Quenching an expanding chiral condensate

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(Received 25 July 1994)

We simulate quenching in the O(4) σ model of hadronic matter expanding along the z axis, with randomly generated initial conditions imposed at a given boost invariant time $\tau_0 = \sqrt{t^2 - z^2}$. A comparison of our results with the simulations of Rajagopal and Wilczek for a nonexpanding case shows that the normalized power exhibits approximately the same frequency of oscillations in the laboratory time in both cases. However, the response of the expanding system depends on τ_0: e.g., for $\tau_0 = 1$ fm it is about 2 orders of magnitude smaller than for the nonexpanding system. Also, the relaxation time becomes shorter with expansion present. When $\tau_0 \to \infty$ the two cases become identical. Kinematical windows for the production of a disoriented chiral condensate are also discussed.

PACS number(s): 12.38.Mh, 12.39.Fe, 25.75.+r

I. INTRODUCTION

It has been conjectured recently that some processes of multiparticle production are realizations of a quench from some randomly excited state of hadronic matter to a zero temperature state whose subsequent evolution follows classical equations of motion [1]. The randomly excited initial state gives the initial conditions for the coherent radiation of the chiral fields $(\sigma, \vec{\pi})$ which form an O(4) four-vector and satisfy the equations of motion derived from the Lagrangian

$$L = \int d^4x \left[\tfrac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - \frac{\lambda}{4}(\phi_i\phi_i - v^2)^2 + H\sigma \right] , \quad (1.1)$$

where $\phi_i \equiv (\sigma, \vec{\pi})$ is a vector in the internal space, and λ, v, H are the parameters of the model.

As in [1] we are going to consider classical solutions of the equations of motion obtained from (2.2) below. In terms of multimesonic states they approximate coherent states of mesons. Therefore, we are going to discuss just one of the many possible production mechanisms, a mechanism which is presumably not very common but is of considerable interest because it may lead to some disoriented chiral condensates [2–4].

Without discussing the region of validity of the model of Ref. [1], we shall merely work out some modifications implied by the longitudinal expansion superimposed on such processes (related earlier papers are [3,6–9]). This, we believe, adds a realistic element to the simulations (compare, e.g., [5]). The prospects of observing disoriented chiral condensates are covered in [2–4], and are not discussed in this note. However, at the end of the paper we do make a comment on the existence of a kinematic window for production of Centauros, a window which is implied by the expansion.

II. SIMULATION

We solve numerically the system of equations derived from (1.1). The longitudinal expansion, along the z axis, is introduced through the change of variables [5]

$$z = \tau \sinh\eta, \quad t = \tau \cosh\eta , \quad (2.1)$$

with the transversal coordinates (x, y) unchanged. $\tau = \sqrt{t^2 - z^2}$ is now the invariant time in which the evolution of the system takes place, and $\eta = \frac{1}{2}\ln(t + z)/(t - z)$ is the so called quasirapidity. One may call (τ, η) the comoving coordinates: (2.1) defines a transformation to a local frame which moves with the velocity $\tanh\eta = z/t$ with respect to the Minkowski (lab) frame. The field ϕ_i now depends on the invariant time τ and the local velocity of expansion is determined by η. Note also that the metric of the space-time in which ϕ_i evolves is

$$ds^2 = d\tau^2 - \tau^2 d\eta^2 - dy^2 - dx^2 .$$

The system of equations to be solved is now

$$\left[\frac{1}{\tau}\frac{\partial}{\partial\tau}\left(\tau\frac{\partial}{\partial\tau}\right) - \frac{1}{\tau^2}\frac{\partial^2}{\partial\eta^2} - \Delta_\perp\right]\sigma = -\lambda\sigma(\sigma^2 + \vec{\pi}^2 - v^2) + H ,$$

