

New physics motivated by the low energy approach to electric charge quantization

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The low-energy approach to electric charge quantization predicts physics beyond the minimal standard model. A model-independent approach via effective Lagrangians is used to examine the possible new physics, which may manifest itself indirectly through family-lepton-number-violating rare decays.

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The observed quantization of electric charge has long been a profound puzzle in physics. The fact that hydrogen atoms and neutrons are electrically neutral (to within experimental precision) helps shape the physics of the everyday world. Electric charge quantization is thus a most important fact of nature, and our understanding would be seriously incomplete if we could not fathom why all particles carry integer multiples of the down-quark charge.

Until the last few years, grand unification of the strong, weak, and electromagnetic interactions seemed to be the most likely way electric charge quantization would eventually be understood. Unfortunately, grand unified theories are difficult to test experimentally because of the extremely high energy scales involved (typically 10^{14-16} GeV). It is possible some hint of grand unification such as proton decay may surface at any time if we are fortunate. However, even if proton decay were to be discovered it would really only tell us that baryon number is not conserved, and there are many ways to violate baryon number conservation without invoking grand unification. In general, any such evidence we may find will be at best indirect and suggestive rather than compelling.

It is therefore important to explore ways of understanding electric charge quantization that do not involve physics at largely inaccessible energy scales. In recent years, a simple approach to the problem based on the classical and quantal gauge invariance of the standard model (SM) Lagrangian has been explicated in the literature. This work has shown that the oft-quoted proclamation that the SM sheds no light on electric charge quantization is wrong, and it provides hope that this puzzle can be solved in the foreseeable future. Importantly, the low-energy approach to electric charge quantization predicts that physics beyond the minimal SM is required for our understanding to be complete. The task of this paper is to introduce a model-independent strategy via

effective Lagrangians for thinking about what this new physics might be.

We will not repeat the precise details of the low-energy electric charge quantization calculations here because they can be easily accessed through review articles and the original papers [1]. However, by way of reminder let us go through the main steps in the analysis for the SM.

(i) We first write down the multiplet assignments of all the particles in the theory under the non-Abelian $SU(3)_c \otimes SU(2)_L$ part of the gauge group. All of the weak hypercharge or Y quantum numbers are left as arbitrary parameters.

(ii) We use the arbitrary normalization of $U(1)$ charges to rescale the hypercharge of the Higgs doublet ϕ to be 1. This is purely a matter of convenience. We then use $SU(2)_L$ gauge invariance to write the vacuum expectation value (VEV) of ϕ in the conventional form $\langle \phi \rangle = (0, v)^T$. The electric charge generator Q is then defined to be that linear combination of I_{3L} and Y which annihilates $\langle \phi \rangle$, where I_{3L} is the diagonal generator of $SU(2)_L$. We find that $Q = I_{3L} + Y/2$ where we have again assigned the arbitrary normalization of Q to conform with convention. We now know what electric charge is, so we can begin to discuss its quantization. This means we have to establish the quantization of Y .

(iii) Particle physics up to about 100 GeV is well described by the SM Lagrangian. So we now write down this Lagrangian, which forces us to relate some of the hypercharges of the fermions in order to ensure the gauge invariance of the Yukawa interaction terms. In the case of the SM, some arbitrary hypercharges remain.

(iv) Gauge anomaly cancellation is now imposed in order to protect the gauge symmetry against quantal breaking. This further constrains the hypercharge parameters.

No further sensible constraints exist. If the above procedures were enough to force all hypercharge parameters to take on unique values we would conclude that the construction of the theory would only be possible if hypercharge and hence electric charge quantization were to hold. This is the sense in which electric charge quantization would be understood. If all the parameters were to turn out to be fixed, then a major feature of the theory such as gauge invariant Yukawa coupling terms or

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anomaly cancellation would have to be sacrificed if one wanted to dequantize electric charge. Since this would be too high a price to pay, we would deem electric charge quantization to be understood by consistency with the rest of known particle physics. However, if the procedures above were to fail in determining all of the hypercharge parameters, then electric charge quantization would not be a necessary consequence of the construction of the model.

