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Friedmann universes connected by Reissner-Nordström wormholes: Quantum effects

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In a model of a wormhole consisting of closed Friedmann regions connected by a Reissner-Nordström black hole previously introduced, quantum effects developing in strong curvature regions are investigated. The lack of appropriate theoretical instruments limits the analysis to a two-dimensional section of the original model. We show that inside the wormhole, at the Cauchy horizon, the two-dimensional curvature blows up stronger as compared to the classical case. The resulting divergence is still mild enough for the tidal impulse to stay finite. The extension of this result to four dimensions is, however, rather problematic.

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I. INTRODUCTION

In a previous paper [1] (hereafter referred to as paper I) we introduced a simple model describing a wormhole connecting two large cosmological regions having a closed Friedmann geometry. The wormhole is represented as a part of a Reissner-Nordström space-time. Unlike the well-known Einstein-Rosen bridge, this wormhole does not pinch off, allowing in principle an observer to travel safely from one universe to the other. As we have shown in paper I, this eventuality is rather doubtful as the Reissner-Nordström wormhole is highly unstable against the accretion of cosmological matter onto the black hole. This triggers the formation of a curvature singularity at the Cauchy horizon of the wormhole. However, as the tidal distortions acting near the Cauchy horizon on a free falling observer remain finite, the character of the singularity turns out to be very mild and the wormhole appears to be traversable.

All our analysis, and therefore the conclusion, relied on classical physics (general relativity). But as the curvature grows up near the singularity, towards Planckian values, one expects the influence of quantum effects to become more and more relevant in this scenario. In the absence of a coherent quantum theory of gravity, one cannot hope to follow the evolution of the space-time up to the point where the curvature actually reaches Planck levels. One is confined to a study of quantum effects due to fields other than gravity during the incipiently quantum ("semiclassical") era when a classical description of the space-time geometry still makes approximate sense [2].

Even within this framework (i.e., quantum field theory in curved space-times) the study of vacuum polarization and particle creation effects in our wormhole model (and for black hole interiors in general) seems at the present not yet feasible (see Refs. [3,4]) for a preliminary attempt towards this direction).

In this paper we shall confine ourselves to an investigation of quantum effects in a two-dimensional (2D) reduction (the so-called "s wave sector") of our wormhole space-time, in the hope to get some insight on the real theory. These kinds of 2D models are often proposed as theoretical laboratory for investigating the fundamental issues of black hole physics.

We shall in particular analyze the vacuum polarization induced in the wormhole by the presence of conformally invariant 2D fields coupled to gravity. This is done by computing the expectation values of the associated stress-energy tensor operators which are shown to be regular on the inner horizon but to diverge on the Cauchy horizon of the wormhole.

Finally within the framework of a 2D theory of semiclassical gravity, we analyze in a self-consistent manner the back reaction of the vacuum polarization onto the space-time. Again the resulting geometry turns out to be singular on the Cauchy horizon and the singularity appears stronger than in the classical case.

II. THE WORMHOLE MODEL: CLASSICAL THEORY

In this section we outline the classical analysis of the Reissner-Nordström (RN) wormhole connecting two Friedmann-Robertson-Walker (FRW) closed universes.

The details can be found in paper I.

The space-time under investigation consists of a RN black hole glued to a closed dust-filled FRW universe along a spherical timelike thin shell Σ comoving with respect to the FRW geometry. The shell ends its trajectory on the right-hand side (RHS) singularity $r = 0$ of the RN metric (see Fig. 1).

The space-time admits a Cauchy horizon (CH) which is located inside the inner horizon of the RN black hole, both of them being regular and null hypersurfaces. As Σ is comoving with the cosmological matter there is no energy flow from the cosmological region onto the black hole.

If one allows for the existence of an influx, here modeled as a null fluid perturbation in the FRW region, the RN geometry must be replaced by a charged Vaidya one and dramatic effects have been shown to occur at the CH.

Let us first consider the RN-FRW junction along the comoving shell Σ as depicted in Fig. 1. The metric of the RN region can be written as

$$\begin{aligned} ds^2 &= -f_0 dt^2 + f_0^{-1} dr^2 + r^2 d\Omega^2 \\ &= -f_0 dv^2 + 2dvdr + r^2 d\Omega^2 , \end{aligned} \tag{2.1}$$

where

$$f_0 = 1 - 2m_0/r + e^2/r^2 , \tag{2.2}$$

v is Eddington-Finkelstein advanced time, $d\Omega^2$ is the usual line element on S^2, m_0 and e are the constant mass and charge of the hole, respectively ($m_0 > |e|$). We shall denote by r_0 the black hole inner horizon radius and by k_0 its surface gravity:

$$r_0 = m_0 - (m_0^2 - e^2)^{1/2} , \quad k_0 = \frac{(m_0^2 - e^2)^{1/2}}{r_0^2} . \tag{2.3}$$

The metric of the FRW region is

$$ds_*^2 = -dt_*^2 + a^2(t_*)(d\chi^2 + \sin^2\chi d\Omega^2) \tag{2.4}$$

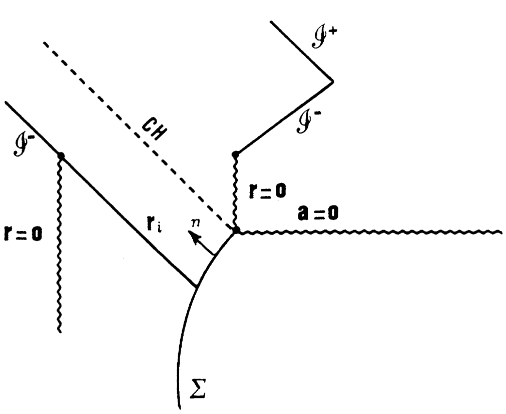

FIG. 1. Conformal diagram of part of the Reissner-Nordström wormhole.

or equivalently

$$ds_*^2 = a^2 dv_*(2d\chi - dv_*) + a^2 \sin^2\chi d\Omega^2, \tag{2.5}$$

where we have introduced a null advanced time v_* as

$$dv_* = \frac{dt_*}{a(t_*)} + d\chi . \tag{2.6}$$

a is the scale factor and according to the Einstein equations it satisfies

$$\left(\frac{da_*}{dt}\right)^2 = \frac{2M}{a} - 1 \tag{2.7}$$

with $M = 4\pi\rho_* a^3/3 =$ const and ρ_* the energy density of the cosmological fluid. The metric on the singular hypersurface Σ is

$$ds^2|_\Sigma = -d\tau^2 + R^2(\tau)d\Omega^2 , \tag{2.8}$$

where τ is the proper time along the shell and $R(\tau)$ its radius.

