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Null gravitational redshift experiment with nonidentical atomic clocks

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A test of the local position invariance LPI principle embodied in the Einstein equivalence principle (EEP) has been performed via a "null" gravitational redshift experiment. The rate of a magnesium frequency standard has been compared with that of a cesium reference clock searching for a dependence on the solar gravitational potential during a period of 430 days. Because of the Earth's orbital motion during the experiment, the solar potential in the laboratory had a peak-to-peak variation of 6.7×10^{-10}, allowing us to set an upper limit on the relative frequency variation of 7×10^{-4} of the external potential. This result represents an improvement of more than one order of magnitude with respect to previous analogous tests of the LPI principle and leads also to a more rigorous limit on a possible spatial variation of the fine-structure constant.

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I. INTRODUCTION

The gravitational redshift of spectral lines is a direct consequence of the Einstein equivalence principle EEP on which the theory of general relativity is based [1]. In addition to its basic importance in several scientific fields such as theoretical physics and astrophysics, it has a direct impact on time and frequency metrology. In fact, the high accuracy level achieved by the atomic frequency standards ($\sim 3 \times 10^{-14}$) requires us to take into account the gravitational redshift even when comparing two clocks located in different places on the Earth or close to its surface, because the frequency of a clock is an increasing function of its altitude and the fractional change is of the order of 1×10^{-13}/km [2]. Moreover, the development under way of frequency standards based on ultracold atoms trapped in optical molasses is expected to improve their accuracy by up to two orders of magnitude [3], requiring an evaluation of the redshift and of any other relativistic shift with a higher degree of precision.

Several tests of the EEP have been reported in the literature (see, for example, [4] and references therein); among them a basic test of the gravitational redshift has been performed by Vessot *et al.* [5]. This experiment was based on the measurement of the frequency shift of a H maser on a spacecraft launched upward to 10 000 km compared with a similar maser on Earth. The agreement between the observed frequency shift and the predicted value turned out to be 7×10^{-5}.

On the other hand, the violation of the EEP was excluded at the 10^{-2} level by Turneaure *et al.* [6], by a "null gravitational redshift experiment," in which the rates of two "nonidentical" clocks at the same location were compared as they moved together in a gravitational field. The results of this last experiment, based on the comparison between the frequencies of H masers and of superconducting cavity stabilized oscillators (SCSO's), can be viewed as a test of the local position invariance (LPI) principle, embodied in the EEP, which states that in local freely falling frames the outcome of any nongravitational test experiment is independent of when and where in the Universe it is performed, and also as an experimental limit on a possible dependence on the gravitational potential of the product of fundamental constants $\alpha^3 g_p m_e/m_p$, where α is the fine-structure constant, g_p the proton gyromagnetic ratio, and m_e/m_p the electron-to-proton mass ratio.

Recently, an atomic frequency standard based on the fine-structure transition ${}^3P_1$-${}^3P_0$ $\Delta m_j = 0$ in the metastable triplet of ^{24}Mg has been developed [7] with a frequency stability and repeatability good enough ($\sim 3 \times 10^{-13}$) to allow a new null gravitational redshift experiment with an improved result with respect to [6] of more than one order of magnitude. The two "nonidentical" clocks used in this experiment, the Cs and Mg atomic frequency standards, differ for the technical implementation and for the kind of transition used as "clock transition" (hyperfine structure and fine structure, respectively). The experiment here described has been performed interpreting the frequency ratio of Cs and Mg clocks as a function of the solar gravitational potential during a period of 430 days (December 1991 to February 1993). In Sec. II we report the theoretical basis of this experiment and in Sec. III we analyze and discuss the experimental results.

II. THEORETICAL DESCRIPTION OF THE EXPERIMENT

In low gravitational fields ($\phi/c^2 \ll 1$) a consequence of the EEP is a redshift effect, which is universal and independent of the nature of the clock, given by

$$\nu = \nu_0(1 + \phi/c^2) , \tag{1}$$

where ν_0 is the clock frequency when $\phi = 0$ and ν the frequency of the clock redshifted by the gravitational potential ϕ. On the other hand, if the EEP were violated, and in particular the LPI principle, (1) could be written [8]

