

Calculation of particle production by Nambu-Goldstone bosons with application to inflation reheating and baryogenesis

Alexandre Dolgov* and Katherine Freese

Physics Department, University of Michigan, Ann Arbor, Michigan 48109

(Received 12 October 1994)

A semiclassical calculation of particle production by a scalar field in a potential is performed. We focus on the particular case of production of fermions by a Nambu-Goldstone boson θ . We have derived a (non)local equation of motion for the θ field with the back reaction of the produced particles taken into account. The equation is solved in some special cases, namely, for purely Nambu-Goldstone bosons and for the tilted potential $U(\theta) \propto m^2\theta^2$. Enhanced production of bosons due to parametric resonance is investigated; we argue that the resonance probably disappears when the expansion of the Universe is included. Application of our work on particle production to reheating and an idea for baryogenesis in inflation are mentioned.

PACS number(s): 98.80.Cq, 11.30.Fs, 14.80.Mz

I. INTRODUCTION

Nambu-Goldstone bosons (NGB's) are ubiquitous in particle physics: they arise whenever a symmetry is spontaneously broken. If there is additional explicit symmetry breaking, these particles become pseudo Nambu-Goldstone bosons (PNGB's). In this paper, we consider particle production by Nambu-Goldstone bosons as they rotate about the bottom of the "Mexican hat" potential, whether or not there is a tilt around the bottom (i.e., with or without explicit symmetry breaking).

The Nambu-Goldstone bosons, hereafter called θ , are assumed to couple to fermions; thus as the θ field moves it is capable of producing these fermions. Here we perform a semiclassical calculation. The θ field is treated classically while the particles produced are quantized. The back reaction of quantum fermions on the evolution of the θ field is calculated. Our motivation is to lay out a general way to perform such a calculation and to carry it out for a specific coupling. The primary results of our calculations in flat spacetime are Eqs. (2.21) and (2.22). We then demonstrate examples and generalize to curved spacetime with massive fermions.

Particle production by Nambu-Goldstone fields may have several applications. One is the QCD axion. Another is particle production by the inflation field [1] in natural inflation [2]. Particle production is, of course, important for estimates of reheating in inflation. Many models of inflation involve "slowly rolling" fields that evolve down a potential. Subsequent to the "slowly rolling" epoch, there must be an epoch of reheating, where vacuum energy is converted into the production of radiation energy. The equation of motion for the inflaton field is taken to be

$$\ddot{\theta} + 3H\dot{\theta} + \Gamma\dot{\theta} = -\frac{dU}{d\theta}. \quad (1.1)$$

The term $\Gamma\dot{\theta}$ is assumed to describe the reheating. Γ is taken to be the decay rate of the inflaton field. This heuristic term really describes much more complicated physics. In fact one should accurately calculate the production of particles and its back reaction on the inflaton field as it rolls down the potential. Previous work on this subject includes [3,4].

In addition, there may be a mechanism for baryogenesis during natural inflation. If the equivalent of the Peccei-Quinn field can be made to carry baryon number, one may be able to do baryogenesis as the inflaton is rolling down its potential. This has several nice features: (i) the same field would be responsible for both inflation and baryogenesis and (ii) the inflaton could reheat to very low temperatures, perhaps as low as nucleosynthesis temperatures of \sim MeV. [In fact, for this mechanism to work one would have to reheat to below the electroweak temperature to avoid sphaleron destruction, if it is operative; alternatively the inflaton could generate nonvanishing $(B - L)$ asymmetry which is preserved by sphalerons.] The approach is similar to proposals of Affleck and Dine [5] and Cohen and Kaplan [6]. It is assumed that the inflaton field Φ is complex and has a nonvanishing baryon number. The corresponding baryon current generated by the classical rolling down of the inflaton field is essentially equal to the angular momentum of the two-dimensional mechanical motion in the plane $(\text{Re}\Phi, \text{Im}\Phi)$. Thus, as the field rolls in one direction, it preferentially creates baryons over antibaryons, while the opposite is true as it rolls in the opposite direction. (We assume that the decays during reheating are baryon number conserving.) Thus, no CP violation is required of the particle physics; instead those regions of the Universe in which the inflaton by chance rolls down the potential in one direction turn out to be baryon dominated, while those that roll down the other direction turn out to be antibaryon dominated. Conveniently, each of these regions is inflated to be very large, so that it makes sense for our baryon-dominated region to be large enough to encompass our observable Universe. The requirements to create a specific particle physics model for this proposal