What happens in the construction of the minimal SM (that is, the SM without right-handed neutrinos)? If only one generation of fermions is considered, then the above procedure is sufficient to fix all of the hypercharge param-

eters. Electric charge quantization is thus understood. However, in the realistic case of three generations, one hypercharge parameter remains undetermined. Electric charge quantization is thus not understood, although the form of electric charge dequantization is severely constrained.

To be specific, weak hypercharge and hence electric charge can be dequantized in three similar but mutually exclusive ways in the three-generation minimal SM. This is conveniently expressed by writing the actual weak hypercharge of the theory Y as a linear combination of standard weak hypercharge Y_{st} and another generator. The three allowed forms for Y are

$$Y = Y_{\text{st}} + \epsilon(L_e - L_\mu), \quad Y = Y_{\text{st}} + \epsilon(L_e - L_\tau), \quad \text{or} \quad Y = Y_{\text{st}} + \epsilon(L_\mu - L_\tau), \quad (1)$$

where ϵ is the arbitrary parameter and $L_{e,\mu,\tau}$ are the family-lepton-number generators.

The presence of family-lepton-number differences can be easily understood *a posteriori*. We know that the three-generation minimal SM has five $U(1)$ invariances which commute with $SU(3)_c \otimes SU(2)_L$. The generators of these $U(1)$ groups are standard hypercharge Y_{st} , baryon number B , and the family-lepton-numbers $L_{e,\mu,\tau}$. Any anomaly-free linear combination of these five charges can be chosen as the generator of the gauged $U(1)$ in the SM gauge group. Apart from Y_{st} , the family-lepton-number differences are the only anomaly-free combinations.¹ Therefore nothing prevents us from gauging any linear combination of Y_{st} and one of these differences, and so a one-parameter dequantization of actual weak hypercharge results. Note that the three family-lepton-number differences are not mutually anomaly-free, which is why there are three distinct one-parameter solutions for Y .

This analysis provides strong motivation for extending the minimal SM in such a way that electric charge quantization can be understood through the steps outlined above. The perspective cast by the preceding paragraph provides the simplest way of stating what characteristics this new physics should have. We must end up with $U(1)_{Y_{\text{st}}}$ being the only anomaly-free $U(1)$ invariance of the Lagrangian. Our task is therefore to explicitly break $L_e - L_\mu$, $L_e - L_\tau$, and $L_\mu - L_\tau$ without introducing any other anomaly-free Abelian invariances of the Lagrangian (such as $B - L$).

Several concrete suggestions for what this new physics might be have been canvassed in the literature. For instance, the introduction of three generations of Majorana right-handed neutrinos is sufficient [2]. (Kobayashi-Maskawa-like mixing in the lepton sector in general explicitly breaks all of the family-lepton-number symme-

tries, but the right-handed neutrinos must also have a Majorana character to explicitly break $B - L$. Although $B - L$ is not anomaly-free in the minimal SM, it is anomaly-free when three right-handed neutrinos are added.) Alternatively, one can introduce only one right-handed neutrino state which need not be Majorana [3]. A different suggestion is that two Higgs doublets be used to explicitly break the troublesome $L_i - L_j$ symmetries [4].

There are many other candidates for the required new physics. Whatever the new physics might be, we know that it must explicitly break all of the $L_i - L_j$ invariances. It therefore makes sense to perform a *model-independent* analysis using effective Lagrangian techniques of all possible higher-dimensional operators that explicitly break family-lepton-number differences. The task of this paper is to begin such an analysis. We will assume that low-energy physics can be described by an effective Lagrangian written in terms of the fields of the minimal SM only (in particular, we will exclude right-handed neutrinos from the low-energy world). Nonrenormalizable operators breaking $L_i - L_j$ will be constructed from these fields, and experimentally relevant processes induced by these operators will be identified and bounds given. We will draw conclusions about what the underlying renormalizable extension of the minimal SM should look like whenever appropriate.

The building blocks of our analysis are the fields of the minimal SM. Each generation of fermions has the $G_{\text{SM}} = SU(3)_c \otimes SU(2)_L \otimes U(1)_{Y_{\text{st}}}$ structure

$$\begin{aligned} \ell_L &\sim (1, 2)(-1), & e_R &\sim (1, 1)(-2), \\ q_L &\sim (3, 2)(1/3), & u_R &\sim (3, 1)(4/3), \\ d_R &\sim (1, 3)(-2/3). \end{aligned} \quad (2)$$

The gauge bosons have their usual transformation properties, while the Higgs doublet ϕ is characterized by $\phi \sim (1, 2)(1)$.