$$\nu = \nu_0[1 + (1 + \beta)\phi/c^2] , \tag{2}$$

β being a dimensionless parameter that measures the extent of LPI violation of the clock considered. A test of (2) may be performed with identical clocks, A and B, if they experience different values of ϕ, ϕ_A, and ϕ_B, respectively; in this case the relative frequency difference should be

$$(\nu_B - \nu_A)/\nu_0 = (1 + \beta)(\phi_B - \phi_A)/c^2 . \tag{3}$$

In Table I we report the principal tests of the EEP based on (3), performed by comparing two identical atomic clocks at different gravity potential values. It is obvious from (3) that if $\phi_A = \phi_B$ nothing can be deduced with identical clocks about the possible violation of the EEP. The use of identical clocks requires that they are submitted to different gravitational potential values in order to observe a possible EEP violation, if any, while the availability of nonidentical (atomic) clocks ($\beta_A \neq \beta_B$) allows us to perform the test with both clocks in the same laboratory, strongly simplifying the experiment and the data evaluation.

In the case of two nonidentical clocks A and B, subjected to the same gravitational potential $\phi_A = \phi_B = \phi$, a possible violation of LPI leads to the following expression for their frequency ratio, under the conditions $\phi/c^2 \ll 1$ and $\beta_{A,B} \ll 1$:

$$\nu_A/\nu_B = (\nu_{A_0}/\nu_{B_0})[1 + (\beta_A - \beta_B)\phi/c^2] , \tag{4}$$

where ν_{A_0} and ν_{B_0} are the gravity-free clock frequencies. Equation (4) is the basis for a "null" gravitational redshift experiment; in fact, a dependence of ν_A/ν_B versus ϕ would imply $\beta_A \neq \beta_B$, that is, a redshift not independent of the nature of the clock, a feature in contrast with the "universality" postulates embodied in the EEP.

The test described in this paper is based on (4) and involves two nonidentical quantum frequency standards, the Cs atomic clock [13] and the Mg frequency standard recently developed [7], observed in the time-dependent gravitational potential provided by the eccentric orbit of the Earth around the Sun. The frequency of the Cs atomic clock ($\nu_0 \approx 9.2$ GHz) corresponds to the $F = 3$, $m_F = 0 \to F = 4$, $m_F = 0$ hyperfine transition in the ^{133}Cs ground state $6\,^2S_{1/2}$ and is adopted for the definition of the second in the International System of units, while the frequency of the Mg clock ($\nu_0 \approx 601$ GHz) corresponds to the $^3P_1$-$^3P_0$ $\Delta m_j = 0$ fine-structure transition in the metastable triplet of ^{24}Mg. The solar gravity potential $\phi_S(t)$ at the laboratory is given with sufficient approximation by [14]

$$\frac{\phi_S(t)}{c^2} = -\frac{3GM_S}{2ac^2} - \frac{2GM_S}{ac^2} e \cos\varphi(t) - A \cos\Omega_D t , \tag{5}$$

where G is the Newtonian constant of gravitation, M_S is the solar mass, a is the semimajor axis of the Earth orbit around the Sun, e is the eccentricity, and $\varphi(t)$ the true anomaly. The first term of (5) is the average value of the solar potential at the geoid surface, the second term takes into account the periodical (1 yr) variation of ϕ_S due to the eccentric orbit of the Earth around the Sun, and the third term is the diurnal component where $2\pi/\Omega_D$ is the period of 1 day and $A \approx 2.7 \times 10^{-13}$ at the laboratory latitude. The gravity potential range spanned by the second term is $\Delta\phi_S/c^2 \approx 6.7 \times 10^{-10}$; such a variation is considered in this paper and allows a more sensitive test of (4) than the third term $\Delta\phi_S/c^2 \approx 5.7 \times 10^{-13}$, mainly considered in the experiment of Turneaure *et al.* [6].

The comparison of the two nonidentical atomic clocks, moreover, provides information about the independence of products of fundamental nongravitational constants with respect to the potential ϕ, as expected from LPI [15]. In our case the following relation holds for the ratio of Cs and Mg frequencies [16]:

$$\nu_{\mathrm{Cs}}/\nu_{\mathrm{Mg}} \propto g_I m_e/m_p , \tag{6}$$

where g_I is the Cs nucleus gyromagnetic ratio and the logarithmic derivatives of (6) with respect to the time and to the gravity potential, namely,

$$\frac{d}{dt}\ln\left[\frac{\nu_{\mathrm{Cs}}}{\nu_{\mathrm{Mg}}}\right] = \frac{d}{dt}\ln\left[g_I\frac{m_e}{m_p}\right], \tag{7a}$$

$$\frac{d}{d\phi}\ln\left[\frac{\nu_{\mathrm{Cs}}}{\nu_{\mathrm{Mg}}}\right] = \frac{d}{d\phi}\ln\left[g_I\frac{m_e}{m_p}\right] , \tag{7b}$$

allow a complementary test of LPI.