As the shell is comoving, $\chi = \chi_0 =$ const on Σ, and the matching relations imply that

$$t_* = \tau , \quad R(\tau) = a(\tau) \sin\chi_0 . \tag{2.9}$$

The advanced times v and v_* are related on Σ by

$$\frac{dv}{dv_*} = a\left(\dot{t} + \frac{\dot{R}}{f_0(R)}\right) , \tag{2.10}$$

where an overdot means $d/d\tau$. Using the normalization condition for the shell velocity this equation can also be written as

$$\frac{dv}{dv_*} = \frac{a}{\epsilon F - \dot{R}} , \tag{2.11}$$

where

$$F = [f_0(R) + \dot{R}^2]^{1/2} \tag{2.12}$$

and $\epsilon = \pm 1$ such that

$$\frac{dt}{d\tau} = \epsilon \frac{F}{f_0} . \tag{2.13}$$

We now allow for a null fluid flow from the FRW region onto the black hole. Let us assume the influx to be small enough so that it can be considered as a negligible perturbation in the FRW region, without any effect on its geometry, and that it does not interact with the shell which thus remains comoving ($\chi = \chi_0$).

Relations (2.4)–(2.9) for the FRW and shell metrics therefore still apply. However the RN geometry of the black hole has to be replaced, because of the influx, by a charged Vaidya solution: namely,

$$ds^2 = dv(2dr - fdv) + r^2 d\Omega^2 , \tag{2.14}$$

where now

$$f = 1 - 2m(v)/r + e^2/r^2 . \tag{2.15}$$

The stress energy tensor of the in-going null fluid perturbation is represented as

$$T^{*\rm rad}_{\alpha\beta} = \frac{L^*(v_*)}{4\pi a^2 \sin^2\chi} l^*_\alpha l^*_\beta \ , \tag{2.16}$$

where $l^*_\alpha = -\partial_\alpha v_*$ and $L^*(v_*)$ is an unspecified luminosity function (later to be taken as a constant for simplicity).

The relation between the advanced times along Σ is now

$$\frac{dv}{dv_*} = \frac{1}{f}\frac{dR}{dv_*}[1+\sqrt{1+a^2 f(dR/dv_*)^{-2}}] \ , \tag{2.17}$$

where

$$\frac{dR}{dv_*} = -R\left(\frac{2M\sin\chi_0}{R} - 1\right)^{1/2} . \tag{2.18}$$

It has been shown, in paper I, that the influx causes the mass function $m(v)$ entering Eq. (2.15) to diverge to $-\infty$ as one approaches the CH ($v = v_0$ or $v_* = 0$). The following relation can be obtained in the above limit for a constant L_*:

$$m = -\gamma^* v_*^{-4} \quad \text{or} \quad m = -\gamma(v - v_0)^{-2/3} \ , \tag{2.19}$$

where $\gamma_* = L_*^2 M^{-3} \sin^{-1}\chi_0$ and $\gamma = L_*^{4/3}(24)^{-2/3} M^{-1}\sin^{1/3}\chi_0$.

We quote also for later use the following relations valid in the limit $v_* \to 0$:

$$R \simeq \frac{M}{2}\sin\chi_0 v_*^2 \tag{2.20}$$

and

$$\frac{dv}{dv_*} \simeq \beta v_*^5 \tag{2.21}$$

or

$$v - v_0 = \frac{\beta}{6} v_*^6 \ , \tag{2.22}$$

where $\beta = M^3 \sin\chi_0 (4L_*)^{-1}$.

In spite of the divergence of the curvature at the CH induced by Eq. (2.19), the metric is regular there and can be approximated as

$$ds^2 = -\frac{2}{r(u,v)} du dv + r^2 d\Omega^2 \ , \tag{2.23}$$

where we have introduced the null retarded coordinate u such that

$$du = -m(v)dv - r dr \ . \tag{2.24}$$

Integration of this leads to

$$\frac{r^2}{2} = 3\gamma(v - v_0)^{1/3} - u \ . \tag{2.25}$$

The regular behavior of the metric is an indication of the mildness of the singularity: although tidal forces diverge on the CH, the tidal distortion remains finite making the wormhole traversable.

III. QUANTUM EFFECTS: INNER HORIZON

For the reason stressed in the Introduction, the study of quantum effects is feasible only in a dimensional reduction of our model. We begin this study by considering the vacuum polarization associated with conformally invariant massless matter fields propagating on a 2D section of our wormhole spacetime, obtained by setting $\theta = \phi = $ const in the line elements.

In this section we concentrate our attention to the region close to the inner horizon of the black hole where the effect of the accretion can be considered small and the space-time metric reasonably approximate by a RN solution.

