"Nonfactorizable" terms in hadronic B-meson weak decays

João M. Soares
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3
(Received 29 September 1994; revised manuscript received 11 November 1994)

The branching ratios for the hadronic B-meson weak decays $B \to J/\psi\, K$ and $B \to D\pi$ are used to extract the size of the "nonfactorizable" terms in the decay amplitudes. It is pointed out that the solutions are not uniquely determined. In the $B \to J/\psi\, K$ case, a twofold ambiguity can be removed by analyzing the contribution of this decay to $B \to Kl^+l^-$. In the $B \to D\pi$ case, a fourfold ambiguity can only be removed if the "nonfactorizable" terms are assumed to be a small correction to the vacuum insertion result.

PACS number(s): 13.25.Hw

I. INTRODUCTION

An increasing sample of B mesons has been gathered from different experiments in recent times, and will tend to increase sharply in the near future with the advent of B factories and, possibly, experiments in hadron colliders targeted at B physics. The major concern in this paper is to clarify how such a wealth of data can be used to study some of the aspects that remain unclear in the hadronic weak decays of the B and the other flavored mesons. The focus shall be on the two-body decays that proceed through the tree level, Cabibbo favored, quark transitions $b \to c\bar{c}s$ and $b \to c\bar{u}d$. The corresponding effective weak Hamiltonian, once QCD corrections have been included, is the sum of two four-quark operators, that only differ in the color indices of their quark fields and the strength of their Wilson coefficients. In the calculation of the decay amplitudes, one is faced with the task of evaluating the matrix elements, between the initial and final hadronic states, of those two four-quark operators. The vacuum insertion (factorization) approximation [1] reduces this problem to that of determining the matrix elements of bilinear quark operators; such matrix elements can then be measured from leptonic and semileptonic decays, or calculated in some model for the mesons. Unfortunately, this is a poor approximation, as can be seen in the strong disagreement between the factorization predictions and the observed rates for the color suppressed D or B decays (see, for example, Ref. [2]).

The standard procedure [2] in dealing with this discrepancy has been to preserve the vacuum insertion result for the hadronic matrix elements, but to replace the Wilson coefficients that multiply them by two free parameters. These parameters are then determined from a fit to the observed values of the branching ratios. For the case of the D- and B-meson decays, the two parameters (for each case) fit the available data quite well. For the D mesons, their values correspond to dropping the contribution of the operator in the weak Hamiltonian which has a color mismatch in the quark fields [2]. This is a procedure that finds some theoretical justification in $1/N_c$ expansion arguments [3]. Quite surprisingly, the recent data on B-meson decays [4] have shown that, in this case, the values of the parameters do not obey the same rule: neglecting the contribution of the color mismatched operator cannot be used as a systematic procedure to obtain the value of the free parameters, as it was the case for the D decays.

In view of the failure of the factorization approximation, and the failure of the standard procedure of dropping the contribution of one of the operators in the Hamiltonian altogether, a different approach is necessary. In particular, a phenomenological picture must be used that includes the "nonfactorizable" terms that appear in the hadronic matrix elements beyond the vacuum insertion approximation. The question of extracting those terms from the experimental data should then be addressed. This program was first advocated by Deshpande, Gronau, and Sutherland [5], and it has been recently applied to both D- and B-meson decays by Cheng [6]. Here, their approach is generalized to include corrections to the factorization approximation, for both operators in the effective weak Hamiltonian. I will concentrate on the case of the B decays, and derive the size of the "nonfactorizable" corrections for the decays $B \to J/\psi\, K$ and $B \to D\pi$. Special attention is paid to the discrete ambiguities that affect the solutions, and possible ways of resolving those ambiguities are discussed. Also, it will be stressed that the corrections corresponding to each of the operators in the Hamiltonian can only be disentangled by making additional assumptions. The results of Cheng [6] are recovered among other possible solutions.

II. THE "NONFACTORIZABLE" TERMS IN $B \to J/\psi\, K$ AND $B \to D\pi$

The tree level, Cabibbo favored, hadronic weak decays of the B mesons correspond to the quark transitions $b \to c\bar{c}s$ and $b \to c\bar{u}d$. The decay amplitudes are derived from the effective weak Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}}\,\{V_{cb}V_{cs}^*[C_1(\mu)\mathcal{O}_1^c + C_2(\mu)\mathcal{O}_2^c] + V_{cb}V_{ud}^*[C_1(\mu)\mathcal{O}_1^u + C_2(\mu)\mathcal{O}_2^u]\}, \tag{1}$$

where

$$\mathcal{O}_1^c = \bar{c}_\alpha\gamma_\mu(1-\gamma_5)c_\beta\ \ \bar{s}_\beta\gamma^\mu(1-\gamma_5)b_\alpha, \tag{2}$$

$$\mathcal{O}_2^c = \bar{c}_\alpha\gamma_\mu(1-\gamma_5)c_\alpha\ \ \bar{s}_\beta\gamma^\mu(1-\gamma_5)b_\beta, \tag{3}$$

and the operators $\mathcal{O}^u_{1,2}$ are obtained from $\mathcal{O}^c_{1,2}$, replacing $\bar{s}$ and c by $\bar{d}$ and u, respectively. The Wilson coefficients $C_1(\mu)$ and $C_2(\mu)$ contain the short distance QCD corrections. In the leading logarithm approximation, they are [7] $C_{1,2} = (C_+ \pm C_-)/2$, with

$$C_\pm(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{\frac{6\gamma_\pm}{33-2n_f}} \tag{4}$$

