Production of polarized τ pairs and tests of CP violation using polarized $e^{\pm}$ colliders near threshold

Yung Su Tsai
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(Received 11 October 1994)

We consider the production of τ pairs by electron-positron colliding beams at the maximum cross section near the threshold. At this energy τ pairs are produced mostly in the s wave which implies that the spin of the τ pairs is almost always pointing in the beam direction independent of the production angle. When both electrons and positrons are longitudinally polarized in the same direction, for example, 90%, one can obtain τ pairs with 99% polarization in the direction of the polarization vectors of the incident beams. Tests of CP violation and study of the structure of weak interactions using such polarized τ pairs are discussed.

PACS number(s): 14.60.Fg, 11.30.Er, 13.10.+q, 13.88.+e

I. INTRODUCTION

CP violation in the standard model is highly *ad hoc* in the sense that it was invented to explain the decay of K_L and it arbitrarily assumes that there is no CP violation in the leptonic and the first generation quark vertices. In this paper we propose to test whether there is CP violation in the τ decay using the proposed τ-charm factory with longitudinally polarized electron and positron beams.

The τ-charm factory [1] is a proposed electron-positron colliding beam machine operating at around 4 GeV in the center of mass where τ and charm particles have maximum cross sections. When a pair of spin-$\frac{1}{2}$ particles are produced near threshold they are produced mostly in the s wave, resulting in polarizations of $\tau^{\pm}$ both pointing in the same direction [2] along either e^+ or e^- depending upon the initial polarization of the incident beams. This is true almost independent of the production angle. We show that at $E = 2.087$ GeV for the incident electron in the colliding beam the cross section is maximum, and the s-wave production is still dominant. For example, if e^+ and e^- are both polarized 90% in the direction of e^- momentum, the τ pair will be 99% polarized in the direction of e^- momentum.

In Sec. II we compute the cross section and the polarization of τ^- and τ^+ using longitudinally polarized e^- and e^+ beams. The cross sections and polarization of τ^- and τ^+ from the τ-charm factory are compared with those obtainable from the B factory.

In Sec. III we discuss how these polarized $\tau^{\pm}$ can be used to test CP violation, CPT violation, and conserved vector current theorem in $\tau^{\pm}$ decays. We constructed a very generic model of CP violation to investigate many salient features of possible CP violation in the semileptonic decay of τ into 2π.

In Sec. IV we generalize the observations made in the previous section and devise a model-independent way to find CP violation in any CP-violating decay mode. We also conclude that assuming equal luminosities and initial $e^{\pm}$ polarizations, the τ-charm factory is a factor 7.7 better than the B factory for checking CP violation in τ.

II. PRODUCTION OF POLARIZED $\tau^{\pm}$ BY POLARIZED $e^{\pm}$ COLLIDING BEAMS

In our problem the mass of the electron can be ignored; the error caused by this approximation can be shown to be $O(m_e^2/E^2)$ by an explicit calculation, which is 10^{-7} in our problem. When the mass is ignored $(1-\gamma_5)/2$ becomes a left- (right-)handed helicity projection operator for an electron (positron), whereas $(1+\gamma_5)/2$ becomes a right- (left-)handed helicity projection operator for an electron (positron). γ_5 commutes with 1, γ_5, and $\sigma_{\mu\nu}$, but anticommutes with γ_μ and $\gamma_\mu\gamma_5$; thus, in the electron positron annihilation the helicity of e^+ and e^- must be opposite to each other in order to annihilate if the current consists of vector and axial vector. The opposite holds for a scalar, pseudoscalar, or tensor. The standard electroweak interaction has only vector and axial vector interactions if we ignore the contribution from neutral Higgs boson exchange and $g-2$ of the electron. The neutral Higgs boson exchange is ignored because its contribution to the cross section is expected to be small for two reasons: (1) the Higgs boson's coupling to the electron is expected to be small and (2) the spin-0 particle exchange (Higgs boson) and spin-1 particle exchange (γ) do not interfere in the limit of zero electron mass.

The anomalous magnetic moment term is negligible at high energy because its contribution to the cross section is

$$O\left(\left(\frac{m_e}{2E}\frac{\alpha}{\pi}\ln\frac{2E}{m_e}\right)^2\right) \sim 10^{-15}$$

of the γ_μ terms. We shall also ignore the possible existence of the electric dipole moment of τ because many people [3] have worked on this problem already. Thus we assume CP conservation in the production of τ pairs and

 ©1995 The American Physical Society

we deal only with possible CP violation in τ decay. In this paper we shall also ignore the Z_0 exchange diagram that contributes 10^{-3} to the polarization. This does not affect the accuracy of our experiment because we cannot measure the polarization of electrons and positrons to this accuracy anyway.

Let H_1 and H_2 be the helicities of e^- and e^+, respectively. Let us write the cross section for $e^+e^- \to \tau^+\tau^-$ as $\sigma(H_1, H_2)$. The argument given above shows that with an accuracy of 10^{-7} we have $\sigma(+,+) = 0$ and $\sigma(-,-) = 0$ and only $\sigma(+,-)$ and $\sigma(-,+)$ are not zero.

Suppose there are N_{1+} electrons with helicity $H_1 = +$, N_{1-} electrons with helicity $H_1 = -$, N_{2+} positrons with helicity $H_2 = +$, and N_{2-} positrons with helicity $H_2 = -$. The total number of events is proportional to

$$N_{1+}N_{2-}\sigma(+,-) + N_{1-}N_{2+}\sigma(-,+) \ . \tag{2.1}$$

The longitudinal polarizations (not helicities) of electrons and positrons are, by definition

$$w_1 = \frac{N_{1+} - N_{1-}}{N_1} \quad \text{where } N_1 = N_{1+} + N_{1-} \ ,$$

$$w_2 = -\frac{N_{2+} - N_{2-}}{N_2} \quad \text{where } N_2 = N_{2+} + N_{2-} \ .$$

From these four equations we have

$$\begin{aligned} \frac{N_{1+}}{N_1} &= \frac{1+w_1}{2}, \quad \frac{N_{1-}}{N_1} = \frac{1-w_1}{2} \ , \\ \frac{N_{2+}}{N_2} &= \frac{1-w_2}{2}, \quad \frac{N_{2-}}{N_2} = \frac{1+w_2}{2} \ . \end{aligned} \tag{2.2}$$

Substituting Eq. (2.2) into (2.1) we obtain

$$\frac{N_1N_2}{4}[(1+w_1w_2)\{\sigma(+,-)+\sigma(-,+)\} + (w_1+w_2)\{\sigma(+,-)-\sigma(-,+)\}] \ . \tag{2.3}$$

From Eq. (2.3), we observed the following.

(1) When both electrons and positrons are unpolarized, the cross section is, by definition,

$$\tfrac{1}{4}\{\sigma(+,-)+\sigma(-,+)\} \ . \tag{2.4}$$

When only the electron beam is polarized, the cross section is

$$\frac{1}{4}\{\sigma(+,-)+\sigma(-,+)\} + \frac{w_1}{4}\{\sigma(+,-)-\sigma(-,+)\} \ . \tag{2.5}$$

When both the electron and positron beams are polarized, the cross section is

$$\frac{1+w_1w_2}{4}\{\sigma(+,-)+\sigma(-,+)\} + \frac{w_1+w_2}{4}\{\sigma(+,-)-\sigma(-,+)\} \ . \tag{2.6}$$

(2) Comparison of Eqs. (2.4), (2.5), and (2.6) shows that no new physics is obtained by polarizing both beams. However when both beams are polarized and when the polarization of e^+ is in the same direction as that of e^-, the total number of counts is increased by a factor $(1+w_1w_2)$ and the effective polarization is increased from w_1 to $(w_1+w_2)/(1+w_1w_2)$. We shall often assume that only the electron is polarized in order to simplify the calculation and discussion. When both $e^\pm$ are polarized all we need to do is to multiply the whole expression by a factor $(1+w_1w_2)$ and change w_1 to $(w_1+w_2)/(1+w_1w_2)$.