*Permanent address: ITEP, Moscow, 117259 Russia.

are restrictive and are discussed below.

In Sec. II, we discuss the sample Lagrangian we consider, and calculate the particle production for this case. First we perform the calculation in the absence of expansion of the Universe, and for production of massless fermions. In Sec. III, we apply the results to two specific examples: (i) a scalar θ rotating in a potential without explicit symmetry breaking, and (ii) the same scalar but now oscillating near the minimum of a quadratic potential produced by explicit symmetry breaking; for example, such a potential may give rise to inflation. In Sec. IV, we include the effects of expansion of the Universe and fermion masses. We also discuss the possibility of parametric resonance [3,7,8] whereby large numbers of scalars might be produced during reheating in inflation. We argue that there is probably no resonance if one includes the expansion of the Universe. In Sec. V, we discuss the possible application to inflation, particularly to the model of natural inflation. The baryogenesis model mentioned above is discussed in this section. In Sec. VI we conclude.

II. PARTICLE PRODUCTION IN A SIMPLE MODEL

A. Lagrangian

We first describe a simple model in which we calculate particle production. Consider the fundamental action for a complex scalar field Φ and two fermions Q and L :

$$S = \int d^4x \sqrt{-g} [g^{\mu\nu} \partial_\mu \Phi^* \partial_\nu \Phi - V(\Phi^* \Phi) + i\bar{Q}\gamma^\mu \partial_\mu Q + i\bar{L}\gamma^\mu \partial_\mu L + (g\Phi\bar{Q}L + \text{H.c.})]. \quad (2.1)$$

Note that Q and L can be any fermions, not necessarily quarks and leptons of the standard model. For example, they can be heavy fermions; they may be given some of the same quantum numbers as particles in the standard model if they couple to ordinary quarks and leptons. For this section of the paper, we will take the intrinsic mass of the Q and L fields to be zero, and will include mass effects in later sections.

This action is invariant under the appropriate U(1) symmetry. For example, in this paper, we will take the Lagrangian to be invariant under

$$\Phi \rightarrow e^{i\alpha} \Phi, \quad Q \rightarrow e^{i\alpha} Q, \quad L \rightarrow L. \quad (2.2a)$$

Equation (2.2a) is the symmetry we will use for the rest of the paper.

We did, however, want to point out that a very similar analysis would apply to the case of global chiral U(1) symmetry in a Lagrangian with Yukawa coupling $g\psi_L\psi_R\Phi$. Here subscripts L and R refer to left- and right-handed projections of the fermion fields, $\psi_{R,L} = (1 \pm \gamma_5)\psi/2$. This Lagrangian is invariant under

$$\psi_L \rightarrow e^{i\alpha/2} \psi_L, \quad \psi_R \rightarrow e^{-i\alpha/2} \psi_R, \quad \Phi \rightarrow e^{i\alpha} \Phi, \quad (2.2b)$$

which is the Peccei-Quinn (PQ) symmetry [9] in axion models. Then the current would be $J^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$. Although we do not explicitly analyze this case, it would be very similar to the one we do look at.