($\gamma_- = -2\gamma_+ = 2$; n_f is the number of active flavors). For $\Lambda^{(5)}_{\overline{\rm MS}} = 200$ MeV [8], and at the scale $\mu = 5.0$ GeV, this gives

$$C_1 = 1.117, \quad C_2 = -0.266, \tag{5}$$

where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme.

For the exclusive decays $B^- \to J/\psi K^-$ or $\overline{B^0_d} \to J/\psi \overline{K^0}$, the amplitude is

$$A(B \to J/\psi K) = -\frac{G_F}{\sqrt{2}} V_{cb} V^*_{cs} (C_1 \langle J/\psi K | \mathcal{O}^c_1 | B\rangle + C_2 \langle J/\psi K | \mathcal{O}^c_2 | B\rangle). \tag{6}$$

The hadronic matrix element of $\mathcal{O}^c_2$ is

$$\langle J/\psi K | \mathcal{O}^c_2 | B \rangle = M(1+Y), \tag{7}$$

where

$$M \equiv \langle J/\psi | \bar{c}_\alpha \gamma_\mu (1-\gamma_5) c_\alpha | 0 \rangle \langle K | \bar{s}_\beta \gamma^\mu (1-\gamma_5) b_\beta | B \rangle \tag{8}$$

is the result in the vacuum insertion (factorization) approximation, and Y parametrizes the correction to this approximation. The hadronic matrix element of $\mathcal{O}^c_1$ is

$$\langle J/\psi K | \mathcal{O}^c_1 | B \rangle = \frac{1}{N_c} M(1+Y) + MX, \tag{9}$$

where the vacuum insertion result corresponds to $Y = X = 0$ (the $1/N_c$ suppression factor stems from the projection of the color mismatched c-$\bar{c}$ quark fields into a color singlet). The parameter Y is the same as in Eq. (7), and X parametrizes the additional deviation from the vacuum insertion result, which is due to the color mismatch in $\mathcal{O}^c_1$. A Fierz transformation of the color structure in this operator gives

$$\mathcal{O}^c_1 = \frac{1}{N_c} \bar{c}_\alpha \gamma_\mu (1-\gamma_5) c_\alpha \; \bar{s}_\beta \gamma^\mu (1-\gamma_5) b_\beta + \frac{1}{2} \bar{c} \gamma_\mu (1-\gamma_5) \lambda^a c \; \bar{s} \gamma^\mu (1-\gamma_5) \lambda^a b, \tag{10}$$

and so X can be written as

$$X = \frac{1}{M} \langle J/\psi K | \tfrac{1}{2} \bar{c} \gamma_\mu (1-\gamma_5) \lambda^a c \; \bar{s} \gamma^\mu (1-\gamma_5) \lambda^a b | B \rangle. \tag{11}$$

In this work, X and Y will be treated as free parameters, to be determined from the experimental data. They are defined by Eqs. (7) and (9), and they describe the "nonfactorizable" terms in the hadronic matrix elements of the two operators in $H_{\rm eff}$. In principle, these parameters can also account for possible effects on the decay amplitude of inelastic final state scatterings. However, it will be assumed that the complex phases due to on-mass-shell intermediate states can be neglected, so that X and Y can be taken to be real.

The decay amplitude can then be written as

$$A(B \to J/\psi K) = -\frac{G_F}{\sqrt{2}} V_{cb} V^*_{cs} M a, \tag{12}$$

with

$$a = C_2 + \frac{1}{N_c} C_1 + \tilde{a}, \tag{13}$$

and the "nonfactorizable" part

$$\tilde{a} = C_1 X + \left(C_2 + \frac{1}{N_c} C_1\right) Y. \tag{14}$$

In the Wirbel-Stech-Bauer (WSB) model [9], $M = 5.84$ GeV^3 $\times$ $f_\psi/(395$ MeV). With $|V_{cb}| = 0.038\sqrt{1.63\ \text{psec}/\tau_B}$ [10], and $f_\psi = 395$ MeV (which corresponds to $\Gamma_{e^+e^-} = (5.26 \pm 0.37)$ keV [8]), the branching ratio is

$$B(B \to J/\psi K) = 1.90|a|^2\%. \tag{15}$$

From the average of the experimental results [8]

$$B(B^- \to J/\psi K^-) = (0.102 \pm 0.014)\%, \tag{16}$$

$$B(\overline{B^0} \to J/\psi \overline{K^0}) = (0.075 \pm 0.021)\%, \tag{17}$$

it follows that $|a| \simeq 0.215$, and so

$$\tilde{a} \simeq 0.11 \quad \text{or} \quad -0.32. \tag{18}$$

The twofold ambiguity in the value of $\tilde{a}$ corresponds to the unknown sign of a; it cannot be resolved by the branching ratio of $B \to J/\psi K$ alone. Each of the two solutions is represented by a line in the (X, Y) plane and it is shown in Fig. 1.