As far as (7a) is concerned, the following upper limit has been recently estimated [16]:

$$\left|\frac{d}{dt}\ln\left(g_I\frac{m_e}{m_p}\right)\right| \le 4.8 \times 10^{-13}\ \mathrm{yr}^{-1} . \tag{8}$$

The experimental results referring to (7b) will be reported and discussed in the next section and, along with other data reported in the literature, will lead to information also on the fine-structure constant α.

TABLE I. Main tests of the gravitational redshift performed with identical atomic clocks.

Clocks	β	Reference
Cs-Cs	$\lvert\beta_{\mathrm{Cs}}\rvert < 10^{-1}$	[9]
Cs-Cs	$\lvert\beta_{\mathrm{Cs}}\rvert < 1.5 \times 10^{-2}$	[10]
Cs-Cs	$\lvert\beta_{\mathrm{Cs}}\rvert < 2 \times 10^{-1}$	[11]
Cs-Cs	$\lvert\beta_{\mathrm{Cs}}\rvert < 6 \times 10^{-2}$	[12]
H-H	$\lvert\beta_{\mathrm{H}}\rvert < 7 \times 10^{-5}$	[5]

III. DATA ANALYSIS AND EXPERIMENTAL RESULTS

The Cs clock used in our experiment is a commercial apparatus with a long term fractional stability of

1×10^{-13} in one year, whose frequency is continuously steered to maintain an agreement within 3×10^{-13} with the international measurement reference. Moreover, it is daily compared, via satellite time comparisons, with respect to the laboratory primary Cs standard of Physikalisch-Tecnische Bundesanstalt (PTB), Germany, whose uncertainty is a few units in 10^{-14}. The Mg clock is a prototype developed in our laboratories: it is based on a thermal source of Mg atoms excited with high efficiency to the metastable triplet ^3P by an electric discharge. The $^3P_1$-$^3P_0$ magnetic-dipole transition is induced with a Ramsey-type optical cavity across the atomic beam and is observed through the fluorescence light due to the $^3P_1$ decay to the ground state (intercombination line); an all-solid-state frequency synthesizer provides the radiation at 601 GHz to the interaction Ramsey cavity and is driven by a 5 MHz quartz crystal oscillator which is frequency locked to the atomic transition via the fluorescence signal. The absolute value of the Mg clock transition is known with a relative uncertainty of $\pm 1 \times 10^{-12}$ ($\nu = 601\,277\,157\,869.1 \pm 0.6$ Hz) and the stability of its output frequency reaches the value 2×10^{-13} for measurement times $\tau = 3000$ s. A complete description of this frequency standard is reported in [7] as well as a detailed error budget which takes into account the main physical effects affecting the Mg output frequency.

The measures of the Mg frequency offset with respect to the Cs reference are reported in Fig. 1 and have been obtained in two steps. First, the mean value and the standard deviation of the frequency measures performed in a day versus the local Cs reference are evaluated. Then, all the central values are corrected for the second order Zeeman and Doppler shifts, taking into account the effective operating parameters which are known to affect the Mg output frequency. Taking into account the possible sources of errors, the final uncertainty of each measurement value amounts to 0.18 Hz (3×10^{-13} in relative value).

In Fig. 1 we have also reported the variations of the solar gravity potential ϕ_S/c^2, curve a, for the same period during which the frequency comparisons have been performed, as obtained from (5) and from the Earth-Sun daily distance data [17].

