PHYSICAL REVIEW D VOLUME 51, NUMBER 6 15 MARCH 1995

Effective light-front quantization of scalar field theories and two-dimensional electrodynamics

E. V. Prokhvatilov,* H. W. L. Naus, and H.-J. Pirner
Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 19, D-69120 Heidelberg, Germany
(Received 3 June 1994)

We introduce a new method to include condensates in the light-cone Hamiltonian. By using a Gaussian approximation to the ordinary vacuum in a theory close to the light front, we derive an effective Hamiltonian on the light cone, which has new terms reflecting the nontriviality of the vacuum. We demonstrate our method for scalar ϕ^4 theory and the massive Schwinger model.

PACS number(s): 11.10.Ef, 11.10.Kk, 11.30.Qc, 12.20.−m

I. INTRODUCTION

The idea of quantizing field theories on the light front (i.e., on the hyperplane tangent to the light cone) was put forward by Dirac [1]. He pointed out that in such a formulation, the part of the Lorentz symmetry described kinematically is maximal. In other words, the number of generators of the Poincaré group, which depends on the dynamics, is minimal. Instead of Lorentz coordinates x^μ ($\mu = 0,1,2,3$) Dirac used the lightlike coordinates

$$x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3), \qquad x^\perp = x^{1,2}. \tag{1}$$

The coordinate x^+ plays the role of time. The subgroup of the Poincaré group consisting of the generators M_{12}, M_{+-}, $M_{-\perp}$, and P^+, $P_\perp$ is dynamically independent. This maximal amount of kinematical symmetry is related to the trivial structure of the vacuum in this formulation [2]. Indeed the vacuum is identified with the lowest eigenstate of the momentum $P^+ \geq 0$. The Fock space constructed over this vacuum [2] can be used to solve the eigenvalue problem for the mass (squared) operator: $m^2 = 2P^+P^- - P_\perp^2$. For states with fixed P^+ and $P_\perp = 0$, one has to solve the Schrödinger equation

$$P^-|m^2, P^+, P_\perp = 0\rangle = \frac{m^2}{2P^+}|m^2, P^+, P_\perp = 0\rangle. \tag{2}$$

This approach appears promising in nonperturbative studies of gauge theories, in particular QCD [3–6].

The quantization surface $x^+ = 0$, however, is a characteristic surface of the field equations. This peculiarity is reflected in infrared singularities, $P^+ \to 0$, in such formulations. Consequently, one is forced to use some regularization. Usually the most simple regularization is chosen: $P^+ \geq \varepsilon > 0$, where ε is a cutoff parameter. The simplicity of the vacuum and of the physical Fock space is related to this choice of regularization.

The question about the equivalence of such a light-front formulation to the usual one arises. To answer this question, results for various two-dimensional models have been considered: sine-Gordon [7,8], φ^4 model [9,10], QED [11,12], QCD [13], etc. The results for the mass spectra agree rather well with the results of the usual approaches, except for some "vacuum effects." These are usually connected with condensates which are zero in the naive light-front formalism. In four-dimensional space-time the spectrum of positronium in QED was considered with similar results [14].

To gain understanding about the equivalence of light-front formulation of the ordinary one, it is useful to consider the theory again on a spacelike plane, close to the light front [15,16], and investigate the limiting transition to the latter. This can be done by introducing the coordinates [15]

$$y^0 = x^+ + \frac{1}{2}\eta^2 x^-, \qquad y^3 = x^-, \qquad y^\perp = x^\perp, \tag{3}$$

with the metric $g_{\mu\nu}(\eta)$ ($g_{0\nu} = 0$, $g_{03} = g_{30} = 1$, $g_{33} = -\eta^2$). The quantization plane is defined by $y^0 = 0$. The parameter η is small and in the limit $\eta \to 0$ the exact light front is approached.

In the studies [15,16] of two-dimensional gauge theories formulated on a finite y^3 interval with periodic boundary conditions, it was explicitly shown that one obtains equivalent results only, when the continuum limit $L \to \infty$ is made first and then followed by the transition to the exact light cone $\eta \to 0$ (or $L\eta \to \infty$, $\eta \to 0$). Taking the limit $\eta \to 0$ at fixed L ($L\eta \to 0$) yields the usual light-front formulation (with $|x^-| \leq L$) with zero condensates.

Attempts have been made to take into account vacuum effects by considering zero ($P^+ = 0$) Fourier modes of the fields [9,10,15,17]. However, in the light-front formulation these zero modes have peculiar dynamics [4,10,17].

*Permanent address: Department of Theoretical Physics, Institute of Physics, St. Petersburg University, Ulyanovskaya 1, St. Petersburg 198904, Russia.

For example, they depend on nonzero modes through some specific canonical constraints related to the choice of the boundary conditions for $|x^-| = L$ [4,10]. This means that the physics at low momenta (zero modes) can depend on high-momentum modes in a complicated fashion.

In this paper another, more efficient, approach to light-front quantization is proposed. It is based on approximations for the vacuum in the ordinary formulation [18] and on the appropriate choice of canonical variables reflecting the nontriviality of the vacuum in the given approximation. In terms of these new variables we then take the naive light-front limit ($\eta \to 0$ at fixed ε); the resulting theory will include information on the approximate, nontrivial, vacuum. This approach is demonstrated by two simple examples: scalar field theory in two dimensions (next section) and the massive Schwinger model (Sec. III).

II. SCALAR FIELD THEORY IN 1+1 DIMENSIONS

For scalar field theory, we define the Lagrangian density as

$$\mathcal{L}(\varphi) = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi(y) \partial_\nu \varphi(y) - \frac{1}{2} m_0^2 \varphi^2(y) - \lambda U(\varphi), \quad (4)$$

where $U(\varphi)$ is an interaction term. The theory is formulated using the y^μ coordinates, Eq. (3); here, in the two-dimensional case, the space coordinate is denoted by y^1. Consequently, we can write

$$\mathcal{L}(\varphi) = \partial_0 \varphi(y) \partial_1 \varphi(y) + \frac{1}{2} \eta^2 [\partial_0 \varphi(y)]^2 - \frac{1}{2} m_0^2 \varphi^2(y) - \lambda U(\varphi). \quad (5)$$

After introducing the canonical variable $\Pi(y)$, the conjugate momentum of $\varphi(y)$,

$$\Pi(y) = \frac{\partial L}{\partial[\partial_0 \varphi(y)]} = \eta^2 \partial_0 \varphi(y) + \partial_1 \varphi(y),$$

the Hamiltonian reads

$$H = \int dy^1 \left\{ \frac{(\Pi - \partial_1 \varphi)^2}{2\eta^2} + \frac{1}{2} m_0^2 \varphi^2 + \lambda U(\varphi) \right\} =: H_0 + \lambda U. \quad (6)$$

