

Some constraints on neutral heavy leptons from flavor-conserving decays of the Z boson

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Small neutrino masses can arise in some grand unified models or superstring theories. We consider a model with an enhanced fermion sector containing Dirac neutral heavy leptons. The dependence on the mass and mixing parameters of these new fermions is investigated for several measurable quantities. We study the flavor-conserving leptonic decays of the Z boson and universality breaking in these decays. We also consider the W boson mass dependence on neutral heavy lepton parameters.

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I. INTRODUCTION

Experimental evidence suggests that neutrino masses are very small, if not zero. However, the way the standard model (SM) of electroweak interactions accommodates massless neutrinos, by the absence of right-handed neutrino fields, is considered unnatural. Interesting solutions to this problem have been suggested in the low energy limit of superstring theories [1], and in grand unified theories (GUT's) [2]. Naturally small neutrino masses may even be accommodated within the framework of the $SU(2)_L \times U(1)_Y$ symmetry. Either the fermion content of the SM or the Higgs sector may be extended. In this work, we concentrate on the former option.

We consider an $SU(2)_L \times U(1)_Y$ based, superstring-inspired model with an extended fermion sector including neutral heavy leptons (NHL's) [3,4]. The NHL's are Dirac particles and $B-L$ (baryon minus lepton number) conservation is imposed as an unbroken symmetry. This is in contrast to seesaw models [2,5] wherein both light neutrinos and NHL's are Majorana particles and $B-L$ is broken.

The model considered allows for the possibilities of lepton-flavor violation, universality violation, and CP violation. Our primary interest here is in obtaining potential constraints on the model from current experimental data on Z leptonic decay widths. We focus on the direct contribution of NHL's to flavor-conserving leptonic Z decays via one-loop diagrams. In addition to the Z partial widths for individual lepton flavors, we also calculate a measure of the leptonic universality violation, as defined in Ref. [6]. Further, we find that we must take into account the impact of NHL's on the mass of the W boson since M_W is an input parameter within the renormalization scheme adopted here. Hence, we display also the dependence of M_W on the parameters of our model.

This paper is organized as follows. In Sec. II we describe the model. Existing experimental constraints on the parameters of the model, namely, the masses and mixings of the neutral leptons, are reviewed in Sec. III. Our one-loop calculation of the Z leptonic decay, $Z \rightarrow l^+ l^-$, is presented in Sec. IV. This section also

contains a discussion of our renormalization scheme, including the consideration of M_W . Many of the detailed results are relegated to the Appendix. In Sec. V, we present our results on the Z leptonic widths, the universality violating measure, and the W mass. We summarize and draw our conclusions in the final section.

II. DESCRIPTION OF THE MODEL

Originally, only broken $B-L$ symmetry and Majorana neutrinos were thought of as providing an understanding of the smallness of neutrino masses. However, as discussed in Ref. [4], superstring-inspired models can have small neutrino masses (in fact, zero) even if $B-L$ symmetry is unbroken and NHL's are Dirac particles. In these models, the SM particle content is extended by two new neutrino fields $N_R(0,0)$ and $S_L(0,0)$ per family; the zeros indicate $SU(2)_L \times U(1)_Y$ quantum numbers. Imposing total lepton number conservation leads to the mass matrix

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} (\bar{\nu}_L \bar{N}_L \bar{S}_L) \begin{pmatrix} 0 & D & 0 \\ D^T & 0 & M^T \\ 0 & M & 0 \end{pmatrix} \begin{pmatrix} \hat{\nu}_R \\ N_R \\ \hat{S}_R \end{pmatrix}, \quad (1)$$

where $\nu_L = (\nu_L^e, \nu_L^\mu, \nu_L^\tau)$ and $\hat{\nu}_R = |CPT\rangle \nu_L$. D and M are 3×3 mass matrices. The diagonalization of the mass matrix yields three massless neutrinos (ν_i) along with three Dirac NHL's (N_a) of mass $M_N \sim M$. Note that this implies there are no time-dependent neutrino oscillations and no neutrinoless double β decays. The weak eigenstates ν_L are mostly massless neutrinos with a small mixing ($\sim D/M$) of NHL's. The NHL mixing in this model is not restricted by small neutrino masses (as is often the case with seesaw models where both the mixing and the masses of light neutrinos are sensitive to the D/M ratio), and hence rates for all interesting phenomena can be large [4,7,8]. This model is thus attractive not only conceptually, but also practically.

The weak interaction eigenstates ν_L are related to six mass eigenstates n_α via a 3×6 mixing matrix K with components K_{ln_α} $l = e, \mu, \tau$, and $n_\alpha = \nu_1, \nu_2, \nu_3, N_4, N_5, N_6$:

$$K_{ln_\alpha} = \begin{pmatrix} K_{e\nu_1} & K_{e\nu_2} & K_{e\nu_3} & K_{eN_4} & K_{eN_5} & K_{eN_6} \\ K_{\mu\nu_1} & K_{\mu\nu_2} & K_{\mu\nu_3} & K_{\mu N_4} & K_{\mu N_5} & K_{\mu N_6} \\ K_{\tau\nu_1} & K_{\tau\nu_2} & K_{\tau\nu_3} & K_{\tau N_4} & K_{\tau N_5} & K_{\tau N_6} \end{pmatrix} \equiv (K_L, K_H). \quad (2)$$

After rotating away redundant degrees of freedom from K , we are left with 3^2 angles and $(3-1)^2$ phases. This allows for possible lepton-flavor violation, universality violation, and CP violation.