It is well known [2] that the inner horizon of a RN black hole is suspected to be highly unstable under perturbations of quantum origin. To show this, consider the 2D section of the RN metric, Eq. (2.1), rewritten in terms of two null Eddington-Finkelstein coordinates u, v:

$$ds^2 = -(1 - 2m_0/r + e^2/r^2) du dv \ , \tag{3.1}$$

where

$$u = t - r_* \ , \quad v = t + r_* \tag{3.2}$$

and

$$r_* = \int f_0^{-1} dr \ . \tag{3.3}$$

The expectation value $\langle T_{ab}\rangle$ of the stress-energy tensor operator for conformally invariant matter field propagating on a 2D spacetime, described by a conformally flat metric

$$ds^2 = -C(\tilde u, \tilde v) d\tilde u d\tilde v \ , \tag{3.4}$$

is determined by the conservation equations $\langle T^a_{b;a}\rangle = 0$ and the trace anomaly. Namely in the conformal gauge of Eq. (3.4) we have (see Ref. [2])

$$\langle T_{\tilde u\tilde u}\rangle = -(12\pi)^{-1} C^{1/2}(C^{-1/2})_{,\tilde u\tilde u} + f(\tilde u) \ , \tag{3.5a}$$

$$\langle T_{\tilde v\tilde v}\rangle = -(12\pi)^{-1} C^{1/2}(C^{-1/2})_{,\tilde v\tilde v} + g(\tilde v) \ , \tag{3.5b}$$

$$\langle T_{\tilde u\tilde v}\rangle = \langle T_{\tilde v\tilde u}\rangle = -(96\pi)^{-1} C R \ . \tag{3.5c}$$

R here is the 2D scalar curvature

$$R + 4C^{-3}(C C_{,\tilde u\tilde v} - C_{,\tilde u} C_{,\tilde v}) \ . \tag{3.6}$$

f and g are arbitrary functions of their arguments, reflecting the nonlocal character of $\langle T_{ab}\rangle$: they depend on the choice of the quantum state in which the expectation values are to be taken.

If the expectation values are taken in a state $|\hat u, \hat v\rangle$, f and g represent the expectation values of $T_{\tilde u\tilde u}$ and $T_{\tilde v\tilde v}$, respectively, evaluated in the $|\hat u\hat v\rangle$ state, but normal ordered with respect to the $|\tilde{u},\tilde{v}\rangle$ state. One can show that f (g) can be simply expressed in terms of the Schwarzian derivative of $\hat{u}$ ($\hat{v}$) with respect to $\tilde{u}$ ($\tilde{v}$).

For the 2D RN metric of Eq. (3.1) we have

$$\langle T_{uu}\rangle = (24\pi)^{-1}\left(-\frac{m_0}{r^3}+\frac{3m_0^2}{2r^4}+\frac{3e^2}{2r^4}-\frac{3m_0e^2}{r^5}+\frac{e^4}{r^6}\right)+f(u) \tag{3.7a}$$

$$\langle T_{vv}\rangle = (24\pi)^{-1}\left(-\frac{m_0}{r^3}+\frac{3m_0^2}{2r^4}+\frac{3e^2}{2r^4}-\frac{3m_0e^2}{r^5}+\frac{e^4}{r^6}\right)+g(v) \tag{3.7b}$$

$$\langle T_{uv}\rangle = (96\pi)^{-1}f_0\left(-\frac{4m_0}{r^3}+\frac{6e^2}{r^4}\right) . \tag{3.7c}$$

Quantum states that appear to contain no quanta according to inertial asymptotic observers in the past are characterized by $g(v)=0$. This because v modes $e^{i\omega v}$ agree asymptotically with standard Minkowski ingoing wave.

The quantum state describing black hole evaporation (Unruh state) has $g(v)=0$ and $f(u)=(k_+)^2(48\pi)^{-1}$ where k_+ is the surface gravity of the event (exterior) horizon of the black hole. If one is dealing instead with black hole in thermal equilibrium with its own quantum radiation (neglecting spontaneous discharge) one has $f(v)=g(u)=(48\pi)^{-1}(k_+)^2$, which characterizes the Hartle-Hawking state.

In order to investigate the behavior of $\langle T_{ab}\rangle$ on the inner horizon ($r=r_0$), one needs to express the stress tensor components in coordinates well behaved on this horizon, namely, Kruskal U,V coordinates related to the u,v one as follows:

$$U=-\exp(-k_0u) , \tag{3.8a}$$

$$V=-\exp(-k_0v) . \tag{3.8b}$$

One then finds that for both the above choices of quantum state, the stress tensor behaves on $r_0(V=0)$ as

$$\langle T_{VV}\rangle \sim -\alpha V^{-2} , \tag{3.9a}$$

$$\langle T_{UV}\rangle \sim \text{regular function} , \tag{3.9b}$$

$$\langle T_{UU}\rangle \sim \text{regular function} , \tag{3.9c}$$

where α is a positive constant, taking the value $(48\pi)^{-1}k_0^2$ for the Unruh state and $(48\pi)^{-1}(k_0^2-k_+^2)$ for the Hartle-Hawking one.

Rewritten in terms of the original u,v coordinates the limiting behaviors are

$$\langle T_{vv}\rangle \simeq -\alpha , \tag{3.10a}$$

$$\langle T_{uv}\rangle \simeq 0 , \tag{3.10b}$$

$$\langle T_{uu}\rangle \sim \text{regular function} . \tag{3.10c}$$

From Eqs. (3.9) one concludes that a free-falling observer measures an (exponential) infinite flux of negative-energy radiation propagating inwards the hole along the inner horizon of the RN black hole. This flux can be regarded as negative-energy Hawking radiation at a temperature proportional to the surface gravity k_0 of the inner horizon.

In general one can show that, for an arbitrary state, $\langle T_{ab}\rangle$ is regular on the inner horizon if the following condition is are satisfied (Ref. [5]): (i) $\langle T_{vv}\rangle$ vanishes at least as $(r-r_0)^2$ for $r\to r_0$; (ii) $\langle T_{vu}\rangle$ vanishes as $(r-r_0)$ for $r\to r_0$; (iii) $\langle T_{uu}\rangle$ regular as $r\to r_0$. It is evident that condition (i) is not satisfied by the Unruh and the Hartle-Hawking state and therefore the inner horizon for an asymptotically flat RN spacetime is highly pathological.

