Matter accretion by cosmic string loops and wakes

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We develop quantitative methods for the study of structure formation with cosmic strings in an expanding universe. The gravitational effects of arbitrary string configurations are calculated using linearized gravity. The growth of density fluctuations is then studied using these gravitational forces as a source term in the Zeldovich approximation. We use either the adhesion modification or an N-body tree code to project this fluctuation growth into the nonlinear regime. These methods are applied to specific loop and long string solutions beginning at equal matter and radiation $t_{\rm eq}$ on scales corresponding to about 10 Mpc today (h=0.5). We reproduce analytic results for spherical and planar collapse. We show that these methods are applicable to accretion about closed oscillating loops and in the wakes of moving long strings which possess significant small-scale structure, quantitatively confirming the wiggly string approximation with a renormalized string energy density $\tilde{\mu}$. We demonstrate the efficiency of the fragmentation of wakes created by wiggly strings by the present day. These methods are sufficiently computationally efficient to employ in the study of an evolving string network. For the cosmic string scenario, we conclude that reliable quantitative predictions must take into account non-Gaussianity, vorticity generation, and nonlinear fragmentation effects.

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I. INTRODUCTION

There has been renewed interest recently in models in which large-scale structure formation is seeded by topological defects such as cosmic strings [1,2]. Observational data are providing increasingly stringent constraints on the simplest inflationary models based on cold dark matter (CDM) and Gaussian fluctuations. Indeed, it can be argued that cosmic string scenarios currently exhibit better consistency between the galactic and microwave background [Cosmic Background Explorer (COBE)] normalizations [3–5], though this is due in part to the large uncertainties associated with predictions in these models. While progress has been slow with strings, this is not because of obvious flaws, rather it is a consequence of their calculational complexity.

Few studies of structure formation with cosmic strings have taken our knowledge quantitatively much further than the original proposals of Zeldovich [1] and Vilenkin [2]. Unfortunately, most work occurred during the initial flurry of interest when string network evolution was inadequately understood and oversimplified. The superseded picture of scale-invariant evolution then prevalent, with a few loops being produced per Hubble volume per Hubble time, is now often termed the "old string scenario" (though it may have some validity for models with loops nucleated during inflation). Subsequent high-resolution numerical simulations of string evolution demonstrated scaling densities, but revealed the presence of significant small-scale structure on strings or "wiggliness" (Bennett and Bouchet (BB) [6,7]; Allen and Shellard (AS) [8]; see also Albrecht and Turok (AT) [9]). This substructure has important dynamical consequences and also causes loop formation on scales much smaller than the horizon. These results suggest that long strings are more important than loops in seeding density perturbations [10–12]. We should point out a significant caveat here concerning the effect of the small-scale structure on the nature of the scaling solution, although this should be limited by gravitational radiation back reaction. This has been the focus of recent analytic study (see, for example, [13,14]).

In more recent work, Albrecht and Stebbins [3,4] have studied the string power spectrum taken from AT simulations, projecting this forward to the present day using linear theory transfer functions for both cold and hot dark matter. This was an invaluable first step, but further developments are necessary because strings create nonlinear objects at early times and the power spectrum provides an incomplete description of non-Gaussian perturbations. As emphasized in the formal synchronous gauge treatment of Veeraraghavan and Stebbins (VS) [15], there are also a variety of motivations for studying string perturbations in position space. Note that the VS approach in the comoving synchronous gauge is being pursued by another group [16].

The central theme of this paper is the development of accurate and practical methods for the study of string structure formation, with work currently proceeding to link these with an expanding universe network simulation (see also [17]). We determine the time-dependent metric in linearized gravity for arbitrary evolving string configurations. Using the resulting gravitational forces as a source term in the Zeldovich approximation, we grow the

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induced perturbations in a cold dark matter universe. We can project these into the nonlinear regime using the adhesion approximation. Effectively, a viscosity-dependent term restricts the shell crossing which otherwise would occur at early times. Alternatively, late-time evolution is also studied using an N-body tree code. Here, we consider relatively small scales (simulation box size about $5h^{-1}$ Mpc with h=0.5) in a cold dark matter universe beginning at the equal matter-radiation transition $t_{\rm eq}$. In this limit, we are able to avoid a number of subtleties including defect compensation and the use of retarded time potentials (refer to VS). The specific string realizations we study in this paper are accretion about oscillating loops and wake formation behind moving straight and wiggly strings. Having made quantitative comparisons of our methods against well-known analytic results, we proceed to consider more general cases such as the nonlinear fragmentation of a wiggly string wake.

II. FORMALISM

A. Cosmic strings: The source

This discussion is motivated by the possibility of cosmic vortex strings forming at a phase transition in the early Universe [18]. Galaxy formation scenarios with strings generally require values of the dimensionless parameter $G\mu/c^2 \sim 10^{-6}$, where μ is the string mass per unit length (we employ units in which c=1). This corresponds to energy scales naturally associated with a grand unified phase transition. For local gauge strings, the microphysical width of the string will be many orders of magnitude smaller than its typical curvature radius, so we can take a zero thickness limit. In this case, the string sweeps out a two-dimensional world sheet in spacetime $x^\mu = x^\mu(\sigma,\tau)[= (x^0, \mathbf{x})]$, with τ timelike and σ spacelike parameters. In flat space, the variation of the Nambu action (corresponding to the area of this world sheet) yields the equations of motion

$$\ddot{\mathbf{x}} - \mathbf{x}'' = 0 \ , \tag{1}$$

where we have taken $\tau = x^0 = t$ and the "transverse" gauge,

$$\dot{\mathbf{x}}^2 + \mathbf{x}'^2 = 1 \ , \tag{2}$$

$$\dot{\mathbf{x}} \cdot \mathbf{x}' = 0 \ , \tag{3}$$

with overdots and primes denoting derivatives with respect to t and σ, respectively. Except for the constraints (2) and (3), solutions to (1) would be trivial, being simply a superposition of left- and right-moving modes. A simple closed-loop solution we will use is [19]