The usual (equal y^0) commutation relations are imposed:

$$[\varphi(y^1), \Pi(y^{1'})] = i\delta(y^1 - y^{1'}).$$

We make a Fourier decomposition of the canonical fields φ and Π in terms of the "bare" operators b and b^+ ($b|0_b\rangle = 0$, with $|0_b\rangle$ as the free-field vacuum):

$$\varphi(y) = \frac{1}{\sqrt{4\pi}} \int_{-\infty}^{+\infty} \frac{dk_1}{\sqrt{E_0(k_1)}} [b(k_1) + b^\dagger(-k_1)] e^{-ik_1 y^1},$$

$$\Pi(y) = \frac{-i}{\sqrt{4\pi}} \int_{-\infty}^{+\infty} dk_1 \sqrt{E_0(k_1)} [b(k_1) - b^\dagger(-k_1)] e^{-ik_1 y^1}, \quad (7)$$

where $E_0(k_1) = \sqrt{k_1^2 + \eta^2 m_0^2}$. For the operators b and $b^\dagger$ we have standard commutation relations

$$[b(k_1), b^\dagger(k_1')] = \delta(k_1 - k_1'),$$

$$[b(k_1), b(k_1')] = 0 = [b^\dagger(k_1), b^\dagger(k_1')].$$

In terms of b and $b^\dagger$, H_0 is diagonal by construction:

$$H_0 = \int_{-\infty}^{+\infty} dk_1 \frac{E_0(k_1) - k_1}{\eta^2} b^\dagger(k_1) b(k_1). \quad (8)$$

Since η appears also in the energy E_0, only terms in H_0 with $k_1 \le 0$ are singular in the limit $\eta \to 0$. In order to make the energy finite in this limit we consider the restricted Fock space $\mathcal{F}_{(\varepsilon)}$:

$$\mathcal{F}_{(\varepsilon)} =: \left\{ \prod_i b^\dagger(k_i)|0_b\rangle, k_i \ge \varepsilon > 0 \right\},$$

where ε is the cutoff parameter. If we now take $\eta \to 0$ at fixed $\varepsilon > 0$, we obtain a finite result for the energy, because $[E_0(k_1) - k_1]/\eta^2 \to m_0^2/2k_1$, for $k_1 \ge \varepsilon > 0$ and $\eta \to 0$. The limiting form of the Hamiltonian on the subspace $\mathcal{F}_{(\varepsilon)}$ reproduces the light-cone Hamiltonian P^-,

$$P^- = \lim_{\eta \to 0} H_\eta \quad (\text{acting on } \mathcal{F}_{(\varepsilon)}) = \int_{-\infty}^{+\infty} dx^- \left\{ \frac{1}{2} m_0^2 \varphi_\varepsilon^2(x) + \lambda U(\varphi_\varepsilon) \right\}, \quad (9)$$

where $\varphi_\varepsilon(x)$ is the parametrization of the field in light-front coordinates:

$$\varphi_\varepsilon(x^-, x^+ = 0) = \frac{1}{\sqrt{4\pi}} \int_\varepsilon^\infty \frac{dp^+}{\sqrt{p^+}} [b(p^+) e^{-ip^+ x^-} + b^\dagger(p^+) e^{ip^+ x^-}]. \quad (10)$$

Note that we would get the same result in the theory formulated on a finite interval, $-L \le y^1 \le L$, with periodic boundary conditions in y^1. In this case the role of the cutoff parameter ε would be taken over by the parameter π/L. Furthermore, the result (9) can be obtained via time-independent perturbation theory in η [15,16].

At this point we want to introduce a better way to formulate the light-cone limit $\eta \to 0$. Before the limiting transition, i.e., still for finite η, we approximate the vacuum by a Gaussian trial state [18,19] using the limit

$\varepsilon \to 0$. This trial state is parametrized by a Bogoliubow-type transformation:

$$|0_a\rangle = \exp\left[-\frac{1}{2}\int dk f(k)[b^\dagger(k)b^\dagger(-k) - b(k)b(-k)] + f_0(b^\dagger(0) - b(0))\right]|0_b\rangle, \quad (11)$$

where $f(k)$ and f_0 are real, and $f(k) = f(-k)$. The trial vacuum can be easily defined with new operators $a(k_1)$, $a^\dagger(k_1)$ such that $a(k_1)|0_a\rangle = 0$. As follows from Eq. (11), these new operators $a(k_1)$ and $a^\dagger(k_1)$ are linear combinations of the old operators $b^\dagger(k_1)$ and $b(k_1)$. Therefore one can rewrite the Fourier decompositions of φ and Π in terms of a, $a^\dagger$:

$$\varphi(y) = \varphi_0 + \frac{1}{\sqrt{4\pi}}\int \frac{dk_1}{\sqrt{E(k_1)}}[a(k_1) + a^\dagger(-k_1)]e^{-ik_1y^1},$$
$$\Pi(y) = \frac{-i}{\sqrt{4\pi}}\int dk_1\sqrt{E(k_1)}[a(k_1) - a^\dagger(-k_1)]e^{-ik_1y^1}. \quad (12)$$

Identifying these expressions with the corresponding ones in terms of the b and $b^\dagger$ operators [Eq. (7)] yields the linear transformations between the sets $(a, a^\dagger)$ and $(b, b^\dagger)$ in terms of $E(k_1)$, $E_0(k_1)$, and φ_0. Then the condition $a(k_1)|0_a\rangle = 0$ determines the relation between $[E(k_1), \varphi_0]$ and $[f(k_1), f_0]$ to be

$$E(k_1) = E_0(k_1)\exp[2f(k_1)],$$
$$\varphi_0 = \frac{f_0}{\sqrt{\pi\eta m_0}}\frac{1 - \exp[-f(0)]}{f(0)}. \quad (13)$$

In the following we will consider $E(k_1)$ and φ_0 as parameters of the transformation (or, equivalently, of the trial state).

