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B_c spectroscopy

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In the framework of potential models for heavy quarkonium the mass spectrum for the system $(\bar{b}c)$ is considered. Spin-dependent splittings, taking into account a change of a constant for the effective Coulomb interaction between the quarks, and widths of radiative transitions between the $(\bar{b}c)$ levels are calculated. In the framework of QCD sum rules, the masses of the lightest vector B_c^* and pseudoscalar B_c states are estimated, the scaling relation for leptonic constants of heavy quarkonia is derived, and the leptonic constant f_{B_c} is evaluated.

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INTRODUCTION

Recently, theoretical interest has risen in the study of the B_c meson, the heavy $(\bar{b}c)$ quarkonium with open charm and b quarks. This interest is stimulated by the experimental B_c search, being performed at Fermilab and the CERN e^+e^- collider LEP.

On the one hand, similar to D and B mesons with the open charm and b quarks, respectively, B_c is a long-living particle, decaying due to weak interaction. On the other hand, B_c consists of heavy quarks, and, therefore, it can be reliably described by the use of methods developed for the $(\bar{c}c)$ charmonium and the $(\bar{b}b)$ bottomonium.

As for B_c production, exact analytic expressions have been recently derived for the functions of heavy quark fragmentation into heavy quarkonium [1–3], within the scaling limit $M^2/s \ll 1$. The functions depend on the ratios of the b- and c-quark masses to the B_c mass. The normalization of the $b \to B_c^{(*)-} c$ fragmentation functions also depends on the leptonic constant value for the B_c meson.

When describing the B_c decays, it is important to know the spectroscopic characteristics of the B_c meson.

Some preliminary estimates of the bound state masses of the $(\bar{b}c)$ system have been made in [4, 5], devoted to the description of the charmonium and bottomonium properties, as well as in Ref. [6]. Recently in Refs. [7] and [8] the revised analysis of the B_c spectroscopy has been performed in the framework of the potential approach and QCD sum rules.

In the present paper we consider the $(\bar{b}c)$ spectroscopy with an account of the change of the effective Coulomb interaction constant, defining spin-dependent splittings of the quarkonium levels. We calculate the widths of radiative transitions between the levels and analyze the leptonic constant f_{B_c} in the framework of the QCD sum rules in the scheme, allowing one to derive the scaling relation for the leptonic constants of the heavy quarkonia.

So, in addition to our previous considerations of the B_c family, the spin-dependent splittings in the $(\bar{b}c)$ system with the Martin potential are recalculated with the improved choice of the effective coupling with gluons. The consistency condition for the confining potential and one-gluon exchange in the spin-dependent forces is taken into account, and some numerical errors are canceled. Explicit expressions for the splittings are given. The phenomenological regularities in the heavy quarkonium spectra are discussed in detail. The tables for the radiative decay widths in the $(\bar{b}c)$ system are essentially expanded to include the P- to D-wave $E1$ transitions and suppressed $M1$ transitions. Calculations for some hadronic transitions are reviewed. The detailed comparison of numerical results, obtained by Eichten and Quigg, and in the present paper, is performed. In the part concerning the evaluation of leptonic constant for the basic B_c state, the QCD sum rule analysis is modified to generalize the scaling relation for the leptonic constants of heavy quarkonia with the hidden flavors to the case of B_c.

In Sec. I we calculate the mass spectrum of the $(\bar{b}c)$ system with an account of spin-dependent forces. In Sec. II the widths of the radiative transitions in the B_c meson family are evaluated. In Sec. III the leptonic constant of B_c is calculated. In the conclusion we discuss the obtained results.

I. MASS SPECTRUM OF B_c MESONS

The B_c meson is the heavy $(\bar{b}c)$ quarkonium with open charm and b quarks. It occupies an intermediate place in the mass spectrum of the heavy quarkonia between the $(\bar{c}c)$ charmonium and the $(\bar{b}b)$ bottomonium. The approaches, applied to the charmonium and bottomonium study, can be used to describe the B_c meson properties; experimental observation of B_c could also serve as a test for these approaches, and it could be used for the detailed quantitative study of the mechanisms of the heavy quark production, hadronization, and decays.

In the present section we obtain the results on the B_c meson spectroscopy. We will show that below the threshold for the hadronic decay of the $(\bar{b}c)$ system into the BD meson pair, there are 16 narrow bound states, cascadely decaying into the lightest pseudoscalar $B_c^+(0^-)$ state with the mass $m(0^-) \simeq 6.25$ GeV.

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A. Potential

The mass spectra of the charmonium and the bottomonium are experimentally studied in detail [9], and they are properly described in the framework of phenomenological potential models of nonrelativistic heavy quarks [4, 5, 10–12]. To describe the mass spectrum of the $(\bar{b}c)$ system, one would prefer to use the potentials, whose parameters do not depend on the flavors of the heavy quarks, composing a heavy quarkonium; i.e., one would use the potentials, which rather accurately describe the mass spectra of $(\bar{c}c)$ as well as $(\bar{b}b)$, with one and the same set of potential parameters. The use of such potentials allows one to avoid an interpolation of the potential parameters from the values, fixed by the experimental data on the $(\bar{c}c)$ and $(\bar{b}b)$ systems, to the values in the intermediate region of the $(\bar{b}c)$ system.

As has been shown in Ref. [13], with an accuracy up to an additive shift, the potentials, independent of heavy quark flavors [4, 5, 10–12], coincide with each other in the region of the average distances between heavy quarks in the $(\bar{c}c)$ and $(\bar{b}b)$ systems, so

$$0.1 \text{ fm} < r < 1 \text{ fm} , \tag{1}$$

although those potentials have different asymptotic behavior in the regions of very low $(r \to 0)$ and very large $(r \to \infty)$ distances.

In the model of Eichten *et al.* [4], in accordance with asymptotic freedom in QCD, the potential has the Coulomb-like behavior at low distances, and the term, confining the quarks, rises linearly at large distances,

$$V_C(r) = -\frac{4}{3}\frac{\alpha_S}{r} + \frac{r}{a^2} + c_0 , \tag{2}$$

so that

$$\begin{aligned} \alpha_S &= 0.36 , \\ a &= 2.34 \text{ GeV}^{-1} , \\ m_c &= 1.84 \text{ GeV} , \\ c_0 &= -0.25 \text{ GeV} . \end{aligned} \tag{3}$$

The Richardson potential [10] and its modifications in Refs. [12] and [14] also correspond to the behavior, expected in the framework of QCD, so

$$V_R(r) = -\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{r}\cdot\mathbf{q}}\, \frac{4}{3}\, \frac{12\pi}{27}\, \frac{1}{q^2 \ln(1+q^2/\Lambda^2)} , \tag{4}$$

so that

$$\Lambda = 0.398 \quad \text{GeV} . \tag{5}$$

In the region of the average distances between heavy quarks (1), the QCD-motivated potentials allow the approximations in the forms of the power (Martin) or logarithmic potentials.

The Martin potential has the form [11]

$$V_M(r) = -c_M + d_M(\Lambda_M r)^k , \tag{6}$$

so that

$$\begin{aligned} \Lambda_M &= 1 \quad \text{GeV} , \\ k &= 0.1 , \\ m_b &= 5.174 \quad \text{GeV} , \\ m_c &= 1.8 \quad \text{GeV} , \\ c_M &= 8.064 \quad \text{GeV} , \\ d_M &= 6.869 \quad \text{GeV} . \end{aligned} \tag{7}$$

The logarithmic potential is equal to [15]

$$V_L(r) = c_L + d_L \ln(\Lambda_L r) , \tag{8}$$

so that

$$\begin{aligned} \Lambda_L &= 1 \quad \text{GeV} , \\ m_b &= 4.906 \quad \text{GeV} , \\ m_c &= 1.5 \quad \text{GeV} , \\ c_L &= -0.6635 \quad \text{GeV} , \\ d_L &= 0.733 \quad \text{GeV} . \end{aligned} \tag{9}$$

The approximations of the nonrelativistic potential of heavy quarks in the region of distances (1) in the form of the power (6) and logarithmic (8) laws, allow one to study its scaling properties.

In accordance with the virial theorem, the average kinetic energy of the quarks in the bound state is determined by the expression

$$\langle T \rangle = \frac{1}{2}\left\langle \frac{r dV}{dr} \right\rangle . \tag{10}$$

Then, the logarithmic potential allows one to conclude that for the quarkonium states one gets

$$\langle T_L \rangle = \text{const} , \tag{11}$$

independently of the flavors of the heavy quarks, composing the heavy quarkonium:

$$\text{const} = d_L/2 \approx 0.367 \quad \text{GeV} .$$

In the Martin potential, the virial theorem (10) allows one to obtain the expression

$$\langle T_M \rangle = \frac{k}{2+k}\,(c_M + E) , \tag{12}$$

where E is the binding energy of the quarks in the heavy quarkonium. Phenomenologically, one has $|E| \ll c_M$ [for example, $E(1S, c\bar{c}) \simeq -0.5$ GeV], so that, neglecting the binding energy of the heavy quarks inside the heavy quarkonium, one can conclude that the average kinetic energy of the heavy quarks is a constant value, independent of the quark flavors and the number of the radial or orbital excitation. The accuracy of such approximation for $\langle T \rangle$ is about 10%, i.e., $|\Delta T/T| \sim 30$–40 MeV.

