$SU(4)_L \otimes U(1)_N$ model for the electroweak interactions

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(Received 6 September 1994)

Assuming the existence of right-handed neutrinos, we consider an electroweak model based on the gauge symmetry $SU(4)_L \otimes U(1)_N$. We study the neutral currents coupled to all neutral vector bosons present in the theory. There are no flavor-changing neutral currents at the tree level, coupled with the lightest neutral vector boson.

PACS number(s): 12.60.Cn

Symmetry principles have been used in elementary particle physics at least since the discovery of the neutron. A symmetry is useful to both issues: the classification of particles and the dynamics of the interactions among them. The point is that there must be a part of the particle spectrum in which the symmetry manifests itself at least in an approximate way. This is the case for quarks u and d and in the leptonic sector for the electron-neutrino and electron. For instance, the SU(2) appears as an approximate symmetry in the doublets $(\nu_e, e)^T$. If one assumes this symmetry among these particles and in the sequential families as well, almost all the model's predictions are determined.

The full symmetry of the so called standard model is the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. This model spectacularly explains all the available experimental data [1]. Usually it is considered that this symmetry emerges at low energies as a result of the breaking of higher symmetries. Probably, these huge symmetries are an effect of grand unified scenarios and/or their supersymmetric extensions.

Considering the lightest particles of the model as the sector in which a symmetry is manifested, it is interesting that the lepton sector could be the part of the model determining new approximate symmetries. For instance, ν, e, and e^c could be in the same triplet of an $SU(3)_L \otimes U(1)_N$ symmetry. This sort of model has been proposed recently [2]. In this case neutrinos can remain massless in arbitrary order in perturbation theory, or they get a mass in some modifications of the models [3]. If we admit that right-handed neutrinos do exist, it is possible to build a model in which ν^c, ν, and e are in the same multiplet of SU(3) [4]. In fact, if right-handed neutrinos are introduced it is a more interesting possibility to have ν, e, ν^c, and e^c in the same multiplet of an $SU(4)_L \otimes U(1)_N$ electroweak theory.

Notice that using the lightest leptons as the particles which determine the approximate symmetry, *if each generation is treated separately*, SU(4) is the highest symmetry group to be considered in the electroweak sector. A model with the $SU(4) \otimes U(1)$ symmetry in the lepton sector was suggested some years ago in Ref. [5]. However, quarks were not considered there. This symmetry in both quarks and leptons was pointed out recently [6] and here we will consider the details of such a model.

Hence, our model has the full symmetry $SU(3)_C \otimes SU(4)_L \otimes U(1)_N$. These sort of models are anomaly-free only if there are equal number of **4** and **4*** (considering the color degrees of freedom), and furthermore requiring the sum of all fermion charges to vanish. Two of the three quark generations transform identically and one generation, it does not matter which one, transforms in a different representation of $SU(4)_L \otimes U(1)_N$ [7]. This means that in these models as in the $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ ones [2], in order to cancel anomalies, the number of families (N_f) must be divisible by the number of color degrees of freedom (n). Hence the simplest alternative is $n = N_f = 3$. On the other hand, at low energies these models are indistinguishable from the standard model.

The electric charge operator is defined as

$$Q = \frac{1}{2}\left(\lambda_3 - \frac{1}{\sqrt{3}}\lambda_8 - \frac{2}{3}\sqrt{6}\lambda_{15}\right) + N, \tag{1}$$

where the λ matrices are [8]

$$\lambda_3 = \mathrm{diag}(1,-1,0,0), \quad \lambda_8 = \tfrac{1}{\sqrt{3}}\mathrm{diag}(1,1,-2,0),$$
$$\lambda_{15} = \tfrac{1}{\sqrt{6}}\mathrm{diag}(1,1,1,-3).$$