We assume the global symmetry is spontaneously broken at the energy scale f in the usual way, e.g., via a potential of the form

$$V(|\Phi|) = \lambda [\Phi^* \Phi - f^2/2]^2. \quad (2.3)$$

The resulting scalar field vacuum expectation value (VEV) is

$$\langle \Phi \rangle = f e^{i\phi/f} / \sqrt{2}. \quad (2.4)$$

Below the scale f , we can neglect the superheavy radial mode of Φ ($m_{\text{radial}} = \lambda^{1/2}f$) since it is so massive that it is frozen out. The remaining light degree of freedom is the angular variable ϕ , the Goldstone boson of the spontaneously broken U(1) [one can think of this as the angle around the bottom of the Mexican hat described by Eq. (2.3)]. For simplicity of notation, we introduce the dimensionless angular field $\theta \equiv \phi/f$. We thus study the effective Lagrangian [10,11] for θ :

$$\mathcal{L}_{\text{eff}} = \frac{f^2}{2} \partial_\mu \theta \partial^\mu \theta + i\bar{Q}\gamma^\mu \partial_\mu Q + i\bar{L}\gamma^\mu \partial_\mu L + (gf\bar{Q}Le^{i\theta} + \text{H.c.}) - U(\theta). \quad (2.5)$$

The global symmetry is now realized in the Goldstone mode: \mathcal{L}_{eff} is invariant under

$$Q \rightarrow e^{i\alpha} Q, \quad L \rightarrow L, \quad \theta \rightarrow \theta + \alpha. \quad (2.6)$$

At this stage, θ is massless because we have not yet explicitly broken the symmetry.

With a rotation of the form in Eq. (2.6) with $\alpha = -\theta$, the Lagrangian can alternatively be written

$$\mathcal{L}_{\text{eff}} = \frac{f^2}{2} \partial_\mu \theta \partial^\mu \theta + i\bar{Q}\gamma^\mu \partial_\mu Q + i\bar{L}\gamma^\mu \partial_\mu L + (gf\bar{Q}L + \text{H.c.}) + \partial_\mu \theta J^\mu - U(\theta), \quad (2.7)$$

where the fermion current derives from the U(1) symmetry; here, $J^\mu = \bar{Q}\gamma^\mu Q$.

B. Explicit symmetry breaking

Our subsequent analysis of particle production applies whether or not the symmetry is further broken explicitly. Several options exist for explicitly breaking the global symmetry and generating a PNGB potential at a mass scale $\sim \Lambda$. Models include the schizon models of [12]. Another possibility is the QCD axion [13]: dynamical chiral symmetry breaking through strongly coupled gauge fields. When QCD becomes strong at a scale $\Lambda_{\text{QCD}} \sim \text{GeV}$, instanton effects become important. Chiral dynamics induces a fermion condensate, $\langle \bar{\psi}\psi \rangle \sim \Lambda^3$, and the potential for the angular PNGB field becomes

$$U(\theta) = \Lambda^4 [1 \pm \cos \theta]. \quad (2.8)$$

Such a potential is used in the case of natural inflation, although at higher mass scales, and will be discussed later.

C. Equations of motion

The equations of motion for the Q fields are

$$i\gamma^\mu \partial_\mu Q + \partial_\mu \theta \gamma^\mu Q + gfL = 0 \quad (2.9a)$$

and

$$i\partial_\mu (\bar{Q}\gamma^\mu) - \partial_\mu \theta \bar{Q}\gamma^\mu - gf\bar{L} = 0. \quad (2.9b)$$

The combination of these two equations can be written

$$\partial_\mu J_Q^\mu = \partial_\mu (\bar{Q}\gamma^\mu Q) = -igf(\bar{Q}L - \bar{L}Q). \quad (2.10)$$

The equation of motion for θ is

$$\mathcal{D}^2\theta + U'(\theta)/f^2 = \frac{-1}{f^2}\partial_\mu J^\mu = \frac{ig}{f}(\bar{Q}L - \bar{L}Q). \quad (2.11)$$

The equation of motion for the L fields, which we assume does not transform under the symmetry, is

$$i\gamma^\mu \partial_\mu L = -gfQ. \quad (2.12)$$

As above, if we add the equation of motion for the \bar{L} field, we find

$$\partial_\mu (\bar{L}\gamma^\mu L) = -igf(\bar{Q}L - \bar{L}Q). \quad (2.13)$$

D. Particle production

Here we calculate the production of Q and L particles by the angular θ field as it rotates around the Mexican hat (which may or may not be tilted). For the moment we will neglect expansion of the Universe.