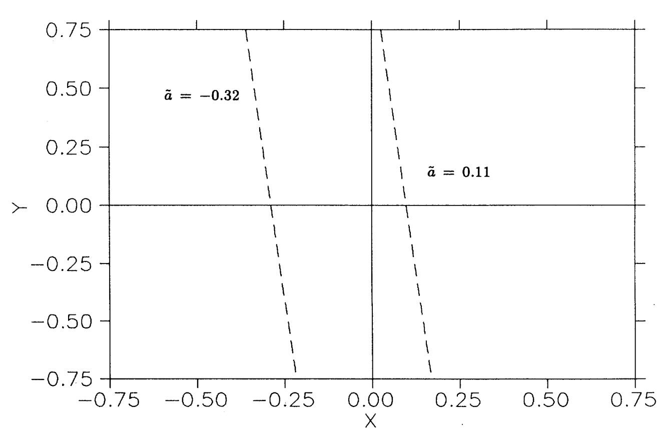

FIG. 1. The "nonfactorizable" terms in $B \to J/\psi K$. The two dashed lines correspond to the two solutions in Eq. (18).

Proceeding in a similar way for the exclusive processes $\overline{B^0_d} \to D^+\pi^-$, $\overline{B^0_d} \to D^0\pi^0$, and $B^- \to D^0\pi^-$, the weak decay amplitudes are

$$A^{+-} = -\frac{G_F}{\sqrt{2}} V_{cb} V^*_{ud} M_1 a_1, \tag{19}$$

$$A^{00} = -\frac{G_F}{\sqrt{2}} V_{cb} V^*_{ud} \frac{1}{\sqrt{2}} M_2 a_2, \tag{20}$$

and

$$A^{0-} = -\frac{G_F}{\sqrt{2}} V_{cb} V^*_{ud} M_1 a_1 \left(1 + \frac{M_2}{M_1}\frac{a_2}{a_1}\right), \tag{21}$$

respectively (nonspectator contributions are very small and they have been neglected). In the vacuum insertion approximation, the hadronic matrix elements are

$$M_1 \equiv \langle D^+|\bar{c}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha|\overline{B^0_d}\rangle \langle \pi^-|\bar{d}_\beta \gamma_\mu (1-\gamma_5) u_\beta|0\rangle \tag{22}$$

and

$$M_2 \equiv \sqrt{2}\langle \pi^0|\bar{d}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha|\overline{B^0_d}\rangle \times \langle D^0|\bar{c}_\beta \gamma_\mu (1-\gamma_5) u_\beta|0\rangle. \tag{23}$$

In the WSB model [9], $M_1 = 1.85$ GeV^3 and $M_2 = 2.28$ GeV^3 (for $f_D = 220$ MeV). As in the previous case, the parameters

$$a_1 = C_1 + \frac{1}{N_c} C_2 + \tilde{a}_1 \tag{24}$$

and

$$a_2 = C_2 + \frac{1}{N_c} C_1 + \tilde{a}_2 \tag{25}$$

include the quantities

$$\tilde{a}_1 = C_2 X_1 + \left(C_1 + \frac{1}{N_c} C_2\right) Y_1, \tag{26}$$

$$\tilde{a}_2 = C_1 X_2 + \left(C_2 + \frac{1}{N_c} C_1\right) Y_2, \tag{27}$$

which account for the "nonfactorizable" terms in the decay amplitudes. The $X_{1,2}$ and $Y_{1,2}$ parameters are defined by

$$\langle D^+\pi^-|\mathcal{O}^u_1|\overline{B^0_d}\rangle = M_1(1+Y_1), \tag{28}$$

$$\langle D^+\pi^-|\mathcal{O}^u_2|\overline{B^0_d}\rangle = \frac{1}{N_c} M_1(1+Y_1) + M_1 X_1, \tag{29}$$

and

$$\sqrt{2}\langle D^0\pi^0|\mathcal{O}^u_2|\overline{B^0_d}\rangle = M_2(1+Y_2), \tag{30}$$

$$\sqrt{2}\langle D^0\pi^0|\mathcal{O}^u_1|\overline{B^0_d}\rangle = \frac{1}{N_c} M_2(1+Y_2) + M_2 X_2. \tag{31}$$

As for the $B \to J/\psi\, K$ case, it is assumed that the effects of inelastic final state scatterings, if at all significant, are such that $X_{1,2}$ and $Y_{1,2}$ can be taken to be real.

