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Reliability of the estimation of CP asymmetries for nonleptonic decays of B^0 and $\bar{B}^0$ into non-CP eigenstates

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CP asymmetries for two-body nonleptonic decay of B^0 and $\bar{B}^0$ into non-CP eigenstates are calculated using two different methods: (i) the Bauer-Stech-Wirbel factorization method to compute the decay amplitudes directly; (ii) using $B^0, \bar{B}^0$ decay amplitude ratios to avoid the direct computation of the decay amplitudes. A comparison of the results are made.

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I. INTRODUCTION

The CP asymmetries for two-body nonleptonic decays of B^0 and $\bar{B}^0$ have been systematically estimated [1]. As proved in Ref. [1] (see its Appendix), for CP eigenstates f the amplitude ratios

$$\zeta = A(\bar{B}^0 \to f)/A(B^0 \to f) ,$$
$$\bar{\zeta} = A(B^0 \to \bar{f})/A(\bar{B}^0 \to \bar{f})$$

depend only on Kobayashi-Mashawa (KM) matrix elements. But if the final state f is not a CP eigenstate, ζ, $\bar{\zeta}$ are not pure KM factors. In the estimation of CP asymmetries in Ref. [1], the pure KM factor approximation for ζ and $\bar{\zeta}$ is also used for non-CP eigenstates, such as $D^{\pm}\pi^{\mp}$, etc. But how reliable are these estimations? In this Brief Report, we first compute the CP asymmetries for the decays of B^0 and $\bar{B}^0$ into non-CP eigenstates using the Bauer-Stech-Wirbel factorization method [2]. Then we compare these results with those by using the amplitude ratios ζ and $\bar{\zeta}$. In Sec. II we present all the results of the two different methods. Section III is devoted to the discussions and conclusions.

II. COMPUTATION OF THE PARTIAL-DECAY-RATE ASYMMETRIES

For simplicity of comparison, we consider only incoherent B_d^0, $\bar{B}_d^0$ mesons. For instance, sitting on the Z^0 resonance, $b\bar{b}$ pairs will be produced in the form of $B_d^0 B_u^-$ and $\bar{B}_d^0 B_u^+$. Here B_d^0, $\bar{B}_d^0$ are produced incoherently and observing the charge of $B_u^-(B_u^+)$ would confirm the decayed neutral meson to be $B_d^0(\bar{B}_d^0)$. In the decays of incoherent B_d^0, $\bar{B}_d^0$ mesons, we can define the CP asymmetry parameter as

$$a_f(t) = \frac{\Gamma(B^0_{d,\rm phys}(t) \to f) - \Gamma(\bar{B}^0_{d,\rm phys}(t) \to \bar{f})}{\Gamma(B^0_{d,\rm phys}(t) \to f) + \Gamma(\bar{B}^0_{d,\rm phys}(t) \to \bar{f})} \qquad (2.1)$$

where

$$|B^0_{d,\rm phys}(t)\rangle = f_+(t)|B_d^0\rangle + \frac{q}{p} f_-(t)|\bar{B}_d^0\rangle ,$$
$$|\bar{B}^0_{d,\rm phys}(t)\rangle = \frac{p}{q} f_-(t)|B_d^0\rangle + f_+(t)|\bar{B}_d^0\rangle , \qquad (2.2)$$

$$|B_L\rangle = p|B_d^0\rangle + q|\bar{B}_d^0\rangle ,$$
$$|B_H\rangle = p|B_d^0\rangle - q|\bar{B}_d^0\rangle , \qquad (2.3)$$

$$f_\pm(t) = \tfrac{1}{2}(e^{-i\lambda_L t} \pm e^{-i\lambda_H t}) ,$$
$$\lambda_{L,H} = m_{L,H} - \frac{i}{2}\Gamma_{L,H} . \qquad (2.4)$$

Integrating with time t from zero to infinity we can obtain the integrated CP asymmetry

$$\mathcal{A}_f = \int_0^\infty dt \;\; a_f(t) . \qquad (2.5)$$

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Now we first compute the asymmetry parameter $\mathcal{A}_f$ in the Bauer-Stech-Wirbel (BSW) scheme.

We take $B_d^0 \to D^+\pi^-$ as an example for the purpose of illustration.

