be

$$\mathcal{V}_\mu(k,q)|_{q=0} = -\frac{2n}{\Lambda^2}eQ\left(\frac{\Lambda^2}{\Lambda^2-k^2}\right)^{n+1} k_\mu = -2\mathcal{F}'(k^2)k_\mu, \tag{6}$$

$$\begin{aligned}\frac{\partial}{\partial q_\rho}\frac{\partial}{\partial q_\lambda}\mathcal{V}_\mu(k,q)|_{q=0} &= -\frac{n(n+1)eQ}{\Lambda^4}\left(\frac{\Lambda^2}{\Lambda^2-k^2}\right)^{n+2}(2g_{\rho\lambda}k_\mu + g_{\mu\rho}k_\lambda + g_{\mu\lambda}k_\rho) \\ &\quad - \frac{8n(n+1)(n+2)eQ}{3\Lambda^6}\left(\frac{\Lambda^2}{\Lambda^2-k^2}\right)^{n+3} k_\rho k_\lambda k_\mu \\ &= -\mathcal{F}''(k^2)(2g_{\rho\lambda}k_\mu + g_{\mu\rho}k_\lambda + g_{\mu\lambda}k_\rho) - \frac{8}{3}\mathcal{F}'''(k^2)k_\rho k_\lambda k_\mu.\end{aligned} \tag{7}$$

This completes the proof of (3).

The self-energy parts $\Pi_{ll}^{\mu\nu}$ of the W_l^μ ($l = 1,\ 2,\ 3$) and the mixing amplitude $\Pi_{3Q}^{\mu\nu}$ between W_3^μ and A^ν arise from the diagrams in Fig. 1, and are given by

$$\Pi_{ll}^{\mu\nu} = -\frac{g^2}{4}\int\frac{id^4k}{(2\pi)^4}\mathrm{Tr}\left[\tau_l\gamma^\mu\gamma_L\frac{\mathcal{F}^2((k+q)^2)}{\not k+\not q-m}\tau_l\gamma^\nu\gamma_L\frac{\mathcal{F}^2(k^2)}{\not k-m}\right], \tag{8}$$

$$\Pi_{3Q}^{\mu\nu} = -\frac{eg}{2}\int\frac{id^4k}{(2\pi)^4}\mathrm{Tr}\left[\tau_3\gamma^\mu\gamma_L\frac{\mathcal{F}((k+q)^2)}{\not k+\not q-m}Q\gamma^\nu\frac{\mathcal{F}(k^2)}{\not k-m} + 2\tau_3\gamma^\mu\gamma_L\frac{Q\mathcal{V}^\nu(k,q)}{\not k-m}\right], \tag{9}$$

where the second term in the trace in Eq. (9) arises from the second term in the A_μ expansion of $\mathcal{F}(-\mathcal{D}^2)$ [Eq. (3)]. It is necessary to guarantee the electromagnetic gauge invariance. Owing to the fact that the form factors are suppressed at high momenta, (8) and (9) are convergent integrals. The self-energy parts $\Pi^{\mu\nu}_{ij}$ ($i,\ j = 1,\ 2,\ 3,\ Q$) are represented by the form

$$\Pi^{\mu\nu}_{ij} = -g^{\mu\nu}\Pi_{ij}(q^2) + (q^\mu q^\nu \ \text{ terms}). \tag{10}$$

To extract the oblique correction parameters S, T, and U defined in Ref. [9], we perform Wick rotation in the complex k_0 plane, and integrate them over three angular variables in the four-dimensional Euclidian k space. After a lengthy calculation, we obtain the vacuum polarization amplitude $\Pi_{ij}(0)$:

$$\Pi_{11}(0) = -\frac{1}{32\pi^2}\int_0^\infty ds \frac{\mathcal{F}_u^2(-s)\mathcal{F}_d^2(-s)}{(1+m_u^2/s)(1+m_d^2/s)}\ , \tag{11}$$

$$\Pi_{33}(0) = -\frac{1}{64\pi^2}\int_0^\infty ds\left[\left(\frac{\mathcal{F}_u^2(-s)}{1+m_u^2/s}\right)^2 + \left(\frac{\mathcal{F}_d^2(-s)}{1+m_d^2/s}\right)^2\right]\ , \tag{12}$$

$$\Pi'_{11}(0) = -\frac{1}{48\pi^2}\int_0^\infty ds\ s^3\left(\frac{d}{ds}\frac{\mathcal{F}_u^2(-s)}{s+m_u^2}\right)\left(\frac{d}{ds}\frac{\mathcal{F}_d^2(-s)}{s+m_d^2}\right)\ , \tag{13}$$

$$\Pi'_{33}(0) = -\frac{1}{96\pi^2}\int_0^\infty ds\ s^3\left[\left(\frac{d}{ds}\frac{\mathcal{F}_u^2(-s)}{s+m_u^2}\right)^2 + \left(\frac{d}{ds}\frac{\mathcal{F}_d^2(-s)}{s+m_d^2}\right)^2\right]\ , \tag{14}$$

$$\Pi'_{3Q}(0) = -\frac{1}{32\pi^2}\int_0^\infty ds\left\{Q_u\left[\frac{s^2}{s+m_u^2}\left(\frac{d}{ds}\mathcal{F}_u^2(-s)\right) - \frac{m_u^4\mathcal{F}_u^2(-s)}{(s+m_u^2)^3} + \frac{2\mathcal{F}_u^2(-s)}{3(s+m_u^2)}\right] - Q_d\left[u\leftrightarrow d\right]\right\}. \tag{15}$$