(3) The w_1 and w_2 dependence of the cross section given here is applicable also to $e^+e^- \to Z_0 \to \tau^+ + \tau^-$.

(4) If we let $w_1 = w_2 = 0.9$ we obtain $(w_1+w_2)/(1+w_1w_2) = 0.994$.

In this paper we shall not assume the existence of the electric dipole moment of τ; thus, T is conserved in the production. When T is not violated, the polarization of $\tau^\pm$ cannot have components perpendicular to the production plane, i.e., terms proportional to $(\vec{p}_1 \times \vec{p}_-)\cdot\vec{w}$ must be zero, where $\vec{p}_1$ and $\vec{p}_-$ are the momenta of e^- and τ^-, respectively, and $\vec{w}$ is the polarization vector of τ^-, because $\vec{p}_1$, $\vec{p}_-$, and $\vec{w}$ all change signs under T. We ignore the complex phase associated with the final state interaction to allow the existence of such a T-violating term. Under CP transformation the polarization of τ^- turns into polarization of τ^+ denoted by $\vec{w}'$, $w_1 \to w_2$, $\vec{p}_- \to -\vec{p}_+$, and $\vec{p}_1 \to -\vec{p}_2$. Since Eq. (2.6) shows that the cross section is invariant under the exchange $w_1 \leftrightarrow w_2$ and the experimental setup is invariant under interchanges $\vec{p}_- \leftrightarrow -\vec{p}_+$ and $\vec{p}_1 \leftrightarrow -\vec{p}_2$ in the center-of-mass system, we have

$$\vec{w} = \vec{w}' \tag{2.7}$$

in the center-of-mass system. For experiment in the non-center-of-mass system, such as the asymmetric B factory, the experimental data can easily be converted to those in the center-of-mass system by computer software, so there should not be any difficulty. The higher order electromagnetic corrections do not affect the argument given here. This statement is true even when Z_0 is exchanged.

In this paper we use the convention of Ref. [2] (see Sec. IV of that paper). We use the three-dimensional vectors $\vec{s}$ and $\vec{w}$ in the rest frame of τ^- to represent its spin and polarization vectors, respectively. $\vec{s}$ is a unit vector whereas $\vec{w}$ is defined as

$$w_i = \frac{\begin{array}{l}\text{No. of } \tau^- \text{ with } \vec{s} = \hat{e}_i \\ -\text{No. of } \tau^- \text{ with } \vec{s} = -\hat{e}_i\end{array}}{\begin{array}{l}\text{No. of } \tau^- \text{ with } \vec{s} = \hat{e}_i \\ +\text{No. of } \tau^- \text{ with } \vec{s} = -\hat{e}_i\end{array}} \ . \tag{2.8}$$

$(s_-)_\mu$ is the four-vector which becomes $(0, \vec{s})$ in the rest frame of τ^-. We define similar vectors $\vec{s}'$, $(s_+)_\mu$ and $\vec{w}'$ for τ^+. The cross section for producing τ^- with spin $\vec{s}$ and τ^+ with spin $\vec{s}'$ with initial polarization w_1 for e^- and w_2 for e^+ can be written as

$$\frac{d\sigma}{d\Omega}(w_1,w_2,\vec{s},\vec{s}\,') = \frac{e^4}{(2\pi)^2}\frac{1}{4(p_1\cdot p_2)}\int\frac{d^3p_+}{2E}\int\frac{d^3p_-}{2E}\delta^4(p_1+p_2-p_--p_+)\tfrac{1}{4}\mathrm{Tr}(1+\gamma_5 w_1)\not{p}_1\gamma_\mu(1+\gamma_5 w_2)\not{p}_2\gamma_\nu$$
$$\times\tfrac{1}{4}\mathrm{Tr}(1+\gamma_5\not{s}_-)(\not{p}_-+M)\gamma_\nu(1+\gamma_5\not{s}_+)(\not{p}_+-M)\gamma_\mu/(4E^2)^2$$
$$=\frac{\alpha^2}{64E^2}\beta(1+w_1w_2)\left[\left\{1+\cos^2\theta+\frac{\sin^2\theta}{\gamma^2}\right\}+\left\{\left(1-\frac{1}{\gamma^2}\right)\sin^2\theta(s_-\cdot s_+)\right.\right.$$
$$\left.+\frac{1}{E^2}[2(p_1\cdot s_-)(p_1\cdot s_+)-(p_1\cdot s_-)(p_-\cdot s_+)(1+\beta x)-(p_1\cdot s_+)(p_+\cdot s_-)(1-\beta x)]\right\}$$
$$\left.+\frac{w_1+w_2}{1+w_1w_2}\frac{1}{\gamma E}\{2(p_1\cdot s_-)+2(p_1\cdot s_+)-(p_-\cdot s_+)-(p_+\cdot s_-)\}\right], \quad (2.9)$$

where $x = \cos\theta$, $\gamma = E/M$, and $\beta = (1-\gamma^{-2})^{0.5}$. We notice that $w_1, w_2, (p_i\cdot s_-)$ and $(p_i\cdot s_+)$ are pseudoscalars; therefore, these quantities have to occur an even number of times in our expression because we are dealing with parity-conserving electromagnetic interactions in the production. Parity conservation is violated when Z_0 exchange is included. At our energy the correction due to weak interaction is $O(4E^2/M_z^2) = 10^{-3}$. The first set of curly brackets in Eq. (2.9) represents the cross section when the final polarizations are not measured; the second represents the spin correlation and it was first discussed by the author [2] in 1971 and treated subsequently by many people, so we shall not discuss it here. The third set of curly brackets contains terms which produce polarization. Since we do not have to observe both polarizations at the same time we let $s_+ = 0$. We can obtain the polarization vector $\vec{w}$ for τ^- using Eqs. (2.8) and (2.9). For this calculation we shall use the coordinate system shown in Fig. 1. In this frame, for $\vec{s} = \hat{e}_{Z'}$, we have

$$s_- = (\beta\gamma, 0, 0, \gamma)\ . \quad (2.10)$$

For $\vec{s} = \hat{e}_{x'}$ we have

$$s_- = (0,1,0,0)\ , \quad (2.11)$$

$$p_1 = E(1,\sin\theta,0,\cos\theta)\ , \quad (2.12)$$

$$p_- = E(1,0,0,\beta)\ , \quad (2.13)$$

$$p_+ = E(1,0,0,-\beta)\ . \quad (2.14)$$

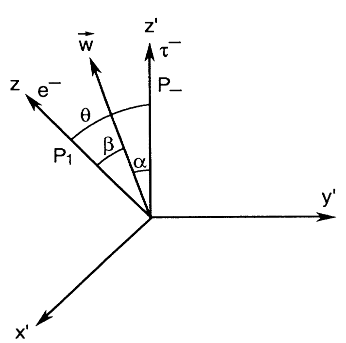

FIG. 1. Coordinate system used in calculating the polarization vector $\vec{w}$ for τ^-. $w_y = 0$ because of T invariance in the production of τ pairs.