$$\left[\frac{1}{\tau}\frac{\partial}{\partial\tau}\left(\tau\frac{\partial}{\partial\tau}\right) - \frac{1}{\tau^2}\frac{\partial^2}{\partial\eta^2} - \Delta_\perp\right]\vec{\pi} = -\lambda\vec{\pi}(\sigma^2 + \vec{\pi}^2 - v^2) , \quad (2.2)$$

with $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$. Incidentally, an alternative expression for the left-hand side (LHS) of (2.2) can be used because

$$\frac{\partial^2}{\partial\eta^2} = (1-\mathcal{V}^2)\frac{\partial}{\partial\mathcal{V}}\left((1-\mathcal{V}^2)\frac{\partial}{\partial\mathcal{V}}\right) ,$$

where $\mathcal{V} = z/t = \tanh\eta$. It shows that *very* close to the light cone, and at fixed $\tau = \tau_0$, the second term in (2.2) is negligible in nonpathological cases.

Now, the evolution follows the invariant time τ; thus the random initial conditions for ϕ_i are

$$\langle\phi_i\rangle = \langle\dot{\phi}_i\rangle = 0, \quad \langle\phi_i^2\rangle = \tfrac{v^2}{4} ,$$
$$\langle\dot{\phi}_i^2\rangle = v^2, \quad \dot{\phi}_i = \tfrac{\partial\phi_i}{\partial\tau} , \tag{2.3}$$

at a given initial τ_0. (2.3) says that we choose ϕ_i and $\dot{\phi}_i$ on each site of the lattice $(\Delta x, \Delta y, \Delta\eta)$ independently and randomly from the Gaussian distributions centered around zero having variances $v^2/4$ and v^2, respectively. Indeed, in terms of the Minkowski coordinates, the random beginning of the process is now set on a hyperboloid $\tau_0 =$ const, not on the hyperplane $t =$ const, as in Ref. [1]. Also the condition $\langle\dot{\phi}_i^2\rangle = v^2$ imposed on the initial random process means different things depending on whether $\dot{\phi}_i$ is $\partial\phi_i/\partial\tau$ or, as in Ref. [1], $\partial\phi_i/\partial t$. In all our simulations we take, as in [1], $v = 84.7$ MeV, $H = (119$ MeV$)^3$, and $\lambda = 20.0$.

There are several important physical consequences of these new initial conditions. In contrast with [1], where the results do not depend on the choice of the initial time, in our case the results do depend on τ_0 [e.g., at $\tau_0 = 0$, on the light cone, (2.2) become singular]. As τ_0 increases the hyperbolic surface becomes more and more like the plane $t =$ const and, in the limit $\tau_0 \to \infty$, our results must coincide with those of Ref. [1]. Therefore τ_0 is a parameter which should be given a physical interpretation. Perhaps it marks in the evolution the end of the quark-gluon era and the beginning of the meson era. In fact, it is quite tempting to reduce the connection between the chiral evolution and the QCD perturbative quark-gluon evolution to the random boundary conditions at their interface.

One should stress also a different physical meaning of the initial correlation lengths $(\Delta x, \Delta y, \Delta\eta)$. We introduce the initial correlations within the transverse *distances* $(\Delta x, \Delta y)$ and within segments of the longitudinal *velocities* $\Delta\eta$ ($\tanh\eta = z/t =$ velocity at a given space-time point) with the condition $\Delta x = \Delta y = \tau_0\Delta\eta$. This corresponds to the natural assumption that the system is isotropic in the local rest frame (comoving frame). However, the longitudinal expansion implies that the initial correlation lengths as viewed from the laboratory system are asymmetric; Δx and Δy may be taken as in [1], but

$$\Delta z = \tau_0 \cosh\eta\Delta\eta , \tag{2.4}$$

and a constant correlation length in η, i.e., $\Delta\eta$, results in a whole spectrum of correlation lengths Δz. *This remains generally true for all correlations which may appear in the evolution of our system system;* e.g., short range correlations in $\Delta\eta$ may, for the space-time points close to the light cone (large η), result in observations of long range correlations in Δz.