In Eqs. (11)–(15), we have assumed the following behaviors of the form factors for $s\to\infty$:

$$\mathcal{F}_i(s) < O(s^0),\ \ \frac{d}{ds}\mathcal{F}_i(s) < O(s^{-1})\ . \tag{16}$$

Then the oblique correction parameters are given by the general expressions involving the arbitrary form factors:

$$T = \frac{N_c}{16\pi\sin^2\theta_W\cos^2\theta_W M_Z^2}\int_0^\infty ds\left[\frac{\mathcal{F}_u^2(-s)}{1+m_u^2/s} - \frac{\mathcal{F}_d^2(-s)}{1+m_d^2/s}\right]^2\ , \tag{17}$$

$$U = \frac{N_c}{6\pi}\int_0^\infty ds\ \ s^3\left[\frac{d}{ds}\frac{\mathcal{F}_u^2(-s)}{s+m_u^2} - \frac{d}{ds}\frac{\mathcal{F}_d^2(-s)}{s+m_d^2}\right]^2\ , \tag{18}$$

$$S = \frac{N_cQ_u}{6\pi}\int_0^\infty ds\left[s^2(m_u^2-s)\left(\frac{d}{ds}\frac{\mathcal{F}_u(-s)}{m_u^2-s}\right)^2 - \frac{\mathcal{F}_u^2(-s)}{m_u^2-s}\right] - \left\{u\leftrightarrow d\right\}\ , \tag{19}$$

where $Q_u = 2/3$, $Q_d = -1/3$ for quarks, $Q_u = 0$, $Q_d = -1$ for leptons, θ_W and M_Z are the Weinberg angle and the Z boson mass, respectively, and N_c is the color factors ($N_c = 3$ for quarks and $N_c = 1$ for leptons). From Eqs. (17) and (18), we can conclude that $T \geq 0$, $U \geq 0$ for the arbitrary form factors while S can be either positive or negative. Equations (17), (18), and (19), however, involve the contributions from the corresponding part of the ordinary radiative corrections in the standard model. After subtraction of them, T and U can be slightly negative.

III. SINGLE-POLE FORM FACTOR

As a simple example, let us see how the oblique correction parameters T, U, and S depend on the new physics scale Λ for the case of the following covariant single-pole form factor

$$\mathcal{F}(-\mathcal{D}^2) = \frac{\Lambda^2}{\mathcal{D}^2+\Lambda^2}\ . \tag{20}$$

We expand $\mathcal{F}(-\mathcal{D}^2)$ in terms of the electromagnetic coupling constant e:

$$\mathcal{F}(-\mathcal{D}^2) = \frac{\Lambda^2}{\partial^2+\Lambda^2}\sum_{n=0}^{\infty}\left[e\left(i\partial^\mu A_\mu + iA_\mu\partial^\mu - eA_\mu^2\right)\frac{-1}{\partial^2+\Lambda^2}\right]^n$$
$$= \frac{\Lambda^2}{\partial^2+\Lambda^2} - e\frac{\Lambda^2}{\partial^2+\Lambda^2}i(\partial^\mu A_\mu + A_\mu\partial^\mu)\frac{1}{\partial^2+\Lambda^2}$$
$$+e^2\frac{\Lambda^2}{\partial^2+\Lambda^2}\left\{A_\mu^2\frac{1}{\partial^2+\Lambda^2} - \left[(\partial^\mu A_\mu + A_\mu\partial^\mu)\frac{1}{\partial^2+\Lambda^2}\right]^2\right\} + O(e^n)(n \geq 3)\ . \tag{21}$$

In Fig. 2, we illustrate the Feynman rules for each term in the form factor (21). Then the self-energy parts $\Pi_{ll}^{\mu\nu}$ and $\Pi_{3Q}^{\mu\nu}$ are given by the diagrams in Fig. 3. It is laborious but straightforward to calculate the oblique correction parameters T, U, and S. We derive them in the Appendix. The analytic results are very complicated, as are given by (A11)–(A13) with (A14)–(A22).

To see the behavior of T, U, and S for large cutoff, we expand them in inverse powers of Λ_i $(i = u, d)$. Equation (A11) is expanded in terms of Λ_i as

$$T \approx \frac{N_c}{8\pi\sin^2\theta_W\cos^2\theta_W M_Z^2}\left[\mathcal{T}_2 + \mathcal{T}_0 + \mathcal{T}_{-2} + O(\Lambda^{-4})\right], \tag{22}$$

where $\mathcal{T}_2$, $\mathcal{T}_0$, and $\mathcal{T}_{-2}$ are the $O(\Lambda^2)$, $O(\Lambda^0)$, and $O(\Lambda^{-2})$ contributions, respectively:

$$\mathcal{T}_2 = -\frac{2\Lambda_u^4\Lambda_d^4}{(\Lambda_u^2-\Lambda_d^2)^3}\ln\left(\frac{\Lambda_d^2}{\Lambda_u^2}\right) + \frac{(\Lambda_u^2+\Lambda_d^2)(\Lambda_u^4 - 8\Lambda_u^2\Lambda_d^2 + \Lambda_d^4)}{6(\Lambda_u^2-\Lambda_d^2)^2}\ , \tag{23}$$