For convenience we will define

$$Q = Q_0 e^{i\theta} \quad (2.14)$$

so that Eq. (2.9) becomes

$$\gamma^\mu \partial_\mu Q_0 = igf e^{-i\theta} L. \quad (2.15)$$

We will solve perturbatively the Heisenberg equations of motion presented above. We will take the free field Q_{in} to satisfy $\gamma^\mu \partial_\mu Q_{\text{in}} = 0$, and make a perturbation expansion $Q_0 = Q_{\text{in}} + gQ_1$. Equation (2.15) is solved by

$$Q_0(x) = Q_{\text{in}}(x) + igf \int d^4y G_Q(x, y) L(y) e^{-i\theta(y)}, \quad (2.16a)$$

where to lowest order we will take $L = L_{\text{in}}$ inside the integral. Here G_Q is the retarded Green's function for the

Q field and satisfies $\partial_\mu \gamma^\mu G_Q(x, y) = \delta^4(x - y)$. Similarly, the solution to Eq. (2.12) is

$$L(x) = L_{\text{in}}(x) + igf \int d^4y G_L(x, y) Q(y) \\ \approx L_{\text{in}}(x) + igf \int d^4y G_L(x, y) Q_{\text{in}}(y) e^{i\theta(y)}. \quad (2.16b)$$

We take the vacuum expectation value (in Heisenberg picture)

$$\langle \mathcal{D}^2\theta(x) + U'(\theta(x))/f^2 \rangle \\ = \frac{1}{f^2} \langle \partial_\mu J^\mu \rangle_{\text{vac}} = \frac{ig}{f} \langle (\bar{Q}L - \bar{L}Q) \rangle_{\text{vac}}. \quad (2.17)$$

We perform a semiclassical treatment, where the θ field is treated classically while the fermion fields are quantized and via the equations of motion determine the evolution of θ . To first order in g , the right-hand side of Eq. (2.17) is $\langle \bar{L}_{\text{in}} Q_{\text{in}} \rangle = 0$. Working to second order in g , we substitute Eq. (2.16a) into Eq. (2.14), and plug that into the right-hand side of (2.17) to obtain

$$\langle \mathcal{D}^2\theta(x) + U'(\theta(x))/f^2 \rangle \\ = g^2 \left\langle \int d^4y \bar{L}_{\text{in}}(x) G_Q(x, y) L_{\text{in}}(y) e^{-i\theta(y)} e^{i\theta(x)} \right. \\ \left. - \bar{Q}_{\text{in}}(x) G_L(x, y) Q_{\text{in}}(y) e^{i\theta(y)} e^{-i\theta(x)} + \text{H.c.} \right\rangle. \quad (2.18)$$

Here, G_Q and G_L are the Green's functions for the Q and L fields. For the case of massless fermions, $G_L = G_Q$.

We now quantize the free-fermion fields:

$$Q_{\text{in}}(x) = \sum_s \int \frac{d^3k}{(2\pi)^{3/2}} \sqrt{\frac{m_Q}{E_k}} [u_k^s b_k^s e^{-ik \cdot x} + v_k^s d_k^{s\dagger} e^{ik \cdot x}], \quad (2.19)$$

where b_k^s and $d_k^{s\dagger}$ are annihilation and creation operators at momentum k and spin s for particles and antiparticles, respectively. The quantization for the free L_{in} field is similar. Now we evaluate the right-hand side of (2.17). The details of the calculation are discussed in Appendix A. As shown there, we find that

$$\langle \bar{L}(x) G(x, y) L(y) \rangle \\ = 4 \int \frac{d^3p}{(2\pi)^7} \int \frac{d^4l}{2E_p} \left[\frac{i}{l^2 + m_Q^2} \right] e^{[i(p+l) \cdot (y-x)]} l \cdot p, \quad (2.20)$$

and a similar expression for the quarks.