Let us come back to our original model. Asymptotic flatness is not a characteristic of our space-time, as the RN wormhole is immersed in a closed FRW universe. We thus have to decide which is the appropriate state for discussing the quantum matter fields in the wormhole. A careful analysis of the boundary conditions will help us to answer this question. We shall show that for a large class of quantum state, which we argue to contain the relevant one for our model, the corresponding $\langle T_{ab}\rangle$, unlike the previous cases, satisfy all the above requirements.

The basic point to note is that in our spacetime the ingoing modes entering the wormhole originate in the cosmological region and then propagate through Σ in the wormhole. It appears therefore quite natural to assume that the relevant quantum state for the matter fields in our model should be one of the conformal vacuum defined in the cosmological region.

As a first try let us consider the "v_* vacuum," i.e., the one constructed in terms of ingoing v_* modes. This implies that the corresponding $g(v)$ function specifying the state in Eq. (3.7b) should be expressed in terms of the Schwarzian derivative of V_* with respect to v.

For computational reasons it appears easier to introduce the inverse function $v=B(v_*)$ and to use well-known properties of the Schwarzian derivative. We write $g(v)$ as

$$g(v) = -(24\pi)^{-1}\left[\frac{3}{2}\left(\frac{B''}{B'}\right)^2-\frac{B'''}{B'}\right](B')^{-2} , \tag{3.11}$$

where a prime means d/dv_*.

All quantities appearing in Eq. (3.11) should be calculated on the shell Σ separating the RN region from the cosmological one. In particular B' is given by Eq. (2.11). Explicit calculation then yields

$$\frac{B''}{B'^2} = H(\epsilon F-\dot{R})-(\epsilon\dot{F}-\ddot{R}) , \tag{3.12a}$$

$$\frac{B'''}{B'^3} = \left(\frac{\ddot{R}}{R}+H^2\right)(\epsilon F-\dot{R})^2-3H(\epsilon\dot{F}-\ddot{R})(\epsilon F-\dot{R}) +2(\epsilon\dot{F}-\ddot{R})^2-(\epsilon\ddot{F}-\dddot{R})(\epsilon F-\dot{R}) , \tag{3.12b}$$

where we have introduced the Hubble parameter $H \equiv \dot{a}/a$.

The complete expression of $\langle T_{ab}\rangle$ is thus given by Eqs. (3.7) where $g(v)$ is determined by (3.11) and (3.12) and $f(u)$ can be chosen such that $\langle T_{ab}\rangle$ is regular on the event horizon f_+ like in the Unruh and Hartle-Hawking state, i.e., $f(u) = (48\pi)^{-1}k_+^2$. The striking feature is that the quantum state so defined gives a $\langle T_{ab}\rangle$ which is regular on the inner horizon r_0, as we shall see.

Taking the limit $r \to r_0$ in Eq. (3.7b), we have that the first term in $\langle T_{vv}\rangle$ approaches $-(48\pi)^{-1}k_0^2$. In order to take the same limit in $g(v)$, one has to note that since $f_0 < 0$ and $\epsilon = -1$, $\epsilon F - \dot{R} \to 0$, then $B''/B'^2 \to k_0$ and $B'''(B')^{-3} \to 2k_0^2$. It follows that $g(v) \to (48\pi)^{-1}k_0^2$ which cancels exactly the contribution of the other term. One can then show that $\langle T_{vv}\rangle \to 0$ like $(r - r_0)^2$.

As also $\langle T_{uv}\rangle \to 0$ like $(r - r_0)$ and $\langle T_{uu}\rangle$ is regular in the above limit, we can conclude that $\langle T_{ab}\rangle$ calculated in the above state is regular on the inner horizon of the wormhole as the regularity conditions (i), (ii), and (iii) are all satisfied. This should not be regarded as a miraculous cancellation which makes the "v_* vacuum" rather peculiar. The same result holds for any conformal vacuum defined in the cosmological region.

Suppose for instance that we define a "cosmological vacuum state" with respect some other null advanced time $\tilde{v}$ in the FRW region related to v_* by $v_* = C(\tilde{v})$, where C is a function of its argument regular for $r \to r_0$. This state differs from the previous v_* one by the contribution of a conserved massless radiation flowing inwards. The function $g(v)$, entering $\langle T_{vv}\rangle$, which characterizes this state is now given by

$$g(v) = -(24\pi)^{-1}\left[\frac{3}{2}\left(\frac{B''^2}{B'}\right) - \frac{B'''}{B'}\right](B')^{-2} - (24\pi)^{-1}\left[\frac{3}{2}\left(\frac{C''^2}{C'}\right) - \frac{C'''}{C'}\right](C')^{-2}(B')^{-2} . \tag{3.13}$$

In the limit $r \to r_0$, $(B')^{-1} \to 0$ and the additional term vanishes, recovering the previous result.

IV. QUANTUM EFFECTS: CAUCHY HORIZON

In the previous section we established the regular behavior of $\langle T_{ab}\rangle$ on the inner horizon for a large variety of quantum states. Here we make a similar analysis for the Cauchy horizon of the wormhole and the disappointing conclusion we reach is that quantum effects will not remove the singular behavior existing in classical theory.

It has been shown in Sec. II that the influx from the cosmological region has a dramatic effect on the gravitational field near the CH: it drives the mass function of the metric to diverge to minus infinity. Nevertheless the metric is regular there and the 2D section is given by

$$ds^2 = -\frac{2}{r(u,v)}du dv , \tag{4.1}$$

where [see Eqs. (1.23)–(2.25)]

$$\frac{r^2}{2} = 3\gamma(v - v_0)^{1/3} - u \tag{4.2}$$

and the mass function behaves as

$$m(v) = -\gamma(v - v_0)^{-2/3} , \tag{4.3}$$

$v = v_0$ being the location of the CH.