$$\begin{aligned}\mathbf{x}(\sigma,t) = \frac{L}{4\pi}\{&\hat{\mathbf{e}}_1[(1-\kappa)\sin\sigma_- + \tfrac{1}{3}\kappa\,\sin 3\sigma_- + \sin\sigma_+] - \hat{\mathbf{e}}_2[(1-\kappa)\cos\sigma_- + \tfrac{1}{3}\kappa\,\cos 3\sigma_- + \cos\theta\,\cos\sigma_+]\\ &-\hat{\mathbf{e}}_3[2\sqrt{\kappa(1-\kappa)}\cos\sigma_- + \sin\theta\,\cos\sigma_+]\} \ ,\end{aligned} \tag{4}$$

where $\sigma_\pm = (2\pi/L)(\sigma \pm t)$, the loop has length L and period $L/2$, and the parameters κ, θ take values in the range $0< \kappa < 1$ and $-\pi < \theta < \pi$. Note that string motion is generally relativistic, with a rms velocity for points on a loop $\langle v^2\rangle^{1/2} = 1/\sqrt{2}$. The energy-momentum tensor of the string in flat space is given by

$$T^{\mu\nu}(\mathbf{y},t) = \mu \int d\sigma(\dot{\mathbf{x}}^\mu\dot{\mathbf{x}}^\nu - \mathbf{x}'^\mu\mathbf{x}'^\mu)\delta^{(3)}(\mathbf{y} - \mathbf{x}(\sigma,t)) \ . \tag{5}$$

In an expanding Friedmann-Robertson-Walker (FRW) universe, we cannot apply the gauge condition (2) as well as $\tau = t$, so the equations of motion become

$$\ddot{\mathbf{x}} + \frac{\dot{a}}{a}(1-\dot{\mathbf{x}}^2)\dot{\mathbf{x}} = \epsilon^{-1}(\epsilon^{-1}\mathbf{x}')' \ , \tag{6}$$

$$\dot{\epsilon} = -2\frac{\dot{a}}{a}\epsilon\dot{\mathbf{x}}^2 \ , \tag{7}$$

where $\mathbf{x}$ is the comoving string position, τ is conformal time, and the σ-energy density along the string is given by

$$\epsilon = \left(\frac{\mathbf{x}'^2}{1-\dot{\mathbf{x}}^2}\right)^{1/2} \ . \tag{8}$$

The evolution is quite different from that in flat space on large scales comparable to horizon. However, if a loop is much smaller than the horizon then, because the period of the loop motion is much less than one Hubble time ($T \approx L/2 \ll H^{-1}$), we have

$$\ddot{\mathbf{x}} >> H\dot{\mathbf{x}} \ . \tag{9}$$

It is a good approximation, therefore, to treat small loops as if they were in flat space. For infinite strings, the same will apply to small-scale oscillations or wiggles. However, on large scales Hubble damping will be significant and must be taken into account.

For the purposes of these small-scale simulations, we employ flat space evolution; a loop remains at the same physical size, while shrinking in the comoving coordinates characterizing the surrounding matter. Numerically, this allows us to use either an explicit analytic solution such as (4) or else straightforward evolution algorithms.

B. Linearized gravity: The force

The forces that the string network exerts on nearby matter can be found using linearized gravity provided $G\mu \ll 1$. We consider metric perturbations $h^{\mu\nu}$ about the flat space Minkowski metric $\eta^{\mu\nu}$,

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}, \ |h^{\mu\nu}| \ll 1 \ , \tag{10}$$

and then the Einstein equations can be linearized in $h^{\mu\nu}$. If we choose the harmonic gauge

$$\partial_\nu (h^\nu_\mu - \tfrac{1}{2}\delta^\nu_\mu h^\lambda_\lambda) = 0 \ , \tag{11}$$

then the field equations reduce to

$$\partial_\lambda \partial^\lambda h^{\mu\nu} = -16\pi G S^{\mu\nu} \ , \tag{12}$$

where $S^{\mu\nu}$ is given in terms of the string energy-momentum tensor (5):

$$S^{\mu\nu} = T^{\mu\nu} - \tfrac{1}{2}\eta^{\mu\nu} T^\sigma_\sigma \ . \tag{13}$$

The retarded solution of the wave equation (12) can then be written as (see, for example, [20])

$$h^{\mu\nu}(\mathbf{y},t) = -4G\mu \int \frac{S^{\mu\nu}(\mathbf{y}, t - |\mathbf{y}-\mathbf{x}|) d^3x}{|\mathbf{y}-\mathbf{x}|} \ . \tag{14}$$

Using (5) and the solution of the wave equation (14), we can integrate over $\mathbf{x}$ to obtain

$$h^{\mu\nu}(\mathbf{y},t) = -4G\mu \int \frac{F^{\mu\nu}(\sigma, t_{\rm ret}) d\sigma}{|\mathbf{y}-\mathbf{x}(\sigma,t_{\rm ret})|[1-\mathbf{n}\cdot\dot{\mathbf{x}}(\sigma,t_{\rm ret})]} \ , \tag{15}$$

where

$$\mathbf{n} = \frac{\mathbf{y}-\mathbf{x}(\sigma,t_{\rm ret})}{|\mathbf{y}-\mathbf{x}(\sigma,t_{\rm ret})|} \ ,$$
$$t_{\rm ret} = t - |\mathbf{y}-\mathbf{x}(\sigma,t_{\rm ret})| \ , \tag{16}$$

$$F^{\mu\nu} = \dot{x}^\mu \dot{x}^\nu - x'^\mu x'^\nu + \eta^{\mu\nu} x'^\sigma x'_\sigma \ . \tag{17}$$