Leptons transform as $(\mathbf{1},\mathbf{4},0)$, one generation, say Q_{1L}, transforms as $(\mathbf{3},\mathbf{4},+2/3)$ and the other two quark families, say $Q_{\alpha L}$, $\alpha = 2,3$, transform as $(\mathbf{3},\mathbf{4}^*,-1/3)$:

$$f_{iL} = \begin{pmatrix} \nu_i \\ l_i \\ \nu_i^c \\ l_i^c \end{pmatrix}_L, \quad Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ u' \\ J \end{pmatrix}_L, \quad Q_{\alpha L} = \begin{pmatrix} j_\alpha \\ d'_\alpha \\ u_\alpha \\ d_\alpha \end{pmatrix}_L, \tag{2}$$

where $i = e, \mu, \tau$; u' and J are new quarks with charge $+2/3$ and $+5/3$, respectively; j_α and d'_α are new quarks with charge $-4/3$ and $-1/3$, respectively. We recall that in Eq. (2) all fields are still symmetry eigenstates. Right-handed quarks transform as singlets under $SU(4)_L \otimes U(1)_N$.

Quark masses are generated by introducing the following Higgs $SU(3)_C \otimes SU(4)_L \otimes U(1)_N$ multiplets: $\eta \sim (\mathbf{1},\mathbf{4},0)$, $\rho \sim (\mathbf{1},\mathbf{4},+1)$, and $\chi \sim (\mathbf{1},\mathbf{4},-1)$,

$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta_1^- \\ \eta_2^0 \\ \eta_2^+ \end{pmatrix}, \rho = \begin{pmatrix} \rho_1^+ \\ \rho^0 \\ \rho_2^+ \\ \rho^{++} \end{pmatrix}, \chi = \begin{pmatrix} \chi_1^- \\ \chi^{--} \\ \chi_2^- \\ \chi^0 \end{pmatrix}. \tag{3}$$

In order to obtain massive charged leptons it is necessary to introduce a $(\mathbf{1},\mathbf{10}^*,0)$ Higgs multiplet, because the lepton mass term transforms as $\overline{f_L^c}f_L \sim (\mathbf{1},\mathbf{6}_A\oplus\mathbf{10}_S)$. The $\mathbf{6}_A$ will leave one lepton massless two others degenerate for three generations. Therefore we will choose the $H=\mathbf{10}_S$. Explicitly, up to a convenient normalization

$$H=\begin{pmatrix} H_1^0 & H_1^+ & H_2^0 & H_2^- \\ H_1^+ & H_1^{++} & H_3^+ & H_3^0 \\ H_2^0 & H_3^+ & H_4^0 & H_4^- \\ H_2^- & H_3^0 & H_4^- & H_2^{--} \end{pmatrix}. \tag{4}$$

If $\langle H_3^0\rangle \neq 0$, $\langle H_{1,2,4}^0\rangle = 0$ the charged leptons get a mass but neutrinos remain massless, at least at tree level. In order to avoid mixing among primed and unprimed quarks we can introduce another multiplet η' transforming as η but with different vacuum expectation value (VEV). The corresponding VEV's are $\langle\eta\rangle = (v/\sqrt{2},0,0,0)$, $\langle\rho\rangle = (0,u/\sqrt{2},0,0)$, $\langle\eta'\rangle = (0,0,v'/\sqrt{2},0)$, $\langle\chi\rangle = (0,0,0,w/\sqrt{2})$, and $\langle H\rangle_{24} = \langle H_3^0\rangle = v''/\sqrt{2}$ for the decouplet. In this way we have that the symmetry breaking of the $\mathrm{SU}(4)_L\otimes \mathrm{U}(1)_N$ group down to $\mathrm{SU}(3)_L\otimes \mathrm{U}(1)_{N'}$ is induced by the χ Higgs boson. The $\mathrm{SU}(3)_L\otimes \mathrm{U}(1)_{N'}$ symmetry is broken down into $\mathrm{U}(1)_{\mathrm{em}}$ by the ρ,η,η', and H Higgs boson. As in model I of Ref. [4], it is necessary to introduce some discrete symmetries which ensure that the Higgs fields give an appropriate quark mass matrix in the charge $-1/3$ and $2/3$ sectors of the direct sum form in order to avoid general mixing among quarks of the same charge. In this case the quark mass matrix can be diagonalized with unitary matrices which are themselves direct sum of unitary matrices. These symmetries also ensure the appropriate VEV's chosen above.

