Interaction of low-energy induced gravity with quantized matter and phase transition induced by curvature

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At a high energy scale the only quantum effect of any asymptotic-free and asymptotically conformal invariant GUT is the trace anomaly of the energy-momentum tensor. The anomaly generates a new degree of freedom, that is, the propagating conformal factor. At lower energies the conformal factor starts to interact with the scalar field because of the violation of conformal invariance. We estimate the effect of such an interaction and find the running of the nonminimal coupling from the conformal value of $\frac{1}{6}$ to 0. Then we discuss the possibility of the first order phase transition induced by curvature in a region close to the stable fixed point and calculate the induced values of Newtonian and cosmological constants.

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I. INTRODUCTION

The cosmological constant problem remains the most mysterious one in modern high energy physics. A number of different approaches have been proposed to solve this problem (see, for example, [1–11] and references therein), but no approach is able to give a completely consistent scheme of which makes this constant vanish.

Here we shall consider the influence of vacuum quantum effects of matter fields in curved space-time to the value of the induced cosmological constant. In a region of asymptotic freedom, that is, in accordance with the modern point of view, at energies beyond the grand unification scale, all interactions between matter fields are weakened and the only quantum effect is the appearance of the trace anomaly of the energy-momentum tensor. The purpose of the present paper is to explore the back reaction of this vacuum effect to the matter fields with respect to the induced value of the cosmological constant. The trace anomaly generates a new dynamical degree of freedom, which is usually named the conformal factor or dilaton. Not so long ago Antoniadis and Mottola considered the theory of the conformal factor as an infrared version of quantum gravity [12], and found this theory a useful tool for the exploration of the cosmological constant problem. Since then this approach (with various modifications) has been developed in a few papers (see, for example, [13–20]). In particular, in Ref. [20] the contributions of the quantized conformal factor to the effective potential of a scalar field were calculated. At the same time in Ref. [20] the semiclassical approximation has been used and the quantum effects of the matter fields were not taken into account. A complete investigation of the system of interacting matter and dilaton field meets some difficulties, because in the corresponding theory there are fourth as well as second derivative terms. Here we consider the complete case and derive one-loop divergences with the use of the method proposed in [21] (see also [22]). Then we point out the one-loop renormalizability of the theory and use renormalization-group method to analyze the running of the coupling and to derive the effective potential of the scalar field. After this we consider the possibility of a first-order phase transition induced by curvature, and discuss a possible way to fine-tune the scale parameter in order to provide a small value of the induced cosmological constant.

The paper is organized as follows. In Sec. II we give an overview of the basical concepts of [20] and formulate the structure of the interaction between the conformal factor and matter fields. Section III is devoted to the calculation of the one-loop counterterms and to the renormalization structure. In Sec. IV the renormalization-group method is applied for the analysis of the running of coupling constants. Here we also obtain the expression for the effective potential. In Sec. V the possibility of a first-order phase transition is stated and the induced values of Newtonian and cosmological constants are calculated.

II. ACTION OF THE CONFORMAL FACTOR AND COUPLING STRUCTURE

The starting point of our investigation is the theory of asymptotically free massless fields of spin 0, $\frac{1}{2}$, and 1 in an external gravitational field. One can find the review of quantum field theory in an extermal gravita-

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tional field, for example, in Refs. [23,24,22]. In particular, in [22] it is the theory of interacting fields in curved space-time is also presented. Multiplicative renormalizability requires nonminimal terms to be included into the action as well as vacuum ones. When radiational corrections are taken into account, the parameter of nonminimal coupling obeys the corresponding renormalization-group equations. As pointed out in [25,26] (see also [22]), in some asymptotically free models asymptotic conformal invariance takes place. This means that the nonminimal coupling ξ (we suppose the nonminimal term to have the form $\xi R\phi^2$) is arbitrary at low energies while at high energies it has the conformal value $\frac{1}{6}$.

The next important quantum effect, in an external gravitational field, is the appearance of the anomaly trace of the energy-momentum tensor which allows us to calculate, with accuracy to some conformal invariant functional, the effective action of the vacuum [27,28] (see also [22]). This effective action originally arises as a nonlocal functional, but it can be written in a local form with the help of an extra dimensionless field, which is named the dilaton, in analogy with string theory, or as the conformal factor.