It is now straightforward to evaluate, with the techniques of the previous section, $\langle T_{ab}\rangle$ using Eqs. (3.5) and (3.6). One finds, as leading behavior,

$$\langle T_{vv}\rangle = (24\pi)^{-1}\left(\frac{m'}{r^2} + \frac{3m^2}{2r^4}\right) + g(v) , \tag{4.4a}$$

$$\langle T_{uu}\rangle = (16\pi)^{-1}r^{-4} + f(u) , \tag{4.4b}$$

$$\langle T_{uv}\rangle = -(12\pi)^{-1}m/r^4 , \tag{4.4c}$$

where $m' \equiv dm/dv$.

Let us evaluate the expectation values in the v_* vacuum. One then needs the Schwarzian derivative of v_* with respect to v. The relation between the two can be approximated near the CH by Eq. (2.22): namely,

$$v - v_0 = \frac{\beta}{6}v_*^6 . \tag{4.5}$$

Using Eq. (3.11) one gets, for this quantum state,

$$g(v) = -\frac{35}{48\pi}\beta^{-2}v_*^{-12} , \tag{4.6}$$

and the stress tensor has the expression

$$\langle T_{vv}\rangle = (24\pi)^{-1}\frac{\gamma_*^2}{r^2}\left(\frac{4}{\gamma_*\beta v_*^{10}} + \frac{3}{2r^2v_*^8}\right) - \frac{35}{48\pi}\frac{1}{\beta^2 v_*^{12}} , \tag{4.7a}$$

$$\langle T_{uu}\rangle = (16\pi)^{-1}r^{-4} + f(u) , \tag{4.7b}$$

$$\langle T_{uv}\rangle = (12\pi)^{-1}\frac{1}{r^4}\gamma_* v_*^{-4} . \tag{4.7c}$$

The function $f(u)$ differs from the value given in the previous section by some finite normal ordering term, since the coordinate u here introduced differs from the Eddington-Finkelstein one used in Sec. III. This fact will not affect our conclusions.

The leading behavior on the CH $v = v_0$ is therefore

$$\langle T_{vv}\rangle \sim -D(v-v_0)^{-2} \ , \tag{4.8a}$$

$$\langle T_{uu}\rangle \simeq \text{regular function} \ , \tag{4.8b}$$

$$\langle T_{uv}\rangle \sim G(v-v_0)^{-2/3} \tag{4.8c}$$

with D and G some positive constants. These results indicate that an observer crossing the horizon measures an infinite negative-energy ingoing flux and also an infinite energy density. By comparing naively this to the classical value $T_{vv}^{\rm clas} \sim m' \sim (v-v_0)^{-5/3}$ we see that the divergence induced by the quantum fluctuations in the v_* vacuum is stronger than the classical one. Note also the sign difference in the vv component (classical versus quantum). Is this divergence strong enough to destroy an observer traveling through the wormhole?

We cannot answer this question on the basis of the previous material. As all our analysis was performed in the framework of quantum field theory in a fixed (mass inflation type) space-time, one cannot evaluate the backreaction of these large quantum fluctuations on the space-time geometry. The fate (and the traversability) of the wormhole remains therefore undetermined.

We will examine in the next paragraph a 2D model of "quantum gravity" which allows the backreaction of the quantum fields on the space-time to be evaluated, at least at the semiclassical level. In this way an answer, though not definitive, to the above question can be given.

V. BACKREACTION IN 2D: A TOY MODEL

Two-dimensional theories of gravity are presently quite popular theoretical laboratories for the investigation of the fundamental issues of quantum gravity. These theories afford considerable simplification for the study of semiclassical problems, since in 2D, as we have seen, the trace anomaly and the conservation law allow a complete determination of the expectation values of $\langle T_{ab}\rangle$ [5] [see Eqs. (3.5)].

The basic idea is to use these objects as a source in some kind of 2D analogue of Einstein equations. This will then account for the backreaction of the quantum field on the geometry.

We do not claim that the following material necessarily bears on a real 4D theory of quantum gravity; however, it provides a useful model in which we can investigate the effects of the quantum fluctuations on our wormhole space-time (and on the inner structure of black holes in general). There are a number of classical actions which have been investigated in 2D gravity, all of which take a similar form. We use here a model inspired by string theory which, as we shall see, leads to a space-time which possesses the same basic features of the one discussed in the previous section. The classical action is given by [6]

$$S = S_g + S_{\rm em} + S_m \ , \tag{5.1}$$

where

$$S_g = \frac{1}{2\pi}\int d^2x\sqrt{-g}e^{-2\phi}[R+4(\nabla\phi)^2+4\lambda^2] \ , \tag{5.2a}$$

$$S_{\rm em} = -\frac{2}{2\pi}\int d^2x\sqrt{-g}e^{-2\phi}F_{ab}F^{ab} \ , \tag{5.2b}$$

$$S_m = -\frac{1}{4\pi}\int d^2x\sqrt{-g}\sum_{i=1}^{N}(\nabla f_i)^2 \ . \tag{5.2c}$$

In the standard notation, R is the Ricci scalar of the 2D metric g_{ab}, λ^2 is the cosmological constant, F_{ab} the electromagnetic field tensor. The "matter fields" are 2D conformally coupled scalar fields f_i, ϕ is the dilaton.

The gravitational action S_g differs from the standard Einstein-Hilbert 4D action dimensionally reduced to a 2D integral under the assumption of spherical symmetry (integrating over the angles θ, φ; in this case $e^{-2\phi}\equiv r^2$), by the term representing the dilaton potential. The Maxwell part of the action, $S_{\rm em}$, is the same as the dimensionally reduced one. The matter part S_m on the contrary does not derive from dimensional reduction. It describes the "natural" coupling of 2D matter fields to 2D gravity.