The Yukawa interactions are

$$\begin{aligned}-\mathcal{L}_Y = {}& \tfrac{1}{2}G_{ij}\overline{f_{iL}^c}f_{jL}H + F_{1k}\bar{Q}_{1L}u_{kR}\eta + F_{\alpha k}\bar{Q}_{\alpha L}u_{kR}\rho^* \\ &+F'_{1k}\bar{Q}_{1L}d_{kR}\rho + F'_{\alpha k}\bar{Q}_{\alpha L}d_{kR}\eta^* \\ &+h_1\bar{Q}_{1L}u'_R\eta' + h_{\alpha\beta}\bar{Q}_{\alpha L}d'_{\beta R}\eta'^* \\ &+\Gamma_1\bar{Q}_{1L}J_R\chi + \Gamma_{\alpha\beta}\bar{Q}_{\alpha L}j_{\beta L}\chi^* + \mathrm{H.c.},\end{aligned} \tag{5}$$

where $i,j=e,\mu,\tau$; $k=1,2,3$; and $\alpha,\beta=2,3$. We recall that up to now all fields are weak eigenstates.

The electroweak gauge bosons of this theory consist of a $\mathbf{15}$, W_μ^i, $i=1,\ldots,15$ associated with $\mathrm{SU}(4)_L$ and a singlet B_μ associated with $\mathrm{U}(1)_N$.

The gauge bosons $-\sqrt{2}W^+ = W^1 - iW^2$, $-\sqrt{2}V_1^- = W^6 - iW^7$, $-\sqrt{2}V_2^- = W^9 - iW^{10}$, $-\sqrt{2}V_3^- = W^{13} - iW^{14}$, $-\sqrt{2}U^{--} = W^{11} - iW^{12}$, and $\sqrt{2}X^0 = W^4 + iW^5$ have masses

$$\begin{aligned} M_W^2 &= \frac{g^2}{4}(v^2+u^2+2v''^2), \\ M_{V_1}^2 &= \frac{g^2}{4}(v'^2+u^2+2v''^2), \end{aligned} \tag{6a}$$

$$\begin{aligned} M_{V_2}^2 &= \frac{g^2}{4}(v^2+w^2+2v''^2), \\ M_{V_3}^2 &= \frac{g^2}{4}(v'^2+w^2+2v''^2), \end{aligned} \tag{6b}$$

$$M_X^2 = \frac{g^2}{4}(v^2+v'^2), \quad M_U^2 = \frac{g^2}{4}(u^2+w^2+8v''^2). \tag{6c}$$

The mass matrix for the neutral vector bosons (up to a factor $g^2/4$) in the W^3, W^8, W^{15}, B basis is

$$\begin{pmatrix} v^2+u^2+2v''^2 & \frac{1}{\sqrt{3}}(v^2-u^2-2v''^2) & \frac{1}{\sqrt{6}}(v^2-u^2+4v''^2) & -2tu^2 \\ \frac{1}{\sqrt{3}}(v^2-u^2-2v''^2) & \frac{1}{3}(v^2+4v'^2+u^2+2v''^2) & \frac{1}{3\sqrt{2}}(v^2-2v'^2+u^2-4v''^2) & \frac{2}{\sqrt{3}}tu^2 \\ \frac{1}{\sqrt{6}}(v^2-u^2+4v''^2) & \frac{1}{3\sqrt{2}}(v^2-2v'^2+u^2-4v''^2) & \frac{1}{6}(v^2+v'^2+u^2+9w^2+8v''^2) & \frac{2}{\sqrt{6}}t(u^2+3w^2) \\ -2tu^2 & \frac{2}{\sqrt{3}}tu^2 & \frac{2}{\sqrt{6}}t(u^2+3w^2) & 4t^2(u^2+w^2) \end{pmatrix} \tag{7}$$

where $t\equiv g'/g$. The matrix in (7) has determinant equal to zero as it must be in order to have a massless photon. There are four neutral bosons: a massless γ and three massive ones Z,Z',Z'' such that $M_Z < M_{Z'} < M_{Z''}$. The lightest one, say Z, corresponds to the neutral boson of the standard model.