To determine how the strings affect particle motion, we employ the geodesic equation

$$\frac{d^2 r^\alpha}{ds^2} + \Gamma^\alpha_{\beta\gamma} \frac{dr^\beta}{ds}\frac{dr^\gamma}{ds} = 0 \ , \tag{18}$$

for which the linearized connections are

$$\Gamma^\alpha_{\beta\gamma} = \tfrac{1}{2}\eta^{\alpha\zeta}(h_{\zeta\beta,\gamma} + h_{\zeta\gamma,\beta} - h_{\beta\gamma,\zeta}) \ . \tag{19}$$

If the velocities of the particles are much smaller than the speed of light (c=1), then we have

$$\frac{dr^0}{dt} \approx 1, \ \ \frac{dr^i}{dt} \ll 1 \ , \tag{20}$$

and the geodesic equation becomes simply

$$\frac{d^2 r^i}{dt^2} = -\Gamma^i_{00} = -\tfrac{1}{2}h^{00}_{,i} - h^{0i}_{,0} \ . \tag{21}$$

In evaluating this motion numerically, we proceed by evaluating $h^{00}_{,i}$ and h^{0i} at each point in space $\mathbf{y}$ by integrating (15) along the string. The previous time step is stored for h^{0i} so that appropriate differences yield the derivatives in (21). We also apply force softening, as in N-body codes, if points approach the string too closely and near cusps. This is to prevent unphysical "kicks" due to the time discretization—the alternative would entail sophisticated variable time stepping. Given that we are most concerned about small scales we ignore retarded time effects and integrate over all points on the string simultaneously. Similar simulations were performed by Vachaspati [21] for some special loop trajectories. The difference from this work is that expression (15) was differentiated analytically before numerical evaluation, force softening does not appear to have been employed, and the motion of test particles was studied, not the growth of density perturbations. However, the linking of string source terms directly to N-body codes has been discussed by Bertschinger [22], though by apparently approximating the string as a velocity-dependent linear mass source rather than with (15).

C. The Zeldovich approximation

To study string-induced density fluctuations in a flat Robertson-Walker background we employ the Zeldovich approximation. The cold dark matter particle trajectories r^i are given by

$$r^i = a(t)[q^i + \psi^i(\mathbf{q},t)] \ , \tag{22}$$

where q^i is the unperturbed comoving position of the particle and ψ^i is its comoving displacement. In the presence of perturbing string seed, the particle will obey

$$\frac{d^2 r^i}{dt^2} = F^i_{\rm seed} + F^i_{\rm matter} \ , \tag{23}$$

where the force $F^i_{\rm matter}$ due to the surrounding matter is given by

$$F^i_{\rm matter} = -\frac{\partial \Phi}{\partial r^i} \tag{24}$$

with the gravitational potential Φ satisfying the Poisson equation

$$\nabla^2 \Phi = 4\pi G \rho_{\rm matter} \ . \tag{25}$$

In the linear regime with ($|\vec{\psi}| \ll |\mathbf{q}|$), it is possible to rewrite (23) using (25) as an equation for the comoving displacement [23]:

$$a\left(\frac{\partial^2}{\partial t^2} + 2\frac{\dot a}{a}\frac{\partial}{\partial t} + 3\frac{\ddot a}{a}\right)\psi^i = F^i_{\rm seed} \ . \tag{26}$$

We solve this driven ordinary differential equation using a standard fourth-order Runge-Kutta scheme.

Finally, we evaluate the source term due to the strings $F^i_{\rm seed}$ using the weak-field gravity accelerations found in (21). While this is reasonable, it depends on a number of assumptions. First, we assume that the self-gravity of the strings and the gravity of the surrounding matter does not significantly affect string motion; we have verified this assumption analytically [24]. As a consequence we can treat strings as a weak external source. Second, there is a gauge choice ambiguity in matching motion for linearized gravity (21) to that in a FRW background (22). However, the harmonic gauge (11) appears to be a natural (nonunique) identification because for weak fields we obtain the Newtonian limit with the gravitational potential given by $\Phi = -\frac{1}{2}h_{00}$. Note that the gravitational effects induced by defects are manifestly causal, so we need not be too concerned about effects such as global deficit angles in realistic string simulations.

Compensation is another issue that we justifiably ignore in these preliminary small-scale simulations. When strings form, there is a corresponding underdensity created in the background energy density. This original underdensity (along with subsequent radiation from the strings) will spread out and thereafter always compensate the string overdensity on scales approaching the horizon, effectively "cutting off" defect gravity on large scales. A long-distance cutoff for the gravitational force at some fixed fraction of the horizon is an approximate means to implement this compensation near the horizon. Here, however, we are considering simulations which initially are much smaller than the horizon.

D. The adhesion modification

Comparisons with N-body codes have demonstrated that the Zeldovich approximation can be projected somewhat into the nonlinear regime. Ultimately, however, shell crossing occurs and the particles continue unaffected along their trajectories. For defect models, shell crossing occurs relatively early on small scales, so it is a problem which must be addressed if density fluctuations are not to be unrealistically "washed out." Here, in the first instance, we introduce a viscosity term into the displacement equation (26) making what is known as the adhesion approximation which simulates gravitational viscosity between the particles [25]. This has been studied in the context of Gaussian initial density fluctuations in order to successfully reproduce the many features of N-body simulations at low computational cost (see, for example, Weinberg and Gunn [26]). The equation of motion of the particles (26) is merely altered by a second derivative of $\dot{\psi}^i$ with a coefficient ν which parametrizes the intensity of the viscosity. On the one hand, we can tune ν to minimize shell crossing while, on the other, ensuring that evolution in the linear regime is not significantly affected. Ideally, we anticipate choosing ν to give well-known virialization results for the pertinent scales under study. For example, we would typically want to obtain the final virialized radius to be half the turnaround radius. In practice, we choose ν to replicate wake thickness results from N-body simulations.