The mixing factor which typically governs flavor-conserving processes, is given by

$$l_{\text{mix}} = \sum_{n_\alpha=N_4, N_5, N_6} |K_{ln_\alpha}|^2, \quad l = e, \mu, \tau, \quad (3)$$

and the flavor-violating mixing factor $l_a l_{b\text{mix}}$ is defined as:

$$l_a l_{b\text{mix}} = \sum_{n_\alpha=N_4, N_5, N_6} K_{l_a n_\alpha} K_{l_b n_\alpha}^*, \quad l_{a,b} = e, \mu, \tau, l_a \neq l_b. \quad (4)$$

Further, an important inequality holds:

$$|l_a l_{b\text{mix}}|^2 \leq l_a l_{a\text{mix}} l_b l_{b\text{mix}}, \quad a \neq b. \quad (5)$$

This implies that one might observe nonstandard effects in flavor-conserving processes even if they are absent in flavor-violating processes.

For reference, the charged current Lagrangian is given by

$$\mathcal{L}_{\text{cc}} = \frac{1}{2\sqrt{2}} g W^\mu \sum_{l=e,\mu,\tau} \sum_{n_\alpha} \bar{l} \gamma_\mu (1 - \gamma_5) K_{ln_\alpha} n_\alpha; \quad n_\alpha = \nu_1, \nu_2, \nu_3, N_4, N_5, N_6, \quad (6)$$

and the neutral current Lagrangian as (the ZNN part is obtained by analogy)

$$\mathcal{L}_{\text{nc}} = \frac{g}{4c_W} Z^\mu \sum_{i=1,2,3; a=4,5,6} \bar{\nu}_i (K_L^\dagger K_H)_{ia} \gamma_\mu (1 - \gamma_5) N_a, \quad (7)$$

where $c_W = \cos\theta_W$, θ_W being the Weinberg angle.

III. REVIEW OF EXISTING CONSTRAINTS ON NEUTRAL HEAVY LEPTONS

Constraints on neutral heavy lepton masses and mixings come from three different sources. First, there is the possibility of direct production of NHL's. For instance, if an NHL is light enough, it could be produced in some decays, e.g., $Z \rightarrow N_\alpha + \nu$, and subsequently decay itself. The rate for Z decays into an NHL and a light neutrino has been given previously [7] as

$$\Gamma(Z \rightarrow N_\alpha + \nu) = a_{\text{mix}} \left(1 - \frac{M_{N_\alpha}^2}{M_Z^2}\right) \left(1 + \frac{M_{N_\alpha}^2}{2M_Z^2}\right) \times \Gamma(Z \rightarrow \nu + \nu), \quad (8)$$

where

$$a_{\text{mix}} = \sum_{l=e,\mu,\tau} |K_{ln_\alpha}|^2. \quad (9)$$

The subsequent NHL decay rate (for $M_N \leq M_W$) is then given by

$$\Gamma_N = a_{\text{mix}} \left(\frac{M_N}{m_\mu}\right)^5 \Phi_l \Gamma_\mu, \quad (10)$$

where Γ_μ is the muon decay rate and Φ_l is the effective number of decay channels available to the NHL [9]. Given the absence of experimental evidence for such direct production, we will consider only NHL's with mass greater than the Z mass.

Second, there are constraints on NHL mixing parameters from a variety of low energy experiments and from experiments at the CERN Large Electron Positron Collider I (LEP I). Because of unitarity properties of the mixing matrix K , a nonzero NHL mixing slightly reduces the couplings of light neutrinos from their standard model values, thus affecting rates for nuclear β decays, τ and π decays, and for Z decays. The following upper limits are consistent with experiment [10]

$$\begin{aligned} ee_{\text{mix}} &\leq 0.0071, \\ \mu\mu_{\text{mix}} &\leq 0.0014, \\ \tau\tau_{\text{mix}} &\leq 0.033, \text{ or } \leq 0.024 \text{ including LEP I.} \end{aligned} \quad (11)$$

These limits are model independent and also independent of the NHL mass. They arise from a global analysis of results including lepton universality experiments, Cabibbo-Kobayashi-Maskawa (CKM) unitarity tests, W mass measurements, and results from LEP I experiments. Note that the LEP constraints presented above do not include NHL loop effects but, rather, only coupling constant modifications due to mixing. We consider NHL loop effects in this work.

Finally, the NHL masses and mixings can be constrained via their contribution in loops to various processes. Such constraints are NHL mass dependent. Flavor-violating processes have previously been studied at low energies; these include $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ [4,8,11], and flavor-violating decays of the τ [8,12]. For instance, in the context of the model considered here, with mass degenerate NHL's, the $\mu \rightarrow e\gamma$ branching ratio is [4,8]

$$B(\mu \rightarrow e\gamma) = \frac{3\alpha}{32\pi} |e\mu_{\text{mix}}|^2 |F_\gamma(x)|^2, \quad (12)$$

where $x = \frac{M_N^2}{M_W^2}$, and $F_\gamma(x)$ is an NHL mass-dependent form factor. For NHL masses $M_N > 500$ GeV, which we will ultimately consider, $F_\gamma(x) \rightarrow -2$. Given the current experimental limit on the $\mu \rightarrow e\gamma$ branching ratio ($\leq 4.9 \times 10^{-11}$) [13], this yields an upper limit on the mixing of

$$|e\mu_{\text{mix}}| \leq 0.00024. \quad (13)$$

By combining the constraints obtained from the global analysis [Eq. (11)] with the inequality relations of Eq. (5) one obtains the following upper limits on the mixing factors

$$\begin{aligned} |e\mu_{\text{mix}}| &\leq 0.0032, \\ |\mu\tau_{\text{mix}}| &\leq 0.0068, \\ |e\tau_{\text{mix}}| &\leq 0.015. \end{aligned} \quad (14)$$

For the mixings $\mu\tau_{\text{mix}}$ and $e\tau_{\text{mix}}$, these are the strongest available constraints. In addition, flavor-violating leptonic Z decays, $Z \rightarrow e\mu, e\tau, \mu\tau$, have also been studied [4,14]. In this work, we consider the flavor-conserving decays $Z \rightarrow ee, \mu\mu, \tau\tau$.