We will assume that the new physics can be assigned an energy scale Λ which is higher than the electroweak scale of 300 GeV. The scale Λ will provide the ultravio-

¹If cancellation of the mixed hypercharge-gravitational anomaly is not imposed, then there are other gauge anomaly-free combinations [1].

let cutoff for the effective theory and the influence of all nonrenormalizable operators on low-energy physics will be suppressed by powers of some typical SM energy or mass divided by Λ . Dimension-5 operators will have a $1/\Lambda$ suppression in the Lagrangian, while dimension-6 operators will be suppressed by $(1/\Lambda)^2$, and so on.

We will restrict our analysis to dimension-5 and -6 operators in this paper. This is logical because of the general expectation that the higher the dimension of the operator the more it is suppressed by powers of mass/ Λ . However, we should be aware of important ways this procedure may be misleading. First, it is possible that a symmetry of the underlying renormalizable theory may forbid all operators from dimension-5 up to some higher dimension, say dimension-7 by way of illustration. In that case, the new physics responsible for ensuring charge quantization will manifest itself first at the dimension-8 level, and the analysis of this paper will be irrelevant. Although this is an interesting possibility, we will for simplicity not focus on it here. Second, although the underlying dynamics may generate dimension-5 and -6 terms, they may necessarily come in with a larger suppression factor than the naive “mass over Λ to some power” expectation. For instance, the topology of Feynman diagrams can prevent certain dimension-6 operators being generated at tree level by *any* underlying gauge theory [5]. In this case there is always an additional loop suppression factor of at least $1/16\pi^2$ to the amplitude of the process, provided the underlying physics does not have a non-perturbative way of generating the dimension-6 operator in question.² It is then possible for dimension-7 and -8 operators to be more important than some dimension-6 operators.

The tedious task of writing down all dimension-5 and -6 operators for the minimal SM has been performed [6]. We will be concerned only with the subset that violates $L_i - L_j$.

Only one set of dimension-5 operators can be constructed under the stated assumptions, and since they happen to break family (and total) lepton number they are relevant. They are given by

$$O_{ij}^5 = \overline{(\ell_{iL})^c} i\tau_2 \phi \ell_{jL} i\tau_2 \phi \quad (3)$$

together with their Hermitian conjugates, where $i, j = 1, 2, 3$ are generation indices. Each of these operators breaks two of the $L_i - L_j$ charges and preserves another. Therefore at least two suitably chosen operators from the O_{ij}^5 set must be simultaneously present. After electroweak symmetry breakdown, these operators induce Majorana terms for the left-handed neutrinos. The coefficients of these operators can be very severely bounded by experiment. The most stringent bound applies to the coefficient a_{11}/Λ of O_{11}^5 because this operator induces a Majorana mass for the left-handed electron-neutrino, given by

$$m_{\nu_e}^{\text{Majorana}} = |a_{11}| \frac{v^2}{\Lambda}. \quad (4)$$

The experimental upper bound is about 1 eV, which leads to the constraint

$$\Lambda > |a_{11}| \times 10^{14} \text{ GeV}. \quad (5)$$

Adopting the general expectation that $|a_{11}| \simeq 1$, we see that Λ should be greater than the very large value of about 10^{14} GeV. The other operators in this set will not provide such stringent bounds, but the typical lower bounds on Λ will be very high nevertheless.

This is an unsatisfactory result with regard to charge quantization, because we were after all endeavoring to find the required new physics at relatively low energies such as 1 TeV. So, the underlying dynamics must either forbid these dimension-5 terms, or suppress them sufficiently. The obvious way the underlying renormalizable theory could forbid these terms is to insist that total lepton number L (or some linear combination of baryon and lepton number such as $3B + L$) be conserved. This is interesting information.