$$\mathcal{T}_0 = \frac{\Lambda_u^6 m_u^2 - 3\Lambda_u^4\Lambda_d^2 m_u^2 - 3\Lambda_u^2\Lambda_d^4 m_d^2 + \Lambda_d^6 m_d^2}{(\Lambda_u^2-\Lambda_d^2)^3}\ln\left(\frac{\Lambda_d^2}{\Lambda_u^2}\right)$$
$$+\frac{(5\Lambda_u^4 - 22\Lambda_u^2\Lambda_d^2 + 5\Lambda_d^4)(m_u^2+m_d^2)}{6(\Lambda_u^2-\Lambda_d^2)^2} + \frac{m_u^4 - m_d^4 + 2m_u^2 m_d^2\ln(m_d^2/m_u^2)}{2(m_u^2-m_d^2)}\ , \tag{24}$$

$$\mathcal{T}_{-2} = (m_u^4 + m_u^2 m_d^2 + m_d^4)\left[\frac{\Lambda_u^6+\Lambda_d^6}{2\Lambda_u^2\Lambda_d^2(\Lambda_u^2-\Lambda_d^2)^2} + \frac{2(\Lambda_d^6 - 2\Lambda_u^2\Lambda_d^4)}{\Lambda_u^2(\Lambda_u^2-\Lambda_d^2)^3}\ln(\Lambda_u^2)\right]$$
$$+2\frac{m_u^6\ln(m_u^2) - m_d^6\ln(m_d^2)}{\Lambda_u^2(m_u^2-m_d^2)} - \frac{17m_u^4}{2\Lambda_u^2} + \frac{6m_u^4}{\Lambda_u^2}\ln\left(\frac{\Lambda_u^2}{m_u^2}\right) + \left\{u \leftrightarrow d\right\}\ . \tag{25}$$

The third term in $\mathcal{T}_0$ corresponds to the standard-model contribution, and should be subtracted to get the pure new-physics contribution. The higher order terms of $O(\Lambda^{-n})$ $(n \geq 4)$ are negligiblly small because the masses of ordinary fermions are very small compared with the new physics scales. If $\Lambda_u \neq \Lambda_d$, $\mathcal{T}_2$ diverges quadratically with increasing Λ_i, while if $\Lambda_u = \Lambda_d$, $\mathcal{T}_2$ vanishes. This realizes the general statement that the oblique correction parameter T vanishes when the global isospin symmetry (custodial symmetry) holds [10]. The breaking of this symmetry is parametrized by the difference $\Delta = \Lambda_u - \Lambda_d$ of the scales. We naturally expect that the symmetry is broken at the order of $\Delta \approx \Lambda_W$ (Λ_W is the weak interaction scale). If it is the case, its effects on T are well observable. Since the top quark mass m_t is large, we cannot ignore the higher order terms $\mathcal{T}_0$ and $\mathcal{T}_{-2}$ of the top-bottom quark generation.

Equation (A12) is expanded in terms of Λ_i as

$$U \approx \frac{N_c}{6\pi}\left[\mathcal{U}_0 + \mathcal{U}_{-2} + O(\Lambda^{-4})\right]\ , \tag{26}$$

where $\mathcal{U}_0$ and $\mathcal{U}_{-2}$ are the $O(\Lambda^0)$ and $O(\Lambda^{-2})$ contributions, respectively:

$$\mathcal{U}_0 = -\frac{\Lambda_u^{10} - 5\Lambda_u^8\Lambda_d^2 + 28\Lambda_u^6\Lambda_d^4 + 28\Lambda_u^4\Lambda_d^6 - 5\Lambda_u^2\Lambda_d^8 + \Lambda_d^{10}}{(\Lambda_u^2-\Lambda_d^2)^5}\ln\left(\frac{\Lambda_d^2}{\Lambda_u^2}\right) - \frac{48\Lambda_u^4\Lambda_d^4}{(\Lambda_u^2-\Lambda_d^2)^4} - \frac{19}{15}$$
$$+\frac{m_u^6 - 3m_u^4 m_d^2 - 3m_u^2 m_d^4 + m_d^6}{(m_u^2-m_d^2)^3}\ln\left(\frac{m_u^2}{m_d^2}\right) + \frac{22m_u^2 m_d^2 - 5(m_u^4+m_d^4)}{3(m_u^2-m_d^2)^2}, \tag{27}$$

$$\mathcal{U}_{-2} = 2m_u^2\left\{\frac{2\Lambda_u^4(\Lambda_u^4 - 5\Lambda_u^2\Lambda_d^2 + 16\Lambda_d^4)}{(\Lambda_u^2-\Lambda_d^2)^5}\ln\left(\frac{\Lambda_u^2}{\Lambda_d^2}\right) - \frac{\Lambda_d^4(31\Lambda_u^4 - 8\Lambda_u^2\Lambda_d^2 + \Lambda_d^4)}{\Lambda_u^2(\Lambda_u^2-\Lambda_d^2)^4}\right.$$
$$\left.+\frac{2}{\Lambda_u^2}\left[\frac{m_d^4(m_d^2 - 3m_u^2)}{(m_u^2-m_d^2)^3}\ln\left(\frac{m_u^2}{m_d^2}\right) + \frac{2m_u^2 m_d^2}{(m_u^2-m_d^2)^2}\right] + \frac{41}{30\Lambda_u^2} - \frac{4}{\Lambda_d^2}\right\} + 2m_d^2\{u \leftrightarrow d\}\ . \tag{28}$$

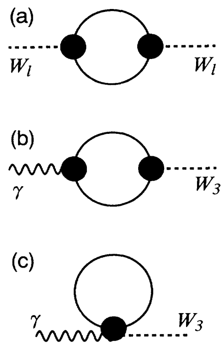

FIG. 1. (a) The self-energy diagram of W_l^μ ($l = 1, 2, 3$), and (b),(c) the mixing diagrams between W_3^μ and A^μ. The blobs indicate the general form factors. The solid, the dashed, and the wavy lines indicate the fermion, the weak boson, and the photon propagators, respectively. The diagram (c) guarantees the electromagnetic gauge invariance.