From the Feynman-Hellmann theorem for the system with the reduced mass μ, one has

$$\frac{dE}{d\mu} = -\frac{\langle T \rangle}{\mu} , \tag{13}$$

and, in accordance with condition (11), it follows that the difference of the energies for the radial excitations

TABLE I. The mass difference for the two lightest vector states of different heavy systems, $\Delta M = M(2S) - M(1S)$ in MeV.

System	Υ	ψ	B_c	ϕ
ΔM	563	588	585	660

of the heavy quarkonium levels does not depend on the reduced mass of the $Q\bar{Q}'$ system

$$E(\bar{n},\mu) - E(n,\mu) = E(\bar{n},\mu') - E(n,\mu') . \tag{14}$$

Thus, in the approximation of both the low value for the binding energy of quarks and the zero value for the spin-dependent splittings of the levels, the heavy quarkonium state density does not depend on the heavy quark flavors

$$\frac{dn}{dM_n} = \varphi(n). \tag{15}$$

The given statement has been also derived in Ref. [16] using the Bohr-Sommerfeld quantization of the S-wave states for the heavy quarkonium system with Martin potential [11].

Relations (14) and (15) are phenomenologically confirmed for the vector S levels of the $b\bar{b}$, $c\bar{c}$, $s\bar{s}$ systems [9] (see Table I).

Thus, the structure of the nonsplit S levels of the $(\bar{b}c)$ system must repeat not only qualitatively, but also quantitatively the structure of the S levels for the $\bar{b}b$ and $\bar{c}c$ systems, with an accuracy up to the overall additive shift of masses.

Moreover, in the framework of the QCD sum rules, the universality of the heavy quark nonrelativistic potential [the independence on the flavors and the scaling properties (11), (14), and (15)] allows one to obtain the scaling relation for the leptonic constants of the S-wave quarkonia [16],

$$\frac{f^2}{M} = \text{const} , \tag{16}$$

independently of the heavy quark flavors in the regime, when

$$|m_Q - m_{Q'}| \text{ is restricted} , \ \Lambda_{\rm QCD}/m_{Q,Q'} \ll 1 ,$$

i.e., when one can neglect the heavy quark mass difference, and, in the regime, when the mass difference is not low, one has

$$\frac{f^2}{M}\left(\frac{M}{4\mu}\right)^2 = \text{const} , \tag{17}$$

where

$$\mu = \frac{m_Q m_{Q'}}{m_Q + m_{Q'}} .$$

Consider the mass spectrum of the $(\bar{b}c)$ system with the Martin potential [11].

Solving the Schrödinger equation with the potential (6) and the parameters (7), one finds the B_c mass spectrum and the characteristics of the radial wave functions $R(0)$ and $R'(0)$, shown in Tables II and III, respectively.

The average kinetic energy of the levels, lying below the threshold for the $(\bar{b}c)$ system decay into the BD pair, is presented in Table IV, where one can see that the term, added to the radial potential due to the orbital rotation,

$$\Delta V_l = \frac{\mathbf{L}^2}{2\mu r^2} \tag{18}$$

weakly influences the value of the average kinetic energy, and the binding energy for the levels with $L \neq 0$ is essentially determined by the orbital rotation energy, which is approximately independent of the quark flavors (see Table V), so that the structure of the nonsplitted levels of the $(\bar{b}c)$ system with $L \neq 0$ must quantitatively repeat the structure of the charmonium and bottomonium levels, too.

B. Spin-dependent splitting of the $(\bar{b}c)$ quarkonium

In accordance with the results of Refs. [17, 18], one introduces the additional term to the potential to take into the account the spin-orbital and spin-spin interactions, causing the splitting of the nL levels (n is the principal quantum number, L is the orbital momentum), so it has the form

$$V_{\rm SD}(\mathbf{r}) = \left(\frac{\mathbf{L}\cdot\mathbf{S}_c}{2m_c^2} + \frac{\mathbf{L}\cdot\mathbf{S}_b}{2m_b^2}\right)\left(-\frac{dV(r)}{rdr} + \frac{8}{3}\,\alpha_S\,\frac{1}{r^3}\right) + \frac{4}{3}\,\alpha_S\,\frac{1}{m_c m_b}\,\frac{\mathbf{L}\cdot\mathbf{S}}{r^3} + \frac{4}{3}\,\alpha_S\,\frac{2}{3m_c m_b}\,\mathbf{S}_c\cdot\mathbf{S}_b\,4\pi\,\delta(\mathbf{r})$$
$$+\frac{4}{3}\,\alpha_S\,\frac{1}{m_c m_b}\,[3(\mathbf{S}_c\cdot\mathbf{n})\,(\mathbf{S}_b\cdot\mathbf{n}) - \mathbf{S}_c\cdot\mathbf{S}_b]\,\frac{1}{r^3} , \quad \mathbf{n} = \frac{\mathbf{r}}{r} , \tag{19}$$

TABLE II. The energy levels of the $\bar{b}c$ system, calculated without taking into account relativistic corrections, in GeV.

n	[6]	[17]	[14]	n	[6]	[17]	[14]	n	[6]	[17]	[14]
$1S$	6.301	6.315	6.344	$2P$	6.728	6.735	6.763	$3D$	7.008	7.145	7.030
$2S$	6.893	7.009	6.910	$3P$	7.122	–	7.160	$4D$	7.308	–	7.365
$3S$	7.237	–	7.024	$4P$	7.395	–	–	$5D$	7.532	–	–

TABLE III. The characteristics of the radial wave functions $R_{nS}(0)$ (in $\text{GeV}^{3/2}$) and $R'_{nP}(0)$ (in $\text{GeV}^{5/2}$), obtained from the Schrödinger equation.

n	Martin	[7]
$R_{1S}(0)$	1.31	1.28
$R_{2S}(0)$	0.97	0.99
$R'_{2P}(0)$	0.55	0.45
$R'_{3P}(0)$	0.57	0.51

where $V(r)$ is the phenomenological potential confining the quarks. The first term takes into account the relativistic corrections to the potential $V(r)$; the second, third, and fourth terms are the relativistic corrections, coming from the account of the one gluon exchange between the b and c quarks; α_S is the effective constant of the quark-gluon interaction inside the $(\bar{b}c)$ system.

The value of the α_S parameter can be determined in the following way.

The splitting of the S-wave heavy quarkonium $(Q_1\bar{Q}_2)$ is determined by the expression

$$\Delta M(nS) = \alpha_S \, \frac{8}{9m_1 m_2} \, |R_{nS}(0)|^2 \, , \tag{20}$$

where $R_{nS}(0)$ is the value of the radial wave function of the quarkonium, at the origin. Using the experimental value of the S state splitting in the $c\bar{c}$ system [9],

$$\Delta M(1S,\, c\bar{c}) = 117 \pm 2 \quad \text{MeV} \, , \tag{21}$$

and the $R_{1S}(0)$ value, calculated in the potential model for the $c\bar{c}$ system, one gets the model-dependent value of the $\alpha_S(\psi)$ constant for the effective Coulomb interaction of the heavy quarks [in the Martin potential, one has $\alpha_S(\psi) = 0.44$].

In Ref. [7] the effective constant value, fixed by the described way, has been applied to the description of not only the $c\bar{c}$ system, but also the $\bar{b}c$ and $\bar{b}b$ quarkonia.

In the present paper we take into account the variation of the effective Coulomb interaction constant versus the reduced mass of the system (μ).

In the one-loop approximation at the momentum scale p^2, the "running" coupling constant in QCD is determined by the expression

$$\alpha_S(p^2) = \frac{4\pi}{b \ln(p^2/\Lambda^2_{\text{QCD}})} \, , \tag{22}$$

where $b = 11 - 2n_f/3$, and $n_f = 3$, when one takes into account the contribution by the virtual light quarks, $p^2 < m^2_{c,b}$.

In the model with the Martin potential, for the kinetic energy of quarks $(c\bar{c})$ inside ψ, one has

TABLE IV. The average kinetic and orbital energies of the quark motion in the $(\bar{b}c)$ system, in GeV.

nL	$1S$	$2S$	$2P$	$3P$	$3D$
$\langle T \rangle$	0.35	0.38	0.37	0.39	0.39
ΔV_l	0.00	0.00	0.22	0.14	0.29

TABLE V. The average energy of the orbital motion in the heavy quarkonia, in the model with the Martin potential, in GeV.

System	$\bar{c}c$	$\bar{b}c$	$\bar{b}b$
$\Delta V_l(2P)$	0.23	0.22	0.21

$$\langle T_{1S}(c\bar{c}) \rangle \simeq 0.357 \quad \text{GeV} \, , \tag{23}$$

so that, using the expression for the kinetic energy,

$$\langle T \rangle = \frac{\langle p^2 \rangle}{2\mu} \, , \tag{24}$$

one gets

$$\alpha_S(p^2) = \frac{4\pi}{b \ln[2\langle T \rangle \mu/\Lambda^2_{\text{QCD}}]} \, , \tag{25}$$

so that $\alpha_S(\psi) = 0.44$ at

$$\Lambda_{\text{QCD}} \simeq 164 \quad \text{MeV} \, . \tag{26}$$

As has been noted in the previous section, the value of the kinetic energy of the quark motion weakly depends on the heavy quark flavors, and it, practically, is constant, and, hence, the change of the effective α_S coupling is basically determined by the variation of the reduced mass of the heavy quarkonium. In accordance with Eqs. (25) and (26) and Table IV, for the $(\bar{b}c)$ system one has

nL	$1S$	$2S$	$2P$	$3P$	$3D$
α_S	0.394	0.385	0.387	0.382	0.383.