Could the coefficients somehow receive a large enough suppression? We can give a qualified answer of “yes” to this question. It is probably fair to say that the most natural candidate thus far proposed as new physics to ensure low-energy charge quantization is the addition of three Majorana right-handed neutrinos. In particular, it is then natural to use the seesaw mechanism to explain why left-handed neutrinos are so light. Recall that all of the mass eigenstate neutrinos in the seesaw model are Majorana, including the light left-handed neutrinos which have masses generically given by m_D^2/M , where m_D is a Dirac mass and M is a Majorana mass with $M \gg m_D$. Furthermore, the seesaw mass matrix induces precisely the dimension-5 operators we have been discussing once the heavy right-handed Majorana neutrinos are integrated out. Why is this such a popular candidate given the pessimistic result of Eq. (5)? The answer is that we generally expect the Dirac masses m_D to be many orders of magnitude smaller than the electroweak scale v , simply because this is so for all observed quark and charged-lepton masses except for the top quark. The Yukawa coupling constants for quarks and charged leptons are unexplained small numbers in the SM, and we simply assume that the Yukawa coupling constants involved in neutrino Dirac masses are similarly unexplained small parameters. In effective Lagrangian language, this means that the dimensionless coefficients a_{ij} are actually many orders of magnitude smaller than 1. This means the lower bound on Λ can be lowered to a respectable level. For instance, if we desire $\Lambda \simeq 1$ TeV then we need $a_{11} \simeq 10^{-11}$. Since in the seesaw model a_{11} is the product of two Dirac neutrino Yukawa coupling constants, we see that values comparable in smallness to the electron Yukawa coupling constant are needed. This “explanation” of the suppression of the a_{ij} ’s is of course highly unsatisfactory, but this just reflects the highly unsatisfactory status of fermion mass generation in the SM.

If Majorana right-handed neutrinos constitute the new physics then our story ends with the dimension-5 terms,

²This amounts to a suppression factor of $1/4\pi$ for Λ .

and the smallness of the neutrino Dirac Yukawa coupling constants is left to be explained by a hypothetical theory of flavor. In this paper we will instead consider other possible explanations. To forbid the O_{ij}^5 operators we will suppose that total lepton number is conserved by the underlying theory³ and we now move on to dimension-6

terms.

There are many dimension-6 operators which conserve L and B but violate $L_i - L_j$. They can be gleaned from the list given in Buchmüller and Wyler in Ref. [6]. Using their notation they are the following.

Four-fermion operators:

$$\begin{aligned}
O_{\ell\ell}^{(1)} &= \frac{1}{2}(\bar{\ell}_L\gamma_\mu\ell_L)(\bar{\ell}_L\gamma^\mu\ell_L), & O_{\ell\ell}^{(3)} &= \frac{1}{2}(\bar{\ell}_L\gamma_\mu\tau^I\ell_L)(\bar{\ell}_L\gamma^\mu\tau^I\ell_L), \\
O_{\ell q}^{(1)} &= \frac{1}{2}(\bar{\ell}_L\gamma_\mu\ell_L)(\bar{q}_L\gamma^\mu q_L), & O_{\ell q}^{(3)} &= \frac{1}{2}(\bar{\ell}_L\gamma_\mu\tau^I\ell_L)(\bar{q}_L\gamma^\mu\tau^I q_L), \\
O_{ee} &= \frac{1}{2}(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R), & O_{eu} &= (\bar{e}_R\gamma_\mu e_R)(\bar{u}_R\gamma^\mu u_R), \\
O_{ed} &= (\bar{e}_R\gamma_\mu e_R)(\bar{d}_R\gamma^\mu d_R), & O_{\ell e} &= (\bar{\ell}_L e_R)(\bar{e}_R \ell_L), \\
O_{\ell u} &= (\bar{\ell}_L u_R)(\bar{u}_R \ell_L), & O_{\ell d} &= (\bar{\ell}_L d_R)(\bar{d}_R \ell_L), \\
O_{qe} &= (\bar{q}_L e_R)(\bar{e}_R q_L), & O_{qde} &= (\bar{\ell}_L e_R)(\bar{d}_R q_L), \\
O_{\ell q} &= (\bar{\ell}_L e_R)(\bar{q}_L e_R).
\end{aligned} \tag{6}$$