IV. CALCULATION OF THE ONE-LOOP LEVEL CONTRIBUTION OF NEUTRAL HEAVY LEPTONS TO LEPTONIC FLAVOR-CONSERVING Z DECAYS

As noted previously, the limits on mixing parameters extracted from LEP I observables [10] do not include NHL's in the one-loop diagrams; only mixing factor modifications are made to the tree level results. We consider here the direct contribution to flavor-conserving leptonic Z decay of NHL's via one-loop diagrams. The importance of studying these processes is enhanced by the implication of Eq. (5); one may observe flavor-conserving (but universality-breaking) Z decays even in the absence of flavor-violating processes.

For Z leptonic decay, NHL's contribute directly to Z oblique corrections, as shown in Fig. 1, to lepton wave function renormalizations and to vertex corrections, as in Fig. 2. These one-loop contributions of NHL's can be incorporated into the framework of the full standard model one-loop electroweak corrections. The one-loop corrected leptonic width can be parametrized as [15]

$$\Gamma_Z = \frac{\Gamma_0 + \delta\Gamma_Z}{1 + \hat{\Pi}_Z(M_Z^2)} (1 + \delta_{\text{QED}}), \quad (15)$$

where the tree level leptonic width of the Z boson is given by

$$\Gamma_0 = \frac{\alpha}{3} M_Z (v_f^2 + a_f^2), \quad (16)$$

v_f and a_f being, respectively, the vector and axial vector couplings of charged leptons to Z . The one-loop electroweak corrections include $\delta\Gamma_Z$, which represent vertex

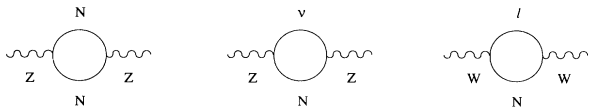


FIG. 1. Diagrams for oblique corrections due to neutral heavy leptons N.

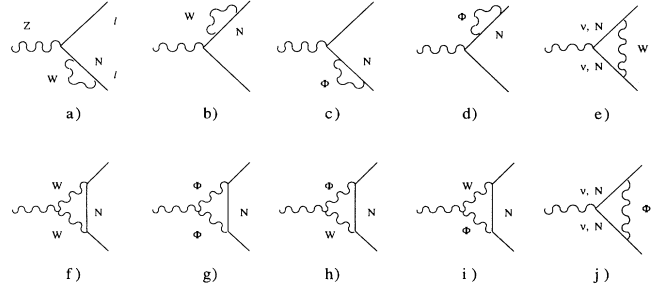


FIG. 2. Diagrams for one-loop vertex correction to flavor-conserving leptonic Z decays due to neutral heavy leptons N.

loops, and $\hat{\Pi}_Z$ and δ_{QED} which represent the Z -oblique corrections and QED corrections, respectively.

Our calculation is done within the framework of an on-shell renormalization scheme as detailed in Ref. [15]. All the SM one-loop diagrams were calculated using standard routines from the CERN electroweak library [15] modified by appropriate mixing factors. The vertex parameter $\delta\Gamma_Z$ actually includes also fermion wave function renormalization and counterterm contributions in the scheme we adopt. This on-shell renormalization scheme takes α , M_Z , and M_W as input parameters. However, the direct measurement of M_W is not yet precise enough for its use as an input parameter; hence, it is replaced by G_μ via the one-loop relation

$$M_W^2 s_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu(1 - \Delta r)} \left(1 - \frac{1}{2} ee_{\text{mix}} - \frac{1}{2} \mu\mu_{\text{mix}} \right), \quad (17)$$

where Δr is an equivalent of the SM quantity Δr^{SM} , and $s_W = \sin\theta_W$. As a result, we also have to consider muon decay loops with NHL's. The types of corrections involved in muon decay are pictured schematically in Fig. 3. They include boxes and vertices with NHL's as well as the lepton wave function renormalizations and W oblique corrections. The NHL diagrams contributing to the W oblique corrections are shown in Fig. 1 along with the Z oblique corrections. These corrections all feed into the Z leptonic decay calculation indirectly via the dependence of M_W on the parameter Δr and the overall factor modified by mixings.

Referring to Eq. (15) now, the QED corrections, δ_{QED} , are not modified from the SM. The factor $[1 + \hat{\Pi}_Z(M_Z^2)]^{-1}$ represents the wave-function renormalization of the Z boson. It depends on all the unrenormalized propagator corrections ($\Sigma_Z, \Sigma_W, \Sigma_{\gamma Z}, \Sigma_\gamma$) [15]. Of these, $\Sigma_{\gamma Z}$ and Σ_γ are not modified within our model



FIG. 3. Schematic representation of the one-loop muon decay diagrams with neutral heavy leptons.

while Σ_Z , Σ_W both contain nonstandard terms. Those nonstandard terms, denoted as $\Sigma_Z^{\nu,N}(s)$ and $\Sigma_W^{\nu,N}(s)$, are given by Eqs. (A2) and (A4), respectively, in the Appendix. They consist of M_N dependent terms representing the direct contribution from NHL's in loops in Fig. 1 and of SM terms modified by mixing factors (the indirect effect of NHL's reducing the mixings of light neutrinos through the unitary matrix K).

The vertex parameter $\delta\Gamma_Z$ includes γ - Z mixing, external fermion wave function renormalization, and counterterm contributions in addition to the vertex loops involving NHL's. Those fermion self energy and $Zf\bar{f}$ vertex loops which contain NHL's are shown in Figs. 2(a)-(j). The individual contributions of Figs. 2(a)-(j) are given in Eq. (A6) in the Appendix.