We prefer to work with the action (5.1) instead of the dimensionally reduced one, simply because the former, when quantum corrections are included, admits (when $F_{ab}=0$) solutions to be obtained in analytic form. The results obtained from both theories for the problem we are interested in (Cauchy horizon structure) are in complete agreement as indicated by numerical computations [7]. The field equations following from Eq. (5.1) are

$$e^{-2\phi}[2\nabla_a\nabla_b\phi - F_{ac}F_b^c + \tfrac{1}{2}g_{ab}R] = \sum_{i=1}^{N}[\tfrac{1}{2}\nabla_a f_i\nabla_b f_i - \tfrac{1}{4}g_{ab}(\nabla f_i)^2] \ , \tag{5.3a}$$

$$R - 4(\nabla\phi)^2 + 4\lambda^2 - 2F_{ab}F^{ab} + 4\Box\phi = 0 \ , \tag{5.3b}$$

$$\Box f_i = 0 \ , \tag{5.3c}$$

$$\nabla_a(e^{-2\phi}F^{ba}) = 0 \ . \tag{5.3d}$$

The solutions of these equations have been discussed in detail in Ref. [8]. Here we just review some basic material for later use.

Equations (5.3) admit charged black hole solutions. They are best understood in the light cone gauge

$$ds^2 = -B(\sigma,v)dv^2 + 2dvd\sigma \ , \tag{5.4}$$

where v is an advanced time. The electromagnetic field tensor solution of Eq. (5.3d) is

$$F_{ab} = Qe^{2\phi}e_{ab} \ , \tag{5.5}$$

where $e_{ab}=e_{[a,b]}$ and $e_{01}=\sqrt{-g}$. Q is a constant representing the charge of the hole.

A purely ingoing f wave is solution of Eq. (5.3c) which is independent of σ and allows an exact solution to the equations of motion to be found: namely,

$$B(\sigma, v) = 1 - \frac{2m(v)}{\lambda} e^{2\phi} + \frac{Q^2}{\lambda^2} e^{4\phi} , \tag{5.6}$$

$$\phi = -\lambda\sigma , \tag{5.7}$$

where $m(v)$ satisfies

$$\frac{dm}{dv} = \frac{1}{4} \left(\frac{df}{dv} \right)^2 . \tag{5.8}$$

We assume $m(v) > |Q|$, so that the space-time is that of a black hole and not a naked singularity. The function $m(v)$ is such that

$$(\nabla\phi)^2 = \lambda^2 - 2m\lambda e^{2\phi} + Q^2 e^{4\phi} \tag{5.9}$$

so that it can be regarded as the mass function of the black hole. When $f_i(v) = 0$, the solution describes a static charged black hole with mass $m(v) = m_0 =$ const. The causal structure of this space-time (see Fig. 2) is the same as the 2D section of the RN solution discussed in the previous section. There are two horizons, located at values of ϕ for which $B(\sigma, v) = 0$, i.e.,

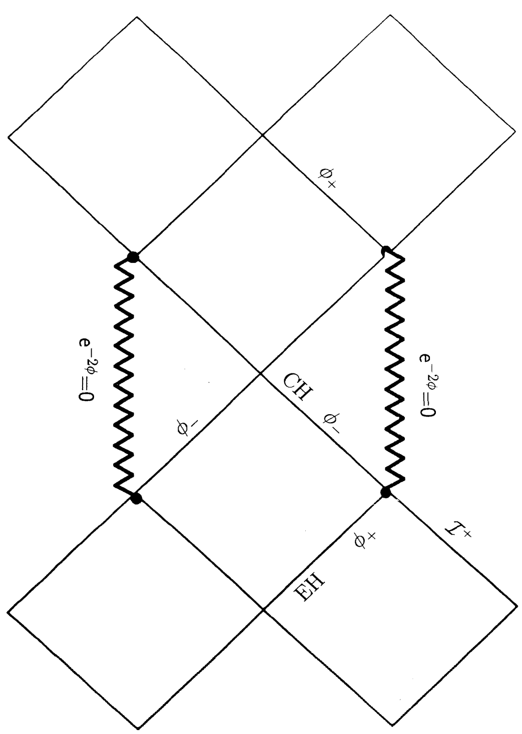

FIG. 2. Conformal diagram of the dilatonic charged black hole.

$$e^{-2\phi}\Big|_{\pm} = \frac{m_0 \pm \sqrt{m_0^2 - Q^2}}{\lambda} , \tag{5.10}$$

where a plus sign refers to the outer horizon and a minus sign to the inner. Let us define $k \equiv (\lambda/4)|\partial_\phi B|$; this corresponds to the "surface gravity" of the event horizon when evaluated at $\phi = \phi_+$ for a static black hole. One easily checks that $k_- \equiv k|_{\phi-} > k_+ \equiv k|_{\phi+}$ provided $m_0 > |Q|$.

The timelike (curvature) singularity is located at $\phi = +\infty$ whereas the asymptotic flat regions are at $\phi \to -\infty$. The role of the dilaton as a gravitational coupling strength is evident from Eqs. (5.2a) and (5.2b). The singularities appear in the strong-coupling region where a full quantum treatment is necessary, whereas the asymptotic regions are in the weak-coupling regime.

The analogy with 4D charged black hole can be further pursued by showing that in this theory the Cauchy horizon ϕ_- is unstable against perturbations and this triggers a mass inflation behavior [9]. We think it is therefore reasonable to regard the space-time of Eq. (5.6), the charged dilatonic black hole, as a 2D version of our wormhole spacetime.

The cosmological setting is taken into account by requiring that the mass function $m(v)$ of Eq. (5.8) behaves, according to Eqs. (2.16)–(2.19), as the CH ($v = v_0$) is approached like

$$m = -\gamma(v - v_0)^{-2/3} \tag{5.11}$$

and diverges as a consequence of the inflow of cosmological radiation. Despite this divergence, the dilaton (which is obtained by integrating twice the ingoing flux [9]) is regular in the above limit [compare this to Eq. (2.25)]; hence the 2D scalar curvature diverges like m.