In our simulations we use, as in Ref. [1], the standard finite difference leapfrog scheme, and the values of the parameters used are given in the figure captions. We checked our program by repeating some of the simulations of Ref. [1]; e.g., we reproduced the results shown in Fig. 1(a) of [1]. While comparing various results with each other one must remember that the amplitudes of the modes depend on the random initial conditions; two randomly generated inputs of an ensemble satisfying the same conditions (2.3) may produce outputs differing in size by a factor 2, 3, or even more. Nevertheless, as we shall see, there are a few more robust characteristics, which, hence, make the scheme less sensitive to the random initial conditions.

The evolution of our system has cylindrical symmetry and we calculate the normalized power of the transversal and the longitudinal modes separately:

$$M_{k_\perp}(\tau) = \frac{1}{V_2\Delta}\left|\int d^2r_\perp d\eta\, e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\pi(\mathbf{r}_\perp,\eta;\tau)\right|^2 , \tag{2.5}$$

$$M_\nu(\tau) = \frac{1}{V_2\Delta}\left|\int d^2r_\perp d\eta\, e^{i\nu\eta}\pi(\mathbf{r}_\perp,\eta;\tau)\right|^2 , \tag{2.6}$$

where $\mathbf{k}_\perp = (k_x, k_y)$, $\mathbf{r}_\perp = (x, y)$, V_2 is the two-dimensional volume of the expanding system (η is dimensionless), and $\Delta = \eta_{\max} - \eta_{\min}$ is the total length of the quasirapidity interval. (2.5) may be compared with the normalized power of the transversal modes of the static case [1]:

$$M_{k_\perp}^{\mathrm{RW}}(t) = \frac{1}{V_{\mathrm{RW}}}\left|\int d^2r_\perp dz\, e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\pi(\mathbf{r}_\perp,z;t)\right|^2 , \tag{2.7}$$

where V_{RW} is the three-dimensional volume of the non-expanding system.

Before showing some of the results let us comment on the problems one has to face while comparing the expanding system with the static one. The laboratory system resides in the (t,z) not the (τ,η) space and it is important to know how the evolution generated in (τ,η) looks to the observer in (t,z). To this end we calculate the quantity

$$M_{k_\perp}(t) = \frac{1}{V_3}\left|\int d^2r_\perp dz\, e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\pi(\mathbf{r}_\perp,\eta;\tau)\right|^2 , \tag{2.8}$$

where η and τ under the integral are expressed in terms of t and z, and $V_3 = 2V_2 t\tanh\eta_{\max}$ (in a symmetric case $\eta_{\max} = -\eta_{\min}$) is the three-dimensional volume of the expanding system.

Figure 1 illustrates some of the relevant points of such an operation. The main effects of presenting in the (t,z) space a mode generated in the (τ,η) space are large cancellations. Since the integrand in (2.8) is oscillatory either in τ or in t, the integration over z in (2.8) leads to