It is characteristic that NHL's in loops generally do not decouple (violation of the Appelquist-Carazzone theorem) [16]; rather they often show a quadratic mass dependence $\sim \frac{M_N^2}{M_W^2}$. This is a common feature for theories based on the spontaneous symmetry-breaking mechanism. We are already familiar with a similar result for the top quark mass in SM loops [17]. Indirect bounds on the top quark mass arise from the Z and W polariza-

tion diagrams and $Zb\bar{b}$ vertex loop since these corrections come in with the amplitude

$$\mathcal{M}_t \sim |V_{tb}|^2 \frac{m_t^2}{M_W^2} \doteq \frac{m_t^2}{M_W^2}. \quad (18)$$

While the top quark mixes with the full strength ($|V_{tb}|^2 \sim 1$), the NHL mixings are limited by Eqs. (11) and (14). As a result, we find sensitivity only for $M_N \sim 10 m_t$. Thus we will only present numerical results for NHL masses greater than about 500 GeV.

Given the nondecoupling feature, the vertex correction $\delta\Gamma_Z$ is dominated for large NHL masses by the diagrams (in decreasing order of importance) 2(j), 2(f), and 2(e), while 2(a) and (b), 2(c) and (d), 2(g), 2(h), and 2(i) are negligible [the largest unrenormalized contribution comes from 2(g) and 2(c) and (d); however, diagrams 2(c) and (d) enter the renormalized vertex correction $\delta\Gamma_Z$ as a part of a counterterm that cancels out the large amplitude of the graph 2(g)].

To illustrate how $\hat{\Pi}_Z$ and Δr depend on M_N (for M_N large), we separate M_N dependent terms as $\hat{\Pi}_{M_N}$, Δr_{M_N} , and find in the limit $\mathcal{X}^{-1} = \frac{M_W^2}{M_N^2} \rightarrow 0$ (large NHL mass)

$$\begin{aligned} \Delta r_{M_N} &= -\frac{\alpha}{\pi s_W^4} \left[l_{HH} \frac{c_W^2}{16} \mathcal{X} + (l_{LL} - 1) \frac{1}{24} \sum_{l=e,\mu,\tau} \ln \frac{M_N^2}{m_l^2} \right], \\ \hat{\Pi}_{M_N} &= \frac{\alpha}{\pi} \left[l_{HH} \frac{c_W^2 - s_W^2}{16 s_W^4} \mathcal{X} + (l_{LL} - 1) \frac{1}{24 s_W^4} \sum_{l=e,\mu,\tau} \ln \frac{M_N^2}{m_l^2} \right], \end{aligned} \quad (19)$$

where

$$l_{HH} = \sum_{l=e,\mu,\tau} (|le_{\text{mix}}|^2 + |l\mu_{\text{mix}}|^2 + |l\tau_{\text{mix}}|^2), \quad (20)$$

$$l_{LL} = 1 - 2(ee_{\text{mix}} + \mu\mu_{\text{mix}} + \tau\tau_{\text{mix}}) + l_{HH}.$$

With the NHL mass of the order of several TeV, one has to worry about the perturbative unitarity bound. A good way to demonstrate this is to bring about the Higgs analogy. The width of the Higgs boson of mass m_H is given by

$$\Gamma_H = \frac{3\alpha}{32M_W^2 s_W^2} m_H^3, \quad (21)$$

which can be compared with the width of the NHL. For NHL mass $M_N \gg M_W, M_Z, M_H$, the width is [18]

$$\Gamma_N = \frac{\alpha}{4M_W^2 s_W^2} M_N^3 a_{\text{mix}}. \quad (22)$$

Demanding that $\Gamma_H \leq \frac{1}{2}M_H$ we get the well-known bound on the Higgs boson mass $M_H \leq 1$ TeV. Similarly, demanding $\Gamma_N \leq \frac{1}{2}M_N$ for the NHL, with the current constraints on mixings, [Eq. (11)], one obtains $M_N \leq 3.5$ TeV.

V. RESULTS

In this section, we present our numerical results. As input parameters, we used $M_Z = 91.173$ GeV, $M_H = 200$ GeV, $\alpha^{-1} = 137.036$, and $A \equiv \frac{\pi\alpha}{\sqrt{2}G_\mu} = 37.281$ GeV. We have assumed degenerate masses for the three NHL's and present results for the NHL mass range $0.5 \text{ TeV} \leq M_N \leq 5 \text{ TeV}$, as motivated by the nondecoupling and perturbative unitarity arguments given in the last section. We have also imposed restrictions on the mixing parameters. We assume that ee_{mix} and $\mu\mu_{\text{mix}}$ are very small relative to $\tau\tau_{\text{mix}}$. The model and NHL mass independent limits quoted in Eq. (11) are more stringent for e and μ than for τ . In addition, our assumption is also partially supported by the smallness of $e\mu_{\text{mix}}$, as determined from $\mu \rightarrow e\gamma$, in combination with the inequality Eq. (5). This neglect of ee_{mix} and $\mu\mu_{\text{mix}}$ proves useful practically in that many of the muon decay loops (boxes and vertex corrections, but not W oblique correction) are eliminated as a result.