The magnitude of the polarization can be obtained readily from Eqs. (2.8)–(2.14):

$$|\vec{w}| = (w_{x'}^2 + w_{Z'}^2)^{1/2} = \left|\frac{w_1+w_2}{1+w_1w_2}\right|\frac{2E\sqrt{p^2\cos^2\theta+M^2}}{E^2+M^2+p^2\cos^2\theta}, \quad (2.15)$$

where $p^2 = E^2 - M^2$. The component of $\vec{w}$ along the τ^- direction is

$$w_{Z'} \equiv |\vec{w}|\cos\alpha = |\vec{w}|\frac{E\cos\theta}{\sqrt{p^2\cos^2\theta+M^2}}\ . \quad (2.16)$$

The component $\vec{w}$ along the incident electron direction is

$$w_Z \equiv |\vec{w}|\cos\beta = |\vec{w}|\frac{E\cos^2\theta+M\sin^2\theta}{\sqrt{p^2\cos^2\theta+M^2}}\ . \quad (2.17)$$

Equation (2.15) shows that at $\theta = 0$ or 180°, the magnitude of the polarization is always maximum independent of energy:

$$|\vec{w}|_{\max} = \left|\frac{w_1+w_2}{1+w_1w_2}\right|\ . \quad (2.18)$$

In Fig. 2(a), the magnitudes of the $\tau^\pm$ polarization are plotted assuming $|\vec{w}|_{\max} = 1$ for the τ-charm factory energy $E = 2.087$ GeV and the B-factory energy $E = 6.0$ GeV. It is seen that at energy $E = 2.087$ GeV where the cross section is maximum, the polarization is almost complete but at the B-factory energy the polarization is less complete even if the incident electron is completely polarized.

In Fig. 2(b), the cosine of angle between τ^- and its direction of polarization is plotted for $E = 2.087$ and 6.0 GeV. $\vec{w}$ is almost parallel to the e^- direction if w_1 is positive for $E = 2.087$ GeV whereas for $E = 6.0$ GeV $\vec{w}$ is no longer so parallel to the initial electron polarization because the production is no longer dominated by the s wave.

In Fig. 3(a) we plot the cosine of the angle between the $\tau^\pm$ polarization vector and the incident electron assuming it to have positive helicity. At $\theta = 0°$, 90°, and 180°, $\tau^\pm$ polarization is always parallel to the electron polarization at all energies. cosβ is almost equal to 1 for $E = 2.087$ GeV but not quite so for the B-factory energy.

In Fig. 3(b) we plot components of $\tau^\pm$ polarization

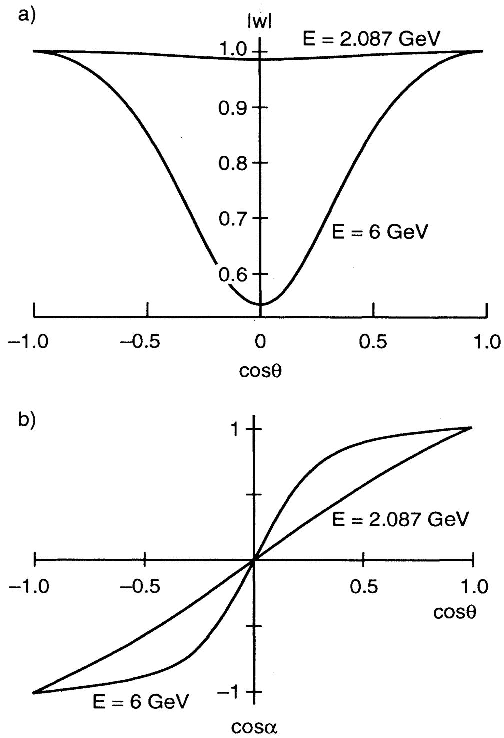

FIG. 2. (a) Magnitude of τ polarization $|\vec{w}|$ as a function of cosθ assuming completely polarized electron beam. (b) cosα versus cos θ, where α is the angle between $\vec{w}$ (polarization of τ) and $\vec{p}_-$.

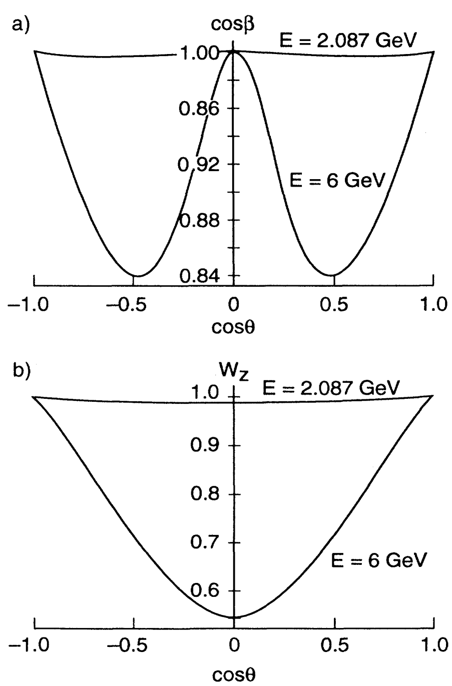

FIG. 3. (a) cosβ versus cosθ; (b) w_Z is the component of τ polarization vector along the electron beam direction.

along the electron direction assuming the electron to be completely right-handed polarized.

A. Total cross section and production rate

The first set of curly brackets in Eq. (2.9) gives the differential cross section. Integrating it with respect to the solid angle and summing over the final spins we obtain the total cross section

$$\sigma(e^+e^- \to \tau^+\tau^-)$$

$$= \frac{r_e^2\pi}{6}\left(\frac{m_e}{M}\right)^2 \beta(1-\beta^2)(3-\beta^2)(1+w_1w_2)\ . \quad (2.19)$$

The cross section has a maximum at $\beta = \sqrt{1.5-\sqrt{1.5}} = 0.5246$ or $E = 2.087$ GeV for $M = 1.777$ GeV. When $\beta = 0.5246$ we have $\beta(1-\beta^2)(3-\beta^2) = 1.036$. Let us therefore write $f(\beta) = (1/1.036)\beta(1-\beta^2)(3-\beta^2)$ and $\sigma(e^+e^- \to \tau^+\tau^-) = \sigma_{\rm max} f(\beta)(1+w_1w_2)$ where

$$\sigma_{\rm max} = \frac{r_e^2\pi}{6}\left(\frac{m_e}{M}\right)^2 1.036 = 3.562 \times 10^{-33}\ {\rm cm}^2\ .$$

Table I gives the numerical value of $f(\beta)$. We notice that at B-factory energies the cross section is 1/6 that of the maximum cross section at $E = 2.087$ GeV.

The factor $(1+w_1w_2)$ is the spin dependence of the total cross section. When either $w_1 = 0$ or $w_2 = 0$ this factor is one. When $w_1 = w_2 = \pm 1$ this factor is 2. When $w_1 = -w_2 = \pm 1$ this factor is zero. In the circular ring if one waits long enough, positrons (electrons) will be polarized parallel (antiparallel) to the magnetic field, reaching the value 0.924 if the guiding field is uniform. These transverse polarizations can be rotated 90° so that polarizations become longitudinal. In the ideal case we have $w_1 = w_2 = \pm 0.924$. In this case we have $(1+w_1w_2) = 1.85$. The time necessary to reach this maximum possible radiative beam polarization is too long with the existing design of the τ-charm factory. The time dependence of the polarization is [4,5] $p(t) = 0.924(1-e^{-t/T_{\rm pol}})$, where $T_{\rm pol}$ in sec is given by

$$T_{\rm pol}({\rm sec}) = \frac{98.7r^2R}{E^5}\ ,$$

where $E = 2.087$ GeV is the beam energy, $r = 12$ m is

TABLE I. Energy dependence of the cross section for $e^+e^- \to \tau^+\tau^-$.