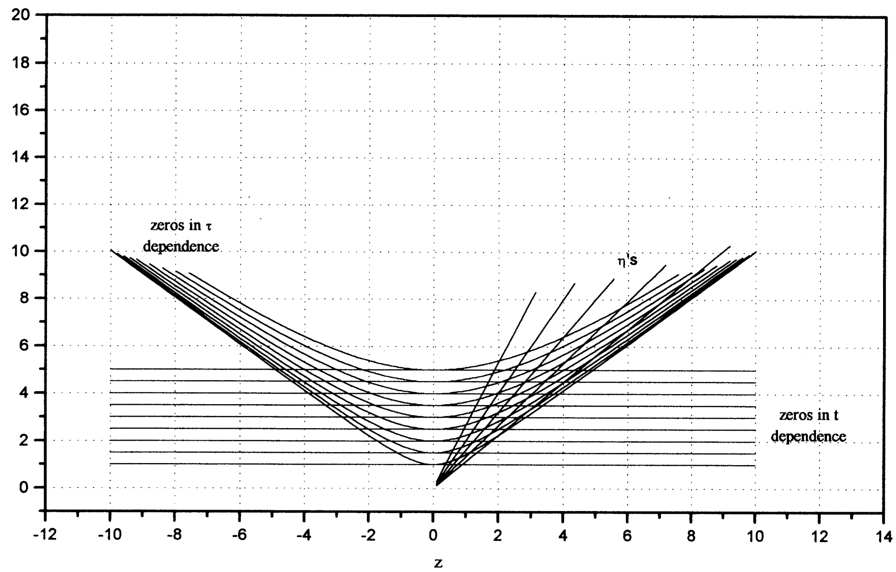

FIG. 1. A schematic map of the zeros of the pion field in the (t, z) and (τ, η) spaces.

substantial cancellations; indeed, integrating over z at a constant t we have to cross several values of τ which are zeros of $\pi(\mathbf{r}_\perp, \eta; \tau)$. The other factor which also reduces $M_{k_\perp}(t)$ is that the process of integration over z takes us right up to the τ_0 hyperboloid (compare Fig. 1), where the initial random conditions take place and help to decrease $M_{k_\perp}$. Because of the complexity of this procedure (the whole evolution in τ and η has to be remembered at each step) we do it only for the transversal modes.

A remark on the longitudinal modes of the expanding system is in order here. The expression we want to discuss is

$$M_{k_z,k_\perp}(t) = \frac{1}{V_3}\left|\int d^2r_\perp dz\, e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp + ik_z z}\pi(\mathbf{r}_\perp, \eta, \tau)\right|^2 . \tag{2.9}$$

While integrating along the z direction at fixed t we cross the zeros of π at several (their number depends on t) values of τ. The distances between the consecutive zeros decrease as z increases. Thus there is no fixed period in the z dependence of π. Hence no possibility of a resonance at a specific k_z occurs and, consequently, a similar reduction in the size of $M_{k_z,k_\perp}$ as in $M_{k_\perp}$ takes place.