The anomaly trace of the energy-momentum tensor has the form [29,23,24]

$$T = \langle T_\mu{}^\mu \rangle = k_1 C^2 + k_2 E + k_3 \Box R\,, \tag{1}$$

where the values of $k_{1,2,3}$ are determined by the number of fields of different spin in a starting grand unified theory (GUT) model. The trace anomaly (1) leads to an equation for the effective action,

$$-\frac{2}{\sqrt{-g}} g_{\mu\nu} \frac{\delta W}{\delta g_{\mu\nu}} = T\,,$$

which has the nonlocal solution [27,28]

$$W[g_{\mu\nu}] = S_c[g_{\mu\nu}] + \int d^4x\sqrt{-g}\left(k_3 + \frac{2}{3}k_2\right) R^2 + \int d^4x\sqrt{-g_x} \int d^4y\sqrt{-g_y} \times \left\{k_1 C^2 + \frac{1}{2}k_2\left(E - \frac{2}{3}\Box R\right)\right\}_x G(x,y) \left\{k_2\left(E - \frac{2}{3}\Box R\right)\right\}_y\,, \tag{2}$$

where $G(x,y)$ is the Green function for the Hermitian conformal covariant fourth-order operator (4), C^2 is the square of the Weyl tensor, and E is Gauss-Bonnet invariant. This effective action can be written in a local form with the help of an auxiliary dimensionless field σ [27]. It reads

$$W[g_{\mu\nu},\sigma] = S_c[g_{\mu\nu}] + \int d^4x\sqrt{-g}\left\{\frac{1}{2}\sigma\Delta\sigma + \sigma\left[k_1' C^2 + \frac{k_2'}{2}\left(E - \frac{2}{3}\Box R\right)\right] + (k_3 + \tfrac{2}{3}k^2)R^2\right\}\,. \tag{3}$$

The conformally covariant self-adjoint operator Δ is defined by

$$\Delta = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu\,. \tag{4}$$

The values of $k'_{1,2,3}$ differ from $k_{1,2,3}$ because of the contribution of Δ to the conformal trace (1) [27]. The solution (3) contains an arbitrary conformal invariant functional S_c, which is the integration constant for Eq. (2). This functional is not essential for our purposes and we shall not take it into account.

Our main supposition is that the quantum effects of induced gravity, that is, of the field σ, are relevant below the scale of asymptotic freedom and asymptotic conformal invariance, where coupling constants in the matter field sector are not equal to zero. We are interested in the cosmological applications, and therefore it is natural to suppose that the transition to low energies (or long distances) corresponds to some conformal transformation in the induced gravity action (3) and hence classical fields and induced gravity appear in different conformal points [32]. To realize this it is necessary to make a conformal transformation of the metric in (3) and then to consider the unified theory. At the same time it is more convenient to make the conformal transformation of metric and matter fields in the action of the last. Such a transformation corresponds to some change of variables in the path integral for the unified theory.

The only source of conformal noninvariance in the action of the fields of spin $0, \frac{1}{2}, 1$ is the nonminimal term in the scalar sector. In the framework of asymptotically conformal invariant models the value of ξ is not equal to $\frac{1}{6}$ at low energies and hence the interaction of the conformal factor with the scalar field arises. Introducing the scale parameter α we obtain the following action for the conformal factor coupled with the scalar field:

$$S = W[g_{\mu\nu},\sigma] + \int d^4x\sqrt{-g}\left\{\frac{1}{2}(1-6\xi)\phi^2\left(\alpha^2 g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + \alpha\Box\sigma\right) + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}\xi R\phi^2 - \frac{1}{24}f\phi^4\right\}\,, \tag{5}$$

where $W[g_{\mu\nu},\sigma]$ is defined in (3). Thus the interaction between the scalar field and conformal factor arises as a result of the conformal transformation of the metric $g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu}\exp(2\alpha\sigma)$ and the matter field $\Phi \to \Phi' = \Phi\exp(d_\Phi\alpha\sigma)$, where d_Φ is the conformal weight of the field Φ. The only kind of field which takes part in such an interaction is the scalar one, where the interaction with the conformal factor appears as a result of nonconformal coupling at low energies. Hence the contributions of other matter fields to the effective potential of the scalar field do not depend on the conformal factor and can be calculated separately. In the next sections we shall concentrate on the theory (5) and estimate the dilaton contributions to the effective potential of the scalar field ϕ.