From now on we specify the interaction as $U(\varphi) = \varphi^4$. We proceed by rewriting the Hamiltonian H in the normal ordered form with respect to the a, $a^\dagger$ operators; :: denotes this normal ordering. The result is

$$H =: \int dy^1\left[\frac{(\Pi - \partial_1\varphi)^2}{2\eta^2} + \frac{1}{2}(m_0^2 + 12\lambda \underbrace{\varphi\varphi})\varphi^2 + \lambda\varphi^4 + \frac{1}{2\eta^2}\underbrace{\Pi\Pi} + \frac{1}{2\eta^2}\underbrace{\partial_1\varphi\partial_1\varphi} + \frac{1}{2}m_0^2\underbrace{\varphi\varphi} + 3\lambda(\underbrace{\varphi\varphi})^2\right]:, \quad (14)$$

where

$$\underbrace{\varphi\varphi}: = \int\frac{dk_1}{4\pi E(k_1)}, \quad \underbrace{\Pi\Pi} := \int\frac{dk_1}{4\pi}E(k_1),$$
$$\underbrace{\partial_1\varphi\partial_1\varphi}: = \int\frac{dk_1(k_1)^2}{4\pi E(k_1)}.$$

These integrals are understood to be regularized by a cutoff parameter Λ, $|k_1| < \Lambda$. In order to fix the parameters $E(k_1)$ and φ_0 we minimize the expectation value of the Hamiltonian density $\mathcal{H}$ in the trial vacuum $|0_a\rangle$. This expectation value is given by

$$\langle 0_a|\mathcal{H}|0_a\rangle = \frac{1}{2\eta^2}(\underbrace{\Pi\Pi} + \underbrace{\partial_1\varphi\partial_1\varphi}) + \frac{1}{2}(m_0^2 + 12\lambda\varphi_0^2)\underbrace{\varphi\varphi} + 3\lambda(\underbrace{\varphi\varphi})^2 + \frac{1}{2}m_0^2\varphi_0^2 + \lambda\varphi_0^4$$
$$= \frac{1}{8\pi\eta^2}\int dk_1\left(E(k_1) + \frac{k_1^2 + \eta^2m_0^2 + 12\eta^2\lambda\varphi_0^2}{E(k_1)}\right) + 3\lambda\left(\int\frac{dk_1}{4\pi E(k_1)}\right)^2 + \frac{1}{2}m_0^2\varphi_0^2 + \lambda\varphi_0^4.$$

At the extremum, $\delta\langle 0_a|\mathcal{H}|0_a\rangle/\delta E(k_1) = 0$, $\delta\langle 0_a|\mathcal{H}|0_a\rangle/\delta\varphi_0 = 0$, we obtain $E^2(k_1)$ in terms of the new mass m:

$$E^2(k_1) = k_1^2 + \eta^2\left[m_0^2 + 12\lambda\varphi_0^2 + \frac{3\lambda}{\pi}\int_{|q_1|\le\Lambda} dq_1E^{-1}(q_1)\right]$$
$$: = k_1^2 + \eta^2m^2, \quad (15)$$

and

$$\varphi_0(m^2 - 8\lambda\varphi_0^2) = 0. \quad (16)$$

Using the equality (15) we get for a large cutoff Λ:

$$\int_{|q_1|\le\Lambda} dq_1E^{-1}(q_1) \underset{(\Lambda/m)\to\infty}{\approx} \ln\frac{4\Lambda^2}{\eta^2m^2}$$
$$= \ln\frac{4\Lambda^2}{\eta^2\lambda} + \ln\frac{\lambda}{m^2} \approx \ln\frac{4\Lambda^2}{\eta^2\lambda}.$$

Since in this limit

$$m^2 \simeq m_0^2 + 12\lambda\varphi_0^2 + \frac{3\lambda}{\pi}\ln\frac{4\Lambda^2}{\eta^2\lambda},$$

we can renormalize the theory by choosing

$$\frac{m_0^2}{\lambda} = \frac{3}{\pi}\ln\frac{\eta^2\lambda}{4\Lambda^2} + \xi,$$

with a parameter ξ, $-\infty < \xi < \infty$. Then we can convert Eq. (15) into a nonlinear equation for m:

$$y + \frac{3}{\pi}\ln y = \xi + 12\varphi_0^2,$$

with

$$y = m^2/\lambda. \quad (17)$$

This equation should be solved together with Eq. (16), which obviously has the solutions

$$(1)\ \ \varphi_0 = 0,$$
$$(2)\ \ \varphi_0^2 = m^2/8\lambda = \frac{1}{8}y. \quad (18)$$

Therefore, there are two different cases:

$$(1)\ \ y + \frac{3}{\pi}\ln y = \xi$$

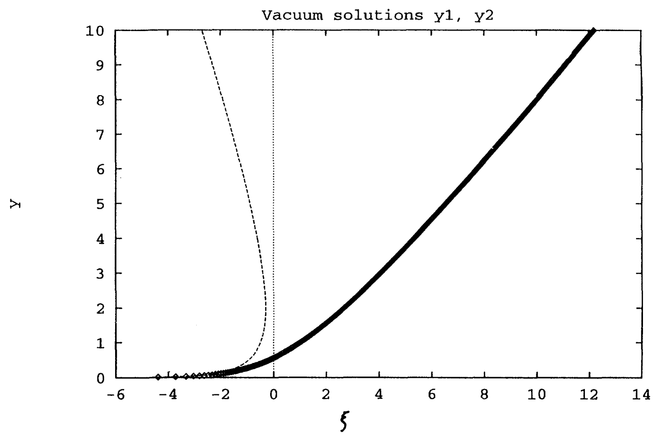

FIG. 1. Solutions $y_1(\xi)$ and $y_2(\xi)$ of Eq. (19).

and

$$(2) \quad -\frac{1}{2}y + \frac{3}{\pi}\ln y = \xi \ . \tag{19}$$

The solutions $y_1(\xi)$ (solid curve), $y_2(\xi)$ (dashed curve) are shown in Fig. 1. Of course, one needs to choose the solution which corresponds to a minimum of the (trial) vacuum energy. The difference of this energy in cases (1) and (2) can be calculated straightforwardly; the result is

$$\langle 0_a|\mathcal{H}|0_a\rangle_{(1)} - \langle 0_a|\mathcal{H}|0_a\rangle_{(2)} = \frac{\lambda}{8\pi}(y_1 - y_2) + \frac{\lambda}{48}\left(y_1^2 + \frac{1}{2}y_2^2\right).$$

At the critical point $\xi_c = -0.503\ldots$, the sign of the energy difference changes and, consequently, the favored solution switches from y_2 to y_1 (for increasing ξ). In other words, we obtain the well-known phase transition in this approximation [18,19]. Moreover, the exact location of the phase transition, i.e., the critical point, agrees with earlier results [18].

For the minimal energy solution of Eqs. (17) and (18) we introduce the notation

$$F(\xi) := \begin{cases} \left(\frac{m^2}{\lambda}\right)_1, & \xi > \xi_c, \\ \left(\frac{m^2}{\lambda}\right)_2^{\text{up}}, & \xi < \xi_c, \end{cases}$$

where $(m^2/\lambda)_2^{\text{up}}$ denotes the upper branch of the curve $(m^2/\lambda)_2$. Now we are in the position to present a renormalized Hamiltonian, which is obtained by subtracting the trial-vacuum energy:

$$H_{\text{ren}} = :\int dy^1 \left[\frac{(\Pi - \partial_1\tilde{\varphi})^2}{2\eta^2} + \frac{1}{2}\lambda F(\xi)\tilde{\varphi}^2 + \lambda\sqrt{2F(\xi)}\theta(\xi_c - \xi)\tilde{\varphi}^3 + \lambda\tilde{\varphi}^4\right]:, \tag{20}$$

with $\tilde{\varphi} := \varphi - \varphi_0$.