The photon field is

$$A_\mu = \frac{1}{(1+4t^2)^{1/2}}\left(tW_\mu^3 - \frac{t}{\sqrt{3}}W_\mu^8 - \frac{2\sqrt{6}}{3}tW_\mu^{15} + B_\mu\right), \tag{8}$$

with the electric charge defined as

$$|e| = \frac{gt}{(1+4t^2)^{1/2}} = \frac{g'}{(1+4t^2)^{1/2}}. \tag{9}$$

In the following, we will use the approximation $v=u=v''\equiv v_1 \ll v'=w\equiv v_2$. In this approximation the three nonzero masses are given by [9]

$$M_n^2 \approx \frac{g^2}{4}\lambda_n v_2^2, \quad n=0,1,2, \tag{10}$$

where

$$\lambda_n = \frac{1}{3}\left[A + 2\left(A^2+3B\right)^{1/2}\cos\left(\frac{2n\pi+\Theta}{3}\right)\right], \tag{11a}$$

$$\begin{aligned} A &= 3+4t^2+(7+4t^2)a^2, \\ B &= -2[1+3t^2+2(4+9t^2)a^2], \end{aligned} \tag{11b}$$

$$C = 8(1+4t^2)a^2, \quad \Theta = \arccos\left[\frac{2A^3+9AB+27C}{2(A^2+3B)^{\frac{3}{2}}}\right], \tag{11c}$$

and we have defined $a \equiv v_1/v_2$. The respective eigenvectors are

$$Z_{n\mu} \approx x_n W^3_\mu + y_n W^8_\mu + z_n W^{15}_\mu + w_n B, \tag{12}$$

with

$$x_n = -\frac{2a^2}{t}\frac{1-3t^2+(1-t^2)a^2-(1-2t^2)\lambda_n}{D_n(t,a)}\,w_n, \tag{13a}$$

$$y_n = \frac{1}{\sqrt{3}t}\frac{2(2+t^2)a^2-10a^4t^2-[1+(1-4t^2)a^2]\lambda_n}{D_n(t,a)}\,w_n, \tag{13b}$$

$$z_n = \frac{1}{\sqrt{6}t}\{8(2+t^2)a^2+4(3+2t^2)a^4 -4[1+2(2+t^2)a^2]\lambda_n+3\lambda_n^2\}w_n/D_n(t,a) \tag{13c}$$

where

$$D_n(t,a) = 2(7+5a^2)-(3+13a^2)\lambda_n+2\lambda_n^2, \tag{13d}$$

and w_n is a function of t and a determined by the normalization condition. The hierarchy of the masses is $M_0 > M_2 > M_1$, i.e., we can make $M_0 \equiv M_{Z''}$, $M_2 \equiv M_{Z'}$, and $M_1 \equiv M_Z$. Hence it is possible to identify the eigenvector with $n=1$ as being the neutral vector boson of the standard electroweak model. Actually, we have checked numerically that $M_Z^2/M_W^2 = 1.303$, when $t = 1.82$ and for any $a \le 0.01$. This is just the value of the standard electroweak model when $\sin^2\theta_W = 0.2325$.

The weak neutral currents have been, up to now, an important test of the standard model. In particular it has been possible to determine the fermion couplings, so far all experimental data are in agreement with the model. In the present model, the neutral currents of a given fermion ψ couple to the Z_n neutral boson are

$$\mathcal{L}^{\rm NC} = -\frac{g}{2c_W}\sum_n[\bar\psi_L\gamma^\mu\psi_L L^\psi_n + \bar\psi_R\gamma^\mu\psi_R R^\psi_n]Z_{n\mu} \tag{14}$$

where $c_W \equiv \cos\theta_W$. From Eq. (9) and assuming that $e = gs_W$ (but $e = g'[c_W^2-3s_W^2]^{1/2}$) it follows that $c_W^2 = (1+3t^2)/(1+4t^2)$. The coefficients in Eq. (14) are