β	E (GeV)	$f(\beta)$
0.1	1.786	0.2857
0.3	1.863	0.7668
0.4	1.939	0.9210
0.5	2.052	0.9953
0.5246	2.087	1.0000
0.55	2.128	0.9988
0.6	2.221	0.9785
0.9951	6.0	0.1688

the bending radius, and $R = 60$ m is the mean radius of the machine. $T_{\rm pol}$ is approximately 6 hours (which is too long). One can reduce this time by reducing r and R and also by inserting wigglers. Another way to obtain the polarized beam is to inject a polarized electron beam which reaches about 80% polarization at SLAC now but eventually may reach almost [6] 100%. Polarized positrons [7] can be obtained by pair production using high energy circularly polarized photons produced by back scattering of polarized laser beams on high energy electrons.

The design luminosity in 1989 was 10^{33} cm^{-2}/sec, but now it probably could [8] reach 3×10^{33} cm^{-2}/sec. Using 10^{33} we obtain a rate of 3.56 $(1 + w_1 w_2)$ τ pairs/sec. Thus we obtain $(1\text{–}6)\times10^8$ τ pairs/year. This means with several years of running one can obtain a sensitivity of 10^{-4} for testing CP violation in the τ decay. If CP violation in τ decay is of order 10^{-3}, similar to the neutral kaon decay, we should be able to investigate the structure of CP violation in τ decay using the τ-charm factory.

III. TESTS OF CP AND CPT VIOLATIONS IN τ DECAY

In quantum mechanics, the time reversal operator T is the least intuitive among T, C, and P operators, because under T i must become $-i$ in addition to changing t into $-t$. The requirement of i going into $-i$ can be seen by applying T to the most important commutators in quantum mechanics:

$$[x_i, p_j] = i\delta_{ij} \ . \tag{3.1}$$

$T[x_i, p_j]T^{-1} = -[x_i, p_j]$. Thus the commutation relation, Eq. (3.1), will not be true unless $TiT^{-1} = -i$.

In order to construct a T noninvariant model, we first construct a T invariant interaction with a real coupling constant and then make this real coupling constant complex with a nonvanishing imaginary part. This may not be the most general prescription for constructing a T-violating model, but it is good enough to bring out many features of T violation for our purpose. The complex phase in the Kobayashi-Maskawa (KM) matrix has a similar structure.

Let $A = |A|e^{i\delta_w}$ be such a coupling constant with $\delta_w \neq 0$ or π for τ^- decay. We have $TAT^{-1} = A^* \neq A$, thus T is violated in the theory. Testing the existence of δ_w in the τ decay is the purpose of this section. In quantum mechanics, the overall phase of the matrix element of any process is undetectable because the transition probability is square of the matrix element. Thus the complex coupling constant must be defined with respect to some other coupling constant whose phase is known. Only the interference between the two will produce a T-violating effect.

The weak Hamiltonian responsible for $\tau^\pm$ decay can be written in general as

$$H_w = \sum_i \left(\frac{j_i^+ J_i^-}{q^2 - M_i^2} + \frac{j_i^- J_i^+}{q^2 - M_i^2} \right) \ , \tag{3.2}$$

where j_i^+ represents a leptonic current whose final minus initial charge is positive, i.e., $\tau^- \to \nu_\tau$, i represents different particles exchanged such as left-handed W's, right-handed W's, charged Higgs bosons, etc. J_i^- is the hadronic or leptonic current whose final minus initial charge is negative. The first term in Eq. (3.2) gives the decay of τ^-, whereas the second term gives the decay of τ^+. One of the requirements of TCP theorem is that H_w be Hermitian, and thus the second term is the Hermitian conjugate of the first. Therefore if there is any complex coupling constant in the decay of τ^-, the corresponding coupling constant for the τ^+ decay must be the complex conjugate of the former.

Let $A = |A|e^{i\delta_w}$ be the complex coupling constants responsible for the T noninvariant decay of τ^-, then TCP invariance demands that the coupling constant $\bar{A}$ responsible for the T noninvariant τ^+ decay must be

$$\bar{A} \equiv |\bar{A}|e^{i\bar{\delta}_w} = |A|e^{-i\delta_w} \ , \tag{3.3}$$

which implies $|\bar{A}| = |A|$ and $\bar{\delta}_w = -\delta_w$. If either of these is violated, TCP is violated.

In the semileptonic decay mode of τ with more than one hadron in the final state, for example, $\tau^- \to \nu_\tau \pi^- \pi^0$, we have complex phase due to final state interactions given by the Breit-Wigner formula for the p-wave resonance (ρ). Because the strong interaction is invariant under charge conjugation this phase is not changed when going from $\tau^- \to \nu_\tau + \pi^- + \pi^0$ to $\tau^+ \to \bar{\nu}_\tau + \pi^+ + \pi^0$. Let the phase shift due to strong interaction be δ_s, we have then for τ^- decay the phase factor $e^{i(\delta_s + \delta_w)}$, but for τ^+ decay we have $e^{i(\delta_s - \delta_w)}$, if TCP is conserved but T is violated. The existence of the strong phase makes it possible to detect the existence of δ_w without violating CPT even from a seemingly T-invariant term such as $\vec{w}\cdot\vec{q}_1$, where $\vec{w}$ is the polarization of τ^- and $\vec{q}_1$ is the momentum of π^-.

In the previous section we showed that τ can be polarized almost 100% and its direction of polarization is almost along the beam direction independent of the production angle (see Figs. 2 and 3) at $E = 2.087$ GeV. We have also shown that the polarization vector for τ^- and τ^+ are parallel to each other and equal in magnitude as long as CP invariance holds in the production. This extra polarization vector $\vec{w}$ of τ^- enables us to construct rotationally invariant dot products such as $c_1\vec{w}\cdot\vec{q}_1$ or $c_2(\vec{w}\times\vec{q}_1)\cdot\vec{q}_2$ where $\vec{q}_1$ and $\vec{q}_2$ are the momenta of decay product of τ^- and similar quantities $c_1'\vec{w}'\cdot\vec{q}_1{}', c_2'(\vec{w}'\times\vec{q}_1{}')\cdot\vec{q}_2{}'$ where $\vec{w}'$ is the polarization vector of τ^+ and $\vec{q}_1{}'$ and $\vec{q}_2{}'$ are the momenta of the charge conjugates of $\vec{q}_1$ and $\vec{q}_2$, respectively.

Under CP we have $\vec{q}_1 \to -\vec{q}_1{}', \vec{q}_2 \to -\vec{q}_2{}'$, and $\vec{w} \to \vec{w}'$. Thus $\vec{w}\cdot\vec{q}_1 \to -\vec{w}'\cdot\vec{q}_1{}', \vec{w}\cdot\vec{q}_2 \to -\vec{w}'\cdot\vec{q}_2{}', (\vec{w}\times\vec{q}_1)\cdot\vec{q}_2 \to (\vec{w}'\times\vec{q}_1{}')\cdot\vec{q}_2{}', \vec{p}_1 \to -\vec{p}_2$, and $w_1 \to w_2$ under CP operation. Thus if CP holds we have

$$c_1 = -c_1' \text{ and } c_2 = c_2' \tag{3.4}$$

and violation of Eq. (3.4) is violation of CP invariance. $\vec{w}\cdot\vec{q}_1$ is T even and CP odd, thus $c_1 + c_1' \neq 0$ means not only CP violation but also CPT violation for any process which does not have a strong interaction phase such as pure leptonic decay model and any semileptonic decay with only one hadron, such as $\nu_\tau + \pi$ and $\nu_\tau + K$. In the leptonic decay of τ there is only one visible final state, thus one cannot construct the triple product $(\vec{w} \times \vec{q}_1) \cdot \vec{q}_2$. Only when the polarization of the final e or μ is measured, one can test the CP violation from the pure leptonic decay of τ unless CPT is violated. Similarly CP-violating effect in the decay $\tau \to \nu_\tau + \pi$ or $\tau \to \nu_\tau + K$ means CPT is also violated.