The Z leptonic width is given as a function of NHL mass in Figs. 4(a), and (b). In Fig. 4(a), we have fixed the mixing $\tau\tau_{\text{mix}} = 0.033$. The width for Z decay to e^+e^- is shown for a top quark mass of 174 GeV. The Z decay rate into $\tau^+\tau^-$ is shown for three values of the top quark mass, 150, 174, and 200 GeV. The dashed lines represent the 1σ variation about the current experimental result for the average Z -leptonic width of $\Gamma_l = 83.96 \pm 0.18$

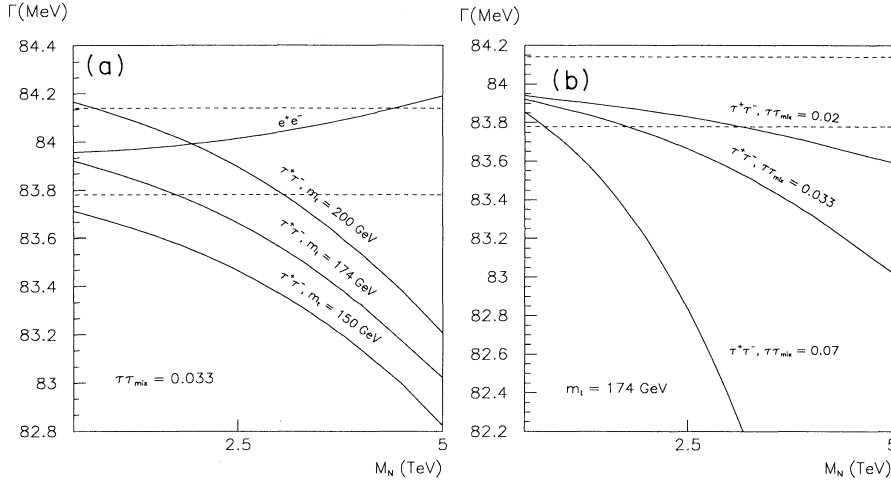


FIG. 4. Z leptonic width as a function of M_N for (a) fixed mixing parameter and different values of m_t (b) fixed m_t and different values of the mixing parameter. The dashed lines represent 1σ band about the current experimental value $\Gamma_l = 83.96 \pm 0.18$ MeV.

MeV [19]. In Fig. 4(b), we fix the top quark mass at 174 GeV and show the Z width to $\tau^+\tau^-$ for three values of the mixing parameter, $\tau\tau_{\text{mix}} = 0.02, 0.033, 0.07$. We also present results for the universality-breaking ratio defined as [6]

$$U_{\text{br}} = \left| \frac{\Gamma(Z \rightarrow \tau^+\tau^-) - \Gamma(Z \rightarrow e^+e^-)}{\Gamma(Z \rightarrow \tau^+\tau^-) + \Gamma(Z \rightarrow e^+e^-)} \right|. \quad (23)$$

This is shown in Fig. 5 as a function of M_N , again with $m_t = 174$ GeV, and the mixing parameter varied about $\tau\tau_{\text{mix}} = 0.033$. The 1σ experimental limit on U_{br} is indicated as the dashed line. Note that the most recently reported values of Z widths into individual lepton flavors [19] have $\Gamma(Z \rightarrow \tau^+\tau^-) > \Gamma(Z \rightarrow e^+e^-)$, $\Gamma(Z \rightarrow \mu^+\mu^-)$, as opposed to the last round of results [13].

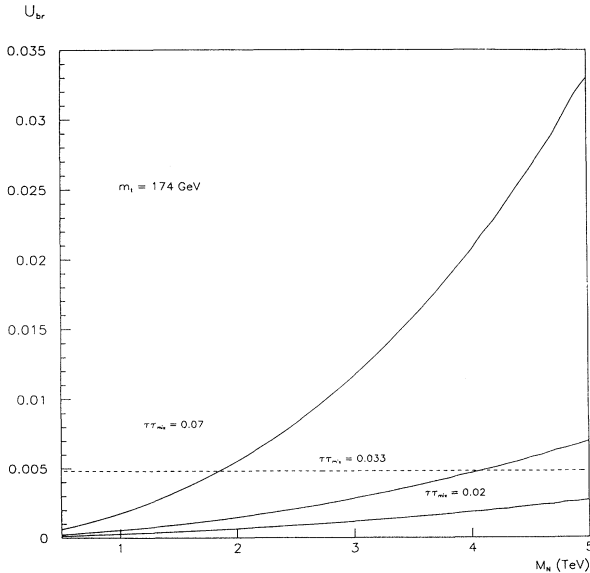


FIG. 5. Universality-breaking parameter U_{br} as a function of M_N for fixed m_t and different values of the mixing parameter. The dashed line represents 1σ experimental limit (< 0.005).

Finally, we present the NHL mass dependence of the W mass in Figs. 6(a) and (b). The top quark mass is varied in Fig. 6(a), while the mixing is held constant at $\tau\tau_{\text{mix}} = 0.033$. In Fig. 6(b), the mixing is varied about $\tau\tau_{\text{mix}} = 0.033$ for a fixed top quark mass $m_t = 174$ GeV.

VI. DISCUSSION AND CONCLUSIONS

Our primary consideration here has been the inclusion of neutral heavy leptons in the calculation of the flavor-conserving Z decays to charged leptons at one-loop level. The dependence of the Z leptonic widths on the NHL mass, M_N , and on the mixing parameter $\tau\tau_{\text{mix}}$ which we retain, was given in Figs. 4(a) and (b). We see for the experimentally allowed upper limit of $\tau\tau_{\text{mix}} = 0.033$, and assuming a top quark mass $m_t = 174$ GeV, the Z decay width to τ leptons is sensitive at the present 2σ level to NHL masses larger than about 2.5 TeV. The top mass dependence is also shown in that figure. The sensitivity to M_N and m_t arises since these heavy fermions generally do not decouple from the one-loop diagrams. Figure 4(b) indicates how the Z width dependence on M_N varies with the mixing parameter. Apart from this comparison of each leptonic width prediction with experiment we can also exploit the flavor universality violation which takes place in the model. The universality-breaking ratio U_{br} , defined in Sec. IV, is sensitive to NHL masses above approximately 3.5 TeV at the 1σ level, assuming $\tau\tau_{\text{mix}} = 0.033$.