Looking for a possible dependence of the Mg-to-Cs frequency ratio with respect to the gravitational potential, we fitted our experimental data with a cosine function (curve b of Fig. 1) having the same period and phase of ϕ_S/c^2, obtaining the following best fit function:

$$\delta_{m_{\rm fit}} = (9.92 \pm 0.06) + (0.067 \pm 0.070)\cos\frac{2\pi}{365}t \ , \qquad (9)$$

where t is expressed in days and $t = 0$ corresponds to the modified Julian date MJD = 48 621. The amplitude of the cosine term, normalized to the Mg frequency, is $(1.1 \pm 1.2) \times 10^{-13}$. Therefore the possible relative variation of $\nu_{\rm Cs}/\nu_{\rm Mg}$ due to ϕ_S/c^2 is limited by the value 2.3×10^{-13}, where the limit has to be intended as |central value| + 1σ. This limit, together with (4) that may be rewritten as

$$|\beta_A - \beta_B| = \left|\frac{d}{d(\phi/c^2)}\ln\frac{\nu_A}{\nu_B}\right|, \qquad (10)$$

yields $|\beta_{\rm Cs} - \beta_{\rm Mg}| \leq 7 \times 10^{-4}$, which represents an improvement of almost a factor of 20 with respect to the analogous experiment by Turneaure *et al.* [6]

Our limit is a conservative estimate for the test of the LPI principle, because the cosinusoidal term of (9), whose amplitude has a mean value lower than its standard deviation, could well be due to a residual temperature effect (Fig. 2, curve b) but, unfortunately, due to the limited number of frequency measurements, it was not possible to determine and hence to remove the effect of the tem-

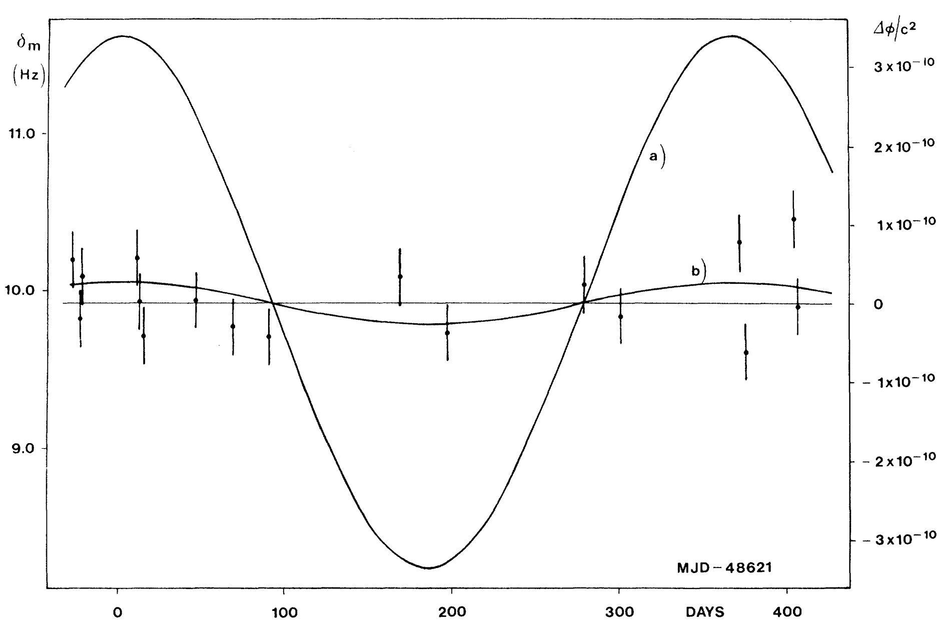

FIG. 1. Mg offset frequency (δ_m) with respect to Cs reference versus the modified Julian date (MJD); $\delta_m = 0$ corresponds to $\nu = 601\,277\,157\,859.2 \pm 0.6$ Hz. (measured values.) Curve a: solar gravitational potential variation $\Delta\phi/c^2$ at the geoid surface. Curve b: maximum amplitude (central value + 1σ) of the cosinusoidal function fitting the experimental data.