We conclude that only the semileptonic decay modes of τ with two or more hadronic final particles can exhibit CP violation without violating CPT at the same time. The best candidate is the decay mode $\tau^\pm \to \nu_\tau + \pi^\pm + \pi^0$. Let us investigate this mode in detail and learn several interesting lessons. The lessons learned can obviously be applied to other decay modes.

A. $\tau^- \to \nu_\tau + \pi^- + \pi^0$ and $\tau^+ \to \bar{\nu}_\tau + \pi^+ + \pi^0$

The energy angle distributions of these two decay modes from polarized τ's had been worked out in detail in Ref. [2] several years before the discovery of the τ. The investigation of possible CP violation using these two decays had been carried out by Nelson *et al.* [9] using spin correlation methods first proposed in Ref. [2]. Since in the τ-charm factory $\tau^\pm$ can be made highly polarized we do not need to use the spin correlation which requires the detection of twice the number of particles and thus is more complicated. We also note that in our method s- and p-wave interferences in the two π state is crucial in untangling the CP violation whereas the Nelson *et al.* paper does not seem to have any s wave.

The two π decay modes have two distinguished advantages. (1) They have the largest branching ratio (25%). (2) It has a two-body (detectable) hadronic final state which has a large phase shift (ρ resonance). This makes it possible to have a coefficient of $\vec{w} \cdot \vec{q}_1$ violating CP invariance without violating TCP invariance. It also enables one to construct a triple product term $(\vec{w} \times \vec{q}_1) \cdot \vec{q}_2$ to test CP invariance. Our investigation is exploratory. We want to known how different types of CP-violating terms in various Lagrangians manifest themselves as the CP-violating effect in the experiment.

We shall assume that the τ neutrino mass is either zero or so small that anything that is of order (m_ν/m_τ) is unobservable experimentally. With this assumption $1-\gamma_5$ and $1+\gamma_5$ are good helicity projection operators for the τ neutrino states and the matrix element containing $(1-\gamma_5)u(\nu_\tau)$ and that containing $(1+\gamma_5)u(\nu_\tau)$ do not interfere. As mentioned previously the complex coupling constant responsible for CP violation can manifest itself only through interference with other terms which have a real coupling constant. This consideration shows that one cannot obtain a CP nonconserving effect through interference of right-handed current with the left-handed current by assuming that the coupling constant of the former has a weak phase compared with the latter.

The consideration given above also shows that if we limit the weak interaction to be transmitted only by exchange of spin-1 and spin-0 particles, then we have only two possible choices of matrix elements denoted by M_1 and M_2 (see Fig. 4) that can interfere with the standard model matrix denoted by M_0:

$$M_0 = \bar{u}(p_2)(\not{q}_1 - \not{q}_2)(1-\gamma_5)u(p_1)L \,, \tag{3.5}$$

$$M_1 = \bar{u}(p_2)\{P(\not{q}_1 - \not{q}_2) + S(\not{q}_1 + \not{q}_2)\}(1-\gamma_5)u(p_1) \,, \tag{3.6}$$

$$M_2 = \bar{u}(p_2)(1+\gamma_5)u(p_1)H \,, \tag{3.7}$$

p_1, p_2, q_1, q_2 are momenta of τ^-, ν_τ, π^-, and π^0, respectively, and [2]

$$L = g_{\tau\rho\nu} g_{\rho\pi\pi} \frac{-1}{(q_1+q_2)^2 - M_\rho^2 + i\Gamma M_\rho} = g_{\tau\rho\nu} g_{\rho\pi\pi} \frac{e^{i\delta_{s1}}}{\sqrt{[(q_1+q_2)^2 - M_\rho^2]^2 + \Gamma^2 M_\rho^2}} \,. \tag{3.8}$$

δ_{s1} is the strong interaction phase shift for the $\pi^-\pi^0$ system in p wave (ρ resonance). Notice that the conserved vector current (CVC) theorem [2] in the standard model says that $\pi^-\pi^0$ cannot be in the S state.

M_1 is another left-handed current due to exchange of a higher mass spin-1 particle called X. For this current, both s and p waves are allowed for the $\pi^-\pi^0$ system because there is no CVC theorem here and we allow T-violating complex coupling constants in M_1. The vector particle X couples to all leptons and quarks, probably obeying some yet to be discovered symmetry principle. In our problem X is coupled to both $\tau\nu_\tau$ and the first generation quarks $\bar{u}d$. Thus we will be seeing the combined effect of CP violation in both the $\tau\nu_\tau$ and $\bar{u}d$ sectors. Let the complex weak phase for the $\tau\nu_\tau X$ vertex be

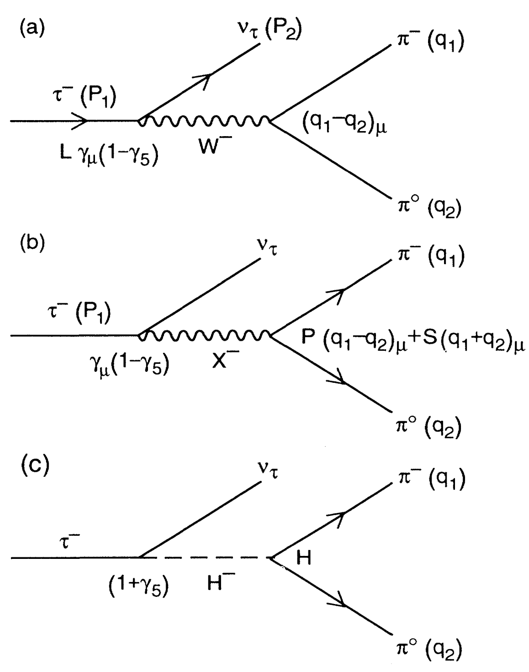

FIG. 4. Feynman diagrams for M_0, M_1, M_2 defined in Eqs. (3.5), (3.6), and (3.7); (a) M_0: W^- exchange; (b) M_1: X^- exchange; (c) M_2: H^- exchange.

$\exp(i\delta_{w\tau X})$ and that for the $\bar{u}dX$ vertex be $\exp(i\delta_{w1X})$. Then in our problem only the combination

$$\delta_{wX} \equiv \delta_{w\tau X} + \delta_{w1X} \tag{3.9}$$

will appear.

The term P in Eq. (3.6) contains the same strong interaction phase factor $\exp(i\delta_{s1})$ defined in Eq. (3.8) and thus in the interference between M_0 and M_1 given by $M_0^+ M_1 + M_1^+ M_0$ this strong interaction phase factor cancels out. Thus the term P does not contribute to the CP-violating effect, only the term S in Eq. (3.6) does. The s-wave part contains the $I = 2, J = 0$ $\pi^-\pi^0$ phase factor $e^{i\delta_{s0}}$ which is different from the p-wave one.

