Natural wormholes as gravitational lenses

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Once quantum mechanical effects are included, the hypotheses underlying the positive mass theorem of classical general relativity fail. As an example of the peculiarities attendant upon this observation, a wormhole mouth embedded in a region of high mass density might accrete mass, giving the other mouth a net *negative* mass of unusual gravitational properties. The lensing of such a gravitationally negative anomalous compact halo object (GNACHO) will enhance background stars with a time profile that is observable and qualitatively different from that recently observed for massive compact halo objects (MACHO's) of positive mass. While the analysis is discussed in terms of wormholes, the observational test proposed is more generally a search for compact negative mass objects of any origin. We recommend that MACHO search data be analyzed for GNACHO's.

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I. INTRODUCTION: WORMHOLES AND NEGATIVE MASS

The work of Morris, Thorne, and Yurtsever [1,2] has led to a great deal of interest in the formation and properties of three-dimensional wormholes (topological connections between separated regions of space-time) that are solutions of Einstein's equations of general relativity. Subsequently, Visser [3] suggested a particular wormhole configuration, a flat-space wormhole that is framed by "struts" of an exotic material, a variant of the cosmic string solutions of Einstein's equations [4,5]. To satisfy the Einstein field equations the cosmic string framing Visser wormholes must have a *negative* string tension [3] of $-1/4G$ and therefore a negative mass density. However, for the total mass of the wormhole system, the negative mass density of the struts should be combined with the effective positive mass density of the wormhole's gravitational field. The overall object could, depending on the details of the model, have positive, zero, or negative net external mass. Note that in hypothesizing the existence of such a wormhole, one has to abandon the averaged null energy condition [1,2]. Indeed, in a generic curved spacetime, it is possible to *prove* that the averaged null energy condition fails [6,7]. Therefore, the hypotheses underlying the positive mass theorem no longer apply and there is nothing, in principle, to prevent the occurrence of negative total mass [7,8]. Some of the Visser wormhole configurations have the shape of a cube or other geometrical solids, but one particularly simple configuration is a flat-space wormhole mouth framed by a single continuous loop of exotic cosmic string.

It has been suggested [4,5] that the inflationary phase of the early Universe might produce closed loops of cosmic string. It is therefore at least plausible that a similar mechanism might produce negative mass string loops framing stable Visser wormholes. Similar conclusions can be reached by relying on inflation to expand the microscopic wormholes commonly believed to exist in the Planck scale spacetime foam to more accessible macroscopic dimensions [9].

If a particle with positive electric charge passes through such a wormhole, its lines of force, threading through the wormhole aperture, give the entrance mouth an effective positive charge (flux lines radiating outward) and give the exit mouth an effective negative charge (flux lines converging inward). Similarly, when a massive object passes through the wormhole, the same back-reaction mechanism might cause the entrance mouth to gain mass and the exit mouth to lose mass [10]. Now let us consider a stable Visser wormhole with near-zero mass residing in the mass-energy-rich environment of the early Universe. The expected density fluctuations of the early Universe suggest that the separated wormhole mouths will reside in regions of differing mass density, leading to a mass flow between the regions they connect. As this mass passes through the wormhole, the entrance mouth will gain mass while the exit mouth will lose mass by the

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same amount. Soon, if the mass flow continues, the exit wormhole mouth will acquire a net negative mass.

This will lead to a gravitational instability, since the positive mass mouth will attract more mass through its aperture while the negative mass mouth will gravitationally repel nearby mass [11]. Thus the positive-negative imbalance will be fed by gravity and will continue to grow. If this process proceeds without interruption, the exit wormhole mouth might develop a stellar-scale negative mass. Visser wormholes, therefore, provide at least one motivation for seriously considering the possible existence of naturally occurring astronomical objects of sizable negative mass.

We have tried to sketch only *one* scenario for the possible existence of negative mass objects. We must, however, point out that the nonexistence of such objects would not rule out the existence of natural wormholes, since their properties are highly model dependent. Also, observations providing evidence of negative mass objects would not require the existence of natural wormholes, since other negative mass objects could conceivably exist, but would indicate a direct violation of the averaged null energy condition mentioned above.

Negative mass objects, while repelling all nearby mass (positive or negative), would themselves be attracted [11] by the mass of a nearby galaxy and might form part of a galactic halo. They would have unusual gravitational properties that could produce detectable gravitational lensing effects. These lensing effects, for the same absolute mass, are of the same magnitude as those recently detected for massive cosmic halo objects (MACHO's) [12,13], but, as we will show below, are qualitatively different in shape. We here examine in detail the lensing effects of gravitationally negative anomalous compact halo objects (GNACHO's).