The fourth and fifth terms in $\mathcal{U}_0$ are the contributions from the standard model, and should be subtracted to get the pure new-physics contribution. If we send one of Λ_u and Λ_d to infinity, $\mathcal{U}_0$ diverges logarithmically. On the other hand if $\Lambda_u = \Lambda_d$, $\mathcal{U}_0$ vanishes. Even though the global isospin symmetry is broken, i.e., $\Lambda_u \neq \Lambda_d$, U is not so large, because U does not involve the terms of $O(\Lambda^2)$.

We also expand Eq. (A13) in terms of Λ_i as

$$S \approx \frac{N_c}{6\pi}\left[\mathcal{S}_0 + \mathcal{S}_{-2} + O(\Lambda^{-4})\right] , \tag{29}$$

where $\mathcal{S}_0$ and $\mathcal{S}_{-2}$ are the $O(\Lambda^0)$ and $O(\Lambda^{-2})$ contributions, respectively:

$$\mathcal{S}_0 = -\frac{16}{15} - Y\ln\left(\frac{\Lambda_d^2}{\Lambda_u^2}\right) + 1 + Y\ln\left(\frac{m_d^2}{m_u^2}\right) , \tag{30}$$

$$\mathcal{S}_{-2} = 2(1-Y)\frac{m_u^2}{\Lambda_u^2}\ln\left(\frac{m_u^2}{\Lambda_u}\right) + 2(1+Y)\frac{m_d^2}{\Lambda_d^2}\ln\left(\frac{m_d^2}{\Lambda_d}\right) + \frac{(64+45Y)m_u^2}{15\Lambda_u^2} + \frac{(64-45Y)m_d^2}{15\Lambda_d^2} , \tag{31}$$

where Y ($= Q_u + Q_d$) is the weak hypercharge of the lefthanded component of the fermion. The third and fourth terms in $\mathcal{S}_0$ correspond to the standard-model contribution, and should be subtracted to get the pure new-physics contribution. Even though the global isospin symmetry is broken, i.e., $\Lambda_u \neq \Lambda_d$, S is not so large, because S does not involve the terms of $O(\Lambda^2)$ as is the case of U. Even if the global isospin symmetry of the new physics scales holds, i.e., $\Lambda_u = \Lambda_d$, the value of S does not vanish but becomes $-8N_c/45\pi$ which is independent of the new physics scales, the fermion masses, and the weak hypercharge.

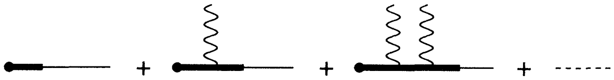

FIG. 2. The Feynman diagrams for each term in the covariant single-pole form factor in Eq. (20). The thick solid lines indicate the single-pole form factors. The thin solid and the wavy lines indicate the fermion and the photon propagators, respectively.

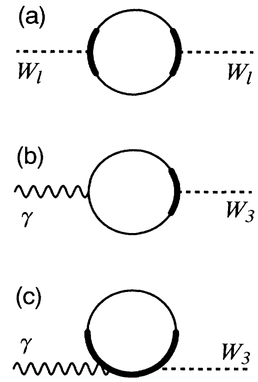

FIG. 3. (a) The self-energy diagram of W_l^μ ($l = 1, 2, 3$), and (b),(c) the mixing diagrams between W_3^μ and A^μ for the case of the covariant single-pole form factor. The thick solid lines indicate the single-pole form factors. The thin solid, the dashed, and the wavy lines indicate the fermion, the weak boson, and the photon propagators, respectively.

IV. *S*-*T* AND *S*-*U* PLANE TRAJECTORIES

In Figs. 4 and 5, we plot the S-T and the S-U plane trajectories of the form factor effects for various values of $\overline{\Lambda} = (\Lambda_u + \Lambda_d)/2$ and $\Delta = \Lambda_u - \Lambda_d$. The solid lines denote the trajectories with $\overline{\Lambda}$ fixed, and the dashed lines denote the trajectories with Δ fixed. In Figs. 4 and 5, (a), (b), and (c) show those for a lepton doublet (ν_e-e, ν_μ-μ, or ν_τ-τ), a light quark doublet (u-d or c-s), and the top-bottom quark doublet, respectively. In this calculation, we used the value m_t=174 GeV for the mass of the top quark [11]. From these diagrams, we can see that T increases rapidly with increasing $|\Delta|$, while S weakly depends on Δ if $\overline{\Lambda}$ is small. For the top-bottom doublet, the higher order terms $\mathcal{T}_0$, $\mathcal{T}_{-2}$, $\mathcal{U}_{-2}$, and $\mathcal{S}_{-2}$ in Eqs. (22), (26), and (29) also have sizable contributions. Furthermore, T and U becomes asymmetric as a function of Δ, and can be slightly negative, because the top quark and the bottom quark have a large mass difference [Fig. 4(c) and Fig. 5(c)]. On the other hand, for the light-fermion doublets, T and U are almost symmetric in Δ and positive definite [Figs. 4(a) and 4(b), Figs. 5(a) and 5(b)]. Hence we can see that the effects of the form factors to the oblique correction parameters become sizable only when the new physics scales break global isospin symmetry, i.e., $\Lambda_u \neq \Lambda_d$. When $\Delta = \Lambda_u - \Lambda_d$ is the order of weak interaction scale Λ_W, the effects of form factor to T can be observed by experiment because $T = O(\Delta^2/M_Z^2) = O(\Lambda_W^2/M_Z^2)$. Notice that T does not strongly depend on $\overline{\Lambda}$ as far as Δ is fixed. On the other hand, it is difficult to detect the effects of form factor on U and S unless $\overline{\Lambda}$ is as small as the weak interaction scale.