M_2 is the matrix element for charged Higgs boson exchange [10]. The part proportional to $(1-\gamma_5)$ in Eq. (3.7) does not interfere with M_0, so we left it out. It has s-wave interaction phase factor $\exp(i\delta_{s0})$ and the weak phase factor $\exp(i\delta_{wH})$, where

$$\delta_{wH} \equiv \delta_{w\tau H} + \delta_{w1H} , \tag{3.10}$$

where $\delta_{w\tau H}$ is the T-violating weak phase associated with $\tau\nu_\tau H$ vertex, while δ_{w1H} is the similar phase for the first generation quarks. In summary the phases associated with L, P, S, and H are

$$L = |L| \exp(i\delta_{s1}) , \tag{3.11}$$
$$P = |P| \exp(i\delta_{s1} + i\delta_{wX}) , \tag{3.12}$$
$$S = |S| \exp(i\delta_{s0} + i\delta_{wX}) , \tag{3.13}$$
$$H = |H| \exp(i\delta_{s0} + i\delta_{wH}) . \tag{3.14}$$

If CPT invariance holds we have for τ^+ decay

$$\bar{L} = |L| \exp(i\delta_{s1}) , \tag{3.15}$$
$$\bar{P} = |P| \exp(i\delta_{s1} - i\delta_{wX}) , \tag{3.16}$$
$$\bar{S} = |S| \exp(i\delta_{s0} - i\delta_{wX}) , \tag{3.17}$$
$$\bar{H} = |H| \exp(i\delta_{s0} - i\delta_{wH}) . \tag{3.18}$$

Since strong interaction is C invariant, the strong interaction phase shifts δ_{s1}, δ_{s0} are not changed when going from τ^- to τ^+ whereas the weak phases δ_{wX} and δ_{wH} change sign because of Hermiticity of the Lagrangian, which results in the TCP theorem. The decay energy-angle distribution of the decay, polarized $\tau^- \to \nu_\tau + \pi^- + \pi^0$ can be written as

$$\Gamma = \frac{1}{2M_\tau}\frac{1}{(2\pi)^5}\int\frac{d^3p_2}{2E_2}\int\frac{d^3q_1}{2w_1}\int\frac{d^3q_2}{2w_2}\delta^4(p_1 - p_2 - q_1 - q_2)|M_0 + M_1 + M_2|^2 . \tag{3.19}$$

We assume M_1 and M_2 to be much smaller than M_0, therefore we compute [11]

$$(M_0 + M_1 + M_2)^\dagger(M_0 + M_1 + M_2) \cong M_0^\dagger M_0 + (M_0^\dagger M_1 + M_1^\dagger M_0) + (M_0^\dagger M_2 + M_2^\dagger M_0) ,$$

$$\begin{aligned} M_0^\dagger M_0 &= 2|L|^2\frac{\mathrm{Tr}}{4}(1+\gamma_5 \not{w})(\not{p}_1 + M)(1+\gamma_5)(\not{q}_1 - \not{q}_2)\,\not{p}_2(\not{q}_1 - \not{q}_2)(1-\gamma_5) \\ &= 2|L|^2[4(w\cdot q_1)M\{(q_1\cdot q_2) - (p_1\cdot q_1) + (p_1\cdot q_2) - m_\pi^2\} + 4(w\cdot q_2)M\{(q_1\cdot q_2) + (p_1\cdot q_1) - (p_1\cdot q_2) - m_\pi^2\} \\ &\quad +4\{-(q_1\cdot q_2)(p_1\cdot q_1 + p_1\cdot q_2 + m_\pi^2) + (p_1\cdot q_1)^2 + (p_1\cdot q_2)^2 \\ &\quad -2(p_1\cdot q_1)(p_1\cdot q_2) + m_\pi^2(p_1\cdot q_1 + p_1\cdot q_2) - m_\pi^2 M^2\}] . \end{aligned} \tag{3.20}$$

This gives the energy-angle distribution of π^- and π^0 in the standard model which was treated in detail in Ref. [2].

$$\begin{aligned} M_0^\dagger M_1 + M_1^\dagger M_0 &= 2|L|e^{-i\delta_{s1}}\frac{\mathrm{Tr}}{4}(1+\gamma_5 \not{w})(\not{p}_1 + M)(1+\gamma_5)(\not{q}_1 - \not{q}_2)\,\not{p}_2\{P(\not{q}_1 - \not{q}_2) + S(\not{q}_1 + \not{q}_2)\}(1-\gamma_5) \\ &\quad +2|L|e^{i\delta_{s1}}\frac{\mathrm{Tr}}{4}(1+\gamma_5 \not{w})(\not{p}_1 + M)(1+\gamma_5)\{P^*(\not{q}_1 - \not{q}_2) + S^*(\not{q}_1 + \not{q}_2)\}\,\not{p}_2(\not{q}_1 - \not{q}_2)(1-\gamma_5) \\ &= 4|L|[4(w\cdot q_1)M\cos\delta_{wX}(q_1\cdot q_2 - p_1\cdot q_1 + p_1\cdot q_2 - m_\pi^2)|P| \\ &\quad +4(w\cdot q_1)M\cos(\delta_{s0} - \delta_{s1} + \delta_{wX})(q_1\cdot q_2 - p_1\cdot q_1 + m_\pi^2)|S| \\ &\quad +4(w\cdot q_2)M\cos\delta_{wX}(q_1\cdot q_2 + p_1\cdot q_1 - p_1\cdot q_2 - m_\pi^2)|P| \\ &\quad +4(w\cdot q_2)M\cos(\delta_{s0} - \delta_{s1} + \delta_{wX})(-q_1\cdot q_2 + p_1\cdot q_2 - m_\pi^2)|S| \\ &\quad +4(\vec{w}\times\vec{q}_1)\cdot\vec{q}_2 M^2\sin(\delta_{s0} - \delta_{s1} + \delta_{wX})|S| \\ &\quad +4\cos\delta_{wX}\{-(q_1\cdot q_2)(p_1\cdot q_1) - (q_1\cdot q_2)(p_1\cdot q_2) + (q_1\cdot q_2)M^2 \\ &\quad +(p_1\cdot q_1)^2 - 2(p_1\cdot q_1)(p_1\cdot q_2) + (p_1\cdot q_1)m_\pi^2 + (p_1\cdot q_2)^2 + (p_1\cdot q_2)m_\pi^2 - m_\pi^2 M^2\}|P| \\ &\quad +4\cos(\delta_{s0} - \delta_{s1} + \delta_{wX})\{-(q_1\cdot q_2)(p_1\cdot q_1) + (q_1\cdot q_2)(p_1\cdot q_2) \\ &\quad +(p_1\cdot q_1)^2 - (p_1\cdot q_1)m_\pi^2 - (p_1\cdot q_2)^2 + (p_1\cdot q_2)m_\pi^2\}|S|] , \end{aligned} \tag{3.21}$$

and

$$M_0^\dagger M_2 + M_2^\dagger M_0 = 2|L|e^{-i\delta_{s1}}\frac{\mathrm{Tr}}{4}(1+\gamma_5 \not{w})(\not{p}_1 + M)(1+\gamma_5)(\not{q}_1 - \not{q}_2)\not{p}_2(1+\gamma_5)H$$
$$+2|L|e^{i\delta_{s1}}\frac{\mathrm{Tr}}{4}(1+\gamma_5 \not{w})(\not{p}_1 + M)(1-\gamma_5)\not{p}_2(\not{q}_1 - \not{q}_2)(1-\gamma_5)H^*$$
$$= 4|H||L|[2(w\cdot q_1)\cos(\delta_{s0}-\delta_{s1}+\delta_{wH})(2(p_1\cdot q_2)-M^2)$$
$$+2(w\cdot q_2)\cos(\delta_{s0}-\delta_{s1}+\delta_{wH})(-2(p_1\cdot q_1)+M^2) + 4(\vec{w}\times\vec{q}_1)\cdot\vec{q}_2 M\,\sin(\delta_{s0}-\delta_{s1}+\delta_{wH})$$
$$+2M\cos(\delta_{s0}-\delta_{s1}+\delta_{wH})(p_1\cdot q_1 - p_1\cdot q_2)]\,. \quad (3.22)$$

B. Observations

(1) The decay energy angle distribution of $\tau^+ \to \bar{\nu}_\tau + \pi^+ + \pi^0$ can be obtained by reversing all momenta of the particles $p_1 \to -p_1'$, $p_2 \to -p_2'$, $q_1 \to -q_1'$, $q_2 \to -q_2'$ and reverse the signs of all weak phases $\delta_{wX} \to -\delta_{wX}, \delta_{wH} \to -\delta_{wH}$. When CP is conserved, i.e., $\delta_{wX} = \delta_{wH} = 0$, the coefficients of $w\cdot q_1$ and $w\cdot q_2$ change sign but the coefficients of $(\vec{w}\times\vec{q}_1)\cdot\vec{q}_2$ remain the same under CP operation in agreement with Eq. (3.4). If $\delta_{wX} \neq 0$ or $\delta_{wH} \neq 0$, then Eq. (3.4) is violated thus CP is violated.