The W boson mass also exhibits some sensitivity to NHL parameters arising from the mixing factor modifications and the presence of one-loop diagrams containing neutral heavy leptons, as described in Sec. III. From Figs. 5(a) and (b) we see that the W mass, currently measured as $M_W = 80.23 \pm 0.18$ GeV [20], is sensitive at the 1σ level to NHL masses greater than about 3.5 TeV, again assuming $\tau\tau_{\text{mix}} = 0.033$ and $m_t = 174$ GeV. The experimental error on M_W might be expected to come down to about 0.05 GeV once LEP II measures W pair production [21].

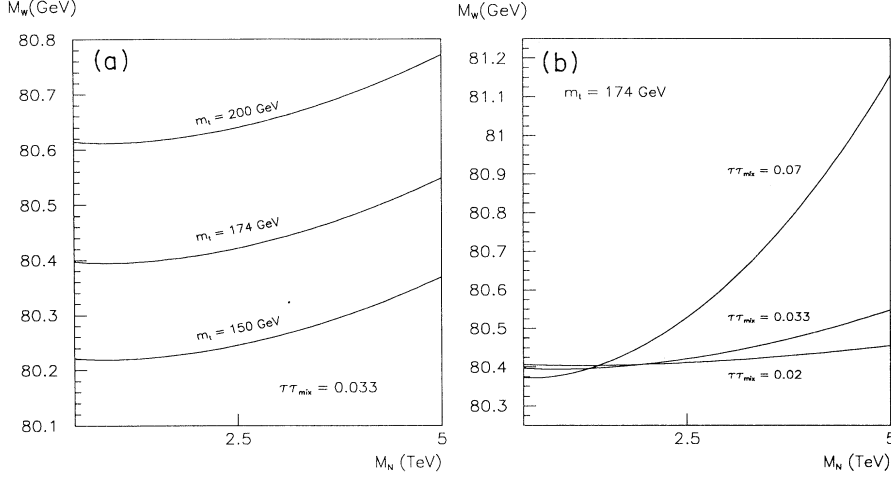


FIG. 6. W mass as a function of M_N for (a) fixed mixing parameter and different values of m_t (b) fixed m_t and different values of the mixing parameter. The dashed lines represents 1σ band about the current experimental value $M_W = 80.23 \pm 0.18$ GeV.

We have considered a model containing isosinglet neutral heavy leptons which can accommodate various phenomena beyond the standard model, such as lepton flavor-violation, CP violation, and lepton universality violation. We have presented the dependence on the mass and mixing parameters of this model for Z decays to charged leptons and for the W boson mass. Because the NHL mass and mixing dependence is different for the Z decay width and the W boson mass, they provide somewhat complementary information on these parameters.

Current data from LEP I on Z leptonic widths and the present Collider Detector at Fermilab (CDF) and D0 Collaboration measurements of M_W are sensitive to NHL masses greater than about 2.5–3.5 TeV. With the accumulation of about 60 (pb)^{-1} at LEP I in 1994 and the prospect of the very precise W mass measurement at LEP II, these sensitivities will certainly be improved considerably. Thus the Z partial width to leptons and W mass measurements can provide, along with the other observables discussed in Sec. III, a consistency check on the possible existence of isosinglet neutral heavy leptons.

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APPENDIX

In this Appendix, we present the parts of our calculation which are necessary to obtain the NHL dependent

modifications of the one-loop Z decay width to charged leptons. We refer the reader to Ref. [15] for further details. We define the following symbols and mixing factors:

$$\begin{aligned}
 \mathcal{X} &= \frac{M_N^2}{M_W^2}, \\
 \Delta &= \frac{2}{\epsilon} - \gamma + \ln 4\pi + \frac{3}{2} \ln \mu^2, \\
 \Delta^m &= \frac{2}{\epsilon} - \gamma + \ln 4\pi - \ln \frac{m^2}{\mu^2}, \\
 l_1 &= ll_{\text{mix}}, \\
 l_2 &= |le_{\text{mix}}|^2 + |l\mu_{\text{mix}}|^2 + |l\tau_{\text{mix}}|^2, \\
 l_3 &= ll_{\text{mix}} - l_2, \\
 l_4 &= 1 - 2ll_{\text{mix}} + l_2, \\
 l_{CH} &= ee_{\text{mix}} + \mu\mu_{\text{mix}} + \tau\tau_{\text{mix}}, \\
 l_{HH} &= \sum_{l=e,\mu,\tau} l_2, \\
 l_{LH} &= 2(l_{CH} - l_{HH}), \\
 l_{LL} &= 1 - 2l_{CH} + l_{HH}.
 \end{aligned} \tag{A1}$$

First we deal with the direct contributions of NHL's to the Z boson wave function renormalization, as parametrized by $\hat{\Pi}_Z(M_Z^2)$. $\hat{\Pi}_Z(M_Z^2)$ depends on two quantities which are modified from the SM by the inclusion of NHL's. The neutral lepton part of the unrenormalized self-energy of the Z boson, which consists of the SM massless ν loop, the mixed νN loop and the NN loop, is given by

$$\begin{aligned}
 \Sigma_Z^{\nu,N}(s) &= \frac{\alpha}{8\pi s_W^2 c_W^2} \left\{ \frac{s}{3} l_{LL} \sum_{l=e,\mu,\tau} \left[\Delta^{m_l} + 2 - \ln \left(-\frac{s}{m_l^2} - i\epsilon \right) - \frac{1}{3} \right] \right. \\
 &\quad + l_{LH} \left[\Delta^{M_N} \left(\frac{s}{3} - \frac{M_N^2}{2} \right) + \frac{2}{9} s - \frac{M_N^2}{6} + F(s, 0, M_N) \left(\frac{s}{3} - \frac{M_N^2}{6} - \frac{M_N^4}{6s} \right) \right] \\
 &\quad \left. + l_{HH} \left[\Delta^{M_N} \left(\frac{s}{3} - M_N^2 \right) - \frac{s}{9} + \frac{1}{3} F(s, M_N, M_N) (s - M_N^2) \right] \right\},
 \end{aligned} \tag{A2}$$

where the function $F(s, m_1, m_2)$ is given by

$$F(s, m_1, m_2) = -1 + \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \ln \frac{m_1}{m_2} - \int_0^1 dx \ln \frac{sx^2 - x(s + m_1^2 - m_2^2) + m_1^2 - i\epsilon}{m_1 m_2}. \quad (\text{A3})$$