After much algebra (presented in Appendix A), we find

$$\langle \mathcal{D}^2\theta + U'(\theta)/f^2 \rangle = -\frac{4g^2}{\pi^2} \int_0^\infty d\omega \omega^2 \int_{-\infty}^0 dt' \sin(2\omega t') \sin[\theta(t+t') - \theta(t)]. \quad (2.21)$$

As shown in Appendix A, after the ω integration is done and with some algebra, this equation becomes

$$\langle \mathcal{D}^2\theta + U'(\theta)/f^2 \rangle = -\frac{g^2}{2\pi^2} \lim_{w \rightarrow \infty} \int_{-\infty}^0 dt' \left[\frac{\cos 2wt' - 1}{t'} \right] \left[\ddot{\theta}(t+t') \cos \Delta\theta - \dot{\theta}^2(t+t') \sin \Delta\theta \right]. \quad (2.22)$$

Here w tends to infinity and we have defined $\Delta\theta \equiv \theta(t+t') - \theta(t)$.

Equations (2.21) and (2.22) are the major results of this section. To reiterate, these equations describe the evolution of the θ field with production of massless fermions taken into account in a semiclassical approximation. So far the expansion of the Universe has not been included.

III. EXAMPLES

In this section we will apply Eq. (2.21) to two examples. First we will consider the case of $U'(\theta) = 0$. This is the case where there is no explicit symmetry breaking. Thus the potential looks like an ordinary Mexican hat, in which every point around the bottom is equivalent. For example, there is no tilt (no cosine potential). Another situation in which this case would be relevant is the case when the rotation proceeds so far up in the Mexican hat that the details around the bottom are irrelevant and can be ignored.

The second example we will consider is one that would be relevant to inflation, namely oscillations around the bottom of a tilt in the potential. In this case $U'(\theta) \neq 0$ and there is explicit breaking of the symmetry.

Case I: $U'(\theta) = 0$: In this first case, we consider a simple Mexican hat with no explicit symmetry breaking. The zeroth order solution to Eq. (2.21) would be obtained by setting the right hand side, which is proportional to g^2 , to zero. The zeroth order solution is $\theta = \text{const}$. In other words, the field is simply rotating around in the Mexican hat with constant angular velocity. We substitute this ansatz $\dot{\theta} = \text{const}$ back into Eq. (2.21) to see what the corrections would be. The t' integral becomes

$$\int_{-\infty}^0 dt' \sin(2\omega t') \sin(\dot{\theta} t') = \frac{\pi}{2} [\delta(2\omega + \dot{\theta}) - \delta(2\omega - \dot{\theta})].$$

Thus, Eq. (2.21) becomes

$$\ddot{\theta} + \frac{g^2}{2\pi} \dot{\theta}^2 \text{sgn}(\dot{\theta}) = 0. \quad (3.1)$$

If the field Φ has been arranged to carry baryon number, then the baryon number is shifted (via a baryon-

conserving decay) to the fermions. The baryon number of the fermions satisfies $\partial_\mu J^\mu = f^2 \ddot{\theta}$ from the equations of motion. Thus $\dot{n}_B = f^2 \dot{\theta}$, where n_B is the baryon number carried by the fermions. The change in baryon number carried by the fermions is thus determined by the change in $\dot{\theta}$ via $\Delta n_B = f^2 \Delta \dot{\theta}$, similar to Ref. [5]. Here, all that has happened is that this mechanism transfers the initial baryon number $B_i = f^2 \dot{\theta}_i$ into the quarks. Subscripts i refer to initial values. Thus, it is an initial value problem: to get the right value today one would need exactly the right value of θ_i . We are not proposing this as a likely explanation for the baryon content of our Universe.