In order to determine the values of the parameters $\tilde{a}_1$ and $\tilde{a}_2$, the decay amplitudes are extracted from the experimental value of the corresponding branching ratios [8],

$$B(\overline{B^0_d} \to D^+\pi^-) = (0.30 \pm 0.04)\%, \tag{32}$$

$$B(\overline{B^0_d} \to D^0\pi^0) < 0.048\% \ \ (90\% \text{ C.L.}), \tag{33}$$

$$B(B^- \to D^0\pi^-) = (0.53 \pm 0.05)\%, \tag{34}$$

and compared to the predictions in Eqs. (19)–(21). The latter are the amplitudes in the absence of final state interaction phases, and so the comparison must be done with care (to be sure, I use the notation $\mathcal{A}^{+-}$, $\mathcal{A}^{00}$, and $\mathcal{A}^{0-}$ for the full amplitudes). Contrary to the $B \to J/\psi\, K$ case, the $B \to D\pi$ decays involve two isospin channels, and an elastic final state interaction phase δ can appear between the two isospin amplitudes $A_{3/2}$ and $A_{1/2}$. In general,

$$A_{3/2} = |A_{3/2}| \qquad A_{1/2} = |A_{1/2}| e^{i\delta'}, \tag{35}$$

where $\delta' = \delta$ or $\delta + \pi$ (according to the relative sign of the two amplitudes in the absence of the final state interaction phase). The full amplitudes $\mathcal{A}^{+-}$, $\mathcal{A}^{00}$, and $\mathcal{A}^{0-}$ are related to the isospin amplitudes $A_{3/2}$ and $A_{1/2}$, in the following way:

$$\mathcal{A}^{+-} = \frac{1}{\sqrt{3}} A_{3/2} + \frac{\sqrt{2}}{\sqrt{3}} A_{1/2}, \tag{36}$$

$$\mathcal{A}^{00} = \frac{\sqrt{2}}{\sqrt{3}} A_{3/2} - \frac{1}{\sqrt{3}} A_{1/2}, \tag{37}$$

$$\mathcal{A}^{0-} = \sqrt{3} A_{3/2}. \tag{38}$$

This allows the determination of the magnitudes $|A_{1/2}|$ and $|A_{3/2}|$, as well as $\cos\delta'$, from the experimental results in Eqs. (32)–(34). In particular, it follows that

$$\cos\delta' > 0.77. \tag{39}$$

The lack of a more precise value for δ' is due to the fact that only an upper limit exists for $B(\overline{B^0_d} \to D^0\pi^0)$. For now, I will take $\delta' = 0$ (i.e., the final state interaction phase is either $\delta = 0$ or π). Then,

$$|A_{3/2}| = \frac{1}{\sqrt{3}} |\mathcal{A}^{0-}|, \tag{40}$$

$$|A_{1/2}| = \frac{\sqrt{3}}{\sqrt{2}} \left(|\mathcal{A}^{+-}| - \frac{1}{3} |\mathcal{A}^{0-}| \right). \tag{41}$$

Since the decay amplitudes in the absence of the final state interaction phase are

$$A^{+-} = \frac{1}{\sqrt{3}} |A_{3/2}| \pm \frac{\sqrt{2}}{\sqrt{3}} |A_{1/2}|, \tag{42}$$

$$A^{00} = \frac{\sqrt{2}}{\sqrt{3}} |A_{3/2}| \mp \frac{1}{\sqrt{3}} |A_{1/2}|, \tag{43}$$

$$A^{0-} = \sqrt{3} |A_{3/2}| \tag{44}$$

(the two signs correspond to $\delta' = \delta$ or $\delta + \pi$), it follows that

$$|A^{+-}| = |\mathcal{A}^{+-}|\,, \qquad \frac{A^{0-}}{A^{+-}} = \frac{|\mathcal{A}^{0-}|}{|\mathcal{A}^{+-}|}, \tag{45}$$

for $\delta = 0$, and

$$|A^{+-}| = |\tfrac{2}{3}|\mathcal{A}^{0-}| - |\mathcal{A}^{+-}||$$

$$\frac{A^{0-}}{A^{+-}} = \frac{|\mathcal{A}^{0-}|}{\frac{2}{3}|\mathcal{A}^{0-}| - |\mathcal{A}^{+-}|}, \tag{46}$$

for $\delta = \pi$. The predictions of Eqs. (19)–(21), in terms of the parameters a_1 and a_2, are replaced on the left-hand side (LHS) of these equations; whereas the experimental input from the branching ratios is used for the RHS. If the experimental result of Eq. (39) is interpreted as showing a negligible phase shift from the final state interaction effects, Eq. (45) gives (with $|V_{cb}|$ as before)