Now we consider the total contributions from all the fermion doublets. In Fig. 6, we show its S-T plane trajectories with varying Δ together with the phenomenological bound on S and T given in Ref. [9] for the top quark mass m_t=150 GeV and the Higgs scalar mass m_H=1 TeV. The phenomenological bound is insensitive to m_H. The reason why we adopt m_t =150 GeV in drawing the phenomenological bound is that it is about the safest value within the experimental error of m_t determined from the Collider Detector at Fermilab (CDF) experiment [11]. Figures 6(a) and 6(b) represent the cases of $\overline{\Lambda}$ = 1 TeV and $\overline{\Lambda}$ = 10 TeV, respectively. In Table I, we show the phenomenological constraint on Δ read off from the diagrams in Fig. 6.

In general, the contributions of the form factors to the oblique correction parameters depend on the functional

TABLE I. The phenomenological constraint on Δ $(= \Lambda_u - \Lambda_d)$.

$\overline{\Lambda}$	68% C.L.	90% C.L.
1 TeV	-55 GeV$< \Delta <$ 76 GeV	-77 GeV$< \Delta <$ 98 GeV
10 TeV	-33 GeV$< \Delta <$ 36 GeV	-66 GeV$< \Delta <$ 69 GeV

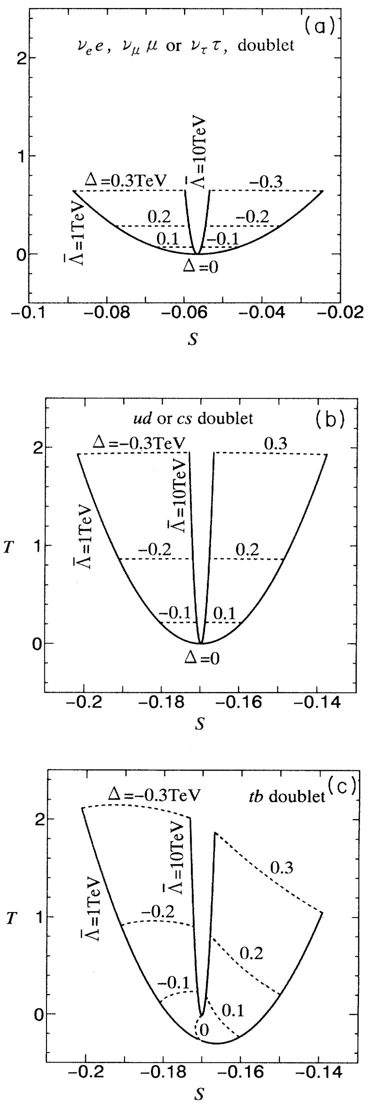

FIG. 4. The S-T plane trajectories of the form factor effects for various values of $\overline{\Lambda} = (\Lambda_u + \Lambda_d)/2$ and $\Delta = \Lambda_u - \Lambda_d$ for each fermion doublet: (a) a lepton doublet (ν_e-e, ν_μ-μ, or ν_τ-τ), (b) a light quark doublet (u-d or c-s), and (c) the top-bottom quark doublet. The solid lines denote the trajectories with $\overline{\Lambda}$ fixed, and the dashed lines denote those with Δ fixed.

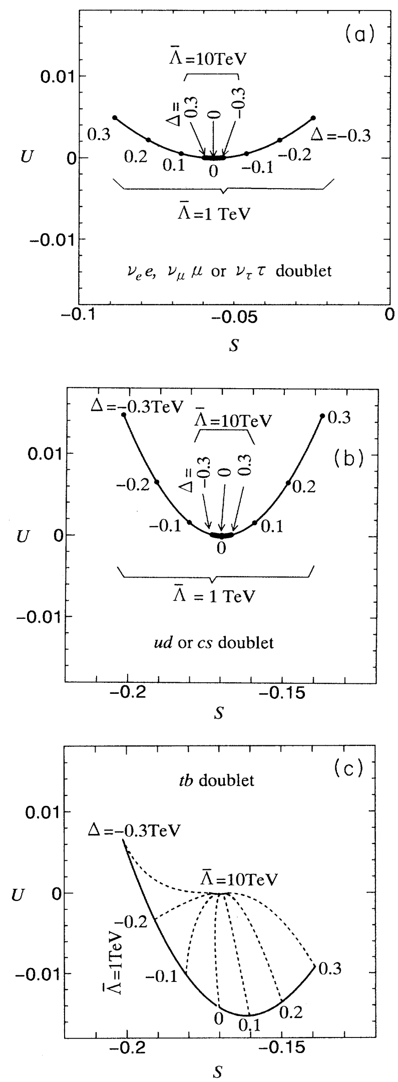

FIG. 5. The S-U plane trajectories of the form factor effects for various values of $\overline{\Lambda} = (\Lambda_u + \Lambda_d)/2$ and $\Delta = \Lambda_u - \Lambda_d$ for each fermion doublet: (a) a lepton doublet (ν_e-e, ν_μ-μ, or ν_τ-τ), (b) a light quark doublet (u-d or c-s), and (c) the top-bottom quark doublet.