For $B_d^0 \to D^-\pi^+$, the effective Hamiltonian is [2]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} \{ a_1 [\bar{b}\gamma_\mu (1-\gamma_5) c]_H [\bar{u}\gamma^\mu (1-\gamma_5) d]_H + a_2 [\bar{b}\gamma_\mu (1-\gamma_5) d]_H [\bar{u}\gamma^\mu (1-\gamma_5) c]_H \} + \text{H.c.} \tag{2.6}$$

Neglecting the contribution of the exchange diagram we have

$$\langle D^-\pi^+ | \mathcal{H}_{\text{eff}}(0) | B_d^- \rangle \cong \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} i f_\pi a_1 p_\pi^\mu \langle D^- | \bar{b}\gamma_\mu (1-\gamma_5) c | B_d^0 \rangle \tag{2.7}$$

where

$$\langle \pi^+ | \bar{u}\gamma^\mu (1-\gamma_5) d | 0 \rangle = i f_\pi p_\pi^\mu$$

has been used.

Using

$$\langle D^- | \bar{b}\gamma_\mu (1-\gamma_5) c | B_d^0 \rangle = \left((p_B + p_D) - \frac{m_B^2 - m_D^2}{q^2} q \right)_\mu F_1(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0(q^2) \tag{2.8}$$

and $q_\mu = (p_B - p_D)_\mu$, we finally obtain

$$\langle D^-\pi^+ | \mathcal{H}_{\text{eff}}(0) | B_d^0 \rangle \approx \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} i f_\pi a_1 (m_B^2 - m_D^2) F_0^{BD}(m_\pi^2) \ . \tag{2.9}$$

Similarly we can get

$$\langle D^-\pi^+ | \mathcal{H}_{\text{eff}}(0) | \bar{B}_d^0 \rangle \approx \frac{G_F}{\sqrt{2}} V_{ub} V_{cd}^* i f_D a_1 (m_B^2 - m_\pi^2) F_0^{B\pi}(m_D^2) \ . \tag{2.10}$$

Also

$$\begin{aligned} \langle D^+\pi^- | \mathcal{H}_{\text{eff}}(0) | B_d^0 \rangle &\approx \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cd} i f_D a_1 (m_B^2 - m_\pi^2) F_0^{B\pi}(m_D^2) \ , \\ \langle D^+\pi^- | \mathcal{H}_{\text{eff}}(0) | \bar{B}_d^0 \rangle &\approx \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} i f_\pi a_1 (m_B^2 - m_D^2) F_0^{BD}(m_\pi^2) \ . \end{aligned} \tag{2.11}$$

The CP asymmetry parameter

$$a_{D^-\pi^+}(t) = \frac{|\langle D^-\pi^+ | \mathcal{H}_{\text{eff}} | B_{d,\text{phys}}^0(t) \rangle|^2 - |\langle D^+\pi^- | \mathcal{H}_{\text{eff}} | \bar{B}_{d,\text{phys}}^0(t) \rangle|^2}{|\langle D^-\pi^+ | \mathcal{H}_{\text{eff}} | B_{d,\text{phys}}^0(t) \rangle|^2 + |\langle D^+\pi^- | \mathcal{H}_{\text{eff}} | \bar{B}_{d,\text{phys}}^0(t) \rangle|^2} .$$

After the time integration we have

$$\mathcal{A}_{D^-\pi^+} = -\frac{0.514 f_\pi f_D m_B^2 (m_B^2 - m_D^2) x_d \text{Im}(V_{tb}^* V_{td} V_{cb} V_{ub} V_{ud}^* V_{cd}^* / V_{tb} V_{td}^*)}{0.476 f_\pi^2 (m_B^2 - m_D^2)^2 |V_{ud}^* V_{cb}|^2 (2 + x_d^2) + 0.138 f_D^2 m_B^4 x_d^2 |V_{cd}^* V_{ub}|^2} \ , \tag{2.12}$$

where the values of the form factors are taken from Ref. [2].

For the KM factors, we use the Wolfenstein parametrization [3]

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} . \tag{2.13}$$

From the updated fit [4]

$$A \sim 0.84, \quad \lambda \sim 0.22, \quad \sqrt{\rho^2 + \eta^2} \sim 0.36 \ . \tag{2.14}$$

The values of ρ and η depend on the top quark mass m_t and $f_{B_d}(B_{B_d})^{1/2}$. For the purpose of illustration, we take $m_t \sim 174$ GeV, $f_{B_d}(B_{B_d})^{1/2} \sim 180$ MeV, and [4]

$$\rho = -0.05, \quad \eta = 0.33 \ . \tag{2.15}$$

The other parameters are taken as

$$\begin{aligned} & f_\pi = 0.13 \text{ GeV}, \quad f_K = f_D = 0.16 \text{ GeV} \ , \\ & m_{B_d} = 5.28 \text{ GeV}, \quad m_{D^+} = 1087 \text{ MeV} \ , \\ & x_d \sim 0.7 \ . \end{aligned} \tag{2.16}$$