II. GRAVITATIONAL LENSING BY NEGATIVE MASSES

Naively, if a gravitationally attractive positive mass acts as a converging lens that brightens a background star, one might expect a gravitationally repulsive negative mass to act as a diverging lens that causes a background star to briefly grow dimmer. Actually the lensing of a negative mass is not analogous to a diverging lens. In certain circumstances it can produce more light enhancement than does the lensing of an equivalent positive mass.

Figure 1 shows the geometry of starlight that is gravitationally lensed by an object of negative mass. Starlight is radiated from the stellar source S and is detected by the observer at D. A gravitationally negative object N lies between source and observer at an impact-parameter distance b from the source-detector axis DS. An off-axis ray of light passes near the negative mass object N, coming within a distance a of it, and is deflected [14,15] by an angle $\delta = (4G|M|/c^2)/a$ where G is Newton's gravitational constant, $|M|$ is the absolute value of the de-

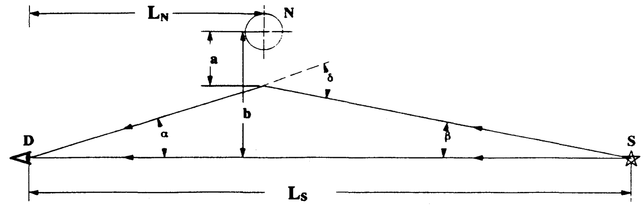

FIG. 1. Geometry for gravitational lensing by a negative mass object. Off-axis light rays from stellar source S are deflected to detector D by the gravitational repulsion of the negative lensing mass (GNACHO) N.

flecting mass, and c is the velocity of light. The angle δ is an external angle of the triangle formed by the direct and deflected rays. This triangle has interior angles α and β; so $\delta = \alpha + \beta$. If the detector-source distance is L_S and the detector-mass distance is L_N, then we have the equations

$$\alpha = (b-a)/L_N \quad (\alpha \ll 1), \tag{1}$$

$$\beta = (b-a)/(L_S - L_N) \quad (\beta \ll 1), \tag{2}$$

$$\delta = \alpha + \beta = \frac{4G|M_N|}{c^2}\frac{1}{a}. \tag{3}$$

These lead to the dimensionless quadratic equation

$$A^2 - AB + 1 = 0 \qquad (B = A + 1/A), \tag{4}$$

where $A \equiv a/a_0$ is the dimensionless distance of closest approach of the deflected ray, $B \equiv b/a_0$ is the dimensionless impact parameter distance of the deflecting mass, and

$$a_0 \equiv \sqrt{\frac{4G|M_N|}{c^2}\frac{L_N(L_S - L_N)}{L_S}} \tag{5}$$

is the characteristic gravitational length scale of the problem. For a positive lensing mass, a_0 would be the radius of the Einstein ring produced when the mass is positioned at zero impact parameter (b=0). To give some feeling for this length scale, if the stellar source S is in the Large Magellanic Cloud and a negative lensing mass N of one solar mass is in the galactic halo, then $L_S = 2 \times 10^{21}$ m, $L_N = 5 \times 10^{20}$ m, and $a_0 = 1.5 \times 10^{12}$ m or about 10 AU.

Solving quadratic (4) for A gives two solutions:

$$A_\pm = \frac{1}{2}[B \pm \sqrt{B^2 - 4}]. \tag{6}$$

When $B > 2$ there are two real solutions of the quadratic, corresponding to two rays that are deflected to the observer. When $B < 2$ there are no real solutions, indicating that the deflection is blocking all rays from reaching the observer. At $B = 2$, $A_+ = A_- = B/2$, and, as will be discussed below, the rainbowlike caustic occurs, al-

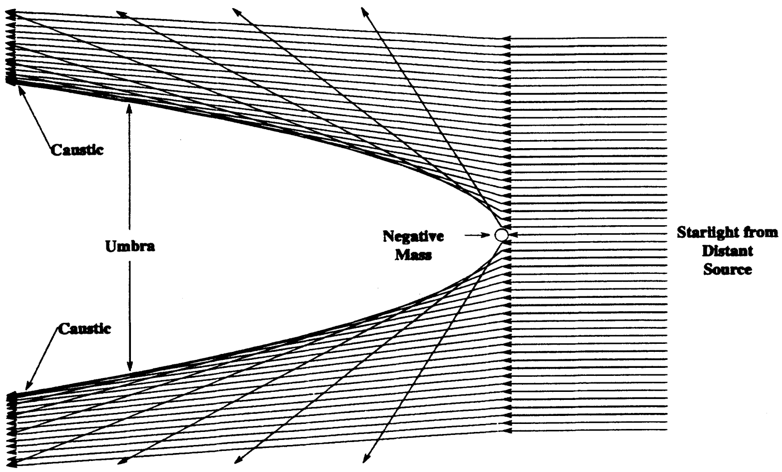

FIG. 2. Light deflection by a negative mass object (horizontal scale highly compressed). Light is swept out of the central region, creating an umbra region of zero intensity. At the edges of the umbra the rays accumulate, creating a rainbowlike caustic and enhanced light intensity.