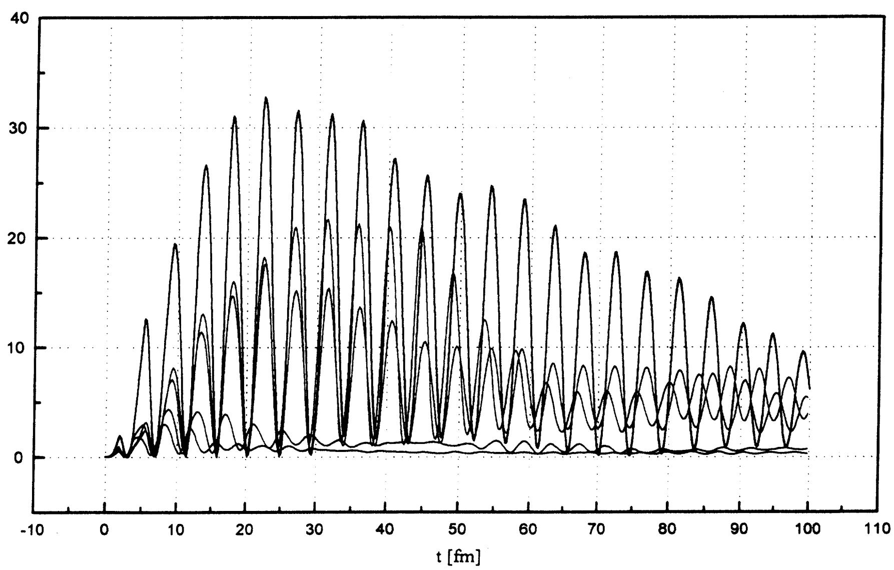

FIG. 2. The power of a few first modes calculated following the procedure given in [1] (the static system). The power of the pion field for the modes $ka = 0.00, 0.20, 0.26, 0.48, 0.60$ is plotted vs time ($k = \sqrt{k_z^2 + k_\perp^2}$, $a = 1$ fm). The simulation was done in a 32^3 box with some additonal test runs in a 64^3 box. The amplitudes of the curves follow the rule that the softer is the mode, the larger the response. The only difference from [1] is that the zero mode is added. Note that the zero mode dominates. This mode is of the same (modulo factor 3) order of magnitude as the zero mode shown in Fig. 3, and hence about two orders of magnitude larger than the one of Fig. 5.

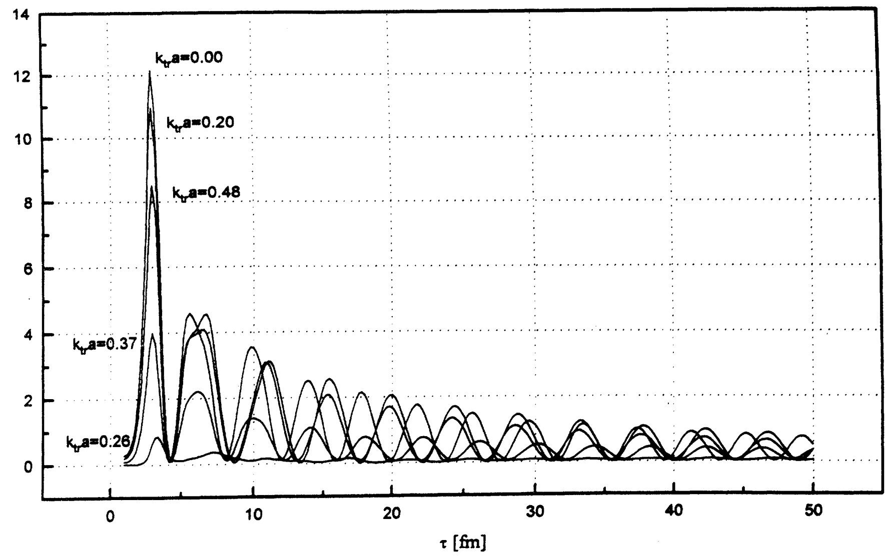

FIG. 3. The power of the five transverse modes $M_{k\perp}(\tau)$ of the expanding system vs τ [compare (2.5)], for the following transverse momenta: $k_\perp a = 0.00, 0.20, 0.26, 0.37, 0.48$; $a = 1$ fm. $\tau_0 = 1$ fm, the steps $d\tau = 0.1$ fm, $d\eta = 0.1$, and $dx = dy = 1$ fm were used in numerical solutions of Eq. (2.2). The random initial conditions were taken in cells $\Delta\eta\Delta x\Delta y = 10\, d\eta\, dx\, dy$, which correspond at $\tau_0 = 1$ fm, $\eta = 0$ to those taken in Ref. [1].