Some remarks are in order. The renormalizability of the theory requires the action of matter fields in an external gravitational field to be supplemented by the vacuum terms (see, for example, [22] for an introduction to renormalization in an external gravitational field). Since renormalization of the vacuum action has been already taken into account in (1)–(3) we shall not consider the details of renormalization in the vacuum sector, but only make some comment after calculations. Our goal is to evaluate the influence of the quantum conformal factor on the physical effects of the ordinary scalar field. That is why we can restrict ourselves to consideration of renormalization in the only sector of the field ϕ.

III. CALCULATION OF ONE-LOOP DIVERGENCES

The calculation of the one-loop divergences of the theory (5) is not a trivial problem because the classical action contains second derivative terms as well as fourth derivative ones. Here we shall use the method of Ref. [21], where the one-loop divergences have been calculated for higher derivative quantum gravity coupled with matter fields. According to [21], we shall use the background field method and hence we start with the separation of fields into background σ, ϕ and quantum τ, η ones, by

$$\sigma \to \sigma' = \sigma + \tau\,, \qquad \phi \to \phi' = \phi + \eta\,. \tag{6}$$

The one-loop effective action is defined as

$$\Gamma = \frac{i}{2}\,\mathrm{Tr}\,\ln\hat{H}\,, \tag{7}$$

where $\hat{H}$ is the bilinear (with respect to the quantum fields τ, η) form of the classical action (5). After some algebra we get the self-adjoint bilinear form

$$\hat{H} = \begin{pmatrix} H_{\tau\tau} & H_{\tau\eta} \\ H_{\eta\tau} & H_{\eta\eta} \end{pmatrix}\,, \tag{8}$$

where

$$H_{\tau\tau} = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu + (6\xi-1)\left[\alpha^2\phi^2\Box + \alpha^2(\nabla^\mu\phi^2)\nabla_\mu\right]\,,$$

$$H_{\tau\eta} = -(6\xi-1)\{\alpha\phi\Box + 2\alpha(\nabla^\mu\phi)\nabla_\mu + \alpha(\Box\phi) - 2\alpha^2\phi(\nabla^\mu\sigma)\nabla_\mu - 2\alpha^2[\nabla_\mu(\phi\nabla^\mu\sigma)]\}\,,$$

$$H_{\eta\tau} = -(6\xi-1)\left[\alpha\phi\Box + 2\alpha^2\phi(\nabla^\mu\sigma)\nabla_\mu\right]\,,$$

$$H_{\eta\eta} = -\Box + \xi R - \frac{1}{2}f\phi^2 - (6\xi-1)\left[\alpha(\Box\sigma) + \alpha^2(\nabla^\mu\sigma)(\nabla_\mu\sigma)\right]\,. \tag{9}$$

After the change of variables $\eta \to i\eta$ we arrive at the following structure of $\hat{H}$:

$$\hat{H} = \begin{pmatrix} \Box^2 + 2V^{\mu\nu}\nabla_\mu\nabla_\nu + N^\mu\nabla_\mu + U & Q_1\Box + Q_2^\mu\nabla_\mu + Q_3 \\ P_1\Box + P_2^\mu\nabla_\mu + P_3 & \Box + E^\mu\nabla_\mu + D \end{pmatrix}\,, \tag{10}$$

where the values of V, N, U, Q_i, P_i, E, D follow from (9). They read

$$V^{\mu\nu} = 2R^{\mu\nu} - \frac{2}{3}Rg^{\mu\nu} + (6\xi-1)\phi^2\alpha^2 g^{\mu\nu}\,,$$

$$N^\mu = \frac{1}{3}(\nabla^\mu R) + (6\xi-1)\alpha^2(\nabla^\mu\phi^2)\,, \qquad U = 0\,,$$

$$Q_1 = -i(6\xi-1)\phi\alpha\,, \qquad Q_2^\mu = -2i(6\xi-1)\left[(\nabla^\mu\phi)\alpha - \phi(\nabla^\mu\sigma)\alpha^2\right]\,,$$