lowing many rays to reach the observer and producing a dramatic brightening of the background star. Figure 2 shows this schematically. The negative mass deflects rays in inverse proportion to their distance of closest approach, creating a shadowed umbra region where light from the source is extinguished. At the edges of the umbra the light rays accumulate to form the caustic and give a very large increase in light intensity. The intensity falls slowly to normal at larger transverse distances.

III. LIGHT MODULATION PROFILES

If the unmodified intensity of the background star is I_0 and the altered intensity of the background star in the presence of the negative lensing mass for each solution is $I_\pm$, then the partial amplification factors $p_\pm \equiv I_\pm/I_0$ are given by [14]

$$p_\pm = \frac{a_\pm}{b}\left|\frac{da_\pm}{db}\right| = \frac{A_\pm}{B}\left|\frac{dA_\pm}{dB}\right| = \frac{(B \pm \sqrt{B^2-4})^2}{4B\sqrt{B^2-4}}. \quad (7)$$

The overall relative intensity $\mathcal{I}_{\rm neg} = p_+ + p_-$ is the modulation in brightness of the background star as detected by the observer, and is given by

$$\mathcal{I}_{\rm neg} = p_+ + p_- = \frac{B^2-2}{B\sqrt{B^2-4}}. \quad (8)$$

It is interesting to compare this with the similar expression for the relative intensity $\mathcal{I}_{\rm pos}$ of a background source that is lensed by an object of *positive* mass:

$$\mathcal{I}_{\rm pos} = \frac{B^2+2}{B\sqrt{B^2+4}}. \quad (9)$$

For the same dimensionless impact parameter B with $B > 2$, it is always true that $\mathcal{I}_{\rm neg} > \mathcal{I}_{\rm pos}$, and so for large impact parameters a negative mass actually provides *more* light enhancement through lensing than an equivalent positive mass. When $B \to 2$, the overall intensity $\mathcal{I}_{\rm neg} \to \infty$, a condition indicating a caustic at which light rays from many trajectories are deflected toward the observer. It provides a distinctive and unusual signature for negative mass lensing.

The intensity modulation that is actually observed occurs when the lensing mass, which is assumed to be moving with transverse velocity V, crosses near the source-detector axis DS with a minimum impact parameter b_0 and a minimum dimensionless impact parameter $B_0 = b_0/a_0$. The time-dependent impact parameter is therefore $b(t) = \sqrt{b_0^2 + (Vt)^2}$, and

$$B(t) = B_0\sqrt{1 + \left(\frac{t}{T_0}\right)^2}, \quad (10)$$

where $T_0 = b_0/V$ is the transit time across the distance of the minimum impact parameter and is the characteristic time scale of the problem.

Equations (8) and (10) are used to calculate the light enhancement profile of the process, taking $\mathcal{I}_{\rm neg} = 0$ when $|B| < 2$, and there are no real solutions to Eq. (4). Figure 3 shows these light enhancement profiles, plotting $\mathcal{I}_{\rm neg}$ vs t/T_0 for a range of minimum dimensionless impact parameters ranging from $B_0 = 0.50$ to 2.20. Figure 4 shows similar light enhancement profiles for a positive mass of the same size and range of B_0 values. As can

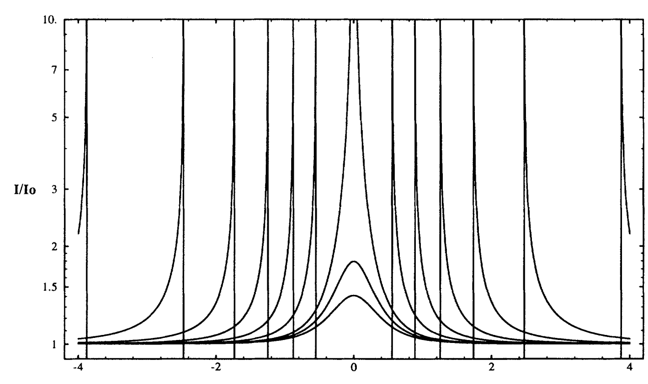

FIG. 3. Intensity profile of a gravitationally negative anomalous compact halo object (GNACHO) as it passes near the source-detector axis DS. The several curves correspond to minimum dimensionless impact parameter values $B_0 = 0.50$ (at edge of plot), 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.10, and 2.20 (small central bump). (See text for definitions of the variables.)

be seen, the light enhancement profiles are qualitatively different for positive and negative lensing masses of the same magnitude and geometry. In particular, the negative mass curves are much sharper, show stronger but briefer light enhancements, and for $B_0 < 2$ show a precipitous drop to zero intensity, i.e., extinction of the light from the source when the lensing mass deflects all light from S away from the observer.