Substituting all these parameters into Eq. (2.12), we get the time-integrated CP asymmetry

$$\mathcal{A}_{D^-\pi^+} = -5.0 \times 10^{-3} \ . \tag{2.17}$$

For the final states involving vector mesons, we use

$$\langle 0 \mid V_\mu \mid V \rangle = \lambda_V m_V^2 \epsilon_\mu(V) \tag{2.18}$$

and take

$$\lambda_{D^*} = 0.14, \quad \lambda_\rho = 0.24 \ . \tag{2.19}$$

Thus, we can use BSW method to compute the asymmetries for different processes. We put all these results in Table I. Note that for the final state f for which $B_d^0 \to f$ can occur but $\bar{B}_d^0 \to f$ cannot, there will be no CP asymmetry.

In order to compare these results with those by use of amplitude ratios, we compute the same CP asymmetries by use of the amplitude ratios as in Ref. [1] but use the same KM parameters as in BSW method. The results are presented also in Table I [denoted by the AR (amplitude ratios) method].

TABLE I. Asymmetries in the BSW and AR schemes. Here BSW means the Bauer-Stech-Wirbel method [2], AR means the amplitude ratio method [1].

Process	$\mathcal{A}_f$(BSW)	$\mathcal{A}_f$(AR)
$B_d^0 \to D^-\pi^+$	-5.0×10^{-3}	-6.6×10^{-3}
$D^+\pi^-$	-2.6×10^{-2}	-3.4×10^{-2}
$D^{*-}\pi^+$	9.1×10^{-3}	6.8×10^{-3}
$D^-\rho^+$	2.3×10^{-3}	6.8×10^{-3}
$\pi^-\rho^+$	0.24	0.37
$D^{*-}D^+$	-0.34	-0.25
D^-D^{*+}	-0.15	-0.26
π^-D^{*+}	4.6×10^{-2}	3.4×10^{-2}
$\pi^+\rho^-$	0.48	0.37
$\bar{D}^0\pi^0$	-7.0×10^{-3}	-6.8×10^{-3}
$\bar{D}^0\eta$	-7.0×10^{-3}	-6.8×10^{-3}
$\bar{D}^0\eta'$	-7.0×10^{-3}	-6.8×10^{-3}
$\bar{D}^{*0}\pi^0$	-6.9×10^{-3}	-6.8×10^{-3}
$\bar{D}^{*0}\eta$	-6.9×10^{-3}	-6.8×10^{-3}
$\bar{D}^{*0}\eta'$	-6.9×10^{-3}	-6.8×10^{-3}
$\bar{D}^0\rho^0$	-7.1×10^{-3}	-6.8×10^{-3}
$\bar{D}^0\omega^0$	-7.1×10^{-3}	-6.8×10^{-3}
$D^0\pi^0$	-3.5×10^{-2}	-3.4×10^{-2}
$D^0\eta$	-3.5×10^{-2}	-3.4×10^{-2}
$D^0\eta'$	-3.5×10^{-2}	-3.4×10^{-2}
$D^{*0}\pi^0$	-3.5×10^{-2}	-3.4×10^{-2}
$D^{*0}\eta$	-3.5×10^{-2}	-3.4×10^{-2}
$D^{*0}\eta'$	-3.5×10^{-2}	-3.4×10^{-2}
$D^0\rho^0$	-3.6×10^{-2}	-3.4×10^{-2}
$D^0\omega^0$	-3.6×10^{-2}	-3.4×10^{-2}

III. DISCUSSIONS AND CONCLUSIONS

In Table I, we show both the results of the BSW and AR methods. From that table we can see that, for most of the processes listed there, the asymmetries of both methods are very close to each other. For very few processes, such as $f = D^-\rho^+$, there is a large discrepancy, but at most a factor 2 difference. So on the whole they agree with each other. At least the order of magnitudes of the CP asymmetries is reliable. But, if we want to use these CP asymmetries to make a precision test of the standard model, it is not good at all. For other purposes, the CP asymmetries computed by both, the BSW and AR methods still can be used. We must remind the reader that we are now talking about the non-CP eigenstates. If the final state is a CP eigenstate, the AR method can give a reliable prediction of the CP asymmetry.

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[1] Donsheng Du, Isard Dunietz, and Dandi Wu, Phys. Rev. D **34**, 3414 (1986), and references therein.
[2] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987).
[3] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
[4] A. Ali and D. London, Phys. Lett. B (to be published).