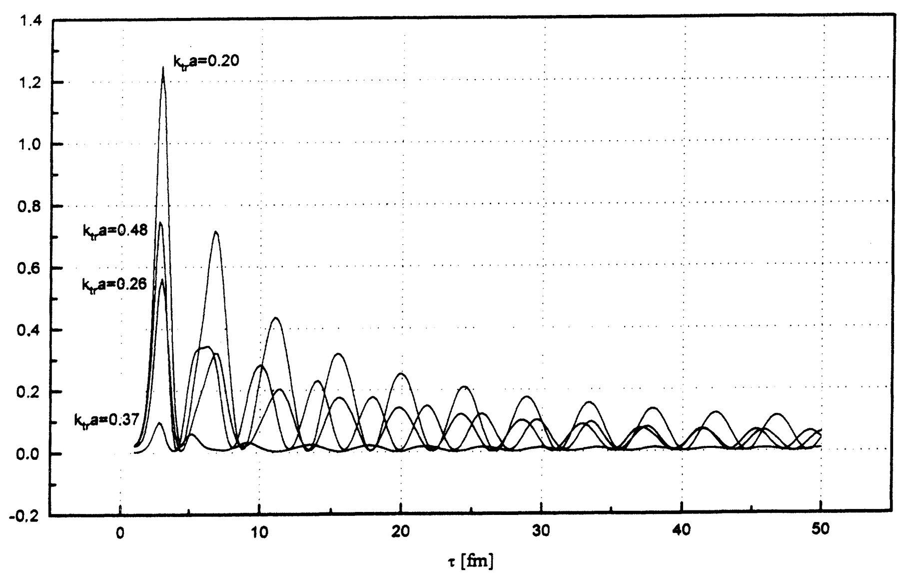

FIG. 4. The transverse modes for $k_\perp a = 0.20, 0.26, 0.37, 0.48$, $a = 1$ fm, of the expanding system vs τ for a random initial condition different from that of Fig. 3. All other parameters are the same as in Fig. 3. As in Fig. 3 we can see that the sequence of the modes does *not* strictly follow the rule that softer modes lead to larger responses (powers of the modes).

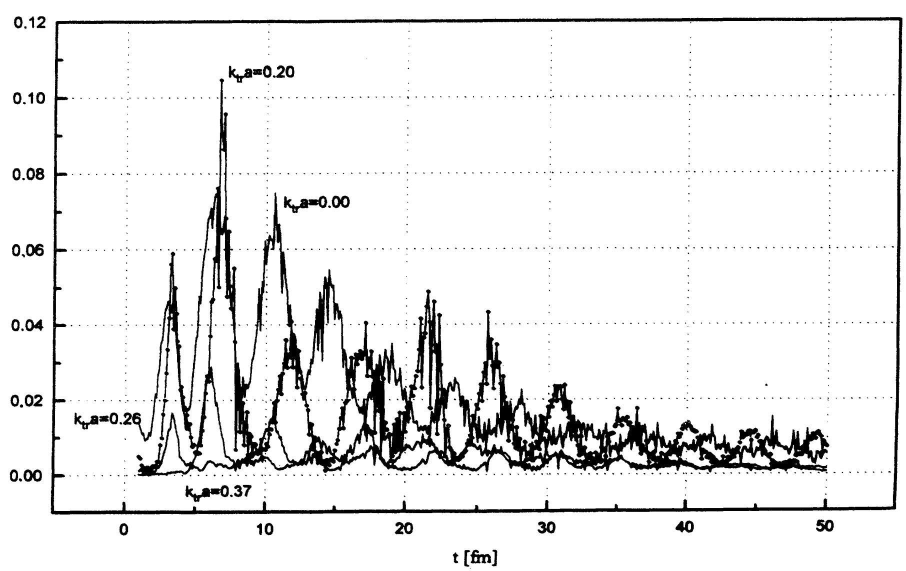

FIG. 5. The power of the first four modes shown in Fig. 3 translated into the (t, z) space. Note the two orders of magnitude reduction of the amplitude. For interpretation of this drastic reduction, see Fig. 1 and the text. To make the figure more transparent the mode $k_\perp a = 0.20$ is marked additionally by dots.