The formation of a wake behind a moving straight string is illustrated in Fig. 1 to illustrate the contrast between the Zeldovich approximation with and without the adhesion term included in (26). Without adhesion,

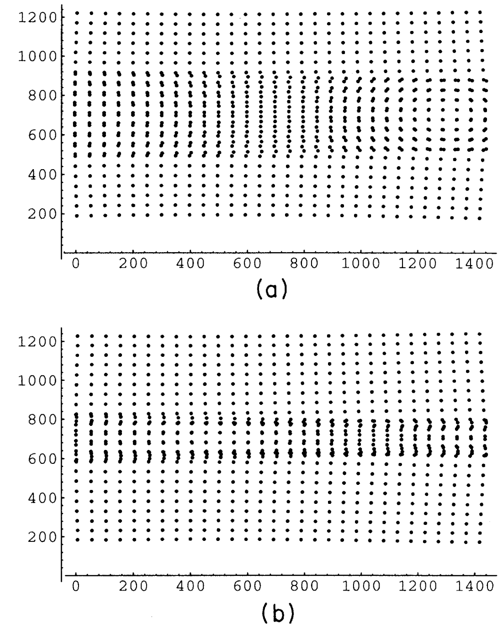

FIG. 1. Comparison of the Zeldovich approximation (a) with the adhesion modification (b) for the wake left behind a moving string. The string, lying perpendicular to the plane of the points, travels from left to right with velocity $v_x = 0.5c$ ($t_i = t_{\rm eq}$ and the box size today $t_f = t_0$ is 10 Mpc). In (a), the shell crossing has broadened the distant "pancake" that can be observed in the adhesion approximation with ν=1.5.

shell crossing is severe and the wake is largely "washed out;" however, its inclusion successfully prevents this and creates a distinct and stable nonlinear feature, as would be expected in realistic collapse. Since early nonlinearities are a feature of string-induced fluctuations, the adhesion modification is essential to the usefulness of the Zeldovich approximation at late times. For this reason, cosmic strings (and other topological defects) may prove to be the most suitable application of the adhesion approximation.

E. The N-body tree code

Although the adhesion approximation gives a good qualitative picture of the main large-scale features formed through gravitational collapse, it fails to give accurate quantitative results on small scales. In order to be able to compute density profiles and peculiar velocity diagrams we have developed an N-body tree code for following nonlinear evolution—this has an $N \ln N$ dependence on the particle number N. This has been thoroughly tested and directly compared with a simple N^2 code. Energy conservation was observed to be better than 1%. A comparison between analytic and N-body results was also performed for spherical collapse (as shown in Fig. 2). These were demonstrably in excellent agreement until the moment when the inner shells began crossing the specific shell under study—a point at which the analytic calculations break down.

The tree code operates by breaking the numerical grid into successively smaller boxes. Each box contains information about the cumulative mass, dipole, and quadrupole moments of all the particles within it. Unlike in an N^2 code, when the force on each particle is calculated it is not necessary to sum over all of the other particle contributions individually. Instead, the program begins at the top of the tree and runs over all boxes at that level. The angle subtended by the boxes, $\theta \approx$(box size)/(box-c.m.-particle distance), determines whether or not it has to go one step down the tree to calculate the force on the test particle with the required precision (θ=0.2). On small scales ($\lesssim 0.5\Delta x$, the initial comoving particle separation), we introduce force softening to ensure energy and momentum conservation. Although this implies some loss in resolution, it is a consequence of our fixed time step.

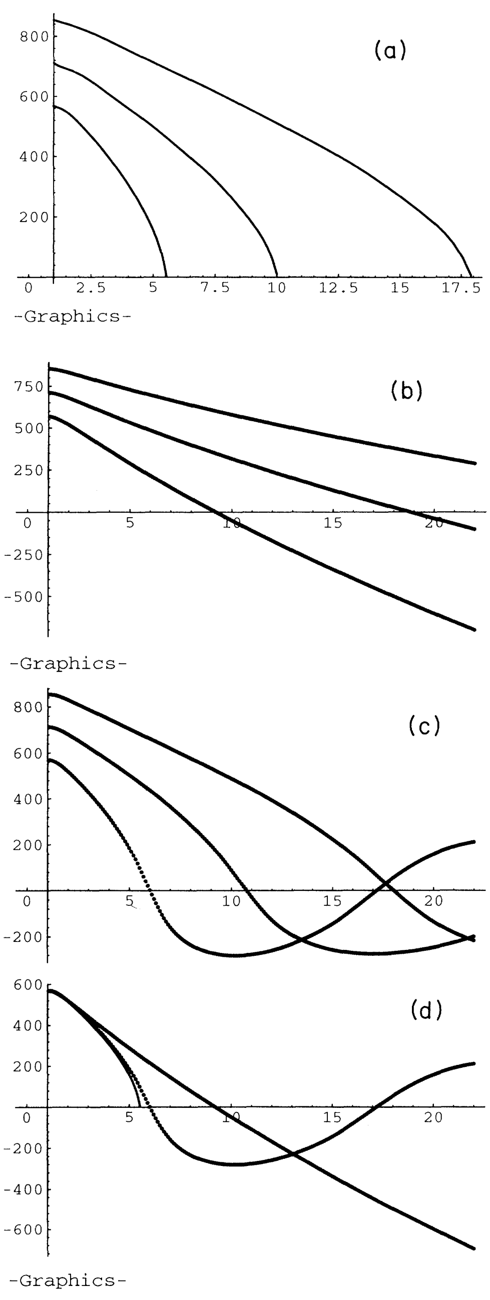

FIG. 2. Comparison of (a) the analytic spherical collapse model, (b) the Zeldovich approximation, and (c) the N-body tree code. The three methods are compared directly for one shell in (d). Deviation between the N-body code and the spherical collapse models occurs with shell crossing.