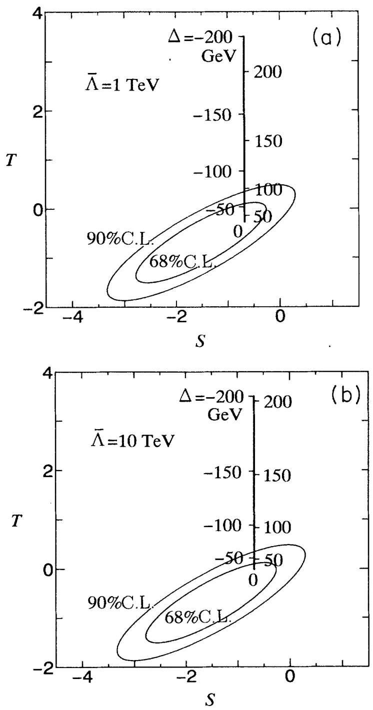

FIG. 6. The S-T plane trajectories of the form factor effects in total, and the phenomenological bound (the ellipses).

form of the form factors $\mathcal{F}(-\mathcal{D}^2)$. We have no way to determine it except for that by high energy experiments in the future. However the explicit form of $\mathcal{F}(-\mathcal{D}^2)$ itself is not important to determine the dependence of the oblique corrections on the compositeness scale, as far as its scale is much larger than the weak interaction scale.

To summarize, we have considered the contributions of the form factors to the oblique correction parameters T, U, and S. The effects of the form factors to the oblique correction parameter T become sizable when the new physics scales Λ_i ($i = u,\ d$) break global isospin symmetry. The effects, if they exist, are so large that they might affect the present day experiments. We found that $\Delta = \Lambda_u - \Lambda_d$ should be restricted in the region given in Table I. We expect that the compositeness, if any, might disclose itself through the electroweak oblique corrections in more accurate experiments in the future.

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APPENDIX

In this appendix, we give the detailed derivation of the oblique correction parameters for the case of the single-pole form factor. The self-energy parts $\Pi^{\mu\nu}_{ll}$ and the mixing amplitude $\Pi^{\mu\nu}_{3Q}$ arise from the diagrams in Fig. 3, and are given by

$$\Pi^{\mu\nu}_{11} = -\frac{g^2}{2}\int\frac{id^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma^\mu\gamma_L\left(\frac{\Lambda_u^2}{(k+q)^2-\Lambda_u}\right)^2\frac{1}{\not k+\not q-m_u}\gamma^\nu\gamma_L\left(\frac{\Lambda_d^2}{k^2-\Lambda_d}\right)^2\frac{1}{\not k-m_d}\right], \tag{A1}$$

$$\Pi^{\mu\nu}_{33} = -\frac{g^2}{4}\int\frac{id^4k}{(2\pi)^4}\mathrm{Tr}\left[\left\{\gamma^\mu\gamma_L\left(\frac{\Lambda_u^2}{(k+q)^2-\Lambda_u}\right)^2\frac{1}{\not k+\not q-m_u}\gamma^\nu\gamma_L\left(\frac{\Lambda_u^2}{k^2-\Lambda_u}\right)^2\frac{1}{\not k-m_u}\right\}+\{u\leftrightarrow d\}\right]. \tag{A2}$$

$$\begin{aligned}\Pi^{\mu\nu}_{3Q} = -\frac{ge}{2}\int\frac{id^4k}{(2\pi)^4}\mathrm{Tr}\Bigg[\Bigg\{&Q_u\gamma^\mu\frac{1}{\not k+\not q-m_u}\left(\frac{\Lambda_u^2}{(k+q)^2-\Lambda_u}\right)\gamma^\nu\gamma_L\left(\frac{\Lambda_u^2}{k^2-\Lambda_u}\right)\frac{1}{\not k-m_u}\\ &+Q_u\gamma^\mu(2k_\mu+q_\mu)\left(\frac{1}{(k+q)^2-\Lambda_u}\right)\gamma^\nu\gamma_L\left(\frac{\Lambda_u^2}{k^2-\Lambda_u}\right)^2\frac{1}{\not k-m_u}\Bigg\}-\{u\leftrightarrow d\}\Bigg].\end{aligned} \tag{A3}$$