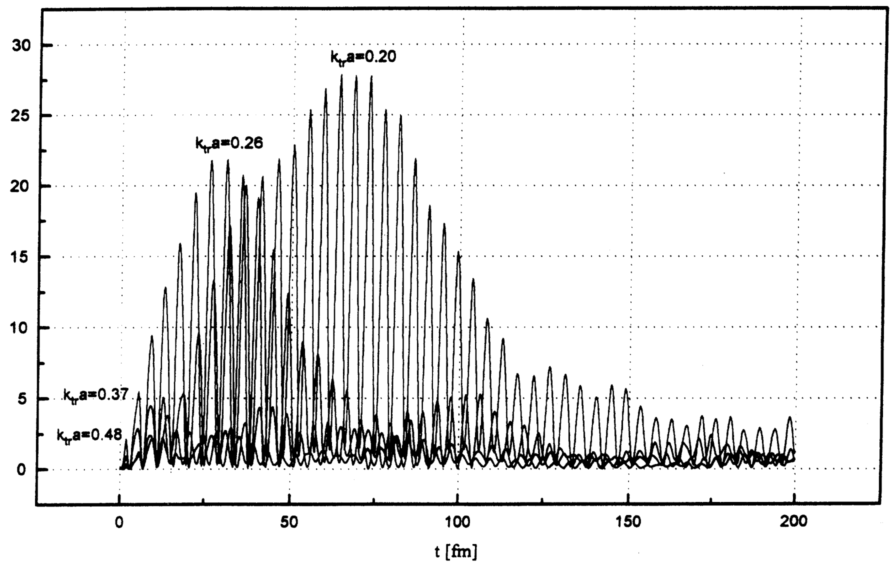

FIG. 6. Four transverse modes of Fig. 3 but generated following the procedures of [1] [the static case, compare Eq. (2.7)]. We see that the rule that softer modes lead to larger responses is satisfied, except for a shift of the maximum of the softest mode ($k_\perp a = 0.20$) towards larger times.

III. RESULTS AND DISCUSSION

We start comparing the evolution in t or in τ of the modes of the expanding and static systems by presenting the static case calculated with the same set of parameters as in Fig. 1 of Ref. [1]. The modes are shown in Fig. 2; the only difference is that we added the zero mode which, as it turns out, dominates. Fig. 2 sets the scales of the response of the system.

Now, we do a similar thing for the expanding system. However, since the longitudinal modes [compare (2.6)] have no counterpart in the static case we restrict our discussion to the transverse modes, Eq. (2.5). The results are shown in Fig. 3. We see that to within a factor of 2 or 3 the size of the response of the expanding system is of the same order of magnitude. There is, however, a marked difference in the time of relaxation: the expanding system relaxes up to ten times faster than the static one. Also, we see that the sizes of the maxima of $M_{k_\perp}(\tau)$ do not necessarily follow the rule found in Ref. [1]: $M_k^{\rm RW} > M_{k'}^{\rm RW}$ when $k < k'$. Although the zero mode and the mode $k_\perp a = 0.20$ dominate, the sequence of the other three ($k_\perp a = 0.26$, $k_\perp a = 0.37$, $k_\perp a = 0.48$) breaks this rule. This can also be seen in Fig. 4 where we present modes resulting from some randomly generated initial conditions different from the ones of Fig. 3.

Now we translate the modes of Fig. 3 into the (t, z) space. The results are shown in Fig. 5. By comparing them with Fig. 3 we see that (a) the sizes of the modes are reduced by two orders of magnitude, (b) the relaxation times stay up to ten times shorter than in the static case, and (c) the evolution in time t recovers some of the characteristics of the static case; as in the case of $M_k^{\rm RW}(t)$ modes we observe first the period of growth of $M_{k_\perp}(t)$'s, then they reach a maximum, and then decrease, whereas the $M_{k_\perp}(\tau)$ modes always start with the maximal fluctuation.

To clinch our claims we present in Fig. 6 the transverse modes for the static case [compare (2.7)]. We see that, qualitatively, the pattern is similar to the one shown in Fig. 2; the size of the response is the same and the sequence of the maxima of modes follows the rule $M_{k_\perp}^{\rm RW} > M_{k'_\perp}^{\rm RW}$ when $k_\perp < k'_\perp$. In Figs. 6 and 3 we also see the slower relaxation of the softest modes relative to the hard ones.