Note, the Martin potential leads to the $R_{1S}(0)$ values, which, with the accuracy up to 15–20%, agrees with the experimental values of the leptonic decay constants for the heavy $c\bar{c}$ and $b\bar{b}$ quarkonia. The leptonic constants are determined by the expression

$$\Gamma(Q\bar{Q} \to l^+ l^-) = \frac{4\pi}{3} \, e_Q^2 \, \alpha^2_{\text{em}} \, \frac{f^2_{Q\bar{Q}}}{M_{Q\bar{Q}}} \, , \tag{27}$$

where e_Q is the heavy quark charge.

In the nonrelativistic model one has

$$f_{Q\bar{Q}} = \sqrt{\frac{3}{\pi M_{Q\bar{Q}}}} \, R_{1S}(0) \, . \tag{28}$$

For the effective Coulomb interaction of the heavy quarks in the basic $1S$ state one has

TABLE VI. The leptonic decay constants of the heavy quarkonia, the values, measured experimentally and obtained in the model with the Martin potential, in the model with the effective Coulomb interaction and from the scaling relation (SR), in MeV.

Model	Expt.[9]	Martin	Coulomb	SR
f_ψ	410 ± 15	547 ± 80	426 ± 60	410 ± 40
f_{B_c}	–	510 ± 80	456 ± 70	460 ± 60
f_Υ	715 ± 15	660 ± 90	772 ± 120	715 ± 70

$$R_{1S}^{\rm Coul}(0) = 2\left(\frac{4}{3}\,\mu\,\alpha_S\right)^{3/2} . \tag{29}$$

One can see from Table VI, that, taking into account the variation of the effective α_S constant versus the reduced mass of the heavy quarkonium [see Eq. (25)], the Coulomb wave functions give the values of the leptonic constants for the heavy $1S$ quarkonia, so that in the framework of the accuracy of the potential models, those values agree with the experimental values and the values obtained by the solution of the Schrödinger equation with the given potential.

The consideration of the variation of the effective Coulomb interaction constant becomes especially essential for the Υ particles, for which $\alpha_S(\Upsilon) \simeq 0.33$ instead of the fixed value $\alpha_S = 0.44$.

Thus, calculating the splitting of the $(\bar{b}c)$ levels, we take into account the α_S dependence on the reduced mass of the heavy quarkonium.

As one can see from Eq. (19), in contrast to the LS coupling in the $(\bar{c}c)$ and $(\bar{b}b)$ systems, there is the jj coupling in the heavy quarkonium, where the heavy quarks have different masses [here, $\mathbf{L}\cdot\mathbf{S}_c$ is diagonalized at the given $\mathbf{J}_c$ momentum, $(\mathbf{J}_c = \mathbf{L}+\mathbf{S}_c,\ \mathbf{J} = \mathbf{J}_c + \mathbf{S}_b)$, $\mathbf{J}$ is the total spin of the system]. We use the following spectroscopic notation for the split levels of the $(\bar{b}c)$ system, $- n^{2j_c}L_J$.

One can easily show that independently of the total spin J projection one has

$$\begin{aligned}
|^{2L+1}L_{L+1}\rangle &= |J = L+1,\ S=1\rangle\ ,\\
|^{2L-1}L_{L-1}\rangle &= |J = L-1,\ S=1\rangle\ ,\\
|^{2L+1}L_{L}\rangle &= \sqrt{\frac{L}{2L+1}}|J=L,\ S=1\rangle + \sqrt{\frac{L+1}{2L+1}}|J=L,\ S=0\rangle\ ,\\
|^{2L-1}L_{L}\rangle &= \sqrt{\frac{L+1}{2L+1}}|J=L,\ S=1\rangle - \sqrt{\frac{L}{2L+1}}|J=L,\ S=0\rangle\ ,
\end{aligned} \tag{30}$$

where $|J,\ S\rangle$ are the state vectors with the given values of the total quark spin $\mathbf{S} = \mathbf{S}_c + \mathbf{S}_b$, so that the potential terms of the order of $1/m_c m_b$, $1/m_b^2$ lead, generally speaking, to the mixing of the levels with the different J_c values at the given J values. The tensor forces [the last term in Eq. (19)] are equal to zero at $L=0$ or $S=0$.

One can easily show, that

$$3\left(n^p n^q - \frac{1}{3}\delta^{pq}\right) S_c^p S_b^q = \frac{3}{2}\left(n^p n^q - \frac{1}{3}\delta^{pq}\right) S^p S^q\ , \tag{31}$$

since for the quark spin one has

$$S_Q^p S_Q^q + S_Q^q S_Q^p = \frac{1}{2}\delta^{pq}\ . \tag{32}$$

The averaging over the angle variables can be represented in the form

$$\langle L,\ m|n^p n^q|L,\ m'\rangle = a(L^p L^q + L^q L^p)_{mm'} + b\delta^{pq}\ , \tag{33}$$

where $\mathbf{L}$ are the orbital momentum matrices in the respective irreducible representation.

Let us use the following conditions.

(1) The normalization of the unit vector:

$$\langle n^p n^q\rangle \delta^{pq} = 1\ . \tag{34}$$

(2) The orthogonality of the radius vector to the orbital momentum:

$$n^p L^p = 0\ . \tag{35}$$

(3) The commutation relations for the angular momentum:

$$[L^p; L^q] = i\epsilon^{pql}L_l\ . \tag{36}$$

Then one can easily find that in Eq. (33) one gets

$$a = -\frac{1}{4\mathbf{L}^2 - 3}\ , \tag{37}$$

$$b = \frac{2\mathbf{L}^2 - 1}{4\mathbf{L}^2 - 3}\ . \tag{38}$$

Thus (see also Ref. [19]),

$$\left\langle 6\left(n^p n^q - \frac{1}{3}\delta^{pq}\right) S_c^p S_b^q \right\rangle = -\frac{1}{4\mathbf{L}^2-3}[6(\mathbf{L}\cdot\mathbf{S})^2 + 3(\mathbf{L}\cdot\mathbf{S}) - 2\mathbf{L}^2\mathbf{S}^2]\ . \tag{39}$$

Using Eqs. (30) and (39), for the level shift, calculated in the perturbation theory at $S=1$, one gets

$$\Delta E_{n^1S_0} = -\alpha_S\,\frac{2}{3m_c m_b}\,|R_{nS}(0)|^2\ , \tag{40}$$

$$\Delta E_{n^1S_1} = \alpha_S\,\frac{2}{9m_c m_b}\,|R_{nS}(0)|^2\ , \tag{41}$$

$$\Delta E_{n^3P_2} = \alpha_S\,\frac{6}{5m_c m_b}\left\langle\frac{1}{r^3}\right\rangle + \frac{1}{4}\left(\frac{1}{m_c^2}+\frac{1}{m_b^2}\right)\left\langle -\frac{dV(r)}{rdr} + \frac{8}{3}\,\alpha_S\frac{1}{r^3}\right\rangle\ , \tag{42}$$

$$\Delta E_{n^1P_0} = -\alpha_S\,\frac{4}{m_c m_b}\left\langle\frac{1}{r^3}\right\rangle - \frac{1}{2}\left(\frac{1}{m_c^2}+\frac{1}{m_b^2}\right)\left\langle -\frac{dV(r)}{rdr} + \frac{8}{3}\,\alpha_S\frac{1}{r^3}\right\rangle\ , \tag{43}$$

$$\Delta E_{n^5D_3} = \alpha_S\,\frac{52}{21m_c m_b}\left\langle\frac{1}{r^3}\right\rangle + \frac{1}{2}\left(\frac{1}{m_c^2}+\frac{1}{m_b^2}\right)\left\langle -\frac{dV(r)}{rdr} + \frac{8}{3}\,\alpha_S\frac{1}{r^3}\right\rangle\ , \tag{44}$$

$$\Delta E_{n^3D_1} = -\alpha_S\,\frac{92}{21m_cm_b}\left\langle\frac{1}{r^3}\right\rangle - \frac{3}{4}\left(\frac{1}{m_c^2}+\frac{1}{m_b^2}\right)\left\langle -\frac{dV(r)}{rdr}+\frac{8}{3}\,\alpha_S\frac{1}{r^3}\right\rangle\,, \tag{45}$$

where $R_{nS}(0)$ are the radial wave functions at $L = 0$, and angular brackets denote the average values calculated under the wave functions $R_{nL}(r)$. The mixing matrix elements have the forms

$$\langle\,{}^3P_1|\Delta E|\,{}^3P_1\rangle = -\alpha_S\,\frac{2}{9m_cm_b}\left\langle\frac{1}{r^3}\right\rangle + \left(\frac{1}{4m_c^2}-\frac{5}{12m_b^2}\right)\left\langle -\frac{dV(r)}{rdr}+\frac{8}{3}\,\alpha_S\frac{1}{r^3}\right\rangle\,, \tag{46}$$

$$\langle\,{}^1P_1|\Delta E|\,{}^1P_1\rangle = -\alpha_S\,\frac{4}{9m_cm_b}\left\langle\frac{1}{r^3}\right\rangle + \left(-\frac{1}{2m_c^2}+\frac{1}{6m_b^2}\right)\left\langle -\frac{dV(r)}{rdr}+\frac{8}{3}\,\alpha_S\frac{1}{r^3}\right\rangle\,, \tag{47}$$