It is straightforward to obtain the following expressions for the oblique correction parameters per one isodoublet fermion:

$$T = \frac{N_c}{8\pi\sin^2\theta_W\cos^2\theta_W M_Z^2}\int_0^1 dx\Big[2C_{ud}(x)-C_{uu}(x)-C_{dd}(x)\Big]\ , \tag{A4}$$

$$U = \frac{N_c}{\pi}\int_0^1 dx\Big[2D_{ud}(x)-D_{uu}(x)-D_{dd}(x)\Big]\ , \tag{A5}$$

$$S = \frac{N_c}{\pi}\int_0^1 dx\Big[I_u(x)+I_d(x)-Q_uJ_u(x)+Q_dJ_d(x)\Big]\ , \tag{A6}$$

where

$$C_{ij}(x) = \frac{\Lambda_i^4\Lambda_j^4}{(\Lambda_i^2-m_i^2)^2(\Lambda_j^2-m_j^2)^2}\left\{(m_i^2x+m_j^2y)\ln\left[\frac{(\Lambda_i^2x+m_j^2y)(m_i^2x+\Lambda_j^2y)}{(\Lambda_i^2x+\Lambda_j^2y)(m_i^2x+m_j^2y)}\right]-\frac{(\Lambda_i^2-m_i^2)(\Lambda_j^2-m_j^2)}{\Lambda_i^2x+\Lambda_j^2y}xy\right\}, \quad \text{(A7)}$$

$$D_{ij}(x) = \frac{\Lambda_i^4\Lambda_j^4xy}{(\Lambda_i^2-m_i^2)^2(\Lambda_j^2-m_j^2)^2}\left\{\ln\left[\frac{(\Lambda_i^2x+\Lambda_j^2y)(m_i^2x+m_j^2y)}{(\Lambda_i^2x+m_j^2y)(m_i^2x+\Lambda_j^2y)}\right]\right.$$
$$\left.+\frac{(\Lambda_i^2-m_i^2)(\Lambda_j^2-m_j^2)}{\Lambda_i^2x+\Lambda_j^2y}xy\left(\frac{1}{\Lambda_i^2x+m_j^2y}+\frac{1}{m_i^2x+\Lambda_j^2y}-\frac{1}{\Lambda_i^2x+\Lambda_j^2y}\right)\right\}, \quad \text{(A8)}$$

$$I_i(x) = \frac{\Lambda_i^8xy}{(\Lambda_i^2-m_i^2)^4}\left\{\ln\left[\frac{\Lambda_i^2m_i^2}{(\Lambda_i^2x+m_i^2y)(m_i^2x+\Lambda_i^2y)}\right]\right.$$
$$\left.-(\Lambda_i^2-m_i^2)\left(\frac{1}{\Lambda_i^2}-\frac{x}{\Lambda_i^2x+m_i^2y}-\frac{y}{m_i^2x+\Lambda_i^2y}\right)-(\Lambda_i^2-m_i^2)^2\frac{xy}{\Lambda_i^4}\right\}, \quad \text{(A9)}$$

$$J_i(x) = \frac{2\Lambda_i^4xy}{(\Lambda_i^2-m_i^2)^2}\ln\left[\frac{\Lambda_i^4m_i^2}{(\Lambda_i^2x+m_i^2y)^2(m_i^2x+\Lambda_i^2y)}\right]$$
$$+\frac{\Lambda_i^4xy}{(\Lambda_i^2-m_i^2)^2}\left[\frac{m_i^2}{\Lambda_i^2}(1+2y)-\frac{m_i^2(\Lambda_i^2+m_i^2)}{(\Lambda_i^2x+m_i^2y)(m_i^2x+\Lambda_i^2y)}+2x-1\right], \quad \text{(A10)}$$

with $y=1-x$ and $i,\ j=u,\ d$. Performing the integration with respect to the Feynman parameter x in the Eqs. (A4)–(A6), we obtain the expressions

$$T = \frac{N_c}{8\pi\sin^2\theta_W\cos^2\theta_W M_Z^2}\Big[T_1(\Lambda_u,\Lambda_d,m_u,m_d)+T_1(\Lambda_d,\Lambda_u,m_d,m_u)-T_2(\Lambda_u,\Lambda_d,m_u,m_d)-T_2(\Lambda_d,\Lambda_u,m_d,m_u)\Big], \quad \text{(A11)}$$

$$U = \frac{N_c}{6\pi}\Big[U_1(\Lambda_u,\Lambda_d)-U_1(\Lambda_u,m_d)-U_1(m_u,\Lambda_d)+U_1(m_u,m_d)+U_2(\Lambda_u,\Lambda_d,m_u,m_d)+U_2(\Lambda_d,\Lambda_u,m_d,m_u)$$
$$+U_3(\Lambda_u,\Lambda_d,m_u,m_d)+U_3(\Lambda_d,\Lambda_u,m_d,m_u)-U_4(\Lambda_u,\Lambda_d,m_u,m_d)-U_5(\Lambda_u,m_u)-U_5(\Lambda_d,m_d)\Big], \quad \text{(A12)}$$

$$S = \frac{N_c}{6\pi}\Big[S_1(\Lambda_u,m_u)+S_1(\Lambda_d,m_d)-Q_uS_2(\Lambda_u,m_u)+Q_dS_2(\Lambda_d,m_d)\Big], \quad \text{(A13)}$$

where

$$T_1(\Lambda_i,\Lambda_j,m_i,m_j) = \frac{\Lambda_i^6\Lambda_j^4}{(\Lambda_i^2-\Lambda_j^2)^2(\Lambda_i^2-m_i^2)(\Lambda_j^2-m_j^2)}\left[1-\left(\frac{2\Lambda_j^2}{\Lambda_i^2-\Lambda_j^2}+\frac{\Lambda_i^2}{\Lambda_i^2-m_i^2}+\frac{\Lambda_i^2}{\Lambda_i^2-m_j^2}\right)\ln(\Lambda_i^2)\right]$$
$$+\frac{\Lambda_i^4\Lambda_j^4m_i^4\ln(m_i^2)}{(\Lambda_i^2-m_i^2)^2(\Lambda_j^2-m_i^2)^2(m_i^2-m_j^2)}, \quad \text{(A14)}$$