Now that we have a reasonable 2D model for our wormhole spacetime metric one can easily compute the quantum (semiclassical) corrections to this classical theory by including in the action a term describing the backreaction of the quantum fields on the geometry. If we assume the number N of matter fields (the scalars) to be very large, in leading $1/N$ order, the quantum fluctuations of the dilaton and the metric can be ignored and one needs only to include one-loop corrections to the energy-momentum tensor of the scalars (there is no analogous contribution from the Maxwell field). This can be computed from the trace anomaly [see Eq. (3.5)]. Equivalently one can add to the classical action, Eq. (5.1), the Polyakov-Liouville term [6]

$$S_L = -\frac{K}{8\pi} \int d^2x \sqrt{-g} R \Box^{-1} R , \tag{5.12}$$

where $K = N/24$. Along with this usual nonlocal term we add a local covariant counterterm [10], which will simplify the subsequent analysis (when $Q=0$ the theory can be solved exactly)

$$S_{\rm ct} = -\frac{K}{8\pi}\int d^2x\sqrt{-g}2\phi R \ . \quad (5.13)$$

The theory is therefore described by an effective action $S = S_{\rm cl} + S_L + S_{\rm ct}$, which includes the backreaction of the scalar fields. We now look for classical solutions of this theory which will describe the "quantum corrected" geometry of the 2D wormhole spacetime.

Introducing null coordinates U, V one can write the metric in the conformal gauge

$$ds^2 = -e^{2\rho}dUdV \ . \quad (5.14)$$

We now perform the field redefinition

$$\Omega = \frac{\sqrt{K}}{2}\phi + \frac{e^{-2\phi}}{\sqrt{K}} \ , \quad (5.15a)$$

$$\chi = \frac{\sqrt{K}}{2}(2\rho - \phi) + \frac{e^{-2\phi}}{\sqrt{K}} \quad (5.15b)$$

and the effective action is

$$S = \frac{1}{\pi}\int d^2x[-\partial_U\chi\partial_V\chi + \partial_U\Omega\partial_V\Omega + \lambda^2 e^{2(\chi-\Omega)/\sqrt{K}}] + S_{\rm em} + \int d^2x\frac{1}{2}\sum_{i=1}^{N}\partial_U f_i\partial_V f_i \ . \quad (5.16)$$

When $Q=0$ there is a residual conformal gauge invariance in Eq. (5.16). In this case one fixes it by the "Kruskal gauge" choice $\Omega = \chi$. The Kruskal coordinate V is related to the usual asymptotic Minkowskian advanced time v by

$$\lambda V = +e^{-k_- v} \quad (5.17)$$

and a similar relation for U and u.

Coming back to our case, $Q \neq 0$, we find the equations of motion for the effective action S,

$$(\chi)_{,UV} = \frac{e^{2(\chi-\Omega)/\sqrt{K}}}{\sqrt{K}}(Q^2e^{4\phi} - \lambda^2) \ , \quad (5.18a)$$

$$(\chi - \Omega)_{,UV} = e^{2(\chi-\Omega)/\sqrt{K}}\frac{Q^2}{\Omega'}e^{4\phi} \ , \quad (5.18b)$$

where the solution (5.5) for the Maxwell field has been introduced and

$$\Omega' \equiv \frac{d\Omega}{d\phi} = \frac{\sqrt{K}}{2} - 2\frac{e^{-2\phi}}{\sqrt{K}} \ . \quad (5.19)$$

The constraints which correspond to the equations of motion of g_{UU} and g_{VV} are

$$-\chi_{,V}\chi_{,V} + \sqrt{K}\chi_{,VV} + \Omega_{,V}\Omega_{,V} + L(V) + Kg(V) = 0 \ , \quad (5.20a)$$

$$-\chi_{,U}\chi_{,U} + \sqrt{K}\chi_{,UU} + \Omega_{,U}\Omega_{,U} + Kf(U) = 0 \ . \quad (5.20b)$$

$L(V)$ is the classical radiation influx, coming from the cosmological region. The functions f and g reflect as before the nonlocality of the quantum theory and are determined by the choice of quantum state for the scalar fields. The choice $f = g = 0$ corresponds to a "Kruskal vacuum." As discussed in the previous paragraph, the natural choice for the quantum state of the matter fields is restricted to the conformal vacua in the cosmological region. With the choice of the v_* vacuum as used earlier, the function $g(V)$ is given in terms of the Schwarzian derivative of v_* with respect to V. The details of the boundary conditions will be given later.

Let us first find a solution of the field equations plus the constraints in the region near the CH. When $Q=0$, this system can be exactly integrated and the solution in the Kruskal gauge is

$$\Omega = \chi = -\frac{\lambda^2}{\sqrt{K}}UV - \sqrt{K}\int^U dU'\int^{U'} dU'' f(U'') - \sqrt{K}\int^V dV'\int^{V'} dV''\left(\frac{L(V'')}{K} + g(V'')\right) . \quad (5.21)$$

In our case $Q \neq 0$, but we shall see that the solution describing the quantum corrected geometry has near the CH ($v = v_0$) the same form as Eq. (5.21), the contribution of the terms which contain the charge becoming negligible. We start by formally integrating Eq. (5.18b):

$$\chi - \Omega = \int^U dU'\int^V dV'\frac{Q^2e^{2(\chi-\Omega)/\sqrt{K}}e^{4\phi}}{\Omega'} \ . \quad (5.22)$$