The solution to Eq. (3.1) is $\dot{\theta} = 2\pi \dot{\theta}_i / (2\pi + g^2 t \dot{\theta}_i)$ and $\theta = 2\pi g^{-2} \ln(2\pi + g^2 t \dot{\theta}_i) + \theta_i$. Here θ_i is the initial value of θ . Thus, we have checked that in the short time or small g limit, $\dot{\theta} \sim \text{const}$ was reasonable. We wish, however, to remind the reader that the expansion of the Universe has not yet been included here.

Case II: Oscillations around the minimum of a potential for θ : Here will consider the case where $U'(\theta) \neq 0$; i.e., there is a tilt as you go around the bottom of the Mexican hat. As a simple example we will consider $U(\theta) = m^2 f^2 \theta^2 / 2$, as would be appropriate near the bottom of a cosine potential.

Then Eq. (2.22) becomes

$$\begin{aligned} \ddot{\theta} + m^2 \theta &= -\frac{g^2}{2\pi^2} \lim_{w \rightarrow \infty} \int_{-\infty}^0 dt' \left[\frac{\cos 2wt' - 1}{t'} \right] \\ &\times \left[\ddot{\theta}(t+t') \cos \Delta\theta - \dot{\theta}^2(t+t') \sin \Delta\theta \right]. \end{aligned} \quad (3.2)$$

Again, the zeroth-order solution is obtained by setting the right-hand side equal to zero. Thus as our ansatz we take $\theta(t) = \theta_0(t) \cos m_R t$ where m_R is the renormalized mass to be defined below. We will assume that $\theta_0(t)$ varies more slowly with time than do the cosine oscillations. We will consider the case of small oscillations around the bottom of the potential. Then we can take $\cos \Delta\theta \approx 1$ and neglect $\dot{\theta}^2$ in Eq. (3.2). [Note that the t' integral should really start from a finite time at which small oscillations begin; here we will approximate the lower limit of the integral as $t'_{\text{initial}} = -\infty$.] Then the t' integral becomes

$$\begin{aligned} &-\frac{g^2}{2\pi^2} \lim_{w \rightarrow \infty} \int_{-\infty}^0 dt' \left[\frac{\cos 2wt' - 1}{t'} \right] [-m_R^2 \theta_0(t) \cos m_R(t+t')] \\ &= -\frac{g^2}{2\pi^2} \lim_{w \rightarrow \infty} \left[-m_R^2 \theta_0 \sin m_R t \int dt' \frac{\sin m_R t'}{t'} + m_R^2 \theta_0 \sin m_R t \int dt' \frac{\sin m_R t'}{t'} \cos 2wt' \right. \\ &\quad \left. + m_R^2 \theta_0 \cos m_R t \int \frac{dt'}{t'} (1 - \cos 2wt') \cos m_R t' \right]. \end{aligned} \quad (3.3)$$

The first term behaves like a friction term, $-g^2 m_R \dot{\theta}/(4\pi)$. The second term is zero after one takes the limit $w \rightarrow \infty$. The third term is a mass renormalization term. After evaluating the integral in this third term as shown in Appendix B, we find that this term is $\frac{g^2}{2\pi^2} m_R^2 \theta \ln(2w/m_R)$. We wish to add this term to the second term on the left-hand side of Eq. (3.2) so that the sum of these terms gives $m_R^2 \theta$. Therefore we define $m_R^2 [1 + \frac{g^2}{2\pi^2} \ln(2w/m_R)] = m^2$. Then the original integro-differential equation is effectively reduced to

$$\ddot{\theta} + m_R^2 \theta + \Gamma \dot{\theta} = 0, \quad (3.4)$$

where $\Gamma \equiv g^2 m_R/4\pi$. The solution to this equation is

$$\theta(t) = \theta_i e^{-\Gamma t/2} \cos(m_R t + \delta) \quad (3.5)$$

where we introduced an arbitrary phase δ which is fixed by initial conditions.