$$Q_3 = -i(6\xi-1)\{\alpha(\Box\phi) - 2\alpha^2[\nabla_\mu(\phi\nabla^\mu\sigma)]\}\,, \qquad P_1 = -i(6\xi-1)\phi\alpha\,,$$

$$P_2^\mu = -2i(6\xi-1)\phi(\nabla^\mu\sigma)\alpha^2\,, \qquad P_3 = 0\,, \qquad E^\mu = 0\,,$$

$$D = -\xi R + \frac{1}{2}f\phi^2 + (6\xi-1)\left[\alpha(\Box\sigma) + \alpha^2(\nabla^\mu\sigma)(\nabla_\mu\sigma)\right]\,. \tag{11}$$

Expressions (10) enable us to use the method and partially also the results of [21] in a direct way. After some algebra we obtain the following general form for the one-loop divergences of the effective action (7):

$$\Gamma_{\text{div}} = \frac{2}{\varepsilon}\int d^4x\sqrt{-g}\left\{\frac{1}{4}P_2^\mu Q_{2\mu} + \frac{1}{4}P_1Q_3 - \frac{1}{4}V_\mu^\mu P_1Q_1 - DP_1Q_1 + \frac{1}{2}(P_1Q_1)^2 + \frac{1}{2}Q_2^\mu\nabla_\mu P_1 - \frac{1}{6}RP_1Q_1\right.$$
$$+\frac{1}{24}V^{\mu\nu}V_{\mu\nu} + \frac{1}{48}(V_\mu^\mu)^2 - \frac{1}{6}V^{\mu\nu}R_{\mu\nu} + \frac{1}{12}V_\mu^\mu R - U$$
$$\left.+\frac{1}{2}\left(D + \frac{1}{6}R\right)^2 + \frac{1}{20}\left(R^{\mu\nu}R_{\mu\nu} - \frac{1}{3}R^2\right) - \frac{1}{36}R^2\right\} + \text{(surface terms)}. \tag{12}$$

Substituting (11) into (12) we arrive at an explicit expression for Γ_{div}:

$$\Gamma_{\text{div}} = \frac{2}{\varepsilon}\int d^4x\sqrt{-g}\left\{\frac{1}{24}\phi^4\left[3f^2 + 12(6\xi-1)^2 f\alpha^2 - 432(6\xi-1)^2\alpha^4\xi^2\right]\right.$$
$$-\frac{1}{2}\phi^2\left(R - 6\alpha\Box\sigma - 6\alpha^2\nabla^\mu\sigma\nabla_\mu\sigma\right)\left[\frac{1}{6}(6\xi-1)f + 2\alpha^2\xi(6\xi-1)^2\right]$$
$$\left.-\frac{7}{120}C^2 + \frac{1}{2}\left(\xi - \frac{1}{6}\right)\left(R - 6\alpha\Box\sigma - 6\alpha^2\nabla^\mu\sigma\nabla_\mu\sigma\right)^2\right\} + \text{(surface terms)} \tag{13}$$

where $\varepsilon = (4\pi)^2(n-4)$ is a parameter of dimensional regularization.

Let us now make some remarks concerning the renormalization structure in the theory under consideration. One can easily see that in this theory a wide cancellation of divergences takes place in both vacuum and matter field sectors. As a result the theory is renormalizable in the matter field sector, at least at the one-loop level. The renormalizability in the vacuum sector is realized in the following sense. As was already pointed out we have to introduce vacuum terms of the form

$$S_{\text{vac}} = \int d^4x\sqrt{-g}\left\{a_1R^2 + a_2C^2\right\}\ . \tag{14}$$

Thus the vacuum counterterms have a structure which differs from the one of the classical action (14) by the same conformal transformation that we have performed when deriving the action (5). If one make this transformation in (14), it will have the same structure as the vacuum divergences in (13), and the last ones can be removed by the renormalization of a_1 and a_2. At the same time the above scheme is not completely consistent, and formally one can provide renormalizability only on the background of constant σ, when the forms of the vacuum terms in (14) and (13) coincide. Note that for our purposes this is quite enough. To construct an entirely renormalizable theory it is necessary to start with the general dilaton gravity [41] taking into account all possible dilaton interactions.