We can use H_{ren} as the starting Hamiltonian for the limiting transition to the light cone. Repeating the steps following Eq. (8), we obtain the effective light-front Hamiltonian

$$P^- =: \int dx^- \left[\frac{\lambda}{2}F(\xi)\tilde{\varphi}_\varepsilon^2(x) + \lambda\sqrt{2F(\xi)}\theta(\xi_c - \xi)\tilde{\varphi}_\varepsilon^3(x) + \lambda\tilde{\varphi}_\varepsilon^4(x)\right] : . \tag{21}$$

This expression differs from the usual one by the presence of the function F describing vacuum effects. In the quadratic term, we see that the effective theory has a renormalized mass term. The cubic term was even completely absent in the usual approach. For $\xi > \xi_c$, i.e., in the phase without zero mode ($\varphi_0 = 0$), the cubic term vanishes identically and the mass renormalization is all that remains. For $\xi < \xi_c$, the reflection symmetry $\varphi \to -\varphi$ is spontaneously broken and a zero mode $\varphi_0 \neq 0$ is present. This zero mode produces in the effective light-cone Hamiltonian an additional interaction term, which explicitly breaks the reflection symmetry. In other words, this formulation converts a spontaneous symmetry breaking into an explicit symmetry breaking in the effective light-cone Hamiltonian. In this way, a rather long-standing defect of light-cone quantization, namely the triviality of the vacuum, can be handled in an approximative way. We emphasize that the proposed approach is very reasonable. The zero modes carry infinite light-cone energy. The strategy to remove high-energy degrees of freedom by effective interactions is the usual strategy of renormalization in equal time field theory.

The effective light-cone Hamiltonian, Eq. (21), can be used for explicit calculation using standard light-cone techniques. We note that this approach can easily be generalized to other scalar field theories in two or more dimensions.

III. MASSIVE SCHWINGER MODEL

The massive Schwinger has also been formulated in the y^μ coordinates, i.e., for $\eta \neq 0$ [15,16]. The Lagrangian density reads

$$\mathcal{L}(A_\mu, \psi) = -\frac{1}{4}g^{\mu\rho}g^{\nu\lambda}F_{\mu\nu}(y)F_{\rho\lambda}(y) + \bar{\psi}(x(y))\left[i\left(\frac{\partial y^\lambda}{\partial x^\mu}\right)\gamma^\mu D_\lambda - M\right]\psi(x(y)), \tag{22}$$

where the covariant derivative,

$$D_\mu = \partial_\mu - ieA_\mu(y),$$

and the field strength tensor, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, are expressed in terms of the vector potential A_μ. The fermion field contains two spinor components,

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} ,$$

and M is the fermion mass. With definition (3) of the coordinates and the γ matrices,

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \gamma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix},$$

we obtain, for the Lagrangian density,

$$\mathcal{L}(y) = \frac{1}{2}F_{01}^2(y) + i\sqrt{2}\psi_+^\dagger D_0\psi_+ + \frac{1}{2}\sqrt{2}i\eta^2\psi_-^\dagger D_0\psi_- + i\sqrt{2}\psi_-^\dagger D_1\psi_- - iM(\psi_-^\dagger\psi_+ - \psi_+^\dagger\psi_-). \quad (23)$$

Note that only the mass term couples the two fermion components ψ_- and ψ_+. Let us consider the theory on a finite y^1 interval: $-L \leq y^1 \leq L$ and impose periodic boundary conditions on the fields A_μ and ψ. We fix the gauge by imposing

$$\partial_1 A_1 = 0, \quad (24)$$

i.e., the "Coulomb gauge."

It has been shown earlier [15,16] that the zero mode of A_1 cannot be gauged away. In the Coulomb gauge the only constraint is Gauss's law

$$\partial_1 F_{01} + e\sqrt{2}\psi_+^\dagger\psi_+ + \frac{1}{2}\sqrt{2}e\eta^2\psi_-^\dagger\psi_- = 0.$$

It can be solved with respect to the nonzero modes of F_{01}:

$$\langle F_{01}\rangle = -\partial_1^{-1}(e\sqrt{2}\psi_+^\dagger\psi_+ + \tfrac{1}{2}\sqrt{2}e\eta^2\psi_-^\dagger\psi_-), \quad (25)$$

where the angular brackets $\langle\,\rangle$ define the nonzero modes

$$\langle f\rangle = f(y^1) - \frac{1}{2L}\int_{-L}^{L} f(y^1)dy^1$$

and ∂_1^{-1} is the periodic Green's function of the differential operator ∂_1 (see, e.g., [20]):

$$\partial_1^{-1}(x) = \sum_{n\neq 0}\frac{1}{2i\pi n}\exp\left(2i\pi n\frac{x}{L}\right). \quad (26)$$

Substituting Eq. (25) into the Lagrangian and performing the Legendre transformation yields the Hamiltonian in terms of the canonical variables $\chi_+ = \eta^{1/4}\psi_+$, $\chi_- = \eta^{-1/4}\eta\psi_-$, $\Pi_1 = \int_{-L}^{L} dy^1 F_{01}(y^1)$, and A_1:

$$H = \int_{-L}^{L} dy^1\left\{\frac{\Pi_1^2}{8L^2} + \frac{1}{2}e^2[\partial_1^{-1}(\chi_+^\dagger\chi_+ + \chi_-^\dagger\chi_-)]^2 - 2i/\eta^2\chi_-^\dagger D_1\chi_- + i\frac{M}{\eta}(\chi_-^\dagger\chi_+ - \chi_+^\dagger\chi_-)\right\}. \quad (27)$$

Moreover, integration of Gauss's law gives a residual constraint, which is to be imposed on the physical states:

$$\int_{-L}^{L} dy^1(\chi_+^\dagger\chi_+ + \chi_-^\dagger\chi_-)|\text{phys}\rangle = 0. \quad (28)$$

Notice that our canonical variables satisfy the commutation relations

$$\left\{\chi_\pm^\dagger(y^1), \chi_\pm(y^{1'})\right\}_{y^0=y^{0'}} = \delta(y^1 - y^{1'}),$$
$$[A_1(y^0), \Pi_1(y^0)] = i. \quad (29)$$

The regularized charge densities of the right and left movers are obtained via point splitting the two densities and connecting the two centers with a string:

$$I_\pm(y^1) = \lim_{\varepsilon\to 0}\left[\chi_\pm^\dagger\left(y^1 \mp \frac{i\varepsilon}{2}\right)\chi_\pm\left(y^1 \pm \frac{i\varepsilon}{2}\right) \times\exp(\pm\varepsilon e A_1) - \frac{1}{2\pi\varepsilon}\right] := \lim_{\varepsilon\to 0}\left(I_\pm(y^1,\varepsilon) - \frac{1}{2\pi\varepsilon}\right). \quad (30)$$