In general, an N-body tree code algorithm has a much larger dynamic range than particle-mesh codes which solve Poisson's equation using fast Fourier transforms. Although there are means to overcome this problem in particle-mesh codes, for example, by using particle-particle interactions on small scales, the computational time can grow dramatically as clustering becomes stronger. The computational time of our tree code was observed to be relatively insensitive to the degree of clustering.

The boundary conditions for an N-body tree code simulation present considerable difficulties only if they are required to be periodic. Periodicity, however, is not a feature expected of the cosmic strings which usually extend well beyond the N-body simulation region. It is far simpler and more appropriate, therefore, to employ "free" boundary conditions. Essentially, we merely carve a spherical region out of an expanding universe and evolve it independently; the fact that this region is unaffected by (sufficiently homogeneous) exterior regions is a well-known Newtonian limit. These boundary conditions have been employed elsewhere (see, for example, [27]). We have also experimented with N-body evolution in a spherical region carved from a larger cubic grid which is concurrently evolved using the Zeldovich or adhesion approximation. While significantly less of the inner region must be discarded in the final analysis, there seemed to be few other advantages from this added complication.

The same treelike structures used in the N-body code are used to calculate other physical quantities, such as density and velocity profiles using appropriate window functions. We have also developed an $N \ln N$ tree code for the evaluation of the gravitational forces induced by the strings.

F. Vorticity and alternative approaches

It is significant that the generation of nonscalar perturbations, such as vorticity, does not appear to be excluded by our formalism. The force term in the Zeldovich displacement equation (26) has two components given in (21). The first term $h^{00}_{,i}$ is irrotational but the second term $h^{0i}_{,0}$ may have a rotational component. This is somewhat puzzling because in linear theory it is widely believed that vorticity does not couple to gravitational source terms. For a nonrelativistic source, it is clear that $h^{0i}_{,0}$ terms will be subdominant, so perhaps this oversight is understandable. However, for relativistic sources such as cosmic strings or other topological defects we can see no *a priori* reason why the generation of nonscalar perturbations might not be significant. Of course, in an expanding background vorticity will die away, while density fluctuations grow, so it might still be justifiably ignored. We note here the distinction from other nonlinear vorticity generation mechanisms using baryonic shocks which have been discussed in the literature [28,29].

With these considerations in mind, we have briefly considered the effect of the source term $h^{0i}_{,0}$ during the initial linear perturbation growth. For stationary loops in flat space, it is well known that the h^{0i} terms average to zero over one oscillation period; we have verified this numerically (refer below to Sec. III) and we see strong cancellation even for small loops in an expanding background. However, for general moving string configurations this cancellation is no longer expected. Preliminary results indicate that these terms may have a substantial impact, reducing the amplitude of some high-density peaks by up to 50%, although the overall large-scale density patterns were not greatly affected. Even if the rotational component of this contribution were small then it might, for example, be invoked to create primordial magnetic fields through an analogue of the Harrison mechanism, along with other wider implications for all defect models. But this possibility is not the subject of this paper and we leave a fuller discussion of vorticity generation, gauge choice issues, modifications to the Zeldovich approximation, and quantitative numerical results for publication elsewhere [30].

It is interesting to note, however, that other work on structure formation with cosmic strings is being pursued by Bouchet *et al.* [16] using the linear VS synchronous gauge formalism ($h^{00} = h^{0i} = 0$), followed by evolution with an N-body code. While the VS formalism is valid up to large (superhorizon) scales, it implicitly treats only the irrotational contributions induced by the strings. A quantitative comparison with our own work on small scales may prove to be illuminating.

III. STRING REALIZATIONS

A cosmic string network in the early Universe evolves toward a scaling regime in which the number of strings per horizon volume remains constant [6–9]. The strings can be divided fairly neatly into two categories: (i) There are *small loops* of length $L \lesssim H^{-1}$ which oscillate relativistically and lose their energy slowly through gravitational radiation (or other preferred channels for global or superconducting strings); (ii) there is also the *long string network* which has slow coherent motions but relativistic substructure—this rapidly loses energy through the formation of small loops. We have successfully applied the methods outlined in Sec. II to both loops and long strings.

A. Loop accretion

Before embarking on simulations of general string configurations, it is important to establish that our numerical code reproduces well-known analytic results. In particular, at large distances the average field of an oscillating loop reduces to that of a point mass with $M = \mu L$. The reason for this is that the time derivatives of the metric tensor vanish when integrated over a complete oscillation period. Hence, only the Newtonian contribution from $h^{00}_{,i}$ will be effective in (21). However, this fact is complicated by strong beams of gravitational radiation from cusp regions when the string moves close to the speed of light. Nevertheless, for very small loops (many times smaller than the grid resolution scale) we were able to demonstrate spherical collapse which accurately replicated the fall-off and density contrast of matter accreting about a stationary point mass. Such spherical collapse about a small loop is shown in Fig. 3.

On scales comparable to the loop, however, accretion can be inhomogeneous and anisotropic because of the asymmetric loop shape and the presence of cusps and kinks. Figure 4 contrasts accretion patterns about the small loop in Fig. 3 and a much larger loop with $L \sim 10\Delta x$ given by (4). The mismatch of the natural time step for string evolution and structure formation meant that the loop source here was only applied for one oscillation period. Nevertheless, the appearance of strong features in the accretion pattern is interesting. An induced quadrupole moment is a useful feature in the "old string scenario" because galaxies forming about loops can acquire angular momentum through near-neighbor torquing. We note, however, that the center of mass of the loops in Fig. 4 is static, but typical loops move with substantial velocities and will leave an elongated wake [31].