$$L^{u_1}_n = -c_W\left(x_n+\frac{1}{\sqrt{3}}y_n+\frac{1}{\sqrt{6}}z_n+\frac{4}{3}w_nt\right), \tag{15a}$$

$$L^{u_\alpha}_n = -c_W\left(x_n-\frac{1}{\sqrt{3}}y_n-\frac{1}{\sqrt{6}}z_n-\frac{2}{3}w_nt\right), \tag{15b}$$

$$L^{u'}_n = -c_W\left(-\frac{2}{\sqrt{3}}y_n+\frac{1}{\sqrt{6}}z_n+\frac{4}{3}w_nt\right), \tag{15c}$$

$$R^{u_1}_n = R^{u_\alpha}_n = R^{u'}_n = -\frac{4}{3}c_W\,w_n\,t\,, \tag{16}$$

for the charge 2/3 quarks, and

$$L^{d_1}_n = -c_W\left(-x_n+\frac{1}{\sqrt{3}}y_n+\frac{1}{\sqrt{6}}z_n+\frac{4}{3}w_nt\right), \tag{17a}$$

$$L^{d_\alpha}_n = -c_W\left(-x_n-\frac{1}{\sqrt{3}}y_n-\frac{1}{\sqrt{6}}z_n-\frac{2}{3}w_nt\right), \tag{17b}$$

$$L^{d'_\alpha}_n = -c_W\left(\frac{2}{\sqrt{3}}y_n-\frac{1}{\sqrt{6}}z_n-\frac{2}{3}w_nt\right), \tag{17c}$$

$$R^{d_1}_n = R^{d_\alpha}_n = R^{d'_\alpha}_n = \frac{2}{3}c_W\,w_n\,t, \tag{18}$$

for the charge $-1/3$ quarks.

Notice that the coefficients L^ψ_n and R^ψ_n given above, Eqs. (15)–(18), summarize all the neutral couplings in the model: $n = 0,1,2$ give the coefficients corresponding to the Z'', Z, and Z', respectively.

We have checked numerically that in fact only for $n=1$ do we have

$$L^{u_1}_1 \approx L^{u_2}_1 \approx L^{u_3}_1 \neq L^{u'}_1 \tag{19a}$$

and

$$L^{d_1}_1 \approx L^{d_2}_1 \approx L^{d_3}_1 \neq L^{d'_2}_1 = L^{d'_3}_1. \tag{19b}$$

Hence, we can introduce a discrete symmetry, as in Model I of Ref. [4], in order to obtain a mass matrix which does not mix u_k with u' and d_k with d'_α, $k = 1,2,3$ and $\alpha = 2,3$. We see that in this case the Glashow-Iliopoulos-Maiani (GIM) mechanism [10] is implemented, at the tree level, in the $Z_1(\equiv Z^0)$ couplings. We must stress that, if the new quarks u' and d'_α are very heavy, the requirements for natural (independent of mixing angles) flavor conservation in the neutral currents to order αG_F [11] break down, and it should be necessary to impose the restriction that the mixing angles between ordinary and the new heavy quarks must be very small [12].

It is useful to define the coefficients $V = \frac{1}{2}(L+R)$ and $A = \frac{1}{2}(L-R)$. In the standard electroweak model, at tree level, we have $V^{\rm SM}_\psi = t_{3L_\psi} - 2Q_\psi\sin^2\theta_W$ and $A^{\rm SM}_\psi = t_{3L_\psi}$, where t_{3L_ψ} is the weak isospin of the fermion ψ: $+1/2$ for u_k and ν_i; $-1/2$ for d_k and l_i. Hence, we have $V^{\rm SM}_U \approx 0.19$ and $A^{\rm SM}_U = 0.5$ for the charge 2/3 sector, and $V^{\rm SM}_D \approx -0.345$ and $A^{\rm SM}_D = -0.5$ for the charge $-1/3$ sector. In our model, also at tree level, using $t = 1.82$ (which is the value for t obtained when $\sin^2\theta_W = 0.2325$) and for any $a \le 0.01$, we obtain $V_U \approx 0.19$, $A_U \approx 0.5$ for u_k, and $V_D \approx -0.345$, $A_D \approx -0.5$ for d_k. We see that the values are in agreement with the values of the standard model. On the other hand $V_{u'} \approx -0.310$, $A_{u'} \approx 0$ and $V_{d'_\alpha} \approx 0.15$, $A_{d'_\alpha} \approx 0$.