Equation (3.4) [or (1.1)] describes the damping of the external field oscillations due to particle production. It was postulated in many papers where the Universe reheating was considered. As we have shown it is indeed correct, but our approach alerts us to several issues that should be considered further with regard to the calculation of the baryon asymmetry. If the spontaneously broken symmetry is associated with the baryon number, the baryon asymmetry generated by the decay of the PNGB field was calculated [6] as $|\dot{n}_B| = \Gamma f^2 |\dot{\theta}|$ which gives

$$|\Delta n_B| = \Gamma f^2 |\Delta \theta|. \quad (3.6)$$

Our first caveat is with regard to energy conservation. The initial energy density of the field θ which creates the baryons is $\rho_\theta(t_i) \sim f^2 m^2 \theta_i^2$. At the end some of this energy density has been converted to baryons, with energy density $\rho_B(t_f) > n_B E_B$ where n_B is the density of the baryonic charge and $E_B \sim m$ is the characteristic energy of the produced fermions. Clearly it must be true that $\rho_B(t_f) < \rho_\theta(t_i)$. If we were to use Eq. (3.6) we would see that this requires $\Gamma < \Delta \theta m$. From the definition of Γ we see that this is satisfied for small values of coupling constant g as long as $\Delta \theta$ is not too small; for extremely small values of $\Delta \theta$, this relationship can never be satisfied.

Our second caveat is as follows: in making the identification $|\dot{n}_B| = \Gamma f^2 |\dot{\theta}|$, one is equating an operator equation (2.11) with a vacuum-averaged equation (3.4). Indeed we started with the operator equation which in our case looks like $\ddot{\theta} + m^2 \theta = \dot{n}_B/f^2$. Comparison with Eq. (3.4) gives the identification mentioned above. However, Eq. (3.4) is not an operator equation but obtained by the vacuum averaging of the operator equation (2.11). As we have seen the average value $\langle \dot{n}_B \rangle$ is not just $\Gamma f^2 \dot{\theta}$ but a more complicated expression (3.3). This issue should be looked at further.

Note that in the case of the spontaneous symmetry breaking without any explicit one, when $U'(\theta) = 0$, the operator equation of motion reads $f^2 \ddot{\theta} = \dot{n}_B$ and $\Delta n_B = f^2 \Delta \theta$ (as was mentioned previously). In this case the characteristic energy of the produced fermions is θ and

their energy density is of the second order in $\dot{\theta}$. This agrees with the energy density of the creating field θ . Thus the final fermion energy is indeed consistent with the original energy in the θ field.

The third caveat is that Eq. (3.3) reduced to Eq. (3.4) only in the approximation that the lower limit of integration was taken to be $-\infty$. This approximation is good as long as there are many oscillations over the course of the integral. For the case of inflation the field oscillates many times during the reheating period, and thus this approximation is probably reasonable.

IV. FURTHER COMPLICATIONS: (i) CURVED SPACETIME (EXPANSION OF THE UNIVERSE), (ii) NONZERO FERMION MASSES, AND (iii) PARAMETRIC RESONANCE

A. Curved spacetime

So far all our results have been for flat spacetime. In order to include the effects of the expansion of the Universe, we now generalize to curved spacetime:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} f^2 (\partial_\mu \theta) (\partial_\nu \theta) g^{\mu\nu} + i \bar{Q} \nabla_\mu \Gamma^\mu Q + i \bar{L} \nabla_\mu \Gamma^\mu L + \partial_\mu \theta \bar{Q} \Gamma_\nu Q g^{\mu\nu} - m_Q \bar{Q} Q - m_L \bar{L} L + g f (\bar{Q} L + \bar{L} Q) - U(\theta) \right]. \quad (4.1)$$