$$\langle\,{}^3P_1|\Delta E|\,{}^1P_1\rangle = -\alpha_S\,\frac{2\sqrt{2}}{9m_cm_b}\left\langle\frac{1}{r^3}\right\rangle - \frac{\sqrt{2}}{6m_b^2}\left\langle -\frac{dV(r)}{rdr}+\frac{8}{3}\,\alpha_S\frac{1}{r^3}\right\rangle\,, \tag{48}$$

$$\langle\,{}^5D_2|\Delta E|\,{}^5D_2\rangle = -\alpha_S\,\frac{4}{15m_cm_b}\left\langle\frac{1}{r^3}\right\rangle + \left(\frac{1}{2m_c^2}-\frac{1}{5m_b^2}\right)\left\langle -\frac{dV(r)}{rdr}+\frac{8}{3}\,\alpha_S\frac{1}{r^3}\right\rangle\,, \tag{49}$$

$$\langle\,{}^3D_2|\Delta E|\,{}^3D_2\rangle = -\alpha_S\,\frac{8}{15m_cm_b}\left\langle\frac{1}{r^3}\right\rangle + \left(-\frac{3}{4m_c^2}+\frac{9}{20m_b^2}\right)\left\langle -\frac{dV(r)}{rdr}+\frac{8}{3}\,\alpha_S\frac{1}{r^3}\right\rangle\,, \tag{50}$$

$$\langle\,{}^5D_2|\Delta E|\,{}^3D_2\rangle = -\alpha_S\,\frac{2\sqrt{6}}{15m_cm_b}\left\langle\frac{1}{r^3}\right\rangle - \frac{\sqrt{6}}{10m_b^2}\left\langle -\frac{dV(r)}{rdr}+\frac{8}{3}\,\alpha_S\frac{1}{r^3}\right\rangle\,. \tag{51}$$

As one can see from Eq. (41), the S-level splitting is essentially determined by the $|R_{nS}(0)|$ value, which can be related to the leptonic decay constants of the S states (0^-, 1^-). Section III is devoted to the calculation of these constants in different ways. We only note here that with enough accuracy the predictions of different potential models on the $|R_{1S}(0)|$ value are in agreement with each other as well as with predictions in other approaches.

For the $2P$, $3P$, and $3D$ levels, the mixing matrices of the states with the total quark spin $S = 1$ and $S = 0$ have the forms

$$|2P,\ 1'^+\rangle = 0.294|S=1\rangle + 0.956|S=0\rangle\,, \tag{52}$$

$$|2P,\ 1^+\rangle = 0.956|S=1\rangle - 0.294|S=0\rangle\,, \tag{53}$$

so that in the 1^+ state the probability of the total quark spin value $S = 1$ is equal to $w(2P) = 0.913$,

$$|3P,\ 1'^+\rangle = 0.371|S=1\rangle + 0.929|S=0\rangle\,, \tag{54}$$

$$|3P,\ 1^+\rangle = 0.929|S=1\rangle - 0.371|S=0\rangle\,, \tag{55}$$

so that $w(3P) = 0.863$,

$$|3D,\ 2'^-\rangle = -0.566|S=1\rangle + 0.825|S=0\rangle\,, \tag{56}$$

$$|3D,\ 2^-\rangle = 0.825|S=1\rangle + 0.566|S=0\rangle\,, \tag{57}$$

so that $w(3D) = 0.680$.

With an account of the calculated splittings, the B_c mass spectrum is shown in Fig. 1 and Table VII.

The masses of the B_c mesons have been also calculated in papers of Ref. [28].

As one can see from Tables II and VIII, the place of the $1S$ level in the $(\bar{b}c)$ system [$m(1S) \simeq 6.3$ GeV] is predicted by the potential models with the rather high

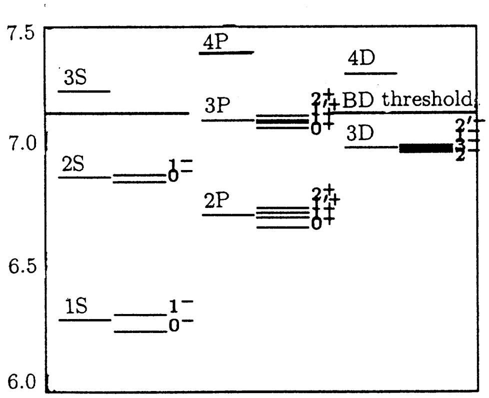

FIG. 1. The mass spectrum of the $(\bar{b}c)$ system with account of splittings.

TABLE VII. The masses (in GeV) of the bound $(\bar{b}c)$ states below the threshold of the decay into the BD meson pair (PP – the present paper).

State	PP	[7]	[14]
$1\,^1S_0$	6.253	6.264	6.314
$1\,^1S_1$	6.317	6.337	6.355
$2\,^1S_0$	6.867	6.856	6.889
$2\,^1S_1$	6.902	6.899	6.917
$2\,^1P_0$	6.683	6.700	6.728
$2P\ 1^+$	6.717	6.730	6.760
$2P\ 1'^+$	6.729	6.736	–
$2\,^3P_2$	6.743	6.747	6.773
$3\,^1P_0$	7.088	7.108	7.134
$3P\ 1^+$	7.113	7.135	7.159
$3P\ 1'^+$	7.124	7.142	–
$3\,^3P_2$	7.134	7.153	7.166
$3D\ 2^-$	7.001	7.009	–
$3\,^5D_3$	7.007	7.005	–
$3\,^3D_1$	7.008	7.012	–
$3D\ 2'^-$	7.016	7.012	–

TABLE VIII. The masses (in GeV) of the lightest pseudoscalar B_c and vector B_c^* states in different models (PP – the present paper).

State	PP	[17]	[14]	[20]	[5]	[21]	[27, 16]
0^-	6.253	6.249	6.314	6.293	6.270	6.243	6.246
1^-	6.317	6.339	6.354	6.346	6.340	6.320	6.319
State	[7]	[22]	[23]	[24]	[25]	[26]	[8]
0^-	6.264	6.320	6.256	6.276	6.286	–	6.255
1^-	6.337	6.370	6.329	6.365	6.328	6.320	6.330

accuracy $\Delta m(1S) \simeq 30$ MeV, and the $1S$-level splitting into the vector and pseudoscalar states is about $m(1^-) - m(0^-) \approx 70$ MeV.

C. B_c meson masses from QCD sum rules

Potential model estimates for the masses of the lightest $(\bar{b}c)$ states are in agreement with the results of the calculations for the vector and pseudoscalar $(\bar{b}c)$ states in the framework of the QCD sum rules [8, 29, 30], where the calculation accuracy is lower than the accuracy of the potential models, because the results essentially depend on both the modeling of the nonresonant hadronic part of the current correlator (the continuum threshold) and the parameter of the sum rule scheme (the moment number for the spectral density of the current correlator or the Borel transformation parameter):

$$m^{\rm SR}(0^-) \approx m^{\rm SR}(1^-) \simeq 6.3 - 6.5 \text{ GeV} . \qquad (58)$$

As has been shown in [31], for the lightest vector quarkonium, the following QCD sum rules take place:

$$\frac{f_V^2 M_V^2}{m_V^2 - q^2} = \frac{1}{\pi} \int_{s_i}^{s_{\rm th}} \frac{{\rm Im}\Pi_V^{\rm QCD\,pert}(s)}{s - q^2}\, ds + \Pi_V^{\rm QCD\,nonpert}(q^2) , \qquad (59)$$

where f_V is the leptonic constant of the vector $(\bar{b}c)$ state with the mass M_V,

$$i f_V M_V \epsilon_\mu^\lambda e^{ipx} = \langle 0|J_\mu(x)|V(p,\, \lambda)\rangle , \qquad (60)$$

$$J_\mu(x) = \bar{c}(x)\gamma_\mu b(x) , \qquad (61)$$

where λ, p are the B_c^* polarization and momentum, respectively, and

$$\int d^4x\, e^{iqx}\langle 0|TJ_\mu(x)J_\nu(0)|0\rangle = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\Pi_V^{\rm QCD} + q_\mu q_\nu\, \Pi_S^{\rm QCD} , \qquad (62)$$

$$\Pi_V^{\rm QCD}(q^2) = \Pi_V^{\rm QCD\,pert} + \Pi_V^{\rm QCD\,nonpert}(q^2) , \qquad (63)$$

$$\Pi_V^{\rm QCD\,nonpert}(q^2) = \sum C_i(q^2) O^i , \qquad (64)$$

where O^i are the vacuum expectation values of the composite operators such as $\langle m\bar{\psi}\psi\rangle$, $\langle \alpha_S\, G_{\mu\nu}^2\rangle$, etc. The Wilson coefficients are calculable in the perturbation theory of QCD. $s_i = (m_c + m_b)^2$ is the kinematical threshold of the perturbative contribution, $M_V^2 > s_i$, $s_{\rm th}$ is the threshold of the nonresonant hadronic contribution, which is considered to be equal to the perturbative contribution at $s > s_{\rm th}$.

Considering the respective correlators, one can write down the sum rules, analogous to Eq. (59), for the scalar and pseudoscalar states.

One believes that the sum rule (59) must rather accurately be valid at $q^2 < 0$.