$$|a_1| \simeq 1.07, \quad 1 + 1.23\frac{a_2}{a_1} \simeq 1.33, \tag{47}$$

and so

$$\tilde{a}_1 \simeq 0.046, \quad \tilde{a}_2 \simeq 0.18 \tag{48}$$

or

$$\tilde{a}_1 \simeq -2.10, \quad \tilde{a}_2 \simeq -0.39. \tag{49}$$

Alternatively, the data can be interpreted as showing a maximal phase shift. Then Eq. (46) gives

$$|a_1| \simeq 0.123, \qquad 1 + 1.23\frac{a_2}{a_1} \simeq -11.6, \tag{50}$$

and so

$$\tilde{a}_1 \simeq -0.905, \quad \tilde{a}_2 \simeq -1.36 \tag{51}$$

or

$$\tilde{a}_1 \simeq -1.15, \quad \tilde{a}_2 \simeq 1.15. \tag{52}$$

The fourfold ambiguity corresponds to the fact that the sign of a_1 cannot be determined from the branching ratios in Eqs. (32)–(34), and the final state interaction phase can only be determined mod π. Each of the four solutions is represented by a line in the (X_1, Y_1) plane and a line in the (X_2, Y_2) plane. The graphs corresponding to Eq. (48) are shown in Fig. 2; the other solutions require larger values of the $X_{1,2}$ or $Y_{1,2}$ parameters, and they are not shown.

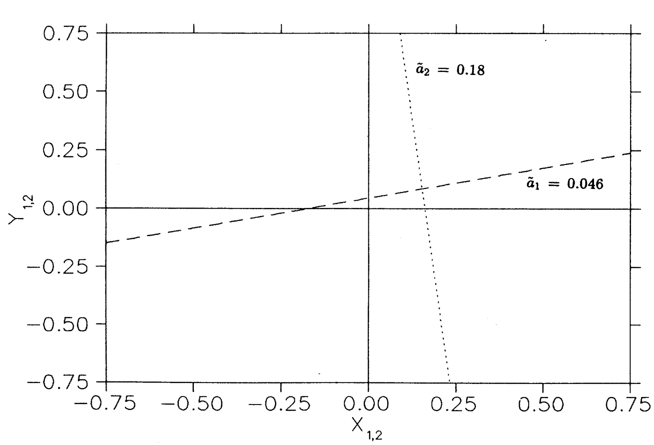

FIG. 2. The "nonfactorizable" terms in $B \to D\pi$. The dashed and dotted lines correspond to the solution in Eq. (50). The other possible solutions are not shown.

At this point, a word should be said about the uncertainties in the results that were presented. The derivation of the parameters $|a|$, $|a_1|$, and a_2/a_1 suffers from the experimental errors in the branching ratios [in particular, $B(\overline{B}^0_d \to D^0\pi^0)$ is still missing], and in $|V_{cb}|$. These will improve with more accumulated data; which will also allow us to derive the hadronic matrix elements of the bilinear quark operators from the semileptonic branching ratios (see the tests of factorization in Ref. [4], for example). At present, the use of the WSB model [9] entails an uncertainty that is hard to quantify. The derivation of the quantities $\tilde{a}$, $\tilde{a}_1$, and $\tilde{a}_2$ suffers from the additional uncertainty on the Wilson coefficients: in particular, the value chosen for the scale μ is important [11]. For μ in the range 4.5–5.5 GeV, the positive solution for $\tilde{a}$ varies by about 20%, whereas the negative solution and the values of $\tilde{a}_{1,2}$ in Eq. (48) vary by about 10%; the other solutions are fairly stable. I have taken $\mu = 5.0$ GeV, which is approximately the constituent b-quark mass.

The X- and Y-type parameters describe different mechanisms by which factorization breaks down, as they correspond to operators that differ significantly in their color structure. However, the analysis of the experimental branching ratios has shown that their values cannot be probed independently, unless further assumptions are made. One reasonable assumption is that the Y-type parameters are small compared to unity, i.e., the vaccum insertion approximation holds well for the hadronic matrix elements of the operators with the correct color assignments, in Eqs. (7), (28), and (30). This was proposed by Bjorken in Ref. [14], on the basis of color transparency arguments: these operators produce a pointlike colorless quark-antiquark pair that does not interact with the hadronic medium, until it hadronizes into one of the mesons in the final state; if the recoil velocity of this colorless object is sufficiently large, it will be beyond the reach of the strong interaction by the time it hadronizes. Then, the colorless quark current inside the operator can be identified with the meson field, and the matrix element of the operator factorizes. This argument should hold best for the two-body B decays such as those considered in here, given the large relative velocity of the mesons in the final state. Within this scenario, some information on the value of the X-type parameters can be extracted from the experimental data. (Notice that the color transparency argument cannot be applied to these terms, as they correspond to operators that produce the pointlike quark-antiquark pair in a color octet.) From the graphs in Figs. 1 and 2 it is clear that small "nonfactorizable" terms of the Y-type alone cannot accommodate the values of $\tilde{a}$ or $\tilde{a}_2$ (from the branching ratios of the color suppressed decays). Nonvanishing X-type terms are necessary (and sufficient) to explain the data. Moreover, when Y and Y_2 are small, it follows from Eqs. (14) and (27) that $X \simeq \tilde{a}/C_1$ and $X_2 \simeq \tilde{a}_2/C_1$. These agree with the results in Ref. [6], up to the fourfold ambiguity in $\tilde{a}_2$, and to small numerical differences due to updated values of the parameters used in here. As for the value of X_1, it is hard to obtain a reliable prediction. For $Y_1 = 0$, Eq. (26) gives $X_1 \simeq \tilde{a}_1/C_2$, but this result is very sensitive to a small value of Y_1. For example, in Fig. 2, a value of Y_1 of about 10% is sufficient to make X_1 small and positive, like X_2.