The leptonic part of the unrenormalized self-energy of the W boson is given by

$$\begin{aligned} \Sigma_W^{\nu, N}(s) = & \frac{\alpha}{12\pi s_W^2} \left\{ \sum_{l=e, \mu, \tau} (1 - l_1) \left[\left(s - \frac{3}{2} m_l^2 \right) \Delta^{m_l} + \left(s - \frac{m_l^2}{2} - \frac{m_l^4}{2s} \right) F(s, 0, m_l) + \frac{2}{3} s - \frac{m_l^2}{2} \right] \right. \\ & + \sum_{l=e, \mu, \tau} l_1 \left[\frac{\Delta^{M_N}}{2} \left(s - \frac{5}{2} M_N^2 - \frac{m_l^2}{2} \right) + \frac{\Delta^{m_l}}{2} \left(s - \frac{5}{2} m_l^2 - \frac{M_N^2}{2} \right) + \left(s - \frac{M_N^2 + m_l^2}{2} - \frac{(M_N^2 - m_l^2)^2}{2s} \right) \right. \\ & \left. \left. \times F(s, M_N, m_l) + \left(s - \frac{M_N^2 + m_l^2}{2} \right) \left(1 - \frac{M_N^2 + m_l^2}{M_N^2 - m_l^2} \ln \frac{M_N}{m_l} \right) - \frac{s}{3} \right] \right\}. \quad (\text{A4}) \end{aligned}$$

Next, we consider the contributions of NHL's to the parameter $\delta\Gamma_Z$. The direct contribution of NHL's in the triangle diagrams of Figs. 2(e)–(j) is given by the sum of amplitudes:

$$\begin{aligned} \mathcal{M} = & i\epsilon\epsilon_\mu\gamma^\mu(1 - \gamma_5)\frac{\alpha}{4\pi} \left\{ l_1 \mathcal{M}_{Z\Phi W} + l_2 \mathcal{M}_{ZNN\Phi} + l_1 \mathcal{M}_{Z\Phi\Phi} + (1 - l_1) \mathcal{M}_{ZWW\nu} \right. \\ & \left. + l_1 \mathcal{M}_{ZWWN} + l_3 \mathcal{M}_{Z\nu W} + l_3 \mathcal{M}_{Z\nu NW} + l_4 \mathcal{M}_{Z\nu\nu W} + l_2 \mathcal{M}_{ZNNW} \right\}, \quad (\text{A5}) \end{aligned}$$

with $(\frac{m_l^2}{M_W^2}$ terms neglected)

$$\begin{aligned} \mathcal{M}_{Z\Phi W} &= + \frac{M_W^2}{2s_W c_W} \mathcal{X} C_0(M_W, M_N, M_W), \\ \mathcal{M}_{ZNN\Phi} &= + \frac{M_W^2}{8s_W^3 c_W} \mathcal{X}^2 C_0(M_N, M_W, M_N), \\ \mathcal{M}_{Z\Phi\Phi} &= - \frac{1}{2s_W^3} \frac{1 - 2s_W^2}{2c_W} \mathcal{X} \left[C_{24}^{\text{fn}}(M_W, M_N, M_W) + \frac{1}{4} \Delta \right], \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{ZWWa} &= - \frac{3c_W}{4s_W^3} \left\{ \frac{2}{3} M_Z^2 [-C_{11}(M_W, M_a, M_W) - C_{23}(M_W, M_a, M_W) - C_0(M_W, M_a, M_W)] \right. \\ & \left. + 4C_{24}^{\text{fn}}(M_W, M_a, M_W) - \frac{2}{3} + \Delta \right\}, \end{aligned}$$

$$\mathcal{M}_{ZabW} = - \frac{1}{8s_W^3 c_W} \left\{ 2M_Z^2 [(C_{23}(M_a, M_W, M_b) + C_{11}(M_a, M_W, M_b))] + 2 - 4C_{24}^{\text{fn}}(M_a, M_W, M_b) - \Delta \right\}$$

where a, b run over N, ν ; $M_\nu = 0$.

(A6)

Here $\mathcal{M}_{Z\Phi W}$ is the sum of equal contributions from diagrams 2(h) and 2(i). Diagram 2(f) comes in both with massless ν 's and NHL's. Diagram 2(e) comes in with four combinations of neutral lepton types. Thus $\mathcal{M}_{ZWW\nu}$ and $\mathcal{M}_{Z\nu\nu W}$ are standard model results but come into the sum [Eq. (A5)] with NHL mixing factor coefficients. Our results in Eq. (A5) are written in terms of the 't Hooft–Veltman integrals [22]. Our conventions are given below, with finite parts indicated by the superscript.