For the nth derivative of Eq. (59) at $q^2 = 0$ one gets

$$f_V^2 (M_V^2)^{-n} = \frac{1}{\pi} \int_{s_i}^{s_{\rm th}} \frac{{\rm Im}\Pi_V^{\rm QCD\,pert}(s)}{s^{n+1}}\, ds + \frac{(-1)^n}{n!} \frac{d^n}{d(q^2)^n} \Pi_V^{\rm QCD\,nonpert}(q^2) , \qquad (65)$$

so, considering the ratio of the nth derivative to the $(n+1)$th one, one can obtain the value of the vector B_c^* meson mass. The calculation result depends on the n number in the sum rules (65), because of taking into the account both the finite number of terms in the perturbation theory expansion and the restricted set of composite operators.

The analogous procedure can be performed in the sum rule scheme with the Borel transform, leading to the dependence of the results on the transformation parameter.

As one can see from Eq. (65), the result, obtained in the framework of the QCD sum rules, depends on the choice of the values for the hadronic continuum threshold energy and the current masses of quarks. Then, this dependence causes large errors in the estimates of the masses for the lightest pseudoscalar, vector, and scalar $(\bar{b}c)$ states.

Thus, the QCD sum rules give the estimates of the quark binding energy in the quarkonium, and the estimates are in agreement with the results of the potential models, but sum rules involve a considerable parametric uncertainty.

II. RADIATIVE TRANSITIONS IN THE B_c FAMILY

The B_c mesons have no annihilation channels for the decays due to QCD and electromagnetic interactions. Therefore, the mesons, lying below the threshold for the B and D mesons production, will, by a cascade way, decay into the $0^-(1S)$ state by emission of γ quanta and π mesons. Theoretical estimates of the transitions between the levels with the emission of the π mesons have uncertainties, and the electromagnetic transitions are quite accurately calculable.

A. Electromagnetic transitions

The formulas for the radiative $E1$ transitions have the form [32, 33]

$$\begin{aligned}
\Gamma(\bar{n}P_J \to n^1S_1 + \gamma) &= \frac{4}{9}\,\alpha_{\rm em}\,Q^2_{\rm eff}\,\omega^3\,I^2(\bar{n}P;nS)\,w_J(\bar{n}P)\;,\\
\Gamma(\bar{n}P_J \to n^1S_0 + \gamma) &= \frac{4}{9}\,\alpha_{\rm em}\,Q^2_{\rm eff}\,\omega^3\,I^2(\bar{n}P;nS)\,[1-w_J(\bar{n}P)]\;,\\
\Gamma(n^1S_1 \to \bar{n}P_J + \gamma) &= \frac{4}{27}\,\alpha_{\rm em}\,Q^2_{\rm eff}\,\omega^3\,I^2(nS;\bar{n}P)\,(2J+1)\,w_J(\bar{n}P)\;,\\
\Gamma(n^1S_0 \to \bar{n}P_J + \gamma) &= \frac{4}{9}\,\alpha_{\rm em}\,Q^2_{\rm eff}\,\omega^3\,I^2(nS;\bar{n}P)\,(2J+1)\,[1-w_J(\bar{n}P)]\;,\\
\Gamma(\bar{n}P_J \to nD_{J'} + \gamma) &= \frac{4}{27}\,\alpha_{\rm em}\,Q^2_{\rm eff}\,\omega^3\,I^2(nD;\bar{n}P)\,(2J'+1)\,w_J(\bar{n}P)w_{J'}(nD)S_{JJ'}\;,\\
\Gamma(nD_J \to \bar{n}P_{J'} + \gamma) &= \frac{4}{27}\,\alpha_{\rm em}\,Q^2_{\rm eff}\,\omega^3\,I^2(nD;\bar{n}P)\,(2J'+1)\,w_{J'}(\bar{n}P)w_J(nD)S_{J'J}\;,
\end{aligned} \tag{66}$$

where ω is the photon energy, $\alpha_{\rm em}$ is the electromagnetic fine structure constant, $w_J(nL)$ is the probability that the spin $S=1$ in the nL state, so that $w_0(nP) = w_2(nP) = 1$, $w_1(nD) = w_3(nD) = 1$, and the $w_1(nP)$, $w_2(nD)$ values have been presented in the previous section.

The statistical factor $S_{JJ'}$ takes the values [33]

J	J'	$S_{JJ'}$
0	1	2
1	1	1/2
1	2	9/10
2	1	1/50
2	2	9/50
2	3	18/25.

The $I(\bar{n}L;nL')$ value is expressed through the radial wave functions:

$$I(\bar{n}L;nL') = \left|\int R_{\bar{n}L}(r)R_{nL'}(r)r^3dr\right|\;. \tag{67}$$

For the set of the transitions one obtains

$$\begin{aligned}
I(1S,2P) &= 1.568\ \ {\rm GeV}^{-1}\;,\\
I(1S,3P) &= 0.255\ \ {\rm GeV}^{-1}\;,\\
I(2S,2P) &= 2.019\ \ {\rm GeV}^{-1}\;,\\
I(2S,3P) &= 2.704\ \ {\rm GeV}^{-1}\;,\\
I(3D,2P) &= 2.536\ \ {\rm GeV}^{-1}\;,\\
I(3D,3P) &= 2.416\ \ {\rm GeV}^{-1}\;.
\end{aligned} \tag{68}$$

In Eq. (66) one uses

$$Q_{\rm eff} = (m_cQ_{\bar b} - m_bQ_c)/(m_c+m_b)\;, \tag{69}$$

where $Q_{c,b}$ are the electric charges of the quarks. For the B_c meson with the parameters from the Martin potential, one gets $Q_{\rm eff} = 0.41$.

For the dipole magnetic transitions one has [4, 32, 33]

$$\Gamma(\bar{n}^1S_i \to n^1S_f + \gamma) = \frac{16}{3}\,\mu^2_{\rm eff}\,\omega^3\,(2f+1)\,A^2_{if}\;, \tag{70}$$

where

$$A_{if} = \int R_{\bar{n}S}(r)R_{nS}(r)j_0(\omega r/2)r^2dr\;,$$

and

$$\mu_{\rm eff} = \frac{1}{2}\,\frac{\sqrt{\alpha_{\rm em}}}{2m_cm_b}\,(Q_cm_b - Q_{\bar b}m_c)\;. \tag{71}$$

Note that, in contrast with the ψ and Υ particles, the total width of the B_c^* meson is equal to the width of its radiative decay into the $B_c(0^-)$ state.

The electromagnetic widths, calculated in accordance with Eqs. (66) and (70), and the frequencies of the emitted photons are presented in Tables IX–XI.

Thus, the registration of the cascade electromagnetic

TABLE IX. The energies (in MeV) and widths (in keV) of the electromagnetic $E1$ transitions in the $(\bar{b}c)$ family.

Transition	ω	Γ	Γ[7]
$2P_2 \to 1S_1 + \gamma$	426	102.9	112.6
$2P_0 \to 1S_1 + \gamma$	366	65.3	79.2
$2P\ 1'^+ \to 1S_1 + \gamma$	412	8.1	0.1
$2P\ 1^+ \to 1S_1 + \gamma$	400	77.8	99.5
$2P\ 1'^+ \to 1S_0 + \gamma$	476	131.1	56.4
$2P\ 1^+ \to 1S_0 + \gamma$	464	11.6	0.0
$3P_2 \to 1S_1 + \gamma$	817	19.2	25.8
$3P_0 \to 1S_1 + \gamma$	771	16.1	21.9
$3P\ 1'^+ \to 1S_1 + \gamma$	807	2.5	2.1
$3P\ 1^+ \to 1S_1 + \gamma$	796	15.3	22.1
$3P\ 1'^+ \to 1S_0 + \gamma$	871	20.1	–
$3P\ 1^+ \to 1S_0 + \gamma$	860	3.1	–
$3P_2 \to 2S_1 + \gamma$	232	49.4	73.8
$3P_0 \to 2S_1 + \gamma$	186	25.5	41.2
$3P\ 1'^+ \to 2S_1 + \gamma$	222	5.9	5.4
$3P\ 1^+ \to 2S_1 + \gamma$	211	32.1	54.3
$3P\ 1'^+ \to 2S_0 + \gamma$	257	58.0	–
$3P\ 1^+ \to 2S_0 + \gamma$	246	8.1	–
$2S_1 \to 2P_2 + \gamma$	159	14.8	17.7
$2S_1 \to 2P_0 + \gamma$	219	7.7	7.8
$2S_1 \to 2P\ 1'^+ + \gamma$	173	1.0	0.0
$2S_1 \to 2P\ 1^+ + \gamma$	185	12.8	14.5
$2S_0 \to 2P\ 1'^+ + \gamma$	138	15.9	5.2
$2S_0 \to 2P\ 1^+ + \gamma$	150	1.9	0.0

TABLE X. The energies (in MeV) and widths (in keV) of the electromagnetic $E1$ transitions in the $(\bar{b}c)$ family.