Operators with fermions and vector bosons:

$$\begin{aligned}
O_{\ell W} &= i\bar{\ell}_L\tau^I\gamma_\mu D_\nu\ell_L W^{I\mu\nu}, & O_{\ell B} &= i\bar{\ell}_L\gamma_\mu D_\nu\ell_L B^{\mu\nu}, \\
O_{eB} &= i\bar{e}_R\gamma_\mu D_\nu e_R B^{\mu\nu}.
\end{aligned} \tag{7}$$

Operator with fermions and scalars:

$$O_{e\phi} = (\phi^\dagger\phi)(\bar{\ell}_L e_R\phi). \tag{8}$$

Operators with fermions, scalars, and vector bosons:

$$\begin{aligned}
O_{\phi\ell}^{(1)} &= i(\phi^\dagger D_\mu\phi)(\bar{\ell}_L\gamma^\mu\ell_L), & O_{\phi\ell}^{(3)} &= i(\phi^\dagger D_\mu\tau^I\phi)(\bar{\ell}_L\gamma^\mu\tau^I\ell_L), \\
O_{\phi e} &= i(\phi^\dagger D_\mu\phi)(\bar{e}_R\gamma^\mu e_R), & O_{D_e} &= (\bar{\ell}_L D_\mu e_R)D^\mu\phi, \\
O_{\overline{D}_e} &= (D_\mu\bar{\ell}_L e_R)D^\mu\phi, & O_{eW} &= (\bar{\ell}_L\sigma^{\mu\nu}\tau^I e_R)\phi W_{\mu\nu}^I, \\
O_{\ell eB} &= (\bar{\ell}_L\sigma^{\mu\nu} e_R)\phi B_{\mu\nu}.
\end{aligned} \tag{9}$$

Generation indices have been suppressed in these equations, while $W^{I\mu\nu}$ and $B^{\mu\nu}$ are the field strength tensors for $SU(2)_L$ and $U(1)_{Yst}$, respectively.⁴

We must now look at which $(L_i - L_j)$ -violating processes these operators can induce. The Particle Data Group [7] lists bounds on the several dozen family-lepton-number violating processes that have been looked for experimentally. It is interesting to classify these processes by examining how many units of $L_e - L_\mu$, $L_e - L_\tau$, and $L_\mu - L_\tau$ they violate. We will denote the number of units violated by $\Delta_{e\mu}$, $\Delta_{e\tau}$, and $\Delta_{\mu\tau}$, respectively. Let us make a few preliminary observations. (i) Most of these processes have nonzero values for each of the Δ_{ij} 's. For instance, the process $\mu \rightarrow e\gamma$ has $\Delta_{e\mu} = 2$, $\Delta_{e\tau} = 1$, and $\Delta_{\mu\tau} = 1$. Therefore, if this rare decay is ever observed

to happen we will be able to conclude on the basis of this single process that all family-lepton-number differences are not conserved and that new physics associated with the charge quantization problem has been found.⁵ (ii) Like $\mu \rightarrow e\gamma$, the majority of the processes listed have one of the Δ_{ij} 's equal to 2, with the other two equal to 1. (iii) Two rare decays of the tau lepton, $\tau^- \rightarrow e^+\mu^-\mu^-$ and $\tau^- \rightarrow \mu^+e^-e^-$, conserve one of the family-lepton-number differences ($L_e - L_\tau$ and $L_\mu - L_\tau$, respectively). Therefore the observation of one of these decays in isolation would not necessarily signal the presence of new physics enforcing charge quantization, even though there would certainly be new physics.

Let us now list a representative selection of interesting processes according to their pattern of $L_i - L_j$ violation.

³Some linear combination of B and L which is conserved will also forbid these operators.

⁴Note that the symbol " O_{eB} " appears in the Buchmüller and Wyler list in both Eqs. (3.31) and (3.60). We have renamed the last of these as " $O_{\ell eB}$."

⁵However, we could not be sure that some other $U(1)$ generator such as $B - L$ was not rendered anomaly-free according to the as yet unknown underlying theory. To be sure of this we would need much more experimental information so that we could construct the entire theory.