(2) Only the interference between s wave in $M_{1,2}$ and p wave in M_0 contributes to the triple product term $(\vec{w}\times\vec{q}_1)\cdot\vec{q}_2$. Experimentally the existence of this term manifests itself as the asymmetry of π^0 distribution with respect to the plane formed by $\vec{w}$ and π^- momenta. CVC is an exact statement in the standard model; thus, the existence of the triple product term shows the existence of weak interaction mechanisms other than the standard model. CP is violated if the asymmetry in $\tau^- \to \nu_\tau + \pi^- + \pi^0$ is different from that for τ^+ decay.

(3) The P-wave part of M_1 does not contribute to the observable CP violation because $\cos\delta_{wX} = \cos(-\delta_{wX})$. From this example we can make a very interesting conclusion: Unless two diagrams have two different strong interaction phases we cannot observe the existence of weak phase using terms involving $w\cdot q_1$ or $w\cdot q_2$. This is because $w\cdot q_1$ and $w\cdot q_2$ are T even in the absence of strong interaction phase differences. Thus we cannot have CP violation without violating CPT using these terms.

(4) When the strong interaction phases in M_0 and M_1 are different the CP violation is proportional to

$$\cos(\delta_{s0}-\delta_{s1}+\delta_{wX}) - \cos(\delta_{s0}-\delta_{s1}-\delta_{wX}) = 2\,\sin(\delta_{s1}-\delta_{s0})\sin\delta_{wX} \quad (3.23)$$

for the coefficients of $w\cdot q_1$ and $w\cdot q_2$, but

$$\sin(\delta_{s0}-\delta_{s1}+\delta_{wX}) - \sin(\delta_{s0}-\delta_{s1}-\delta_{wX}) = 2\,\cos(\delta_{s1}-\delta_{s0})\sin\delta_{wX} \quad (3.24)$$

for the coefficients of $(\vec{w}\times\vec{q}_1)\cdot\vec{q}_2$. We notice that when $\delta_{s1}-\delta_{s0} = 0$, Eq. (3.23) is zero whereas Eq. (3.24) is maximum. The physical reason for the former is already explained in point (3) and the reason for the latter is that $(\vec{w}\times\vec{q}_1)\cdot\vec{q}_2$ is T odd. Thus CP violation in this term does not cause violation of CPT even in the absence of strong interactions.

(5) Exactly the same observation as point (4) can be made for Eq. (3.22).

(6) All observable effects in CP violation can only be produced by the interference between the p wave in M_0 and the s wave in M_1 and M_2 in our model. Our model is generic, so it must be true in general.

(7) Our treatment of the final state interaction is too simplistic. For example, it ignores the possibility that two pions can become four pions and vice versa. This kind of inelastic final state interaction could be different for τ^+ and τ^- causing τ^+ and τ^- to have different branching ratios if CP is violated. This is caused by the difference in energy-angle distributions between $\pi^-\pi^0$ from τ^- and $\pi^+\pi^0$ from τ^+ when CP is violated as can be seen from Eqs. (3.20)–(3.22). This correction is not easy to calculate. Fortunately the method proposed in Sec. IV for testing CP is not affected by this correction.

IV. DISCUSSIONS AND CONCLUDING REMARKS

Since τ^+ and τ^- are not observable directly we have to integrate the production angles and obtain energy-angle distribution of $\pi^-(q_1)$ and $\pi^0(q_1)$ for τ^- decay and $\pi^+(q_1')$ and $\pi^0(q_2')$ distributions for τ^+ decay. Since we are not doing spin correlation experiments, they do not have to come from the same event. We investigate here features of these energy-angle distributions which will exhibit the CP violation after integrating over $\tau^\pm$ momenta. To simplify the argument let us assume that only the incident electron is polarized. As mentioned in Sec. II, this does not change any physics. All we need to change is to increase the overall cross section by a factor $(1+w_1w_2)$ and replace the electron polarization w_1 by $(w_1+w_2)/(1+w_1w_2)$ when positron has a polarization w_2.

Let us choose the direction of polarization of e^- as well as its momentum as the z axis and $\pi^-(q_1)$ lies on the xz plane as shown in Fig. 5.

There are 6 rotationally invariant products involving $\vec{w}_1$:

$$\vec{w}_1\cdot\vec{q}_1 = w_1 q_{1z}\,, \quad (4.1)$$
$$\vec{w}_1\cdot\vec{q}_2 = w_1 q_{2z}\,, \quad (4.2)$$
$$\vec{w}_1\cdot\vec{q}_1{}' = w_1 q_{1z}'\,, \quad (4.3)$$
$$\vec{w}_1\cdot\vec{q}_2{}' = w_1 q_{2z}'\,, \quad (4.4)$$
$$(\vec{w}_1\times\vec{q}_1)\cdot\vec{q}_2 = w_1 q_{1x} q_{2y}\,, \quad (4.5)$$
$$(\vec{w}_1\times\vec{q}_1{}')\cdot\vec{q}_2{}' = w_1 q_{1x}' q_{2y}'\,. \quad (4.6)$$

Under CP we have

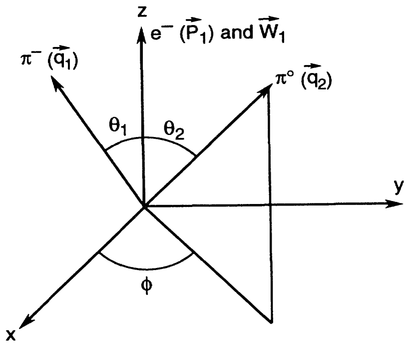

FIG. 5. Coordinate system used in Eqs. (4.1)–(4.10).

$$\vec{w}_1 \to \vec{w}_2, \vec{q}_1 \to -\vec{q}_1{}', \vec{q}_2 \to -\vec{q}_2{}', \vec{p}_1 \leftrightarrow -\vec{p}_2 \ , \qquad (4.7)$$

where $\vec{p}_1$ and $\vec{p}_2$ are momenta of electron and positron, respectively. We note that $(w_1 + w_2)/(1 + w_1 w_2)$ is symmetric with respect to $w_1 \leftrightarrow w_2$. Let $f_1(q_{1z}), f_2(q_{2z}), \bar{f}_1(q'_{1z}), \bar{f}_2(q'_{2z})$ be the longitudinal distribution of π^-, π^0 (from τ^-), π^+, and π^0 (from τ^+), respectively. Let $f_3(q_{1x}, q_{2y})$ and $\bar{f}_3(q'_{1x}, q'_{2y})$ be the transverse momentum distribution of $\pi^-\pi^0$ for τ^- and those of $\pi^+\pi^0$ for τ^+, respectively. If CP is invariant we have

$$f_1(q_{1z}) = \bar{f}_1(-q'_{1z}) \ , \qquad (4.8)$$

$$f_2(q_{2z}) = \bar{f}_2(-q'_{2z}) \ , \qquad (4.9)$$

and

$$f_3(q_{1x}, |q_{2y}|) - f_3(q_{1x}, -|q_{2y}|) = \bar{f}_2(q'_{1x}, |q'_{2y}|) - \bar{f}_2(q'_{1x}, -|q'_{2y}|) \ . \qquad (4.10)$$

Violation of any one of the equalities in Eqs. (4.8), (4.9), and (4.10) signifies the violation of CP. Nonvanishing of either side of Eq. (4.10) signifies the violation of CVC but does not imply the violation of CP unless the equality is violated. The difference in the detection efficiencies of π^+ and π^- may make Eq. (4.8) rather difficult to verify, but Eq. (4.9) does not have this problem.