The function C_0 is defined as

$$\begin{aligned} C_0(m_1, m_2, m_3) &= C_0(p_1, p_2; m_1, m_2, m_3) \\ &= C_0^{\text{fn}}(m_1, m_2, m_3) \end{aligned}$$

$$= - \int \frac{d^n q}{i\pi^2} \frac{1}{D},$$

$$\text{where } D = (q^2 - m_1^2 + i\epsilon)[(q - p_1)^2 - m_2^2 + i\epsilon][(q - p_1 - p_2)^2 - m_3^2 + i\epsilon]. \quad (\text{A7})$$

The functions C_{24}, C_{23}, C_{11} are defined by

$$\begin{aligned} C_\mu &= - \int \frac{d^n q}{i\pi^2} \frac{q_\mu}{D} = -p_{1\mu} C_{11} - p_{2\mu} C_{12}, \\ C_{\mu\nu} &= - \int \frac{d^n q}{i\pi^2} \frac{q_\mu q_\nu}{D} \\ &= p_{1\mu} p_{1\nu} C_{21} + p_{2\mu} p_{2\nu} C_{22} + (p_{1\mu} p_{2\nu} + p_{1\nu} p_{2\mu}) C_{23} - g_{\mu\nu} C_{24}. \end{aligned} \quad (\text{A8})$$

The functions C_{24}, C_{23}, C_{11} are reduced [in the limit $p_1^2 = p_2^2 = m_i^2 \ll (p_1 + p_2)^2 = M_Z^2$] to

$$\begin{aligned} C_{24}(m_1, m_2, m_3) &= \frac{1}{4} \left(\frac{2}{4-n} - \gamma - \ln \pi \right) + C_{24}^{\text{fin}}(m_1, m_2, m_3), \\ C_{24}^{\text{fin}}(m_1, m_2, m_3) &= [m_1^2 C_0(m_1, m_2, m_3) + f_1 C_{11}(m_1, m_2, m_3) + B_1^{\text{fin}}(p_1 + p_2; m_1, m_3)] \left(-\frac{1}{2} \right) + \frac{1}{4}, \\ &\text{with } f_1 = m_1^2 - m_2^2, \\ C_{11}(m_1, m_2, m_3) &= C_{11}^{\text{fin}}(m_1, m_2, m_3) = -\frac{1}{M_Z^2} [f_2 C_0(m_1, m_2, m_3) - B_0^{\text{fin}}(p_1 + p_2; m_1, m_3) + B_0^{\text{fin}}(p_1; m_1, m_2)], \\ &\text{with } f_2 = M_Z^2 + m_2^2 - m_3^2, \\ C_{23}(m_1, m_2, m_3) &= C_{23}^{\text{fin}}(m_1, m_2, m_3) = -\frac{1}{M_Z^2} [B_1^{\text{fin}}(p_1 + p_2; m_1, m_3) + B_0^{\text{fin}}(p_2; m_2, m_3) \\ &\quad + f_1 C_{11}(m_1, m_2, m_3)] + C_{24}^{\text{fin}}(m_1, m_2, m_3) \frac{2}{M_Z^2}. \end{aligned} \quad (\text{A9})$$

The functions B_0, B_1 are defined as

$$\begin{aligned} B_0(p; m_1, m_2) &= \int \frac{d^n q}{i\pi^2} \frac{1}{(q^2 - m_1^2 + i\epsilon)[(q-p)^2 - m_2^2 + i\epsilon]} \\ &= \left(\frac{2}{4-n} - \gamma - \ln \pi \right) + B_0^{\text{fin}}(p; m_1, m_2), \\ B_0^{\text{fin}}(p; m_1, m_2) &= - \int_0^1 dx \ln [p^2 x^2 + m_1^2 - (p^2 + m_1^2 - m_2^2)x], \\ B_1(p; m_1, m_2) &= -\frac{1}{2} \left(\frac{2}{4-n} - \gamma - \ln \pi \right) + B_1^{\text{fin}}(p; m_1, m_2), \\ B_1^{\text{fin}}(p; m_1, m_2) &= \int_0^1 dx \ln [p^2 x^2 + m_1^2 - (p^2 + m_1^2 - m_2^2)x]. \end{aligned} \quad (\text{A10})$$

The remaining diagrams of Fig. 2 contribute NHL dependent terms to the charged lepton wave function renormalization. The contributions to the lepton self-energy of Figs. 2(a) and (b) and Figs. 2(c) and (d) are given as Σ^W and Σ^Φ , respectively.

$$\begin{aligned} -i\Sigma^W &= \frac{i\alpha}{32\pi s_W^2} l_1 [2 - 2 \ln 4\pi - 2 \ln \mu^2 + 2\gamma - \frac{4}{\epsilon} + 4f(\mathcal{X})] p_\alpha \gamma^\alpha (1 - \gamma_5), \\ -i\Sigma^\Phi &= -\frac{i\alpha}{32\pi s_W^2} l_1 \mathcal{X} \left[\frac{2}{\epsilon} + \ln 4\pi + \ln \mu^2 - \gamma - 2f(\mathcal{X}) \right] p_\alpha \gamma^\alpha (1 - \gamma_5), \end{aligned}$$

where

$$f(\mathcal{X}) = \frac{\mathcal{X}^2}{2(\mathcal{X}-1)^2} \ln \mathcal{X} + \frac{\mathcal{X}}{1-\mathcal{X}} - \frac{\mathcal{X}+1}{4(1-\mathcal{X})} + \frac{1}{2} \ln M_W^2. \quad (\text{A11})$$

All these contributions, Eq. (A5) and (A11), along with their corresponding counterterms modify the Zl^+l^- vertex. In addition, the vertex is modified via $\gamma - Z$ mixing. However, the relevant term $\Pi^{\gamma Z}(M_Z^2)$ depends on NHL's only through the Z and W self-energies so we need no other results to determine $\delta\Gamma_Z$.

The results presented in this Appendix are also sufficient to derive the W oblique corrections of Δr , which we have considered.

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