IV. CONCLUSIONS

We use throughout this paper the initial invariant time $\tau_0 = 1$ fm. This is a very important parameter for the expanding system. It determines, among other things, the size of the response of the system to the initial conditions. If we accept that it marks the end of the quark-gluon phase and the beginning of the mesonic evolution, $\tau_0 = 1$ fm looks reasonable. When $\tau_0 \to \infty$ the system goes into the static case of Ref. [1]. We confirmed that through direct simulations at large τ_0's. The convergence is slow, however, and even at $\tau_0 = 100$ fm the expanding system gives a somewhat weaker response than the static one. So, if we settle for $\tau_0 = 1$ fm, the response in the laboratory frame is about 100 times weaker than for the static case.

Clearly, the choice of τ_0 is fairly arbitrary. We wish to stick to the model of the quench where there are even more uncertainties to face, e.g., the randomness of the initial conditions at τ_0 and the choice of the range of the initial correlations. Each of these two factors separately may easily shift the powers of the modes up or down by a factor of 2 or even more. Also, with increasing τ_0 we observe a slow convergence to the static case. Therefore we find that a precise value of τ_0 is not of critical significance provided its value is not much less than 1 fm.

We find that the dominant modes of the expanding system in the (τ, η) space immediately reach their peak values and quickly relax to the equilibrium; their relaxation time is much ($\sim$ ten times) shorter than in the static case. This fast relaxation clearly seen in Figs. 3–5 is, we believe, caused by the expansion which makes, e.g., the amplitudes of the field oscillations go down asymptotically as $\tau^{-1/2}$. This dependence is suggested by a Hartree-like treatment of (2.2), where the nonlinear terms on the RHS of (2.2) are replaced by some averages and the limit of large τ taken. Also, the modes in the (τ, η) space do not necessarily follow the rule found in the static case: the softer the mode the larger its amplitude. Only the softest modes consistently dominate; the sequence of the sizes of the remaining ones depends on the specific randomly generated initial conditions.

We find that the translation from the (τ, η) space to the (t, z) space reduces by two orders of magnitude the size of the response of the system without influencing the relaxation times. Since the (t, z) space is the one where we might observe the evolution of the quenched system, we have to conclude that expansion makes the whole phenomenon much weaker.

We find, as in the static case, an impressive robustness and stability of the oscillatory character of the response of the expanding system to the random initial conditions.

Finally, let us discuss a *kinematical* property of the expanding chiral model, which property may turn out to be important in understanding the conditions under which the disoriented chiral condensates (DCC's) may appear and be detected. If such condensates are identified with the so called Centauro events, this discussion refers to Centauros. From the available papers on the subject [6–8] transpire the difficulties in securing, through dynamical processes, production of large enough objects which would have a better chance to be detected as DCC's (see, however, Ref. [9] for a possible way out of this difficulty). On the other hand, from the discussion of Sec. II of this paper it follows that, if the random initial conditions produce a droplet of DCC in the volume $\Delta x \Delta y \Delta \eta$ at very large $\eta \gg 1$, it may appear in the laboratory frame as, in principle, an object large enough and living long enough, because its lifetime and longitudinal size are

$$\Delta t = \tau_0 \sinh\eta \Delta\eta, \quad \Delta z = \tau_0 \cosh\eta \Delta\eta \ .$$

Indeed, in principle, Δt and Δz are arbitrarily large. However, in order to obtain the available range of sizes of DCC's, it is necessary to study correlations which we did not in this paper. One can find more on this problem in Ref. [6].

In any case, to achieve large Δz we must have *very* large η's. This suggests that Centauros should be composed of the *very fastest* particles of, e.g., the cosmic-ray jets, and they should be hard to observe in routine multihadron production experiments. This seems to be, indeed, the case [10,11]. It may therefore turn out that the difficulties of observing Centauros are of kinematical rather than dynamical origin [12].

ACKNOWLEDGMENTS

This work was supported in part by two KBN Grants No. 2-0054-91-01 and No. 2-0092-91-01. W. Czyz thanks the Institute of Nuclear Physics in Krakow for its financial support.

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