The next point is related to the renormalization in the sector of the scalar field ϕ. One can see that there are no divergences which lead to renormalization of ϕ. All the divergences can be removed by a renormalization of the couplings, including a_1, a_2, and the renormalization-group equations include only β, but not γ functions. Then it follows that the effective potential of the theory depends on β functions only, and therefore it is free of ambiguities which arise, for instance, in gauge theories (see, for example, the discussion in [30] on the higher derivative gravity corrections to the effective potential of the scalar field).

IV. RENORMALIZATION GROUP EQUATIONS

Since the theory defined by Eq. (5) is multiplicatively renormalizable, one can use the renormalization-group method for its study. For our purposes it is more convenient to deal with an arbitrary background metric $g_{\mu\nu}$ and therefore we must use the approach described in [22] (see also the original papers [25,26,31]). The general solution of the renormalization-group equations for the effective action,

$$\left\{\mu\frac{d}{d\mu} + \beta_f\frac{d}{df} + \beta_\xi\frac{d}{d\xi} - \gamma_\phi\frac{\delta}{\delta\phi} - \gamma_\sigma\frac{\delta}{\delta\sigma}\right\}\Gamma[\phi,\sigma,f,\xi,g_{\mu\nu},\mu] = 0\ , \tag{15}$$

has the form

$$\Gamma[\phi,\sigma,f,\xi,g_{\mu\nu}e^{2t},\mu]=\Gamma[\phi(t),\sigma(t),f(t),\xi(t),g_{\mu\nu},\mu], \tag{16}$$

where μ is the dimensional parameter of renormalization. Effective fields and coupling constants obey the equations

$$\frac{d\phi(t)}{dt}=(\gamma_\phi+1)\phi\,, \qquad \phi(0)=\phi\,,$$

$$\frac{d\sigma(t)}{dt}=\gamma_\sigma\sigma, \qquad \sigma(0)=\sigma\,,$$

$$\frac{df(t)}{dt'}=\beta_f, \qquad f(0)=f\,,$$

$$\frac{d\xi(t)}{dt'}=\beta_\xi, \qquad \xi(0)=\xi\,. \tag{17}$$

Here $t'=(4\pi)^{-2}t$, while the γ and β functions are defined as usual. Note that in our case there is no need to renormalize the fields ϕ,σ and therefore all γ functions are equal to zero. The β functions for the effective couplings $f(t),\zeta(t)=1-6\xi(t)$ can be easily obtained from (13). One has

$$\begin{aligned}(4\pi)^2\beta_f&=3f^2+12f\alpha^2\zeta^2+12\alpha^4\zeta^2(\zeta-1)^2\,,\\(4\pi)^2\beta_\zeta&=\zeta[f+2\alpha^2\zeta(\zeta-1)]\,.\end{aligned} \tag{18}$$

Here we introduce ζ instead of ξ for compactness. Let us now consider the asymptotics of the effective couplings $f(t),\zeta(t)$. It is easy to see that both β functions (18) vanish in the physically relevant points $f=0,\zeta=0$ and $f=0,\zeta=1$. There are also three more solutions with negative f, but they do not look so interesting, because the classical potential for ϕ in these cases is not bounded from below. The first solution corresponds to the conformal fixed point while the second one corresponds to the minimal fixed point. The last means that within this solution ξ is equal to zero. One can easily make the analysis of the stability of the minimal fixed point in the framework of standard Lyapunov's method and find it stable in the IR limit $t\to-\infty$. Moreover, one can easily obtain that in this limit $f\sim\xi^6\to 0$ and therefore $f\ll\xi$.

The problem of stability of the first solution cannot be solved in such a way, because if we make an infinitesimal variations of the couplings ζ,f, the linear corrections to this fixed point in β functions are equal to zero. Rejecting the infinitesimal terms of higher order we find

$$\frac{d\delta f}{dt'}=3(\delta f)^2+12\alpha^4(\delta\zeta)^2\,,$$

$$\frac{d\delta\zeta}{dt'}=\delta\zeta(\delta f+2\alpha^2\delta\zeta)\,, \tag{19}$$

where $\delta\zeta,\delta f$ are infinitesimal variations of the couplings. Since we are only interested in the infinitesimal variations, there are three possible cases: (i) $\delta\zeta$ and δf are of the same order, (ii) $\delta\zeta$ is smaller than δf, or (iii)δf is smaller than $\delta\zeta$. A detailed analysis shows that all of these three situations are not possible, and therefore a conformal fixed point is not stable in the IR limit. Moreover, one can easily see that both fixed points are not stable in the UV limit. Note that in case (ii) in the IR limit one formally obtains that $\delta\zeta$ tends to zero. However, condition (ii) does not hold, and therefore this version is also inconsistent.