These chiral charge densities have Fourier expansions ($r = \pm$, $p_n = \pi n/L$):

$$I_r(y) = \frac{1}{2L}\left(Q_r + \sum_{n\neq 0}\sqrt{|n|}I_{n,r}(y^0)\exp(-ip_n y^1)\right). \quad (31)$$

The zero-mode part of the Fourier expansion is defined by the total chiral charges

$$Q_r = \int_{-L}^{+L} I_r(y^1)dy^1, \quad (32)$$

which can be calculated via the ε prescription by inserting the ε-regularized charge density $I_r(y^1,\varepsilon)$ into Eq. (32):

$$Q_r = \lim_{\varepsilon\to 0}[Q_r(\varepsilon) - L/\pi\varepsilon]. \quad (33)$$

The coefficients $I_{n,r}$ obey commutation relations, which are a consequence of the commutation relations of the Fourier coefficients $\chi_{n,r}$ of the fermion fields and of the regularization, Eq. (30):

$$\chi_r(y^1) = \frac{1}{\sqrt{2L}}\sum_{n=-\infty}^{\infty}\chi_{n,r}(y^0)\exp(-ip_n y^1), \quad (34)$$

$$\{\chi_n(y_0), \chi_{n'}^\dagger(y_0)\} = \delta_{nn'}, \quad (35)$$

$$[I_{n,r} I_{n'r'}^\dagger] = r\frac{n}{|n|}\delta_{rr'}\delta_{nn'}, \quad n\neq 0. \quad (36)$$

As usual [21,22], we define subspaces $|l\rangle$ of the total Hilbert space which correspond to sectors $[l = (l_+, l_-)]$ with given edges of occupied energy levels for the right and left movers as follows:

$$\chi_{n,r}|l\rangle = \theta(rl_r - rn)|l\rangle,$$

with

$$\theta(l) = \begin{cases} 1, & l > 0, \\ 0, & l \leq 0\,. \end{cases} \quad (37)$$

Consequently, the operators $I_{n,+}, I_{n,-}^\dagger, n > 0$, annihilate the states $|l\rangle$:

$$I_{n,+}|l\rangle = I^{\dagger}_{n,-}|l\rangle = 0. \tag{38}$$

The charge eigenvalues in sectors $|l\rangle$ depend on the zero-mode gauge field [21,22]

$$Q_r|l\rangle = \left(rl_r + r\frac{eLA_1}{\pi} + 1/2\right)|l\rangle, \quad [Q_r, \Pi_1] = \frac{rieL}{\pi}. \tag{39}$$

We introduce [22] the variables ω_r canonically conjugated to the Q_r such that

$$[\omega_r, Q_{r'}] = i\delta_{rr'}. \tag{40}$$

Then we can represent the fermion fields with the help of the bosonic operators I_r, ω_r, A_1 [22–24]:

$$\chi_r(y^1) = \frac{1}{\sqrt{2L}}\exp(-i\omega_r)\exp\left[\frac{ir\pi}{2}(Q_+ + Q_- - 1) - \frac{ri\pi y^1}{L}\left(Q_r - \frac{reLA_1}{\pi} - \frac{1}{2}\right)\right] \times\exp\left(-\sum_{n>0}\sqrt{n}I^{+}_{n,r}e^{irp_ny^1}\right)\exp\left(+\sum_{n>0}\sqrt{n}I_{n,r}e^{-irp_ny^1}\right). \tag{41}$$

These operators satisfy the commutation relations, Eq. (29), and reproduce the regularized charge densities, Eq. (30). The necessary explanations can be found in the Appendix. The operators I_r link the fermionic to the bosonic description: In the Hamiltonian of Eq. (27) we recognize four terms. The first three terms can be rewritten as a free-boson Hamiltonian in terms of bosonic variables (ϕ, Π_ϕ) constructed from the charge densities I_r:

$$\Pi_\phi = \sqrt{\pi}(I_+ - I_-),$$
$$\phi = -\frac{1}{m}\left(\frac{\Pi_1}{2L} - \partial^{-1}[e(I_+ + I_-)]\right),$$
$$m^2 = e^2/\pi. \tag{42}$$

With the help of the commutation relations (36) one can verify that Π_ϕ and ϕ are canonically conjugate variables. The mass term of the bosonic Hamiltonian is easily calculable using the fact that the zero mode is subtracted in $\langle e(I_+ + I_-)\rangle$:

$$\int_{-L}^{L} dy^1\frac{1}{2}m^2\phi^2 = \int_{-L}^{+L} dy^1\left[\frac{1}{8L^2}\Pi_1^2 + \frac{e^2}{2}[\partial_1^{-1}(I_+ + I_-)]^2\right]. \tag{43}$$

The momentum term can be expressed with the help of Eq. (41) in terms of the chiral charges. The space integral of the square of the zero-mode free chiral charge density $\langle I_r\rangle^2$ is related to the fermionic momentum

$$\pi\int_{-L}^{+L} dy^1\langle I_r\rangle^2 = r\int_{-L}^{+L} dy^1\chi_r^{\dagger}(y^1)(iD_1)\chi_r(y^1). \tag{44}$$

This relation is derived in the Appendix. On the physical subspace defined by

$$Q|\text{phys}\rangle = 0, \quad Q = Q_+ + Q_-, \tag{45}$$

we obtain, with Eqs. (42) and (44),

$$\int_{-L}^{+L} dy^1\frac{(\Pi_\phi - \partial_1\phi)^2}{2\eta^2} = \frac{2}{\eta^2}\int_{-L}^{+L} dy^1\chi_-^{\dagger}(y^1)(-iD_1)\chi_-(y_1). \tag{46}$$

The mass term remains as a last term in the fermionic Hamiltonian of Eq. (27). It is given by direct insertion of the boson representation of the fermion fields Eq. (41) into Eq. (27). After simplifying this expression with the help of normal ordering with respect to $I^{\dagger}_{n,r}$ and $I_{n,r}$ (cf. the Appendix), we obtain

$$\frac{iM}{\eta}\int_{-L}^{+L} dy^1[\chi_-^{\dagger}(y^1)\chi_+(y^1) - \chi_+^{\dagger}(y^1)\chi_-(y^1)] = -\frac{M}{\eta L} : \sin(\omega_+ - \omega_- + \sqrt{4\pi}\langle\phi\rangle) : . \tag{47}$$

In a similar way to the treatment of the scalar field theory in 1+1 dimensions we approximate the vacuum by a trial state $|0_a\rangle$ which is defined as

$$a_n|0_a\rangle = 0, \tag{48}$$

where a_n and $a_n^{\dagger}$ are the normal modes of the boson variables $\phi(y^1)$, $\Pi_\phi(y^1)$:

$$\phi(y^1) = \frac{1}{\sqrt{2L}}\sum_{n=-\infty}^{\infty}\frac{1}{\sqrt{2E_n}}(a_n + a^{\dagger}_{-n})e^{-ip_ny^1},$$
$$\Pi_\phi(y^1) = \frac{-i}{\sqrt{2L}}\sum_{n=-\infty}^{\infty}\sqrt{\frac{E_n}{2}}(a_n - a^{\dagger}_{-n})e^{-ip_ny^1}. \tag{49}$$