III. RESOLVING THE AMBIGUITIES

The sign of the parameter a that appears in the $B \to J/\psi K$ amplitude of Eq. (12) can be determined from the interference between the short distance contribution to $B \to Kl^+l^-$, and the long distance contribution due to $B \to KJ/\psi \to Kl^+l^-$. The short distance amplitude is derived from the effective weak Hamiltonian in Eq. (1) (the operators $\mathcal{O}^c_{1,2}$ contribute at the one-loop level), with the additional electroweak terms:

$$H'_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{tb}V^*_{ts} \sum_{i=7,8,9} C_i(\mu)\mathcal{O}_i, \tag{53}$$

where the operators

$$\begin{aligned} \mathcal{O}_7 &= \frac{e}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu}(1+\gamma_5) b_\alpha F_{\mu\nu}, \\ \mathcal{O}_8 &= \frac{\alpha}{2\pi} \bar{s}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha \bar{l}\gamma_\mu l, \\ \mathcal{O}_9 &= \frac{\alpha}{2\pi} \bar{s}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha \bar{l}\gamma_\mu \gamma_5 l, \end{aligned} \tag{54}$$

contribute to $B \to Kl^+l^-$ at tree level. For $m_t = 175$ GeV, the Wilson coefficients in Eq. (53), in the leading logarithm approximation, are [15] $C_7 = 0.326$, $C_8 = -3.752$, and $C_9 = 4.581$. The decay amplitude is

$$A = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb}V^*_{ts} \left(-\langle K|\bar{s}\gamma^\mu b|B\rangle \frac{1}{2}(C_{8\text{eff}} \bar{u}_l \gamma_\mu v_{\bar{l}} + C_9 \bar{u}_l \gamma_\mu \gamma_5 v_{\bar{l}}) + \langle K|\bar{s} i\sigma^{\mu\nu} q_\nu (1+\gamma_5) b|B\rangle C_7 m_b \frac{1}{q^2} \bar{u}_l \gamma_\mu v_{\bar{l}} \right) \tag{55}$$

($q \equiv p_B - p_K$). The factor

$$C_{8\text{eff}} = C_8 - (3C_2 + C_1) g\left(\frac{4m_c^2}{q^2}, \frac{m_c^2}{m_b^2}\right) + 3a g_{\text{LD}} \tag{56}$$

includes the contribution

$$\begin{aligned} g(x,y) = &-\frac{4}{9}\ln y + \frac{4}{9}x + \frac{8}{27} - \frac{8}{9}\left(1+\frac{1}{2}x\right)\sqrt{x-1}\arctan\frac{1}{\sqrt{x-1}}\theta(x-1) \\ &-\frac{4}{9}\left(1+\frac{1}{2}x\right)\sqrt{1-x}\left(\ln\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} + i\pi\right)\theta(1-x) \end{aligned} \tag{57}$$

from the operators $\mathcal{O}^c_{1,2}$, and the long distance contribution [16]

$$g_{\text{LD}} = \frac{3\pi}{\alpha^2} \sum_{V=J/\psi,\psi'} \frac{m_V \Gamma(V \to l^+l^-)}{q^2 - m_V^2 + i m_V \Gamma_V} \tag{58}$$

from the J/ψ and ψ' resonances. The parameter a that multiplies g_{LD} is the same as in Eq. (12) (for simplicity, I have taken the same parameter for both the J/ψ and the ψ' resonances, but this is not necessary); the relative sign between the long distance and short distance contributions is well determined [17], up to the sign of a. The hadronic matrix elements are parametrized by

$$\begin{aligned} \langle K|\bar{s}\gamma^\mu b|B\rangle &= (p_B + p_K)^\mu f_+(q^2) + q^\mu f_-(q^2), \\ \langle K|\bar{s} i\sigma^{\mu\nu}(1+\gamma_5) b|B\rangle &= s(q^2)[(p_B+p_K)^\mu q^\nu - (p_B+p_K)^\nu q^\mu + i\epsilon_{\mu\nu\alpha\beta}(p_B+p_K)^\alpha q^\beta], \end{aligned} \tag{59}$$

where, in the static b-quark limit [18], $s = -(f_+ - f_-)/2m_B$. The modified WSB model [9] gives