The integral diverges if the dilaton reaches the critical value $\phi_{\rm crit}$, determined by $\Omega' = 0$ and a curvature singularity will result. This singular behavior occurs deep inside the strong-coupling region $[\phi_{\rm crit} = -\frac{1}{2}\ln(K/4)]$ where a semiclassical analysis seems no longer trust worthy. Nevertheless, it has been suggested [10] that $\phi = \phi_{\rm crit}$ should be regarded as a boundary of spacetime, the analogue of the origin of the radial coordinate. As long as this boundary is timelike, boundary conditions on the fields can be imposed but if the boundary becomes spacelike, it is no longer sensible to do so. Thus our asymptotic analysis of the equations of motion makes little sense unless one can show that $\phi = \phi_{\rm crit}$ is not encountered before the CH. We shall give later an argument suggesting that such a scenario can be arranged. Let us first discuss in some detail the boundary conditions on the quantum state for the matter in our wormhole As said before, our choice of v_* vacuum leads, by evaluating the Schwarzian derivative using Eqs. (4.5) and (5.17), to

$$g(V) = -\left[\frac{1}{2} + \frac{35}{72}\left(\ln\frac{V}{V_0}\right)^{-2}\right]\frac{1}{V^2} \ , \quad (5.23)$$

where V_0 is the location of the Cauchy horizon ($v = v_0$). a similar relation can be given for $f(U)$, but the details can be once more ignored since it is sufficient to mention that $f(U)$ remains bounded all the way up to the CH.

Inspection of the constraint (5.20a) shows that the effect of this boundary condition is to introduce an influx of negative energy streaming along the CH. This quantum influx dominates over the classical part $L(V)$ [see Eq. (4.8a) and the following discussion] and produces a strong "defocusing" effect on outgoing geodesics. One therefore expects, if the charge is not small compared to the mass, that $e^{-2\phi}$ (the analogue of the radial coordinate) starting from a value $\phi < \phi_{\rm crit}$ inflates very rapidly and the condition $e^{-\phi} \gg e^{-\phi_{\rm crit}}$ to be satisfied on a portion of the CH which is removed from the cosmological singularity. In other words we investigate the solution of our theory in the so-called "string theory branch."

Coming back to Eq. (5.22), in the region we are interested in $\Omega' \ll 0$ and we need not worry about the vanishing of the denominator. It can be shown that it is consistent to treat the LHS of Eq. (5.22) as vanishing. To leading order as $v \to v_0$ ($V \to V_0$), the CH, the solution to Eqs. (5.18)–(5.20), is

$$\chi = \Omega \simeq -\frac{35}{72}\sqrt{K}\ln[\lambda|V - V_0|] - \sqrt{K}\int^{U} dU' \int^{U'} dU'' f(U'') - \frac{1}{\sqrt{K}}\int^{V} dV' \int^{V'} dV'' L(V'') + \cdots . \quad (5.24)$$

In particular $\Omega \sim e^{-2\phi}/\sqrt{K} \to \infty$ as $V \to V_0$ (the ln term diverging and the others being regular), indicating that the integrand in Eq. (5.22) tends to zero as we approach the CH. This justifies our previous assumption.

At this point it is interesting to examine the Ricci scalar of the quantum corrected wormhole geometry. Keeping only the leading term as $V \to V_0$, we find

$$R \simeq \text{const} \times \frac{e^{2\phi}}{V - V_0}\int^{U} dU' f(U') + \cdots . \quad (5.25)$$

Despite the fact that $e^{2\phi} \to 0$, one finds that the scalar curvature diverges on the CH, the sign depending on the choice of the function $F(U)$. Comparing this result with the classical value of the curvature, which is proportional to the mass function

$$R \sim -(v - v_0)^{-2/3} , \quad (5.26)$$

we see that the singularity along the CH appearing in the quantum corrected geometry is, as expected, stronger than the classical one. The disappointing aspect is that this curvature singularity appears in the weak-coupling region ($e^{\phi} \to 0$). Therefore it is very unlikely that it will be removed in the full (quantization of metric and dilaton) quantum theory. However, as before, the singularity is mild in the sense that although the tidal forces felt by a free falling observer blow up at the CH, the distortions obtained by twice integrating the curvature stay finite, making the 2D wormhole still traversable.

VI. CONCLUSIONS

The Reissner-Nordström space-time has been used here as a wormhole connecting two different universes (closed FRW universes in the case investigated in this work). This wormhole is, unlike the Einstein-Rosen bridge of the Schwarzschild geometry, efficient in the sense that observers can go safely from one universe to the other by traveling through it. This is true classically since although (classical) perturbations cause a null singularity to form inside the wormhole, this singularity is mild enough for the tidal distortion to remain finite.

Our aim in this paper was to examine whether the same conclusion holds when one allows perturbation of the wormhole space-time of quantum origin. Up to now this study cannot be performed in the full 4D context. Quantum field theory in black hole interiors is a rather difficult and almost unexplored subject, not to speak about the backreaction problem, and very little is known about it. Here we only developed a minor program, namely, the study of quantum effects and their backreaction in a simple 2D version of our original model, in the hope to gain some insight into the real 4D problem.

Our analysis reveals that the rise of curvature on the CH of the wormhole which is induced by quantum fluctuations can become quite large as compared to the one caused by classical perturbations. However the curvature singularity that arises on the CH is always mild in the above sense, making the 2D wormhole traversable. The extension of this positive conclusion to 4D is however rather dubious. It is sufficient to say that if the metric and the dilaton are regarded as components of a spherically symmetric metric in 4D, there exist components of the tidal distortion that blow up (although only logarithmically) at the CH. This seems to suggest that a 4D wormhole of the RN type might behave differently. Traversability (hence efficiency) of the wormhole would probably require a very restricted choice of possible quantum states for the matter sector, different from our v_* vacuum, such that the corresponding Schwarzian derivative with respect to v [i.e., the $g(V)$ function entering Eq. (5.20a)] behaves on the CH milder than $(v - v_0)^{-2}$. But of course until a reasonable approach to the back reaction in 4D can be found, no definite answer to the wormhole traversability problem can be given.

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