(A) $\Delta_{e\mu} = 2, \Delta_{e\tau} = 1, \Delta_{\mu\tau} = 1$:

$$\begin{aligned} B(Z \rightarrow e^\pm \mu^\mp) &< 2.4 \times 10^{-5}, & B(\mu^- \rightarrow e^- \gamma) &< 5 \times 10^{-11}, \\ B(\mu^- \rightarrow e^- e^+ e^-) &< 1.0 \times 10^{-12}, & B(\pi^0 \rightarrow \mu^+ e^-) &< 1.6 \times 10^{-8}, \\ B(K^+ \rightarrow \pi^+ e^- \mu^+) &< 2.1 \times 10^{-10}, & B(K_L^0 \rightarrow e^\pm \mu^\mp) &< 9.4 \times 10^{-11}. \end{aligned} \quad (10)$$

(B) $\Delta_{e\mu} = 1, \Delta_{e\tau} = 2, \Delta_{\mu\tau} = 1$:

$$\begin{aligned} B(Z \rightarrow e^\pm \tau^\mp) &< 3.4 \times 10^{-5}, & B(\tau^- \rightarrow e^- \gamma) &< 2.0 \times 10^{-4}, \\ B(\tau^- \rightarrow e^- \pi^0) &< 1.4 \times 10^{-4}, & B(\tau^- \rightarrow e^- \rho) &< 3.9 \times 10^{-5}, \\ B(\tau^- \rightarrow e^- e^+ e^-) &< 2.7 \times 10^{-5}, & B(\tau^- \rightarrow e^- \mu^+ \mu^-) &< 2.7 \times 10^{-5}. \end{aligned} \quad (11)$$

(C) $\Delta_{e\mu} = 1, \Delta_{e\tau} = 1, \Delta_{\mu\tau} = 2$:

$$\begin{aligned} B(Z \rightarrow \mu^\pm \tau^\mp) &< 4.8 \times 10^{-5}, & B(\tau^- \rightarrow \mu^- \gamma) &< 5.5 \times 10^{-4}, \\ B(\tau^- \rightarrow \mu^- \pi^0) &< 8.2 \times 10^{-4}, & B(\tau^- \rightarrow \mu^- \rho) &< 3.8 \times 10^{-5}, \\ B(\tau^- \rightarrow \mu^- \mu^+ \mu^-) &< 1.7 \times 10^{-5}, & B(\tau^- \rightarrow \mu^- e^+ e^-) &< 2.7 \times 10^{-5}. \end{aligned} \quad (12)$$

There are some processes which do not fall into any of these categories. We have already discussed $\tau^- \rightarrow e^+ \mu^- \mu^-$ and $\tau^- \rightarrow \mu^+ e^- e^-$. There is also the result $B(\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu) < 1.8 \times 10^{-2}$ which obeys $\Delta_{e\mu} = 4, \Delta_{e\tau} = \Delta_{\mu\tau} = 2$.

The processes in category A provide the most stringent bounds on Λ . The most severe constraint comes from $\mu \rightarrow e\gamma$. This decay can be induced by the operators $O_{\ell W}, O_{\ell B}, O_{eB}, O_{eW},$ and $O_{\ell eB}$, yielding typically that

$$\Lambda > 10^7 \text{ GeV}. \quad (13)$$

The next most severe constraint comes from $\mu \rightarrow 3e$ which can be induced by $O_{\ell\ell}^{(1)}, O_{\ell\ell}^{(3)},$ and O_{ee} . The typical bound is

$$\Lambda > 10^5 \text{ GeV}. \quad (14)$$

The decays $K^+ \rightarrow \pi^+ e^- \mu^+$ and $K_L^0 \rightarrow \mu^\pm e^\mp$ both yield $\Lambda > 5 \times 10^4 \text{ GeV}$ or so. The bound at the CERN e^+e^- collider LEP on $Z \rightarrow e^\pm \mu^\mp$ implies that $\Lambda > 1 \text{ TeV}$, while $\pi^0 \rightarrow \mu^\pm e^\mp$ implies the very weak bound that $\Lambda > 90 \text{ GeV}$. (The various effective operators containing both quarks and leptons contribute to the processes above that involve hadrons.)

The operator $O_{e\phi}$ induces flavor-changing vertices between the physical Higgs boson and the charged leptons. At tree level this will contribute to $\mu \rightarrow 3e$, while at one-loop level it will contribute to $\mu \rightarrow e\gamma$. However, we find the bounds on Λ due to these Higgs boson effects to be weaker than those derived above.