B. Straight string wakes

The formation of wakes behind moving straight cosmic strings has been discussed by Silk and Vilenkin [10], Steb-

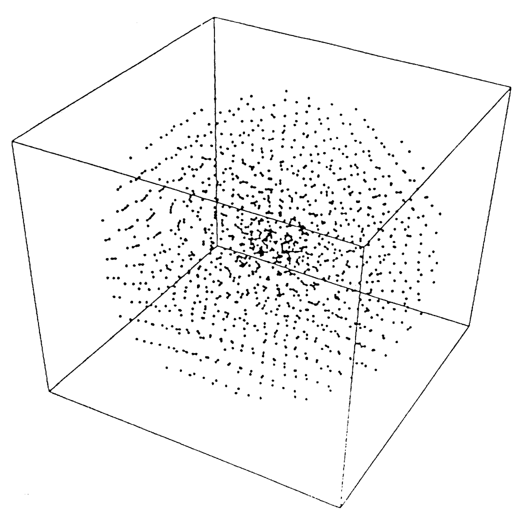

FIG. 3. Spherical collapse about an oscillating Kibble-Turok loop with size initially much smaller than the grid spacing Δx.

bins *et al.* [11], and Vachaspati [12]. The string spacetime is conical, that is, it is locally flat (so a static string exerts no gravitational force) but there is a global deficit angle $\Delta = 8\pi G\mu$. Two particles with velocity v_s moving past a string on opposite sides will acquire a relative velocity towards each other:

$$\delta v = 8\pi G\mu\gamma_s v_s \ , \tag{27}$$

where $\gamma_s = (1 - v_s^2)^{1/2}$. The eventual collision of the particles results in the formation of a "pancake" or wake behind the string. A straightforward planar analysis using (27) gives an initial condition $\dot{\psi}$ for the Zeldovich approximation (26). The thickness of the wake d is defined to be the distance between the turnaround surfaces (where particles break away from the Hubble flow) and is given by

$$d = \frac{48\pi}{5} G\mu v_s \gamma_s t_i \left(\frac{t}{t_i}\right)^{4/3} . \tag{28}$$

The most prominent wakes are those formed when $t_i = t_{\rm eq}$ and, for the parameters $G\mu = 10^{-6}$, v_s=0.5, and h=0.5, the thickness of the wake is approximately 5 Mpc.

For a numerical comparison, we take a straight string which is over four times longer than the box sidelength λ. The string begins its journey at $\Delta x = \lambda/2$ outside the box, enters the grid at $t_{\rm eq}$, and travels through to a distance Δx beyond the opposite side. At this point, the computationally intensive integration of force terms from the string ceases, but the adhesion approximation growth continues. The ensuing wake structure for a string moving at v=0.5 is illustrated in Fig. 5, along with its cross-sectional contours showing a sharp density contrast. From a determination of the turnaround surfaces v_y=0, we find that the outer width of the wake is 5 Mpc, in close agreement with analytic expectations.

In Fig. 6, we compare our three structure formation algorithms: Zeldovich approximation, adhesion approximation, and N-body tree code. This illustrates that the adhesion approximation can provide a good fit to the

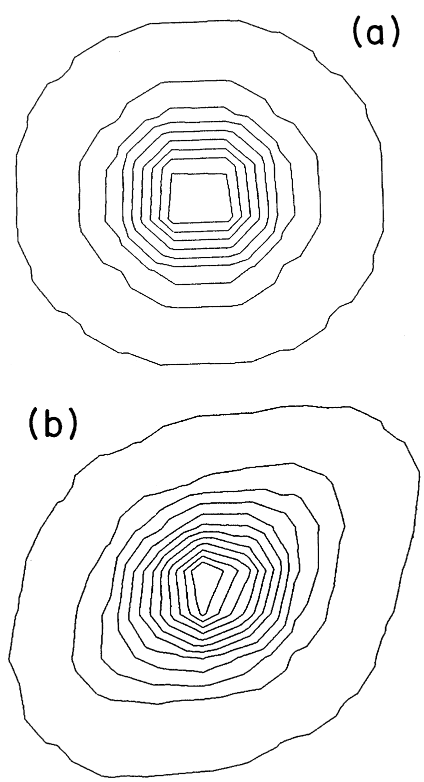

FIG. 4. Collapse about oscillating Kibble-Turok loops using the Zeldovich approximation (26)+an N-body tree code for evolving the perturbations into the nonlinear regime. (a) A stationary small loop with length $L < 0.5\Delta x$ (but enhanced mass density) produces spherical collapse. (b) A large loop with $L \sim 10\Delta x$ produced an elliptical object by the present day.

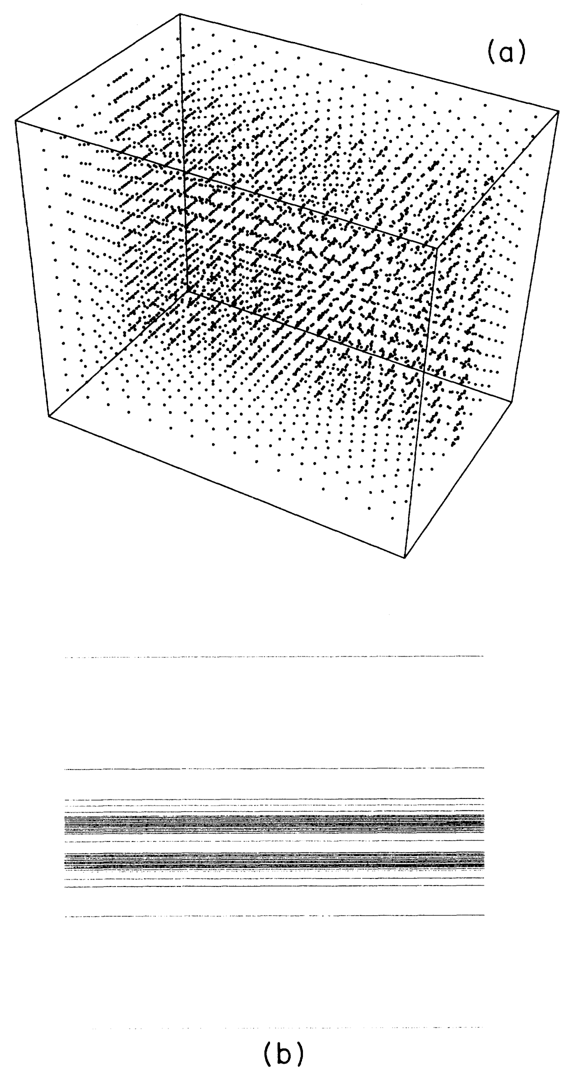

FIG. 5. (a) Three-dimensional plot of the wake left behind a straight cosmic string moving with velocity v=0.5. The adhesion approximation has been employed with $\nu \approx 1.5$ to limit shell crossing. (b) Cross-sectional density profile of this wake.