Transition	ω	Γ	Γ[7]
$3P_2 \to 3D_1 + \gamma$	126	0.1	0.2
$3P_2 \to 3D\ 2'^- + \gamma$	118	0.5	–
$3P_2 \to 3D\ 2^- + \gamma$	133	1.5	3.2
$3P_2 \to 3D_3 + \gamma$	127	10.9	17.8
$3P_0 \to 3D_1 + \gamma$	80	3.2	6.9
$3P\ 1'^+ \to 3D_1 + \gamma$	116	0.3	0.4
$3P\ 1^+ \to 3D_1 + \gamma$	105	1.6	0.3
$3P\ 1'^+ \to 3D\ 2'^- + \gamma$	108	3.5	–
$3P\ 1^+ \to 3D\ 2^- + \gamma$	112	3.9	9.8
$3P\ 1'^+ \to 3D\ 2^- + \gamma$	123	2.5	11.5
$3P\ 1^+ \to 3D\ 2'^- + \gamma$	97	1.2	–
$3D_3 \to 2P_2 + \gamma$	264	76.9	98.7
$3D_1 \to 2P_0 + \gamma$	325	79.7	88.6
$3D_1 \to 2P\ 1'^+ + \gamma$	279	3.3	0.0
$3D_1 \to 2P\ 1^+ + \gamma$	291	39.2	49.3
$3D_1 \to 2P_2 + \gamma$	265	2.2	2.7
$3D\ 2'^- \to 2P_2 + \gamma$	273	6.8	–
$3D\ 2'^- \to 2P_2 + \gamma$	258	12.2	24.7
$3D\ 2'^- \to 2P\ 1'^+ + \gamma$	287	46.0	92.5
$3D\ 2'^- \to 2P\ 1^+ + \gamma$	301	25.0	–
$3D\ 2^- \to 2P\ 1'^+ + \gamma$	272	18.4	0.1
$3D\ 2^- \to 2P\ 1^+ + \gamma$	284	44.6	88.8

transitions in the $(\bar{b}c)$ family can be used for the observation of the higher $(\bar{b}c)$ excitations, having no annihilation channels of the decays.

B. Hadronic transitions

In the framework of QCD the consideration of the hadronic transitions between the states of the heavy quarkonium family is built on the basis of the multipole expansion for the gluon emission by the heavy nonrelativistic quarks [34], with forthcoming hadronization of gluons, independently of the heavy quark motion.

In the leading approximation over the velocity of the heavy quark motion, the action, corresponding to the heavy quark coupling to the external gluon field,

$$S_{\rm int} = -g \int d^4x\ A^a_\mu(x) \cdot j^\mu_a(x) \ , \qquad (72)$$

can be expressed in the form

$$S_{\rm int} = g \int dt\ r^k E^a_k(t,\mathbf{x})\ \frac{\lambda^{ij}_a}{2} \Psi_n(\mathbf{r}) \Psi^{ji}_f(\mathbf{r})\ K(s_n, f)\ d^3\mathbf{r} \ , \qquad (73)$$

TABLE XI. The energies (in MeV) and widths (in keV) of the electromagnetic $M1$ transitions in the $(\bar{b}c)$ family.

Transition	ω	Γ	Γ[7]
$2S_1 \to 1S_0 + \gamma$	649	0.098	0.123
$2S_0 \to 1S_1 + \gamma$	550	0.096	0.093
$1S_1 \to 1S_0 + \gamma$	64	0.060	0.135
$2S_1 \to 2S_0 + \gamma$	35	0.010	0.029

where $\Psi_n(\mathbf{r})$ is the wave function of the quarkonium, emitting gluon, $\Psi^{ij}_f(\mathbf{r})$ is the wave function of the color-octet state of the quarkonium, and $K(s_n, f)$ corresponds to the spin factor (in the leading approximation, the heavy quark spin is decoupled from the interaction with the gluons).

Then the matrix element for the $E1$-$E1$ transition of the quarkonium $nL_J \to n'L'_{J'} + gg$ can be written in the form

$$M(nL_J \to n'L'_{J'} + gg) = 4\pi\alpha_S\ E^a_k E^b_m \times \int d^3r d^3r'\ r_k r'_m\ G^{ab}_{s_{n'}, s_n}(r, r') \times \Psi_{nL_J}(r) \Psi_{n'L'_{J'}}(r'), \qquad (74)$$

where $G^{ab}_{s_n, s_n}(r, r')$ corresponds to the propagator of the color-octet state of the heavy quarkonium

$$G = \frac{1}{\epsilon - H^c_{Q\bar{Q}}} \ , \qquad (75)$$

where $H^c_{Q\bar{Q}}$ is the Hamiltonian of the colored state.

One can see from Eq. (74) that the determination of the transition matrix element depends on both the wave function of the quarkonium and the Hamiltonian $H^c_{Q\bar{Q}}$. Thus, the theoretical consideration of the hadronic transitions in the quarkonium family is model dependent.

In a number of papers of Ref. [35], for the calculation of the values such as (74), the potential approach has been developed.

In papers of Ref. [36] it is shown that nonperturbative conversion of the gluons into the π meson pair allows one to give a consideration in the framework of the low-energy theorems in QCD, so that this consideration agrees with the papers performed in the framework of PCAC (partial conservation of axial vector current) and soft pion technique [37].

However, as it follows from Eq. (74) and the Wigner-Eckart theorem, the differential width for the $E1$-$E1$ transition allows the representation in the form [35]

$$\frac{d\Gamma}{dm^2}(nL_J \to n'L'_{J'} + h) = (2J'+1) \sum_{k=0}^{2} \begin{Bmatrix} k & L & L' \\ s & J' & J \end{Bmatrix}^2 A_k(L, L') \ , \qquad (76)$$

where m^2 is the invariant mass of the light hadron system h, $\{\ \}$ are $6j$ symbols, $A_k(L, L')$ is the contribution by the irreducible tensor of the rang, equal to $k = 0,\ 1,\ 2$, and s is the total quark spin inside the quarkonium.

In the limit of soft pions, one has $A_1(L, L') = 0$.

From Eqs. (74) and (76) it follows, that, with the accuracy up to the difference in the phase spaces, the widths of the hadronic transitions in the $(Q\bar{Q})$ and $(Q\bar{Q}')$ quarkonia are related to the expression [34, 35]

$$\frac{\Gamma(Q\bar{Q}')}{\Gamma(Q\bar{Q})} = \frac{\langle r^2(Q\bar{Q}')\rangle^2}{\langle r^2(Q\bar{Q})\rangle^2} \ . \qquad (77)$$

Then the experimental data on the transitions of $\psi' \to$

TABLE XII. The widths (in keV) of the radiative hadronic transitions in the $(\bar{b}c)$ family.

Transition	Γ [7]	
	$2S_0 \to 1S_0 + \pi\pi$	50
	$2S_1 \to 1S_1 + \pi\pi$	50
	$3D_1 \to 1S_1 + \pi\pi$	31
	$3D_2 \to 1S_1 + \pi\pi$	32
	$3D_3 \to 1S_1 + \pi\pi$	31
	$3D_2 \to 1S_0 + \pi\pi$	32

$J/\psi + \pi\pi$, $\Upsilon' \to \Upsilon + \pi\pi$, $\psi(3770) \to J/\psi + \pi\pi$ [38] allow one to extract the values of $A_k(L, L')$ for the transitions $2S \to 1S + \pi\pi$ and 3D $\to 1S + \pi\pi$ [7].

The invariant mass spectrum of the π meson pair has the universal form [36, 37]

$$\frac{1}{\Gamma}\frac{d\Gamma}{dm} = B\,\frac{|\mathbf{k}_{\pi\pi}|}{M^2}\,(2x^2-1)^2\,\sqrt{x^2-1}\;, \tag{78}$$

where $x = m/2m_\pi$, $|\mathbf{k}_{\pi\pi}|$ is the $\pi\pi$ pair momentum.

The estimates for the widths of the hadronic transitions in the $(\bar{b}c)$ family have been made in Ref. [7]. The hadronic transition widths, having the values comparable with the electromagnetic transition width values, are presented in Table XII. The transitions in the $(\bar{b}c)$ family with the emission of η mesons are suppressed by the low value of the phase space.

Thus, the registration of the hadronic transitions in the $(\bar{b}c)$ family with the emission of the π meson pairs can be used to observe the higher $2S$ and $3D$ excitations of the basic state.

III. LEPTONIC CONSTANT OF B_c MESON

As we have seen in Sec. I, the value of the leptonic constant of the B_c meson determines the splitting of the basic $1S$ state of the $(\bar{b}c)$ system. Moreover, the higher excitations in the $(\bar{b}c)$ system transform, in a cascade way, into the lightest 0^- state of B_c, whose widths of the decays are essentially determined by the value of f_{B_c}, too. In the quark models [39–41], used to calculate the weak decay widths of mesons, the leptonic constant, as the parameter, determines the quark wave packet inside the meson (generally, the wave function is chosen in the oscillator form); therefore, the practical problem for the extraction of the value for the weak charged current mixing matrix element $|V_{bc}|$ from the data on the weak B_c decays can be only solved at the known value of f_{B_c}.

Thus, the leptonic constant f_{B_c} is the most important quantity, characterizing the bound state of the $(\bar{b}c)$ system.

In the present section we calculate the value of f_{B_c} in different ways.

To describe the bound states of the quarks, the use of the nonperturbative approaches is required. The bound states of the heavy quarks allow one to consider simplifications connected to both large values of the quark masses $\Lambda_{\rm QCD}/m_Q \ll 1$ and the nonrelativistic quark motion $v \to 0$. Therefore the value of f_{B_c} can be quite reliably determined in the framework of the potential models and the QCD sum rules [31].