Such a signature might possibly be confused with that of occultation by a dark foreground object. However, an occultation would not be preceded by a dramatic rise in intensity and might, if the object had a significant atmosphere, have different light profiles at different wavelengths. Therefore, the negative gravitational lensing presented here, if observed, would provide distinctive and unambiguous evidence for the existence of a foreground object of negative mass.

Note added. After this paper was written, the OGLE Collaboration released data for a particular microlensing event (OGLE No. 7) that exhibits a caustic [16,17]. The shape of the observed caustic is qualitatively different from that predicted herein for a GNACHO, and instead, this particular event seems to be due to microlensing by a binary system.

IV. CONCLUSION

The calculations presented above show that objects of negative gravitational mass, if they exist, can provide a very distinctive light enhancement profile. Since three groups are presently conducting searches for the gravitational lensing of more normal positive mass objects, we suggest that these searches be slightly broadened so that the signatures of the objects discussed above are not overlooked by overspecific data selection criteria and software cuts. While the analysis of this Brief Report is phrased terms of wormholes, the observational test proposed is more generally a search for compact negative mass objects of any origin. The question of whether quantum field theory is consistent with negative mass in a spacetime that is asymptotically flat and semiclassical near infinity is as yet undecided. Observation of GNACHO's would give an experimental and definitive answer to this question. We recommend that MACHO search data be analyzed for evidence of GNACHO's.

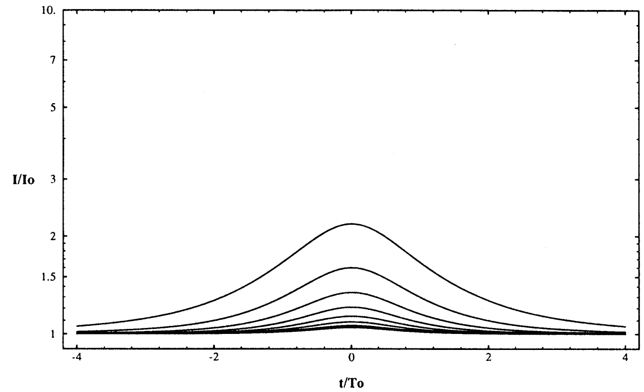

FIG. 4. Intensity profile of an object of equivalent positive mass in the same geometries. Here $B_0 = 0.50$ is the highest curve, and $B_0 = 2.20$ is the lowest.

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[1] M. S. Morris and K. S. Thorne, Am. J. Phys. **56**, 395 (1988).
[2] M. S. Morris, K. S. Thorne, and U. Yurtsever, Phys. Rev. Lett. **61**, 1446 (1988).
[3] M. Visser, Phys. Rev. D **39**, 3182 (1989).
[4] A. Vilenkin, Astrophys. J. **282**, L51 (1984).
[5] J. Richard Gott III, Astrophys. J. **288**, 422 (1985).
[6] R. M. Wald and U. Yurtsever, Phys. Rev. D **44**, 403 (1991).
[7] M. Visser, "Scale anomalies imply violation of the averaged null energy condition," Report No. gr-qc/9409043, 1994 (unpublished).
[8] M. Visser, Nucl. Phys. **B328**, 203 (1989).
[9] T. A. Roman, Phys. Rev. D **47**, 1370 (1993).
[10] V. P. Frolov and I. D. Novikov, Phys. Rev. D **42**, 1057 (1990).
[11] H. Bondi, Rev. Mod. Phys. **29**, 423 (1957).
[12] C. Alcock *et al.*, Nature (London) **365**, 621 (1993).
[13] E. Auburn *et al.*, Nature (London) **365**, 623 (1993).
[14] J. R. Gott III and J. E. Gunn, Astrophys. J. **190**, L105 (1974).
[15] B. Paczynski, Astrophys. J. **304**, 1 (1986).
[16] A. Udalski, *et al.*, "The optical gravitational lensing experiment. Ogle # 7: Binary microlens or a new unusual variable?," Report No. astro-ph/9407084, 1994 (unpublished).
[17] B. Paczynski *et al.*, "Results from the optical gravitational lensing experiment (OGLE)," Report No. astro-ph/9411004, 1994 (unpublished).