$$f_+(q^2) \simeq \frac{h_0}{\left(1 - \frac{q^2}{(m_B+m_K)^2}\right)^2}, \tag{60}$$

$$f_-(q^2) \simeq -f_+(q^2)\frac{m_B - m_K}{m_B + m_K}, \tag{61}$$

with $h_0 = 0.379$ (the pole masses [9] $m_{0^+} = 5.89$ GeV and $m_{1^-} = 5.43$ GeV have been approximated by $m_B + m_K$). The differential branching ratio is then

$$\begin{aligned} \frac{1}{\Gamma}\frac{d\Gamma}{dz} \simeq &\frac{1}{48}\tau_B G_F^2 \alpha^2 |V_{tb}V^*_{ts}|^2 \left(\frac{m_B}{2\pi}\right)^5 (1-z)^3 f_+^2 \\ &\times (|C_9|^2 + |C_{8\text{eff}} + 2C_7|^2) \end{aligned} \tag{62}$$

($z \equiv q^2/m_B^2$), where the lepton and kaon masses were neglected. This is shown in Fig. 3 for a positive and negative. Studying the region of the interference between the short and the long distance contributions will allow one to determine the sign of a, and resolve the ambiguity in Eq. (18). At present, the necessary sensitivity has not been reached yet, and only an upper limit exists on the

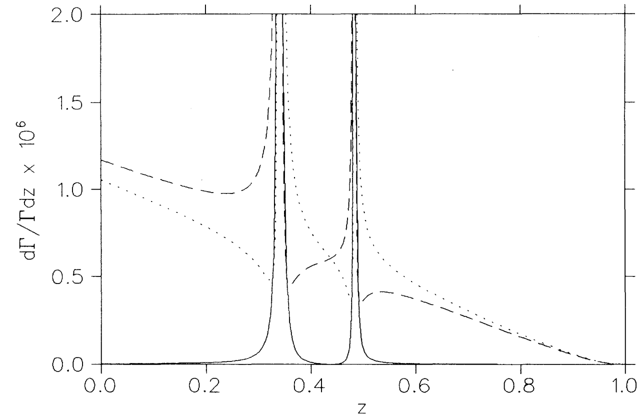

FIG. 3. Differential branching ratio for $B \to Kl^+l^-$ [$z \equiv (p_B - p_K)^2/m_B^2$]. The solid line corresponds to the long distance contribution alone, whereas the other curves include the short distance contribution: with $a > 0$ (dashed line) and $a < 0$ (dotted line).

nonresonant $B \to Kl^+l^-$ decays [19].

The fortunate interference that allows one to determine the sign of a is quite unique, and no similar effect appears for the decays of the type $b \to c\bar{u}d$ that would allow one to determine the sign of a_1 in Eqs. (19) or (21). As for the final state interaction phase δ in Eq. (35), it is known [20] that it should be the same phase that appears in D-π elastic scattering, at the energy $E_{\text{c.m.}} = m_B$. But this is of little use in determining its value. Indeed, it is hard to think of an experimental test that would lift the fourfold ambiguity in the values of the "nonfactorizable" terms $\tilde{a}_1$ and $\tilde{a}_2$, in the $B \to D\pi$ decays. On the other hand, only one of the four solutions (that shown in Fig. 2) is compatible with small values of all $X_{1,2}$ and $Y_{1,2}$ parameters. It has $X_2 \simeq 0.16$, whereas X_1 is more uncertain ($|X_1| \leq 0.5$), as discussed above. In this scenario, the vacuum insertion result provides a reasonable approximation for the matrix elements of both operators in the weak Hamiltonian of Eq. (1). Although there are no theoretical arguments to support small X-type terms, it should be pointed out that such values tend to agree with the theoretical calculations that are presently available. These results, based on QCD sum rule techniques, are still very preliminary and give $X_1 \simeq -0.33$ [21] and X between -0.30 and -0.15 [22]. Corrections to the vacuum insertion result of a similar size have been obtained by lattice calculations [23], and from QCD sum rules [24], in the case of the amplitude for B^0-$\overline{B^0}$ mixing. However, the connection with the "nonfactorizable" terms in the decay amplitudes is not clear.

IV. CONCLUSION

The size of the "nonfactorizable" terms in the amplitudes for the decays $B \to J/\psi K$ and $B \to D\pi$ was derived from the experimental value of the corresponding branching ratios. These terms include two types of corrections to the factorization approximation, which correspond to the two operators in the effective weak Hamiltonian. Their separate sizes cannot be disentangled without further assumptions (for example, that the corrections to factorization are due mainly to the color mismatch in one of the operators). Furthermore, the results can only be determined up to a discrete ambiguity. In the case of $B \to J/\psi K$, the twofold ambiguity can be lifted by determining the sign of the interference between the short distance contribution to $B \to Kl^+l^-$, and the long distance contribution due to $B \to KJ/\psi \to Kl^+l^-$. A similar ambiguity appears in the case of the $B \to D\pi$ decays; because the final state interaction elastic phase between the two isospin amplitudes can only be determined modπ, the ambiguity becomes fourfold. Contrary to the previous case, there is no simple way to determine the correct solution experimentally. However, all but one of the solutions requires large "nonfactorizable" terms that would indicate a severe breakdown of the vacuum insertion approximation.