Clearly, if category A processes are responsible for enforcing charge quantization, then the scale of the new physics is typically at the rather high value of 10^7 GeV . This sort of new physics will therefore be difficult to explore directly. Of course, the fact that the bound on Λ from decays such as $\mu \rightarrow e\gamma$ is so severe reflects our ability to do very high statistics searches for this decay mode. Therefore we may well observe a nonzero rate for this process as statistics improve further, despite a high

value for Λ . This would serve as a dramatic manifestation of the sought after new physics. However, rare decay searches are indirect rather than direct explorations of physics beyond the SM. Ideally, we would like to be able to experiment on the totality of the new physics and not just on its subtle low-energy effects. This would require studying collisions at Λ energies. We therefore conclude that if the non-standard physics induces category A processes at the dimension-6 level then we can realistically only ever expect to study its indirect effects.

According to the analysis of Ref. [5], the dimension-6 operators inducing $f \rightarrow f'\gamma$ cannot be generated at tree level by any underlying gauge theory. Therefore the bound $\Lambda > 10^7 \text{ GeV}$ should in this case be reduced by about $1/4\pi$, since the coefficient of the operator will necessarily have a loop suppression factor. This brings the $\mu \rightarrow e\gamma$ lower bound on Λ into the 800 TeV regime, which is still very high. Machines with center of mass energy beyond that to be available at the CERN Large Hadron Collider (LHC) would then be needed to study the new physics directly. Furthermore, if the new physics has nonperturbative or perhaps even nongauge character then this argument becomes moot.

Let us now turn to category B. The most severe constraint comes once again from radiative lepton decay. The bound on $\tau \rightarrow e\gamma$ yields

$$\Lambda > 40 \text{ TeV}. \quad (15)$$

Other relevant processes are $\tau \rightarrow 3e, \tau \rightarrow e2\mu, Z \rightarrow e\tau,$ and $\tau \rightarrow \rho e$ which all imply that $\Lambda > 1\text{--}2 \text{ TeV}$ or so, while the lower bound from $\tau \rightarrow \pi^0 e$ is a little lower than a TeV. Although 40 TeV is still a little high, the $1/4\pi$ suppression that occurs if the new physics is perturbative yields $\Lambda > 3 \text{ TeV}$ from $\tau \rightarrow e\gamma$. Category C bounds are roughly the same as those from category B. So, it is quite possible for category B and C physics to exist at TeV scale energies, which is a pleasing conclusion.

What have we learned from these observations? First, the underlying dynamics is likely to respect total-lepton-number (or some linear combination of B and L) conser-

vation so that no dimension-5 terms are induced. However, this is far from being a rigorous requirement, as the example of the seesaw model demonstrates. Second, if the new dynamics is to operate at LHC energies, then the $\Delta_{e\mu} = 2$, $\Delta_{e\tau} = 1$, $\Delta_{\mu\tau} = 1$ class of processes must be prevented from occurring at the dimension-6 level. This can happen if the underlying dynamics conserves some linear combination of L_μ (or L_e) and L_τ while at the same time breaking $\Delta_{e\mu}$, $\Delta_{e\tau}$, and $\Delta_{\mu\tau}$. In any case, the observation of such a process would nonetheless be an exciting indirect manifestation of nonstandard dynamics. Third, if processes respecting $\Delta_{e\mu} = 1$, $\Delta_{e\tau} = 2$, $\Delta_{\mu\tau} = 1$, or $\Delta_{e\mu} = 1$, $\Delta_{e\tau} = 1$, $\Delta_{\mu\tau} = 2$ are discovered in the near future, they may be a signal of new dynamics at the TeV scale. Fourth, it is interesting to also contemplate underlying models which do not generate any $(L_i - L_j)$ -violating dimension-5 and -6 terms.

This will serve to lower the bound on the scale of new physics further, and if the model is constructed correctly will allow category A processes such as $\mu \rightarrow e\gamma$ to be induced by TeV scale dynamics. At any rate, effective operators provide a systematic and useful way to classify the phenomenological consequences of underlying theories, and they can even provide hints as to how to build these models.

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