As mentioned previously, for leptonic decays or $\tau \to \nu_\tau + \pi$ (or K) we cannot have violation of equality such as Eq. (4.8) without violating CPT. Thus observation of violation of equality such as Eq. (4.8) for these modes is evidence of violation of CPT in these decay modes.

For decays such as $\tau \to \nu_\tau + \pi + K$, $\tau \to \nu_\tau + 3\pi$ we do not have CVC, thus observation of nonvanishing of either side of Eq. (4.10) does not imply violation of the standard model. However violation of equality in any one of Eqs. (4.8), (4.9), or (4.10) signifies CP violation in these modes.

Since the derivations of Eqs. (4.8), (4.9), and (4.10) are independent of detail mechanisms of CP violation, they bypass the difficulty mentioned at the end of the last section. Experimentalists can go ahead and measure the differences between the left- and right-hand sides of Eqs. (4.8)–(4.10), while theorists can figure out how different models of CP violation will affect the behavior of these functions.

Since the initial $e^\pm$ system is even under CP only in the center-of-mass system, Eqs. (4.8), (4.9), and (4.10) are true only in the c.m. system. However the experimental data in the non-c.m. system can be transformed into those in the c.m. by simple software, so these relations can be used also for the asymmetric B factory.

It should also be noted that w_1 must be equal to w_2 for the initial state to be even under CP. But this is not necessary because we have shown in Eq. (2.6) that the physics depends only on $(1+w_1w_2)$ and (w_1+w_2), which do not require $w_1 = w_2$.

The applications of colliding beams with polarized $e^\pm$ in the production of other particles have not been fully investigated. When hadrons are produced instead of τ's, their production angles can usually be reconstructed because their decays usually do not involve neutrinos. The method used in Sec. III can be used, for example, in the analysis of $\Lambda\bar{\Lambda}$ and $\Xi\bar{\Xi}$ production and their decays. The discussions on physics involved in using the transversely polarized $e^\pm$ machine can be found in Ref. [12].

A. B factory versus τ-charm factory for testing CP in τ decay

Let us compare the B factory and τ-charm factory for testing CP violation as described in this section. Since we are going to integrate with respect to the production angle of τ, we expect the z component of the τ polarization w_z given by Eq. (2.17) averaged over the differential cross section to give the effective polarization. We obtain, from Eqs. (2.17) and (2.9),

$$\begin{aligned} \bar{w}_Z &= \int_{-1}^{1} w_Z \frac{d\sigma}{dx} dx \Big/ \sigma \\ &= \frac{w_1 + w_2}{1 + w_1 w_2} \frac{1 + 2a}{2 + a^2} \\ &\equiv \frac{w_1 + w_2}{1 + w_1 w_2} F(a) \ , \end{aligned} \qquad (4.11)$$

where $a = M/E$, $a = 0.8514$, and 0.2961, respectively, for $E = 2.087$ and 6.0 GeV; and for $E = 2.087$ GeV we have $F(0.8514) = 0.992$, and for $E = 6.0$ GeV we have $F(0.2961) = 0.763$. We note $F(1) = 1$ and $F(0) = 0.5$. The total cross section is given by Eq. (2.19) which has a factor $(1 + w_1 w_2)$ that cancels out with the denominator in Eq. (4.11).

Finally the overall merit factor for each machine is

$$\begin{aligned} \text{merit} &= \text{luminosity} \times \bar{w}_Z \times \text{ total cross section} \\ &\propto \text{luminosity} \times (w_1 + w_2) \\ &\quad \times \sqrt{1 - a^2} a^2 (1 + 2a) \ , \end{aligned} \qquad (4.12)$$

where $a = M/E$. Thus if electron and positron are unpolarized, i.e., $w_1 = w_2 = 0$, it has zero value. Assuming the luminosity and the initial beam polarization to be the same for the two machines, the merit factor is determined by the function $f_m(a) = \sqrt{1 - a^2}a^2(1 + 2a)$. For the τ-charm factory we have $f_m(0.8514) = 1.0276$ whereas for the B factory we have $f_m(0.2961) = 0.1333$. Thus the τ-charm factory is better than the B factory by a factor 7.7 if both have the same luminosity and the initial beam polarizations.

ACKNOWLEDGMENTS

The author wishes to thank Professor T. D. Lee for making a critical remark, and J. D. Bjorken, Richard Prepost, and David Hitlin for useful discussions. This work was supported by the U.S. Department of Energy under Contract No. DE-AC03-76SF00515.

[1] John Jowett, in *Proceedings of the Tau-Charm Factory Workshop*, Stanford, California, 1989, edited by Lydia V. Beers (SLAC Report No. 343, Stanford, 1989), p. 7.
[2] Y. S. Tsai, Phys. Rev. D **4**, 2821 (1971).
[3] F. Hoogeveen and L. Stodolsky, Phys. Lett. B **212**, 505 (1988); S. Goozovat and C. A. Nelson, *ibid.* **267**, 128 (1991); G. Couture, *ibid.* B **272**, 404 (1991); W. Bernreuther and O. Nachtmann, Phys. Rev. Lett. **63**, 2787 (1989); W. Bernreuther, G. W. Botz, O. Nachtmann, and P. Overmann, Z. Phys. C **52**, 567 (1991); W. Bernreuther, O. Nachtmann, and P. Overmann, Phys. Rev. D **48**, 78 (1993); B. Aranthanarayan and S. D. Rindani, Phys. Rev. Lett. **73**, 1215 (1994).
[4] A. A. Sokolov and I. M. Ternov, Sov. Phys.-Dokl. **8** (1964).
[5] J. R. Johnson, R. Prepost, D. E. Wiser, J. J. Murray, R. F. Schwitters, and C. K. Sinclair, Nucl. Instrum. Methods **204**, 261 (1983).
[6] R. Prepost (private communication).
[7] Y. S. Tsai, Phys. Rev. D **48**, 96 (1993).
[8] J. Kirkby (private communication).
[9] Charles A. Nelson *et al.*, Phys. Rev. D **50**, 4544 (1994).
[10] *CP* violation from Higgs boson exchange has been considered by: T. D. Lee, Phys. Rev. D **8**, 1226 (1973); Phys. Rep. **9C**, 143 (1974); S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976).
[11] Throughout this paper our metric is $p_1 \cdot p_2 = p_{10}p_{20} - \vec{p}_1 \cdot \vec{p}_2$. The term $(\vec{w} \times \vec{q}_1) \cdot \vec{q}_2$ is Eps $(w, q_1, q_2, p_1)/M$ evaluated in the rest frame of τ^- $[p_1 = (M, 0, 0, 0)]$.
[12] Y. S. Tsai, Phys. Rev. D **12**, 3533 (1975).