The analysis that was presented can be improved with future experimental results, in particular, with a measurement of the branching ratio for $\overline{B_d^0} \to D^0\pi^0$. Also, other B decays, similar to the ones shown in here, can be considered; the ambiguities in the values of the "nonfactorizable" terms that are extracted from the data will appear in the same fashion.

ACKNOWLEDGMENTS

I would like to thank A. Buras for a very useful discussion, and H.-Y. Cheng and L. Wolfenstein for carefully reading the manuscript and for their comments and corrections. This work was partly supported by the Natural Sciences and Engineering Research Council of Canada.

[1] For a recent review of the factorization procedure in the hadronic weak decays of B mesons, see, for example, M. Neubert *et al.*, in *Heavy Flavors*, edited by A. J. Buras and L. Lindner (World Scientific, Singapore, 1992); also, a thorough discussion can be found in H.-Y. Cheng, Int. J. Mod. Phys. A **4**, 495 (1989).

[2] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987).

[3] A. J. Buras, J.-M. Gérard, and R. Rückl, Nucl. Phys. **B268**, 16 (1986).

[4] For recent results, see CLEO Collaboration, M. S. Alam *et al.*, Phys. Rev. D **50**, 43 (1994).

[5] N. G. Deshpande, M. Gronau, and D. Sutherland, Phys. Lett. **90B**, 431 (1980); M. Gronau and D. Sutherland, Nucl. Phys. **B183**, 367 (1981).

[6] H.-Y. Cheng, Academia Sinica of Taipei, Report No. IP-ASTP-11-94, 1994 (unpublished).

[7] M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33**, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. **52B**, 352 (1974).

[8] Particle Data Group, Phys. Rev. D **50**, 1173 (1994).

[9] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985); see also Neubert *et al.* [1], for a modified version that was used in here.

[10] A. Ali and D. London, CERN Report No. CERN-TH.7398/94, 1994 (unpublished).

[11] The strong μ dependence persists, when QCD corrections are included at the next-to-leading order (NLO) [12, 13]. In Ref. [12] it is shown that the higher order corrections must be included as part of a simultaneous expansion in $C_2 + C_1/N_c$ and α_s, to avoid unphysical results; no large deviation from the leading-logarithm approximation (LLA) result is apparent in the $B \to J/\psi + X_s$ decay. A different approach is advocated in Ref. [13] for the exclusive decays. Use of the NLO results in there would lead to a large renormalization scheme dependence in the "nonfactorizable" terms.

[12] L. Bergström and P. Ernström, Phys. Lett. B **328**, 153 (1994).

[13] A. Buras, Max-Planck-Institut Report No. MPI-PhT/94-60, 1994 (unpublished).

[14] J. D. Bjorken, in *Gauge Bosons and Heavy Quarks*, Proceedings of the 18th SLAC Summer Institute on Particle Physics, Stanford, California, 1990, edited by J. Hawthorne (SLAC Report No. 378, Stanford, 1991).

[15] B. Grinstein, M. J. Savage, and M. B. Wise, Nucl. Phys. **B319**, 271 (1989).

[16] A. Ali, T. Mannel, and T. Morozumi, Phys. Lett. B **273**, 505 (1991).

[17] C. S. Lim, T. Morozumi, and A. I. Sanda, Phys. Lett. B **218**, 343 (1989); P. J. O'Donnell and H. K. Tung, Phys. Rev. D **43**, R2067 (1991); N. Paver and Riazuddin, *ibid.* **45**, 978 (1992).

[18] N. Isgur and M. B. Wise, Phys. Rev. D **42**, 2388 (1990).

[19] CDF Collaboration, Fermilab Report No. Conf-94/145 (unpublished); CLEO Collaboration, R. Balest *et al.*, CLEO Report No. Conf 94-4 (unpublished); UA1 Collaboration, C. Albajar *et al.*, Phys. Lett. B **262**, 163 (1991).

[20] A. Kamal, J. Phys. G **12**, L43 (1986).

[21] B. Blok and M. Shifman, Nucl. Phys. **B389**, 534 (1993); I. Bigi *et al.*, in *B Decays*, 2nd ed., edited by S. Stone (World Scientific, Singapore, 1994).

[22] A. Khodzhamirian and R. Ruckl, Max Planck Inst. Report No. MPI-PHT-94-26 (unpublished).

[23] J. Shigemitsu, talk presented at the International Conference on High Energy Physics, Glasgow, Scotland, 1994 (unpublished).

[24] S. Narison, Phys. Lett. B **322**, 247 (1994); S. Narison and A. Pivovarov, *ibid.* **327**, 341 (1994).