The weights E_n are variational parameters, which then also enter the Fourier coefficients of the chiral charges [cf. Eq. (42)]:

$$I_{n,r} = \frac{-ri}{\sqrt{4E_n|p_n|}}[(E_n + rp_n)a_n - (E_n - rp_n)a^{\dagger}_{-n}]. \tag{50}$$

Inserting these expressions into Eq. (47) and normal ordering with respect to the trial vacuum Eq. (48) we obtain on the physical subspace, Eq. (45), the effective Hamiltonian

$$\mathcal{H} = \int_{-L}^{L} dy^1\left[\frac{(\Pi_\phi - \partial_1\phi)^2}{2\eta^2} + \frac{1}{2}m^2\phi^2 - \frac{M}{\eta L}\times\exp\left\{\frac{\pi}{L}\sum_{n>0}\left(\frac{1}{p_n} - \frac{1}{E_n}\right)\right\} : \cos(\omega + \sqrt{4\pi}\langle\phi\rangle) : \right], \tag{51}$$

with $\omega = \omega_+ - \omega_- - \pi/2$. Note that the normal ordering symbol :: means here to order the operators a_n, $a_n^\dagger$.

In order to fix the variational parameters we look for a minimum of the vacuum energy density using the trial vacuum state $|0_a\rangle$. The calculation proceeds in an analogous way to the calculation of the φ^4 scalar theory. Note that the minimum corresponds to $\omega = 0$. Using condition (49) we obtain the following expression for the vacuum energy density of the Hamiltonian, Eq. (52), at $\omega = 0$:

$$\langle 0_a|\mathcal{H}(y)|0_a\rangle = \frac{1}{4L\eta^2}\sum_{n>0}\left(E_n + \frac{p_n^2+\eta^2 m^2}{E_n}\right) - \frac{M}{\eta L}\exp\left[\frac{\pi}{L}\sum_{n>0}\left(\frac{1}{p_n}-\frac{1}{E_n}\right)\right]. \quad (52)$$

At the minimum of this expression we have ($n > 0$)

$$E_n^2 = p_n^2 + \eta^2 m^2 + \frac{4\pi M\eta}{L}\exp\left\{\frac{\pi}{L}\sum_{n>0}\left(\frac{1}{p_n}-\frac{1}{E_n}\right)\right\} = p_n^2 + \eta^2\mu^2, \quad (53)$$

with

$$\mu^2 = m^2 + \frac{4\pi M}{\eta L}\exp\left\{\frac{\pi}{L}\sum_{n>0}\left(\frac{1}{p_n}-\frac{1}{E_n}\right)\right\}.$$

From this equation μ is to be determined. In order to do that we rewrite the infinite sum in the exponent [25] as

$$\frac{\pi}{L}\sum_{n>0}\left(\frac{1}{p_n}-\frac{1}{E_n}\right) = -2\sum_{k=1}^{\infty}K_0(2\pi ak) + \gamma + \ln\frac{1}{2}a + \frac{1}{2a},$$

where we introduced $a = L\eta\mu/\pi$; K_0 is the modified Bessel function and $\gamma = 0.5772\ldots$ (Euler's constant). In the limit $\eta mL \gg 1$, $a \gg 1$, the sum gives $\gamma + \ln\frac{1}{2}a$ and one readily obtains

$$\mu^2 = m^2 + \frac{4\pi M}{\eta L}\left(\tfrac{1}{2}ae^\gamma\right) = m^2 + 2e^\gamma M\mu, \quad (54)$$

which gives

$$\mu = e^\gamma\left(M + \sqrt{M^2 + e^{-2\gamma}m^2}\right) = \left(M_\gamma + \sqrt{M_\gamma^2 + \frac{e^2}{\pi}}\right), \quad M_\gamma \equiv e^\gamma M. \quad (55)$$

This value of μ corresponds to the effective boson mass parameter in the Hamiltonian (52). Some remarks are in order. Taking $L \to \infty$ corresponds to $\varepsilon \to 0$, ($|k_1| \geq \varepsilon$) in the scalar field theory [Eqs. (15) and (16)]. For obtaining the effects of the nontrivial vacuum one needs to take these limits in the relevant equations [cf. Eqs. (15)–(17)]. Indeed, immediately approaching the light front, $\eta \to 0$ at finite L would not reproduce the boson mass, Eq. (55). This can easily be seen from Eq. (54) in combination with the small-a limit of the infinite sum:

$$\frac{\pi}{L}\sum_{n>0}\left(\frac{1}{p_n}-\frac{1}{E_n}\right) = \frac{1}{2}a^2\xi(3) + O(a^3),$$

where ξ is the Riemann function; $\xi(3) = 1.201\,056\,903\ldots$. Actually, in this limiting case μ diverges as $1/\sqrt{\eta}$.

The next step is to take this Hamiltonian, Eq. (52) with E_n and μ fixed by Eqs. (54) and (55), as the starting one for the transition to the light-cone formulation. Repeating this procedure as outlined in Sec. II for the scalar field theory, we obtain the following effective light-cone Hamiltonian for the massive Schwinger model:

$$P^- =: \int_{-L}^{L} dx^-\left\{\frac{e^2}{2\pi}\varphi_L^2 + \frac{M_\gamma}{2\pi}\left(M_\gamma + \sqrt{M_\gamma^2 + e^2/\pi}\right)\left[1 - \cos\sqrt{4\pi}\varphi_L\right]\right\}:, \quad (56)$$

where $\varphi_L(x)$ is the analogue of light-cone field, defined by the Fourier decomposition

$$\varphi_L(x) = \sum_{n>0}\frac{1}{\sqrt{4Lp_n}}\left(a_n e^{-ip_n x^-} + a_n^\dagger e^{ip_n x^-}\right). \quad (57)$$

Note that the operators a_n, $a_n^\dagger$, $n > 0$ define the light-cone Fock basis. The field $\varphi_L(x)$ can be expressed in terms of light-front fermionic variables:

$$\varphi_L(x) = -\sqrt{\pi}\partial_-^{-1}(\chi_+^\dagger\chi_+).$$

This means that expression (56) can be written on the light cone also in the fermionic basis.

It should be emphasized that the result, Eq. (56), indeed yields a correction to the naive light-cone approach [26]. In the future we hope to address the interesting question how this affects the mass spectrum, in particular for small fermion mass.