For leptons we have

$$L^\nu_n = L^{u_1}_n + \frac{4}{3}c_Ww_nt,\ R^\nu_n = -L^{u'}_n - \frac{4}{3}c_Ww_nt,$$
$$L^l_n = L^{d_1}_n + \frac{4}{3}c_Ww_nt,\ R^l_n = -\frac{3}{\sqrt{6}}z_n. \tag{20}$$

For $a \leq 0.01$ and $t = 1.82$, we obtain $V_\nu \approx 0.5$, $A_\nu \approx 0.5$, and $V_l \approx -0.035$, $A_l \approx -0.5$ which are also in agreement with the values of the standard electroweak model.

On the other hand, there are flavor-changing neutral currents (FCNC's) in the quark sector coupled to Z' and Z''. To show this let us rewrite the neutral current interactions given in Eq. (14) as (for the charge $-1/3$ sector)

$$\mathcal{L}^{\rm NC} = -\frac{g}{c_W}\sum_n \left(\bar{D}'_L\gamma^\mu Y^D_{nL}D'_L + \bar{D}'_R\gamma^\mu Y^D_{nR}D'_R\right) Z_{n\mu} \tag{21}$$

where $D' = (d_1, d_2, d_3)^T$ are the symmetry eigenstates. We recall again that $n = 0, 1, 2$ correspond to Z'', Z, and Z', respectively; we have defined

$$\begin{aligned} Y^D_{nL} &= \mathrm{diag}(L^{d_1}_n, L^{d_2}_n, L^{d_3}_n)\,, \\ Y^D_{nR} &= \mathrm{diag}(R^{d_1}_n, R^{d_2}_n, R^{d_3}_n)\,, \end{aligned} \tag{22}$$

and $L^{d_i}_n$, $R^{d_i}_n$ appear in Eqs. (15) – (18). We have shown above that for $n = 1$, $Y^D_{1L} \propto \mathbf{1}$. However, for $n = 0, 2$ for any value of t and a all L^ψ_n (R^ψ_n) are different. For example, we obtain, using $t = 1.82$, $a = 0.01$,

$$\begin{aligned} L^{d_1}_0 \approx -2.12, \quad L^{d_2}_0 = L^{d_3}_0 \approx 1.12, \\ R^{d_1}_0 = R^{d_2}_0 = R^{d_3}_0 \approx 1.01, \end{aligned} \tag{23}$$

$$\begin{aligned} L^{d_1}_2 \approx -0.61, \quad L^{d_2}_2 = L^{d_3}_2 \approx 0.49, \\ R^{d_1}_2 = R^{d_2}_2 = R^{d_3}_2 \approx 0.12. \end{aligned} \tag{24}$$

Hence, there are FCNC effects due to Z' and Z'' which cannot naturally be excluded. However, when we turn the quarks to be the mass eigenstates by making the biunitary transformations $D'_L = V^D_L D_L$ and $D'_R = V^D_R D_R$ it results in

$$Y^D_{nL} \to V^{D\dagger}_L Y^D_{nL} V^D_L, \quad Y^D_{nR} \to V^{D\dagger}_R Y^D_{nR} V^D_R\ . \tag{25}$$

We recall that a discrete symmetry avoids mixing between d_k and d'_α.