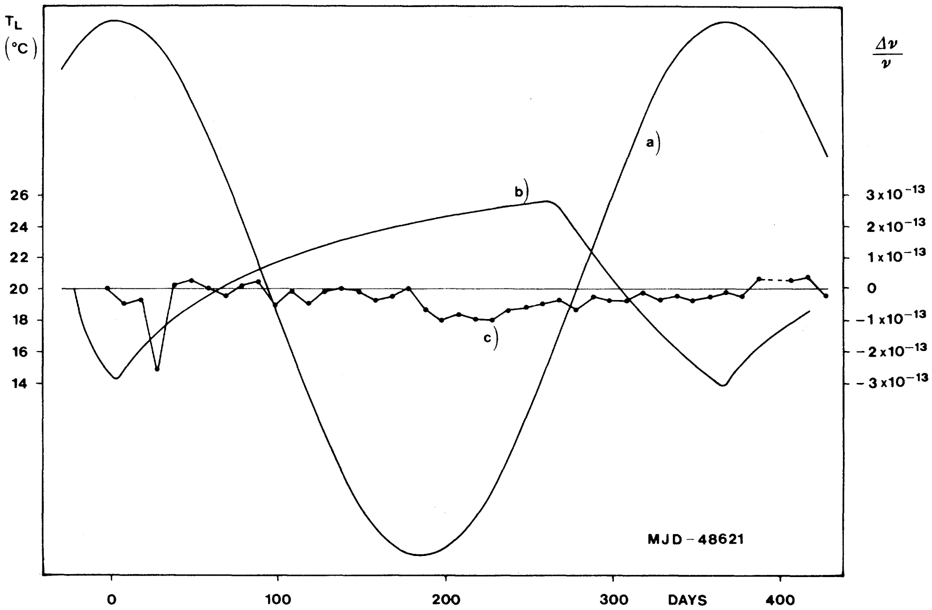

FIG. 2. Curve a is the same as Fig. 1; curve b reports the mean temperature T_L of the laboratory during the period of the experiment; curve c shows the fractional frequency departure of our Cs clock versus the PTB primary standard.

perature on a sound statistical basis. Considering the fact that before every Mg frequency measurement all the parameters were readjusted for the optimum operation, a possible dependence of the output frequency on the temperature can be explained mostly by the thermal changes of the Ramsey cavity acting through the residual first order Doppler effect, which gives the main contribution to the Mg error budget, as discussed in [7]. In the same Fig. 2 we have also reported, curve c, the frequency comparison between our Cs reference clock and the PTB primary standard, confirming the long term stability of the local frequency reference up to the considered level of 10^{-13}.

As far as concerns the fundamental physical constants involved in our experiment, combining the above results with (7b), we have

$$\left|\frac{d}{d(\phi/c^2)}\ln\left(g_1\frac{m_e}{m_p}\right)\right| \leq 7\times10^{-4} . \tag{11}$$

In Table II we reported our results together with analogous results reported in the literature; the data referring to the comparison between Cs and H masers have been deduced from recently published results [18].

Combining all the results of Table II, the following limit on the spatial variation of the logarithm of the fine-structure constant is obtained:

$$\left|\frac{d}{d(\phi/c^2)}\ln\alpha\right| \leq 7\times10^{-3} . \tag{12}$$

This value has been previously reported by Turneaure *et al.* [6], but without an experimental proof of their hypothesis about the time and space stability of $g_p m_e/m_p$.

IV. CONCLUSIONS

The results reported in this paper are consistent with the prediction of the metric theories of gravitation and give the following upper limit of the possible violation of the LPI principle:

$$|\beta_{\mathrm{Mg}} - \beta_{\mathrm{Cs}}| \leq 7\times10^{-4}.$$

Even if the data relative to the Cs-H comparison lead to a limit tighter than ours, the two experiments involve different products of fundamental constants and all together allow one to set a limit to the variation of the fine-structure constant with the gravitational potential.

As a result of possible improvements of the Mg stan-

TABLE II. Null gravitational redshift experiments performed with nonidentical clocks to test the LPI principle. (The limits have to be intended as |central value| $+1\sigma$.)

Clocks	$\gamma \propto \nu_A/\nu_B$	$\max\lvert\beta_A-\beta_B\rvert = \max\left\lvert\frac{d}{d(\phi/c^2)}\ln\gamma\right\rvert$	Reference
H-SCSO	$g_p\frac{m_e}{m_p}\alpha^3$	1.7×10^{-2}	[6]
Cs-H	g_I/g_p	1×10^{-4}	[18]
Cs-Mg	$g_I\frac{m_e}{m_p}$	7×10^{-4}	This work

dard and of developments of new frequency standards based on electronic transitions (Ca, Ca^+, ...) [19,20], tighter tests of the LPI principle will be possible in the near future, based only on the validity of quantum mechanical theory.

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