Some remarks are in order. We start our consideration with some asymptotically free and asymptotically conformal invariant GUT, and consider the scalar field ϕ as part of this GUT. Formally the contribution of the quantum field σ to β_f contradicts the asymptotic freedom and therefore the whole approach looks quite ambiguous, but this is not the case. We suppose the value of α to be close to zero and hence the contributions of the quantum conformal factor are small at short distances. On the other hand, at long distances, the scaling parameter α has some finite value and this leads to some nontrivial dynamics in the far IR. Note that the IR dynamics of α has been recently studied in [41].

V. EFFECTIVE POTENTIAL FOR THE SCALAR FIELD AND PHASE TRANSITION INDUCED BY CURVATURE

Now we follow [31,22] and use the renormalization-group method for the derivation of the effective potential for the scalar field ϕ. According to [31,22] the solution of Eq. (15) for the potential part of the effective action has the form

$$\begin{aligned}V_{\rm eff}=&-\frac{1}{2}\xi R\phi^2+\frac{f}{24}\phi^4+\frac{1}{48}\beta_f\phi^4\left[\ln\frac{\phi^2}{\mu^2}-\frac{25}{6}\right]\\&-\frac{1}{4}\beta_\xi\phi^2R\left[\ln\frac{\phi^2}{\mu^2}-3\right]\,,\end{aligned} \tag{20}$$

where μ is a dimensional parameter of renormalization and both β functions have been defined in Eq. (18).

Here we shall restrict ourselves to consider only the first-order phase transition. Then the equations for the critical values of the curvature R_c and order parameter ϕ_c have the form

$$\begin{aligned}&V_{\rm eff}(R_c,\phi_c)=0, \qquad V'_{\rm eff}(R_c,\phi_c)=0,\\&V''_{\rm eff}(R_c,\phi_c)>0.\end{aligned} \tag{21}$$

Here primes stand for derivatives of V with respect to ϕ. The effective potential in Eq. (20) has a rather complicated form and therefore any relevant analysis of Eqs. (21) needs some restrictions on the values of f,ζ to be imposed. Since we are interested in the phase transition, which has to take place in the far infrared, it is natural to suppose that these couplings obey the renormalization-group equations and have values which are close to the IR stable fixed point $f=0,\xi=0$.

Then the first two equations in (21) have two nontrivial solutions for critical values of the curvature and ϕ. They read

$$\phi_c^2 = q_{1,2}|R_c|, \qquad q_1 = -\frac{\varepsilon}{18\alpha^4\xi^4},$$
$$q_2 = -\frac{\varepsilon}{2}, \quad (\phi_c^{(1)})^2 = \mu^2 \exp\left(\frac{18\alpha^2+1}{6\alpha^2}\right),$$
$$(\phi_c^{(2)})^2 = \mu^2 \exp\left(\frac{25}{6}\right), \tag{22}$$

where $\phi_c^{(1,2)}$ correspond to $q_{1,2}$ and $\varepsilon = R/|R|$ is the sign of the scalar curvature. For q_2 the third condition in Eq. (21) reads as $\xi + O(\xi^2) < 0$, which contradicts the initial condition $\xi \approx \frac{1}{6} > 0$, which must hold at high energies. For q_1 the third condition in Eq. (21) has the form $\frac{\beta_f}{32} + O(\xi^3) > 0$ and so the initial condition is satisfied. Therefore in the framework of the approximation used here the first-order phase transition takes place at critical values of ϕ_c and $|R_c|$ which correspond to the choice q_1 in Eq. (22).