$$T_2(\Lambda_i,\Lambda_j,m_i,m_j) = \frac{\Lambda_i^6}{(\Lambda_i^2-m_i^2)^4}\left[\frac{1}{6}(\Lambda_i^4+10\Lambda_i^2m_i^2+m_i^4)+\frac{\Lambda_i^2m_i^2(\Lambda_i^2+m_i^2)}{\Lambda_i^2-m_i^2}\ln\left(\frac{m_i^2}{\Lambda_i^2}\right)\right], \quad \text{(A15)}$$

$$U_1(\lambda_i,\lambda_j) = \frac{\Lambda_u^4\Lambda_d^4}{(\Lambda_u^2-m_u^2)^2(\Lambda_d^2-m_d^2)^2}\left[\frac{(3\lambda_i^2-\lambda_j^2)\lambda_j^4}{(\lambda_i^2-\lambda_j^2)^3}\ln\left(\frac{\lambda_j^2}{\lambda_i^2}\right)+\frac{2\lambda_i^2\lambda_j^2}{(\lambda_i^2-\lambda_j^2)^2}\right], \quad \text{(A16)}$$

$$U_2(\Lambda_i,\Lambda_j,m_i,m_j) = \frac{\Lambda_u^4\Lambda_d^4}{(\Lambda_u^2-m_u^2)^2(\Lambda_d^2-m_d^2)^2}\left\{\frac{\Lambda_i^2-m_i^2}{(\Lambda_i^2-\Lambda_j^2)^3}\left[-\Lambda_i^4+5\Lambda_i^2\Lambda_j^2+2\Lambda_j^4+\frac{6\Lambda_i^2\Lambda_j^4}{\Lambda_i^2-\Lambda_j^2}\ln\left(\frac{\Lambda_j^2}{\Lambda_i^2}\right)\right]\right\}, \quad \text{(A17)}$$

$$U_3(\Lambda_i,\Lambda_j,m_i,m_j) = \frac{\Lambda_u^4\Lambda_d^4}{(\Lambda_u^2-m_u^2)^2(\Lambda_d^2-m_d^2)^2}\left\{\frac{\Lambda_i^2-m_i^2}{(\Lambda_i^2-m_j^2)^3}\left[\Lambda_i^4-5\Lambda_i^2m_j^4-2m_j^4-\frac{6\Lambda_i^2m_j^4}{\Lambda_i^2-m_j^2}\ln\left(\frac{m_j^2}{\Lambda_i^2}\right)\right]\right\}, \quad \text{(A18)}$$

$$U_4(\Lambda_i,\Lambda_j,m_i,m_j)=\frac{2\Lambda_i^4\Lambda_j^4}{(\Lambda_i^2-\Lambda_j^2)^4(\Lambda_i^2-m_i^2)(\Lambda_j^2-m_j^2)}\left[\Lambda_i^4+\Lambda_j^4+10\Lambda_i^2\Lambda_j^2+\frac{6\Lambda_i^2\Lambda_j^2(\Lambda_i^2+\Lambda_j^2)}{\Lambda_i^2-\Lambda_j^2}\ln\left(\frac{\Lambda_j^2}{\Lambda_i^2}\right)\right], \quad (A19)$$

$$U_5(\Lambda_i,m_i)=\frac{\Lambda_i^8}{2(\Lambda_i^2-m_i^2)^4}\left[\frac{\Lambda_i^6-3\Lambda_i^4m_i^2-15\Lambda_i^2m_i^4+m_i^6}{(\Lambda_i^2-m_i^2)^3}\ln\left(\frac{m_i^2}{\Lambda_i^2}\right)+\frac{2(\Lambda_i^4-7\Lambda_i^2m_i^2-2m_i^4)}{(\Lambda_i^2-m_i^2)^2}+\frac{7m_i^2}{5\Lambda_i^2}-\frac{m_i^4}{5\Lambda_i^4}+\frac{7}{15}\right], \quad (A20)$$

$$S_1(\Lambda_i,m_i)=\frac{\Lambda_i^8}{(\Lambda_i^2-m_i^2)^4}\left[\frac{\Lambda_i^6-3\Lambda_i^4m_i^2-15\Lambda_i^2m_i^4+m_i^6}{(\Lambda_i^2-m_i^2)^3}\ln\left(\frac{m_i^2}{\Lambda_i^2}\right)+\frac{2(\Lambda_i^4-7\Lambda_i^2m_i^2-2m_i^4)}{(\Lambda_i^2-m_i^2)^2}+\frac{7m_i^2}{5\Lambda_i^2}-\frac{m_i^4}{5\Lambda_i^4}+\frac{7}{15}\right], \quad (A21)$$

$$S_2(\Lambda_i,m_i)=\frac{6\Lambda_i^4}{(\Lambda_i^2-m_i^2)^2}\left[\frac{\Lambda_i^6-3\Lambda_i^4m_i^2-12\Lambda_i^2m_i^4+2m_i^6}{3(\Lambda_i^2-m_i^2)^3}\ln\left(\frac{m_i^2}{\Lambda_i^2}\right)-\frac{(3\Lambda_i^2+m_i^2)m_i^2}{(\Lambda_i^2-m_i^2)^2}+\frac{m_i^2}{3\Lambda_i^2}+\frac{5}{6}\right]. \quad (A22)$$

Note that Eqs. (A11)–(A13) involve the contributions from the ordinary radiative corrections in the standard model.

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