N-body simulations from appropriate choices of the viscosity parameter ν.

C. Wiggly string wakes

Long strings in a cosmic string network possess significant small-scale structure (BB, AS), so we cannot expect the resulting structures to have the idealized form illustrated in Fig. 5. Analytic treatments of structure formation with wiggly strings, such as Vachaspati and Vilenkin [28] and Vollick [29], integrate over this substructure to find a renormalized string energy $\tilde{\mu}$ and tension $\tilde{T}$. This introduces a Newtonian force term for the straight string, so the relative velocity δv in (27) becomes modified to

$$\delta v = 8\pi G\mu\gamma_s v_s + \frac{4\pi G(\tilde{\mu} - \tilde{T})}{v_s\gamma_s} . \qquad (29)$$

We can eliminate the string tension by employing the wiggly string equation of state, $\tilde{\mu}\tilde{T} = \mu^2$ [32].

To further test our numerical approach we studied the properties of the wake left behind wiggly strings, possessing a number of randomly superposed harmonics. In a specific example, we directly compared our results for string perturbations centered on the wavelength $\lambda/6$ ($\tilde{\mu}$=1.6 and v=0.5) with those obtained using the Zeldovich approximation but with (29) as an initial condition for the same $\tilde{\mu}$ and v. Agreement was remarkable (better than 1%) for particle distances initially greater than the typical wavelength of the wiggles. Of course, points nearer the string acquire significant inhomogeneities, creating instabilities which affect subsequent infall as we shall discuss below.

The close agreement at large and intermediate distances confirms both the accuracy of our code and the analytic idealization of wiggly string gravitational effects using a renormalized string energy density $\tilde{\mu}$. This proves useful for estimating the mass density of a wiggly string wake in different regimes, but ultimately the analytic approach has some serious inadequacies. Substructure on a realistic cosmic string has a spectrum from a broad range of scales, so it cannot be idealized simply by small-scale wiggles—indeed, power increases in strength up to the string correlation length. Furthermore, an analysis using (29) again produces homogeneous straight string wakes such as those shown in Fig. 5, so important issues such as the actual nonlinear width and fragmentation of the wake cannot be addressed.

In order to understand the formation and fate of wakes, therefore, we need to study realistic wiggly strings which can displace matter in all directions, rather than just those with planar symmetry. To this end, we introduced moving strings possessing a random superposition of lower harmonics (the first, second, third, and fourth) with the fundamental wavelength comparable to the boxsize λ. The effect of a particular realization is illustrated in Figs. 6 and 7 where the string has renormalized energy density $\tilde{\mu} = 1.25\mu$ and velocity v_s=0.5 (these could be realistic values on length scales well below the network correlation length [33]). Despite the restriction of shell crossing with the adhesion approximation, the accretion pattern left behind the strings is considerably less distinct than that shown in Fig. 5. As expected the overall mass of the wake is greater than that for a straight string, but the nonlinear width is also broader. The most notable features, however, are the inhomogeneities that the wiggles have introduced into the ensuing pancake structure.

It should be noted that these preliminary simulations were of fairly low resolution with $\Delta x \sim 0.25$ Mpc and between 32^3 and 64^3 particles. Nevertheless, we were

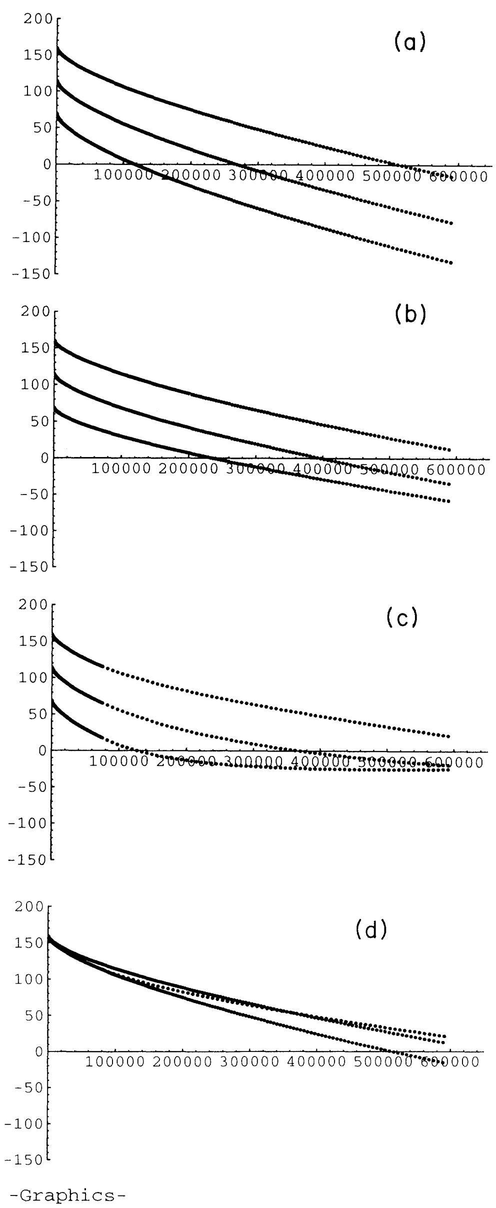

FIG. 6. Comparison of structure formation algorithms in the wake of a straight string. Collapse transverse to the wake is plotted: (a) The Zel'dovich approximation, (b) the adhesion modification with $\nu \approx 1.5$, (c) the N-body code, and (d) a direct comparison of all three approaches for a single shell.