IV. DISCUSSION

The most outstanding advantage of light-cone quantization is the simplicity of the vacuum. However, this advantage also poses problems since nontrivial vacuum effects ought to be present. In this work this question is addressed by approximating the theory close to the light cone and, subsequently, deriving an effective light-cone Hamiltonian. As a result, the canonical light-cone Hamiltonian is seen to be modified.

Much research has been done in the last years in order to account for condensates and spontaneous symmetry breaking on the light cone and similar results as ours have been obtained before. One can distinguish different ways of dealing with this problem. The constrained zero-mode approach [10] explicitly considers zero modes of field operators. They are not dynamical degrees of freedom but appear in constraint equations which are derived in the Dirac-Bergman formalism (or via the equations of motion). This way one gets a nontrivial dependence on the nonzero-mode degrees of freedom, which indeed can yield a nontrivial vacuum structure. For scalar field theories spontaneous symmetry breaking was demonstrated. Moreover, the phase transition was shown to be of second order.

Alternatively, it has been argued in [8] that the zero-mode problem can be avoided by a modification of the canonical light-cone quantization. In this diagrammatic approach generalized tadpole diagrams are taken into account which are usually set to zero. Also in this case the usual light-cone Hamiltonian is changed.

Our work is based on another approach where one quantizes on a spacelike plane close to the light cone. The transition to the exact light cone is governed by a parameter. This procedure has been studied in great detail by several groups [15,16]. Its advantages are the following ones. Infrared singularities are controlled; especially in a combination with a finite volume approach. Massless left moving particles are still present. Constraints are often avoided. Finally, it has been shown that the nontrivial vacuum effects can persist in approaching the light cone.

In our paper, these formal advantages are combined with the simplicity of the canonical light-cone approach. As a first step we make a Gaussian approximation to the vacuum in the frame close to the light cone. Indeed condensates and spontaneous symmetry breaking show up. (Because of the Gaussian approximation the phase transition, however, is of first order.) Second, we make the light-cone transition in order to exploit the usual light-cone advantages and techniques. As a remnant of the nontrivial vacuum new terms in the effective Hamiltonian show up. Scalar field theory in (1+1) dimensions and the massive Schwinger model have been successfully worked out this way.

The generalization of this approach to gauge theories in higher dimensions may be attempted with the help of Hamiltonians where the dependent degrees of freedom have been eliminated after gauge fixing [20,27].

ACKNOWLEDGMENTS

This work was supported by the exchange program between the University of Heidelberg and the University of St. Petersburg. We would like to thank Professor F. Lenz, Professor H. C. Pauli, and Professor J. P. Vary, as well as Th. Gasenzer for useful discussions. H.W.L.N. was supported in part by the Federal Ministry of Research and Technology (BMFT) under Contract No. 06 HD 729. E.V.P. was supported in part by the International Science Foundation, Grant No. R2L000, and the Russian Fundamental Science Foundation, Grant No. 94-02-05520-a.

APPENDIX

In this appendix we give clarifications of Eqs. (41), (44), and (47), based on the more general considerations of [22,23]. Let us demonstrate that the expression in Eq. (41) satisfies canonical anticommutation relations. First of all, notice that the representation (41) acts in the Hilbert space spanned by vectors of the form $\prod_{i,j}\{I^\dagger_{n_i,+}\}\{I_{n_j,-}\}|l\rangle$, $n_i>0$, $n_j>0$, where the $I_{n,+}, I^\dagger_{n,-}$ and $I^\dagger_{n,+}$, $I_{n,-}$ act like annihilation and creation operators with respect to "vacuum" states $|l\rangle$ according to Eqs. (37), (38) at $n>0$. Using for these operators the normal ordering symbol :: we can rewrite Eq. (41) in more compact form:

$$\chi_r(y^1)=\frac{1}{\sqrt{2L}}\exp(-i\omega_r)\exp\left\{ri\pi\left(\frac{1}{2}\bar{Q}_r\right)\right\} \times :\exp\{-r2\pi i\partial_1^{-1}(I_r(y^1))\}:, \tag{A1}$$

where we denote by $\bar{Q}_r$ and $\bar{Q}$ the integer valued parts of the charges Q_r and Q ($\bar{Q}_r=Q_r-reLA_1/\pi-\frac{1}{2}$).

Let us consider the products $\chi_r(y^1)\chi^\dagger_{r'}(y^{1'})$ and $\chi^\dagger_{r'}(y^{1'})\chi_r(y^1)$ as a function of $z=\exp(ri\pi y^1L^{-1})$, $z'=\exp(r'i\pi y^{1'}L^{-1})$, taking the operator products in normal ordered form. We get

$$\chi_r(y^1)\chi^\dagger_{r'}(y^{1'})=F_{rr'}(z,z')\exp\left\{i\frac{\pi}{2}(r-r')\right\} \times(z')^{\bar{Q}_{r'}+1}(z)^{-\bar{Q}_r-\delta_{rr'}} \times\exp\left\{\delta_{rr'}\sum_{n>0}\frac{1}{n}\left(\frac{z'}{z}\right)^n\right\}, \tag{A2}$$

and

$$\chi^\dagger_{r'}(y^{1'})\chi_r(y^1)=F_{rr'}(z,z')(z')^{\bar{Q}_{r'}+\delta_{r',-r}}(z)^{-\bar{Q}_r} \times\exp\left\{\delta_{rr'}\sum_{n>0}\frac{1}{n}\left(\frac{z}{z'}\right)^n\right\}, \tag{A3}$$

with the

$$F_{rr'}(z,z')=\frac{1}{2L}e^{-i(\omega_r-\omega_{r'})}e^{i\frac{\pi}{2}(r-r')\bar{Q}} \times:\exp\{2\pi i[r'\partial_1^{-1}(I_r(y^{1'})) - r\partial_1^{-1}(I_r(y^1))]\}:.$$

Notice that $F_{rr}(z,z)=1/2L$.

We see that for $r\neq r'$ expressions (A2) and (A3) differ only by a sign (due to $\exp[i(\pi/2)(r-r')]=-1$). Hence, $\{\chi_r(y^1),\chi^\dagger_{-r}(y^{1'})\}=0$. For $r=r'$, we use the analytical regularization of the type used in [22]. This yields

$$\chi_r(y^1)\chi^\dagger_r(y^{1'})=\lim_{\epsilon\to 0}\frac{1}{2\pi i}\oint_{|z''|=1-\epsilon}\frac{dz''}{z''}\sum_n\left(\frac{z''}{z'}\right)^n\left(\frac{z''}{z}\right)^{\bar{Q}_r+1} \times\left(1-\frac{z''}{z}\right)^{-1}F_{rr}(z,z''), \tag{A4}$$

and

$$\chi_r^\dagger(y^{1'})\chi_r(y^1) = \lim_{\varepsilon\to 0}\frac{1}{2\pi i}\oint_{|z''|=1+\varepsilon}\frac{dz''}{z''}\sum_n\left(\frac{z''}{z'}\right)^n\left(\frac{z''}{z}\right)^{\bar{Q}_r} \times\left(1-\frac{z}{z''}\right)^{-1}F_{rr}(z,z''). \tag{A5}$$

Adding (A4) and (A5), we get

$$\{\chi_r(y^1),\chi_r^\dagger(y^{1'})\} = \lim_{\varepsilon\to 0}\frac{1}{2\pi i}\left(\oint_{|z''|=1+\varepsilon}-\oint_{|z''|=1-\varepsilon}\right)\frac{dz''}{z''-z} \times\sum_n\left(\frac{z''}{z'}\right)^n\left(\frac{z''}{z}\right)^{\bar{Q}_r}F_{rr}(z,z'') = \frac{1}{2L}\sum_n\exp\left(\frac{ri\pi n}{L}(y^1-y^{1'})\right) = \delta(y^1-y^{1'}). \tag{A6}$$

Analogously, one obtains $\{\chi_r(y^1),\chi_{r'}(y^{1'})\}=0$.