A. f_{B_c} from potential models

In the framework of the nonrelativistic potential models, the leptonic constants of the pseudoscalar and vector mesons [see Eqs. (60) and (61)]

$$\langle 0|\bar{c}(x)\gamma_\mu b(x)|B_c^*(p,\epsilon)\rangle = i f_V\, M_V\, \epsilon_\mu\, e^{ipx}\;, \tag{79}$$

$$\langle 0|\bar{c}(x)\gamma_5\gamma_\mu b(x)|B_c(p)\rangle = i f_P\, p_\mu\, e^{ipx} \tag{80}$$

are determined by expression (28):

$$f_V = f_P = \sqrt{\frac{3}{\pi M_{Bc(1S)}}}R_{1S}(0)\;, \tag{81}$$

where $R_{1S}(0)$ is the radial wave function of the $1S$ state of the $(\bar{b}c)$ system, at the origin. The wave function is calculated by solving the Schrödinger equation with different potentials [4, 5, 10–12, 14] in the quasipotential approach [42] or by solving the Bethe-Salpeter equation with instant potential and in the expansion up to the second order over the quark motion velocity v/c [43, 44].

The values of the leptonic B_c meson constant, calculated in different potential models and effective Coulomb potential with the running α_S constant, determined in Sec. I, are presented in Table XIII.

Thus, in the approach accuracy, the potential quark models give the f_{B_c} values, which are in a good agreement with each other, so that

$$f_{B_c}^{\rm pot} = 500 \pm 80 \quad {\rm MeV}\;. \tag{82}$$

B. f_{B_c} from QCD sum rules

In the framework of the QCD sum rules [31], expressions (59)–(65) have been derived for the vector states. The expressions are considered at $q^2 < 0$ in the schemes of the spectral density moments (65) or with the application of the Borel transform [31]. As one can see from Eqs. (59)–(65), the result of the QCD sum rule calculations is determined not only by physical parameters such as the quark and meson masses, but also by the unphysi-

TABLE XIII. The leptonic B_c meson constant, calculated in the different potential models (the accuracy $\sim 15\%$), in MeV.

Model	Martin	Coulomb	[5]	[7]	[42]	[43, 44]	[45]
f_{B_c}	510	460	570	495	410	600	500

cal parameters of the sum rule scheme such as the number of the spectral density moment or the Borel transformation parameter. In the QCD sum rules, this unphysical dependence of the f_{B_c} value is due to the consideration being performed with the finite number of terms in the expansion of the QCD perturbation theory for the Wilson coefficients of the unit and composite operators. In the calculations, the set of the composite operators is also restricted.

Thus, the ambiguity in the choice of the hadronic continuum threshold and the parameter of the sum rule scheme essentially reduces the reliability of the QCD sum rule predictions for the leptonic constants of the vector and pseudoscalar B_c states.

Moreover, the nonrelativistic quark motion inside the heavy quarkonium $v \to 0$ leads to the α_S/v corrections to the perturbative part of the quark current correlators becoming the most important, where α_S is the effective Coulomb coupling constant in the heavy quarkonium. As it is noted in Refs. [16, 31, 46], the Coulomb α_S/v corrections can be summed up and represented in the form of the factor, corresponding to the Coulomb wave function of the heavy quarks, so that

$$F(v) = \frac{4\pi\alpha_S}{3v} \frac{1}{1-\exp(-4\pi\alpha_S/3v)} , \tag{83}$$

where $2v$ is the relative velocity of the heavy quarks inside the quarkonium. The expansion of the factor (83) in the first order over α_S/v,

$$F(v) \simeq 1 + \frac{2\pi\alpha_S}{3v} , \tag{84}$$

gives the expression, obtained in the first order of the QCD perturbation theory [31].

Note, the α_S parameter in Eq. (83) should be at the scale of the characteristic quark virtualities in the quarkonium (see Sec. I), but at the scale of the quark or quarkonium masses, as sometimes one does it thereby decreasing the value of factor (83).

The choice of the α_S parameter essentially determines the spread of the sum rule predictions for the f_{B_c} value (see Table XIV)

$$f_{B_c}^{\rm SR} = 160 - 570 \;\; \text{MeV} . \tag{85}$$

As one can see from Eq. (85), the ambiguity in the choice of the QCD sum rule parameters leads to the essential deviations in the results from the f_{B_c} estimates (82) in the potential models.

However, as has been noted in Sec. I, (i) the large value of the heavy quark masses $\Lambda_{\rm QCD}/m_Q \ll 1$, (ii) the nonrelativistic heavy quark motion inside the heavy quarkonium $v \to 0$, and (iii) the universal scaling properties of the potential in the heavy quarkonium, when the kinetic energy of the quarks and the quarkonium state density do not depend on the heavy quark flavors [see Eqs. (10)–(15)], allow one to state the scaling relation (17) for the leptonic constants of the S-wave quarkonia

$$\frac{f^2}{M}\left(\frac{M}{4\mu}\right)^2 = \text{const.}$$

Indeed, (i) at $\Lambda_{\rm QCD}/m_Q \ll 1$ one can neglect the quark-gluon condensate contribution, having the order of magnitude $O(1/m_b m_c)$ (the contribution into the ψ and Υ leptonic constants is less than 15%), and (ii) at $v \to 0$ one has to take into account the Coulomb-like α_S/v corrections in the form of factor (83), so that the imaginary part of the correlators for the vector and axial quark currents has the form

$$\text{Im}\Pi_V(q^2) \simeq \text{Im}\Pi_P(q^2) = \frac{\alpha_S}{2}\, q^2 \left(\frac{M}{4\mu}\right)^2 , \tag{86}$$

where

$$v^2 = 1 - \frac{4m_b m_c}{q^2-(m_b-m_c)^2} , \quad v \to 0 .$$

Moreover, condition (15) can be used in the specific QCD sum rule scheme, so that this scheme excludes the dependence of the results on the parameters such as the number of the spectral density moment or the Borel parameter.

Indeed, for example, the resonance contribution into the hadronic part of the vector current correlator, having the form

$$\Pi_V^{(\rm res)}(q^2) = \int \frac{ds}{s-q^2} \sum_n f_{Vn}^2 M_{Vn}^2 \delta(s-M_{Vn}^2) , \tag{87}$$

can be rewritten as

$$\Pi_V^{(\rm res)}(q^2) = \int \frac{ds}{s-q^2}\, s f_{Vn(s)}^2 \,\frac{dn(s)}{ds}\, \frac{d}{dn}\sum_k \theta(n-k) , \tag{88}$$

where $n(s)$ is the number of the vector S state versus the mass, so that

$$n(m_k^2) = k . \tag{89}$$

Taking the average value for the derivative of the steplike function, one gets

$$\Pi_V^{(\rm res)}(q^2) = \left\langle \frac{d}{dn}\sum_k \theta(n-k)\right\rangle \int \frac{ds}{s-q^2} s f_{Vn(s)}^2 \frac{dn(s)}{ds} , \tag{90}$$

and supposing

$$\left\langle \frac{d}{dn}\sum_k \theta(n-k)\right\rangle \simeq 1 , \tag{91}$$

one can, on average, write down

$$\text{Im}\langle\Pi^{(\rm hadr)}(q^2)\rangle = \text{Im}\Pi^{(\rm QCD)}(q^2) , \tag{92}$$

so, taking into the account the Coulomb factor and ne-

TABLE XIV. The leptonic B_c constant, calculated in the QCD sum rules (SR – the scaling relation), in MeV.

Model	[46]	[8]	[29]	[30]	[47]	[48]	[49]	SR
f_{B_c}	375	400	360	300	160	300	450	460

glecting power corrections over $1/m_Q$, at the physical points $s_n = M_n^2$ one obtains

$$\frac{f_n^2}{M_n}\left(\frac{M}{4\mu}\right)^2 = \frac{\alpha_S}{\pi}\,\frac{dM_n}{dn}\ , \tag{93}$$

where one has supposed that

$$m_b + m_c \approx M_{B_c}\ , \tag{94}$$

and

$$f_{Vn} \simeq f_{Pn} = f_n\ . \tag{95}$$

Further, as it has been shown in Sec. I, in the heavy quarkonium the value of dn/dM_n does not depend on the quark masses [see Eq. (15)], and, with the accuracy up to the logarithmic corrections, α_S is the constant value (the last fact is also manifested in the flavor independence of the Coulomb part of the potential in the Cornel model). Therefore, one can draw the conclusion that, in the leading approximation, the right-hand side of Eq. (93) is the constant value, and there is the scaling relation (17) [16]. This relation is valid in the resonant region, where one can neglect the contribution by the hadronic continuum.

Note, scaling relation (17) is in a good agreement with the experimental data on the leptonic decay constants of the ψ and Υ particles (see Table VI), for which one has $4\mu/M = 1$ [16].

The value of the constant in the right-hand side of Eq. (17) is in agreement with the estimate, when we suppose

$$\left\langle\frac{dM_\Upsilon}{dn}\right\rangle \simeq \frac{1}{2}\left[(M_{\Upsilon'} - M_\Upsilon) + (M_{\Upsilon''} - M_{\Upsilon'})\right], \tag{96}$$

and $\alpha_S = 0.36$, as in the model of Eichten *et al.* [4].

Further, in the limit case of B and D mesons, when the heavy quark mass is much greater than the light quark mass $m_Q \gg m_q$, one has

$$\mu \simeq m_q$$

and

$$f^2\,M = \frac{16\alpha_S}{\pi}\,\frac{dM}{dn}\,\mu^2\ . \tag{97}$$

Then it is evident that at one and the same μ one gets

$$f^2\,M = \text{const.} \tag{98}$$

Scaling law (98) is very well known in effective heavy quark theory (EHQT) [50] for mesons with a single heavy quark $(Q\bar{q})$, and it follows, for example, from the identity of the B- and D-meson wave functions within the limit, when an infinitely heavy quark can be considered as a static source of gluon field.