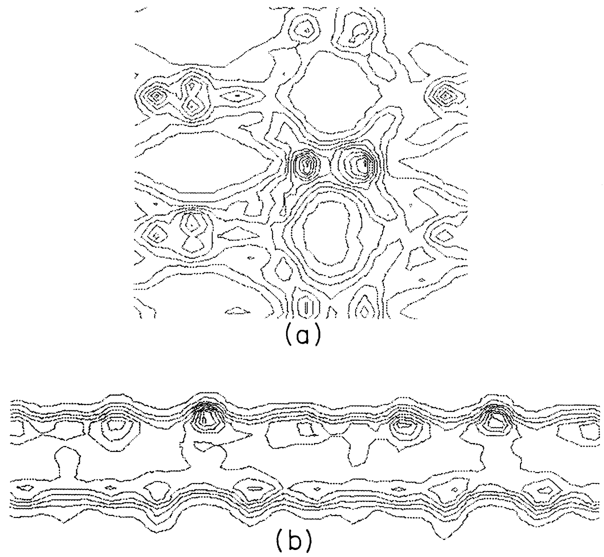

FIG. 7. The adhesion approximation: (a) Cross-sectional density profile in the xy plane of the wake left behind a wiggly cosmic string with renormalized $\tilde{\mu} = 1.25\mu$ and velocity v_x=0.5. (b) Density profile of inhomogeneities within the string wake (the xz plane).

able to measure density contrasts exceeding $\delta \approx 10$ within the string wakes and peculiar velocities near $v_{\rm rms} \sim 300$ km s^{-1}. A more quantitative analysis will only prove to be worthwhile when linked with realistic string network simulations.

"Top-down" models of structure formation which proceed through pancake formation must have mechanisms for their fragmentation into galaxies. This has always been a failing, for example, of hot dark matter models with adiabatic Gaussian perturbations. Here, we observe the first concrete evidence of the potential efficacy of fragmentation mechanisms for realistic string wakes. Not only has the pancake broken up into distinct high-density patches (Fig. 7), these regions possess asymmetries which would favor the acquisition of angular momentum during further collapse. There have been some heuristic discussions of wake fragmentation in the literature, notably in Ref. [28] where it is thought to be caused by rapidly moving small loops or in Ref. [28] (and others) where it is associated with hydrodynamic shocks in the baryonic matter component. From this preliminary analysis, however, it would appear that the seeds for the demise of a wiggly string wake lies within its initial imprint.

IV. DISCUSSION AND CONCLUSIONS

The results in this preliminary study encourage us to believe that the problem of structure formation with cosmic strings is numerically tractable. Integration of the Zeldovich approximation (26) with the adhesion modification is very efficient and linking with an N-body code enables us to obtain more accurate evolution on small scales, particularly wake fragmentation. The most computationally expensive component is the evaluation of the gravitational forces from the strings, since this entails an integration over the entire string length at each point in space. Further developments were required before this could be scaled up to a full string network (with up to a million string points). However, just as an N-body tree code collects distant points together to reduce the number of operations, the same is possible for distant string segments. At the very least, we can study wake formation and fragmentation using wiggly string sections taken from a realistic string simulation. By considering only small scales we have avoided a number of difficult issues associated with string structure formation, such as compensation and retarded time effects. We believe, however, that the approximations we have made are justifiable in this limit (Fig. 8).

The chief physical results we demonstrate in this work are, first, the confirmation of the renormalization idealization for wiggly strings and, second, the inhomogeneous nature of the wakes they produce. The latter indicates a natural and efficient mechanism for the nonlinear fragmentation of these pancakes. We have only considered structure formation with cold dark matter but, while these effects will be somewhat suppressed in hot and mixed dark matter models, they should still operate. We also noted that our formalism apparently allows the generation of vorticity by cosmic strings, an issue we address in greater quantitative detail elsewhere [30].

It remains to make only a general comment. Cosmic-string-seeded structure formation models have often been discussed on the basis of an inadequate understanding of

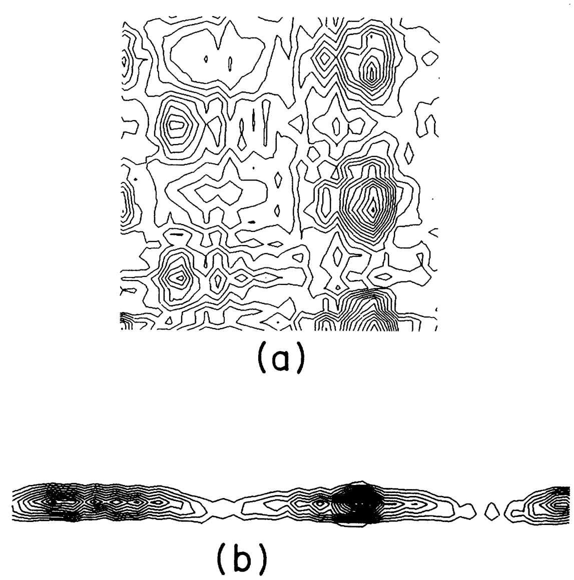

FIG. 8. The N-body tree code: (a) Cross-sectional density profile in the xy plane of the wake left behind a wiggly cosmic string with renormalized $\tilde{\mu}$=1.25μ and velocity v_x=0.5. (b) Density profile of inhomogeneities within the string wake (the xz plane).

either the evolution of a cosmic string network or the complexity of the gravitational processes at work. The lesson of this paper is that to make quantitative predictions in the cosmic-string scenario (and probably other defect models), there appears to be little alternative to examining the full implications of non-Gaussianity and nonlinear fragmentation. We have presented a formalism which we believe makes this problem tractable—at least on small to medium scales—and we are proceeding to a study structure formation with a realistic string network using these methods.

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