To explain Eq. (44) let us consider the ε-regularized charge densities Eq. (30), using Eq. (A3) with the substitutions: $y^1\to y^1+ri\varepsilon/2$, $y^{1'}\to y^1-ri\varepsilon/2$, and expanding in ε up to $O(\varepsilon^2)$. We get

$$I_r(y^1,\varepsilon) = \chi_r^\dagger\left(y^1-\frac{ri\varepsilon}{2}\right)\chi_r\left(y^1+\frac{ri\varepsilon}{2}\right)\exp(r\varepsilon eA_1) = \frac{1}{2\pi\varepsilon}+I_r(y^1)+\pi\varepsilon[I_r(y^1)]^2-\frac{\pi\varepsilon}{48L^2}+O(\varepsilon^2), \tag{A7}$$

in agreement with Eq. (30). Differentiating Eq. (A7) with respect to ε, we obtain

$$\int_{-L}^{L}dy^1\chi_r^\dagger\left(y^1-\frac{ri\varepsilon}{2}\right)iD_i\chi_r\left(y^1+\frac{ri\varepsilon}{2}\right) = -r\left(\frac{L}{\pi\varepsilon^2}+\frac{\pi}{12}\right)+r\pi\int_{-L}^{L}dy^1[I_r(y^1)]^2+O(\varepsilon), \tag{A8}$$

that coincides with Eq. (44) after subtracting the constant and taking the limit $\varepsilon\to 0$.

Equation (47) is a direct consequence of Eq. (A3) and Eq. (41) if it is considered on the physical subspace ($Q=0$). Indeed, from Eq. (A3) we get

$$-\frac{iM}{\eta}(\chi_+^\dagger\chi_- - \chi_-^\dagger\chi_+) = \frac{iM}{2L\eta}(-1)^Q\left[e^{i(\omega_+-\omega_-)}\exp\left(i\frac{\pi y^1}{L}Q\right)e^{2\pi i[\partial_1^{-1}(I_++I_-)]} - e^{-i(\omega_+-\omega_-)}\exp\left(-i\frac{\pi y^1}{L}Q\right)e^{-2\pi i[\partial_1^{-1}(I_++I_-)]}\right], \tag{A9}$$

which indeed coincides with Eq. (47) at $Q=0$.

[1] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
[2] J. R. Klauder, H. Leutwyler, and L. Streit, Nuovo Cimento A **66**, 536 (1970).
[3] A. Casher, Phys. Rev. D **14**, 452 (1976).
[4] V. A. Franke, Yu. V. Novozhilov, and E. V. Prokhvatilov, Lett. Math. Phys. **5**, 239 (1981); **5**, 437 (1981).
[5] R. J. Perry, A. Harindranath, and K. G. Wilson, Phys. Rev. Lett. **65**, 2959 (1990).
[6] A. M. Annenkova, E. V. Prokhvatilov, and V. A. Franke, Phys. At. Nucl. **56**, 813 (1993).
[7] A. M. Annenkova, E. V. Prokhvatilov, and V. A. Franke, Vestn. Leningr. Univ. **4**, 80 (1985).
[8] M. Burkardt, Phys. Rev. D **47**, 4628 (1993).
[9] A. Harindranath and J. P. Vary, Phys. Rev. D **36**, 1141 (1987).
[10] T. Heinzl, S. Krusche, S. Simburger, and E. Werner, Z. Phys. C **56**, 415 (1992); D. G. Robertson, Phys. Rev. D **47**, 2549 (1993); B. van de Sande and S. Pinsky, *ibid.* **49**, 2001 (1994).
[11] A. B. Bylev, E. V. Prokhvatilov, and V. A. Franke, Vestn.

Leningr. Univ. **4**, 8 (1986).
[12] T. Eller, H.-C. Pauli, and S. J. Brodsky, Phys. Rev. D **35**, 1493 (1987).
[13] K. Hornbostel, S. J. Brodsky, and H.-C. Pauli, Phys. Rev. D **41**, 3814 (1990).
[14] A. Tang, H.-C. Pauli, and S. J. Brodsky, Phys. Rev. D **44**, 1842 (1991).
[15] E. V. Prokhvatilov and V. A. Franke, Sov. J. Nucl. Phys. **49**, 688 (1989).
[16] F. Lenz, M. Thies, S. Levit, and K. Yazaki, Ann. Phys. (N.Y.) **208**, 1 (1991); K. Hornbostel, Phys. Rev. D **45**, 3781 (1992).
[17] E. V. Prokhvatilov and V. A. Franke, Sov. J. Nucl. Phys. **47**, 559 (1988).
[18] P. M. Stevenson, Phys. Rev. D **30**, 1712 (1984); **32**, 1389 (1985).
[19] L. Polley and V. Ritschel, Phys. Lett. B **221**, 44 (1989); R. Ibañez-Meier, A. Mattingly, V. Ritschel, and P. M. Stevenson, Phys. Rev. D **45**, 2893 (1992).
[20] F. Lenz, H. W. L. Naus, K. Ohta, and M. Thies, Ann. Phys. (N.Y.) **233**, 17 (1994).
[21] N. S. Manton, Ann. Phys. (N.Y.) **159**, 220 (1985); Y. Nakawaki, Prog. Theor. Phys. **70**, 1105 (1983).
[22] E. V. Prokhvatilov, Theor. Math. Phys. **88**, 685 (1991).
[23] A. Bylev, E. V. Prokhvatilov, and V. A. Franke, Vestn. Leningr. Univ. **4**, 66 (1989).
[24] S. Iso and H. Murayama, Prog. Theor. Phys. **84**, 142 (1990).
[25] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products* (Academic, New York, 1965).
[26] S. Coleman, Ann. Phys. (N.Y.) **101**, 239 (1976).
[27] F. Lenz, H. W. L. Naus, and M. Thies, Ann. Phys. (N.Y.) **233**, 317 (1994).