In our derivation of Eqs. (97) and (98) we have neglected power corrections over the inverse heavy quark mass. Moreover, we have used the presentation about the light constituent quark with the mass equal to

$$m_q \simeq 330\quad \text{MeV}\ , \tag{99}$$

so that this quark has to be considered as nonrelativistic one $v \to 0$, and the following conditions take place:

$$m_Q + m_q \approx M^{(*)}_{(Q\bar{q})}\ ,\quad m_q \ll m_Q\ , \tag{100}$$

and

$$f_V \simeq f_P = f\ . \tag{101}$$

In agreement with Eqs. (97) and (99), one finds the estimates[1]

$$f_{B^{(*)}} = 120 \pm 20\quad \text{MeV}\ , \tag{102}$$

$$f_{D^{(*)}} = 220 \pm 30\quad \text{MeV}\ , \tag{103}$$

that is in agreement with the estimates in the other schemes of the QCD sum rules [31, 51].

Thus, in the limits of $4\mu/M = 1$ and $\mu/M \ll 1$, scaling relation (17) is consistent.

The f_{B_c} estimate from Eq. (17) contains the uncertainty, connected to the choice of the ratio for the b- and c-quark masses, so that (see Table XIV)

$$f_{B_c} = 460 \pm 60\quad \text{MeV}\ . \tag{104}$$

In Ref. [46] the sum rule scheme with the double Borel transform was used. So, it allows one to study effects, related to the power corrections from the gluon condensate, corrections due to nonzero quark velocity, and nonzero binding energy of the quarks in the quarkonium.

Indeed, for the set of narrow pseudoscalar states, one has the sum rules

$$\sum_{k=1}^{\infty}\frac{M_k^4 f_{Pk}^2}{(m_b+m_c)^2(M_k^2-q^2)} = \frac{1}{\pi}\int\frac{ds}{s-q^2}\,\mathrm{Im}\Pi_P(s) + C_G(q^2)\,\left\langle\frac{\alpha_S}{\pi}G^2\right\rangle, \tag{105}$$

where

$$C_G(q^2) = \frac{1}{192 m_b m_c}\,\frac{q^2}{\bar{q}^2}\left(\frac{3(3v^2+1)(1-v^2)^2}{2v^5}\ln\frac{1+v}{1-v} - \frac{9v^4+4v^2+3}{v^4}\right), \tag{106}$$

and

$$\bar{q}^2 = q^2 - (m_b - m_c)^2\ ,\quad v^2 = 1 - \frac{4m_b m_c}{\bar{q}^2}\ . \tag{107}$$

Acting by the Borel operator $L_\tau(-q^2)$ on Eq. (105), one gets

$$\sum_{k=1}^{\infty}\frac{M_k^4 f_{Pk}^2}{(m_b+m_c)^2}\,e^{-M_k^2\tau} = \frac{1}{\pi}\int ds\,\mathrm{Im}\Pi_P(s)\,e^{-s\tau} + C'_G(\tau)\,\left\langle\frac{\alpha_S}{\pi}G^2\right\rangle\ , \tag{108}$$

---

[1] In Ref. [16] the dependence of the S-wave state density dn/dM_n on the reduced mass of the system with the Martin potential has been found by the Bohr-Sommerfeld quantization, so that at the step from $(\bar{b}b)$ to $(\bar{b}q)$, the density changes less than about 15%.

where

$$L_\tau(x) = \lim_{n,x\to\infty} \frac{x^{n+1}}{n!} \left(-\frac{d}{dx}\right)^n , \quad n/x = \tau , \qquad (109)$$

$$C'_G(\tau) = L_\tau(-q^2)\, C_G(q^2) . \qquad (110)$$

For the exponential set in the left-hand side of Eq. (108), one uses the Euler-MacLaurin formula

$$\sum_{k=1}^{\infty} \frac{M_k^4 f_{Pk}^2}{(m_b+m_c)^2}\, e^{-M_k^2\tau} = \int_{m_n}^{\infty} dM_k\, \frac{dk}{dM_k}\, M_k^4 f_{Pk}^2\, e^{-M_k^2\tau} + \sum_{k=0}^{n-1} M_k^4 f_{Pk}^2\, e^{-M_k^2\tau} + \cdots . \qquad (111)$$

Making the second Borel transform $L_{M_k^2}(\tau)$ on Eq. (108) accounting for Eq. (111), one finds the expression for the leptonic constants of the pseudoscalar $(\bar{b}c)$ states, so that

$$f_{Pk}^2 = \frac{2(m_b+m_c)^2}{M_k^3}\, \frac{dM_k}{dk} \left\{ \frac{1}{\pi}\mathrm{Im}\Pi_P(M_k^2) + C''_G(M_k^2) \left\langle \frac{\alpha_S}{\pi} G^2 \right\rangle \right\} , \qquad (112)$$

where we have used the following property of the Borel operator

$$L_\tau(x)\, x^n e^{-bx} \to \delta_+^{(n)}(\tau - b) . \qquad (113)$$

Explicit form for the spectral density and Wilson coefficients can be found in Ref. [46].

Expression (112) is in the agreement with the above-performed derivation of scaling relation (17).

The numerical effect from the mentioned corrections is considered to be not large (the power corrections are of the order of 10%), and the uncertainty, connected to the choice of the quark masses, dominates in the error of the f_{B_c} value determination [see Eq. (104)].

Thus, we have shown that, in the framework of the QCD sum rules, the most reliable estimate of the f_{B_c} value (104) is coming from the use of the scaling relation (17) for the leptonic decay constants of the quarkonia, and this relation agrees very well with the results of the potential models.

IV. CONCLUSION

In the present paper we have considered the spectroscopic characteristics of the bound states in the $(\bar{b}c)$ system.

We have shown that below the threshold of the $(\bar{b}c)$ system decay into the BD meson pair, there are 16 narrow states of the B_c meson family, whose masses can be reliably calculated in the framework of the nonrelativistic potential models of the heavy quarkonia. The flavor independence of the QCD-motivated potentials in the region of average distances between the quarks in the $(\bar{b}b)$, $(\bar{c}c)$, and $(\bar{b}c)$ systems and their scaling properties allow one to find the regularity of the spectra for the levels, nonsplitted by the spin-dependent forces: in the leading approximation the state density of the system does not depend on the heavy quark flavors, i.e., the distances between the nL levels of the heavy quarkonium do not depend on the heavy quark flavors.

We have described the spin-dependent splittings of the $(\bar{b}c)$ system levels, i.e., the splittings, appearing in the second order over the inverse heavy quark masses, $V_{SD} = O(1/m_b m_c)$, accounting for the variation of the effective Coulomb coupling constant of the quarks (the interaction is due to relativistic corrections, coming from the one gluon exchange).

The approaches, developed to describe emission by the heavy quarks, have been applied to the description of the radiative transitions in the $(\bar{b}c)$ family, whose states have no electromagnetic or gluonic channels of annihilation. The last fact means that, due to the cascade processes with the emission of photons and pion pairs, the higher excitations decay into the lightest pseudoscalar B_c meson, decaying in the weak way. Therefore, the excited states of the $(\bar{b}c)$ system have the widths, essentially less (by two orders of magnitude) than those in the charmonium and bottomonium systems.

As for the value of the leptonic decay constant f_{B_c}, it can be the most reliably estimated from the scaling relation for the leptonic constants of the heavy quarkonia, due to the relation, obtained in the framework of the QCD sum rules in the specific scheme. In the other schemes of the QCD sum rules, it is necessary to do an interpolation of the scheme parameters (the hadronic continuum threshold and the number of the spectral density moment or the Borel parameter) into the region of the $(\bar{b}c)$ system, so this procedure leads to the essential uncertainties. The f_{B_c} estimate from the scaling relation agrees with the results of the potential models, whose accuracy for the leptonic constants is notably lower. The value of f_{B_c} essentially determines the decay widths and the production cross sections of the B_c mesons.

The B_c decays have been studied in Refs. [6, 29, 52], where it has been shown, that the B_c lifetime is equal to

$$\tau(B_c) \simeq 0.5 - 0.7 \;\; ps , \qquad (114)$$

and the characteristic decay mode, having the preferable signature for the experimental search, has the branching fraction, equal to

$$B(B_c^+ \to \psi X) \simeq 17\% . \qquad (115)$$

The decay of $B_c^+ \to \psi\pi^+$, having the branching fraction

$$B(B_c^+ \to \psi\pi^+) \simeq 0.2\% , \qquad (116)$$

is chosen by the Collider Detector at Fermilab (CDF) Collaboration for the B_c search, where about 20 events are expected [53].

The B_c production at the Fermilab and LEP colliders has been studied in Refs. [1–3, 6, 53, 54], where it has been shown, that the level of the B_c yield is of the order of

$$\frac{\sigma(B_c X)}{\sigma(\bar{b}b)} \simeq 2\times 10^{-3} .$$

Thus, in the present paper we have described the spectroscopic characteristics of the B_c mesons, whose search at Fermilab and LEP is being carried out, and, as expected, it will be successfully realized in the nearest future.

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