Substituting the values of $\phi_c^{(1)}$ and q_1 from Eq. (22) into the effective potential (20), we obtain an estimate for V_{eff} at the critical point. The corresponding action has the form of the Hilbert-Einstein action

$$S_{\text{ind}} = -\frac{1}{16\pi G_{\text{ind}}} \int d^4x\sqrt{-g}(R - 2\Lambda_{\text{ind}}), \tag{23}$$

where induced values of the Newtonian and cosmological constants are defined from Eqs. (20)–(22) to be

$$\frac{1}{16\pi G_{\text{ind}}} = -\xi\left(\phi_c^{(1)}\right)^2(1-2\alpha^2)^2, \tag{24}$$

$$\Lambda_{\text{ind}} = -\frac{1}{16\pi G_{\text{ind}}}\frac{9\alpha^4}{4(1-2\alpha^2)^2}. \tag{25}$$

If we substitute into (25) the critical value for the order parameter $\phi^{(1)}$, the induced Newtonian and cosmological constants are expressed via the dimensional parameter of the renormalization μ and in place of (24) we get

$$\frac{1}{16\pi G_{\text{ind}}} = -\xi(1-2\alpha^2)^2\mu^2 \exp\left(\frac{18\alpha^2+1}{6\alpha^2}\right). \tag{26}$$

To estimate the induced values of Λ_{ind} and G_{ind} we must remove some arbitrariness related to the value of μ. There are two different ways to do this. One can follow [31,33] and fix the induced value of the Newtonian constant to be equal to its classical value. This means that one chooses μ to be of the same order of the Planck mass $m_P = (8\pi G)^{-1}$. At the same time we are dealing here not with Planck energies as in Ref. [33], but with some energy scale below the unification point M_x. On the other hand we have not any grounds to suppose that the induced values of the Newtonian and cosmological constants were the same at the M_x scale and at the modern epoch. On the contrary, there are some reasons in favor of the effective running of these constants at energies above (see, for example, [34,35]) and below [11] this scale. Therefore it is more reasonable to choose a value of μ close to M_x and thus obtain the values of constants, which are induced by the quantum effects of the conformal factor discussed above. Note that the absence of a cosmological constant in modern observational data needs independent suppression in any part of the energy scale [1]. If one takes the value of α to be close to zero, then the induced value of the cosmological constant will be very small. In fact we suppose that the energy scale of strong matter effects in an external gravitational field is some μ_1 and that the quantum effects of the conformal factor is relevant at another scale $\mu_2 < \mu_1$. Since the close scales correspond to a small value of the scale factor α, we obtain that, if the difference between μ_2 and μ_1 is small enough, the induced value of the cosmological constant is small too. So, if we suppose that the point of the phase transition discussed above is close to M_x, then the induced value of Λ is in good agreement with the observational data.

VI. CONCLUSION

We have considered the interaction between the conformal factor and matter fields in a region close to the scale of asymptotic freedom. In fact the only nontrivial contributions to V_{eff} come from the quantized scalar field. All other fields decouple from the conformal factor and therefore give additive contributions to V_{eff}. These contributions are not so essential, because in the region of asymptotic freedom all interactions between matter fields are weak. The results of our analysis are in good agreement with previous semiclassical results derived in Ref. [20], where the scalar field was treated as pure background. At the same time the physical picture in the considered case is more sophisticated. We meet here the nontrivial scale dependence of the effective nonminimal coupling, which changes from the conformal value of $\frac{1}{6}$ to the IR stable minimal value. The phase transition occurs at a scale close to the stable fixed point, and the resulting induced constants do not depend on ξ, as in the more simple case of Ref. [20].

Now let us briefly discuss some possible ways to extend the above considerations. First of all it would be very interesting to incorporate finite temperature effects into the theory, which have to be very important in the early Universe [35–38]. Thermodynamics properties of quantum fields in static space-times have been studied by many authors [39]. This can be usefully done with the help of a finite-temperature generating functional in "imaginary time" formalism [40]. Unfortunately, in the theory we are considering one encounters some technical difficulties due to the fact that the small disturbance operator is of fourth order.

Next, it is possible to estimate quantum effects of the conformal factor above the unification scale. Since this region is close to Planck energies, it is necessary to take into account the quantum effects of the metric. To achieve renormalizability it is necessary to consider general dilaton gravity [41], which is the direct generalization of higher derivative gravity theory [42–46]. In this case the interaction between gravity (including the conformal factor) and matter fields takes place even without a conformal shift of the induced action. Generally speaking, such an investigation is possible at least at the one-loop level [47], but it is related to a very big volume of calculations.

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