Since $Y^D_{1L} \propto \mathbf{1}$ the respective neutral currents conserve flavor. For $n = 0, 2$ since Y^D_{0L} , Y^D_{2L} are not proportional to the unit matrix there are FCNC effects. However, these couplings involve mixing angles which, in general, do not coincide with the Cabibbo-Kobayashi-Maskawa parameters which are defined as $V_{\rm CKM} = V^{U\dagger}_L V^D_L$ in the W interactions. On the other hand, there are no FCNC effects in the right-handed currents since $Y^{U,D}_{nR} \propto \mathbf{1}$ for any value of n. A similar analysis can be made for the u-like quarks. Hence, no bounds on the Z', Z'' masses arise from FCNC processes without any additional assumption concerning these parameters [7]. The condition $\sin^2\theta_W < 1/4$ will imply masses for Z', Z'' of the order of some TeV's [13].

Finally, we write down the charged current interactions in terms of the symmetry eigenstates. In the leptonic sector they are

$$\begin{aligned} \mathcal{L}^{\rm CC}_l = -\frac{g}{\sqrt{2}}\Big[&\bar{\nu}_L\gamma^\mu l_L W^+_\mu + \overline{\nu^c_L}\gamma^\mu l_L V^+_{1\mu} + \overline{l^c_L}\gamma^\mu \nu_L V^+_{2\mu} \\ &+\overline{l^c_L}\gamma^\mu \nu^c_L V^+_{3\mu} + \overline{l^c_L}\gamma^\mu l_L U^{++}_\mu\Big] + \text{H.c.}, \end{aligned} \tag{26}$$

and the interaction $\frac{1}{2}g\bar{\nu}^c_L\gamma^\mu\nu_L X^0$.

In the quark sector we obtain

$$\begin{aligned} \mathcal{L}^{\rm CC}_Q = -\frac{g}{\sqrt{2}}\Big[&\bar{u}_{kL}\gamma^\mu d_{kL}W^+_\mu + (\bar{u}'_L\gamma^\mu d_{1L} + \bar{u}_{\alpha L}\gamma^\mu d'_{\alpha L})\,V^+_{1\mu} + \left(\bar{J}_L\gamma^\mu u_{1L} + \bar{d}_{\alpha L}\gamma^\mu j_{\alpha L}\right) V^+_{2\mu} \\ &+ \left(\bar{J}_L\gamma^\mu u'_L + \bar{d}'_{\alpha L}\gamma^\mu j_{\alpha L}\right) V^+_{3\mu} + \left(\bar{J}_L\gamma^\mu d_{1L} - \bar{u}_{\alpha L}\gamma^\mu j_{\alpha L}\right) U^{++}_\mu\Big] + \text{H.c.}, \end{aligned} \tag{27}$$

where $k = 1, 2, 3$; $\alpha = 2, 3$. We have also interactions via X^0 among primed and unprimed quarks of the same charge as $\bar{u}'_L\gamma^\mu u_L$ and so on.

Notice that it is not necessary to have a strong fine tuning in the V's and A's coefficients in order to be in agreement with phenomenology. Although both types of coefficients are complicated functions of $\sin^2\theta_W$ and of the ratio $a \equiv v_1/v_2$, once the observed value for $\sin^2\theta_W = 0.2325$ [1] has been chosen, there is a wide range of allowed values for a. We recall that the neutral current data impose the strongest constraints to the physics beyond the standard model. Since in our model the GIM mechanism is implemented, this suggests that the model may be consistent with the charged currents phenomenology as well. At low energies the model coincides with the SU(2)⊗ U(1) model and it will be interesting, from the phenomenological point of view, to study the constraints on the masses of the extra particles and the respective mixing angles which do not exist in the standard model.

At the tree level neutrinos are still massless but they will get a calculable mass through radiative corrections. In this kind of model it is possible to implement the Voloshin's mechanism, i.e., in the limit of exact symmetry, a magnetic moment for the neutrino is allowed, and a mass is forbidden [5].

One of us (F.P.) would like to thank Coordenadoria de Aperfeiçoamento de Pessoal de Ensino Superior (CAPES) for full financial support. This work was also partially financed by Conselho Nacional de Desenvolvimento Científico e Tecnológico. We also would like to thank M. D. Tonasse for useful discussions.

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