We will consider Friedmann-Robertson-Walker metrics and work in conformal time, $ds^2 = a^2(d\tau^2 - d\mathbf{x}^2)$. Since $g_{\mu\nu} = a^2 \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the flat spacetime metric, by making this conformal transformation in the Lagrangian we can reduce the metric to the flat one. With this transformation, we can use the ordinary Minkowski space quantization for the fields and the usual Green's functions. To simplify we redefine the fermion fields as $\psi \rightarrow \psi/a^{3/2}$. Note that the matrices in curved spacetime Γ^μ are now transformed to normal Dirac matrices γ^μ . Then the action is

$$S = \int d^4x \left[\frac{1}{2} f^2 a^2 \partial_\mu \theta \partial^\mu \theta + i \bar{Q} \partial_\mu \gamma^\mu Q + i \bar{L} \partial_\mu \gamma^\mu L + \partial_\mu \theta \bar{Q} \gamma^\mu Q - m_Q a \bar{Q} Q - m_L a \bar{L} L + g f a (\bar{Q} L + \bar{L} Q) - a^4 U(\theta) \right], \quad (4.2)$$

where the summations are now done using $\eta_{\mu\nu}$. With this action the equations of motion are

$$f^2 \partial_\mu (a^2 \partial^\mu \theta) + \partial_\mu (\bar{Q} \gamma^\mu Q) + U'(\theta) a^4 = 0, \quad (4.3)$$

$$i \partial_\mu \gamma^\mu Q + \partial_\mu \theta \gamma^\mu Q - m_Q a Q = -g f a L, \quad (4.4)$$

and

$$i \partial_\mu \gamma^\mu L - m_L a L = -g f a Q. \quad (4.5)$$

For the case $m_Q = m_L = 0$, we know the fermion Green's functions [for nonzero masses there is the complication

that $ma(\tau)$ is time dependent]. In the massless case we can repeat the derivation done in flat spacetime previously and find that the semiclassical equation of motion for the Goldstone field is

$$\begin{aligned} & \langle \partial_\mu (a^2 \partial^\mu \theta) + U'(\theta) a^4 / f^2 \rangle \\ &= -\frac{4g^2}{\pi^2} a(\tau) \int_0^\infty d\omega \omega^2 \int_{-\infty}^0 d\tau' \sin(2\omega\tau') a(\tau + \tau') \\ & \quad \times \sin[\theta(\tau + \tau') - \theta(\tau)]. \end{aligned} \quad (4.6)$$

Of course, the lower limit of the τ' integral should really be some initial time rather than $-\infty$.

B. Nonzero fermion masses

Here we will consider the modifications to the flat spacetime case when the fermion masses are nonzero. To

$$\mathcal{D}^2 \theta + U'(\theta) / f^2 = -\frac{g^2}{\pi^3} \int d^3 l \int_{-\infty}^0 dt' \sin[(E_p + E_l)t'] \sin[\theta(t + t') - \theta(t)] \frac{E_l E_p + l^2 + m_Q m_L}{2E_p E_l}. \quad (4.8)$$

Here $E_l^2 = l^2 + m_Q^2$ and $E_p^2 = l^2 + m_L^2$. Equation (4.8) reduces to Eq. (2.21) when $m_Q = m_L = 0$.

For the case of $\dot{\theta} = \text{const}$ (case I considered above), we can see that only particles with masses $m_1 + m_2 < \dot{\theta}$ can be produced in perturbation theory, according to Eq. (4.8). If we do the t' integration in the $\theta = \text{const}$ case, we get $\delta(\dot{\theta} - E_1 - E_2)$. For $\dot{\theta} < m_1 + m_2$ this can never be satisfied, there is no particle production, and $\ddot{\theta} \equiv 0$ exactly. For $\dot{\theta} > m_1 + m_2$, this δ function can be satisfied for some momentum, and particles are produced.

The question remains what happens if one looks beyond perturbation theory, particularly in an expanding Universe. In the case of e^+e^- production by a slowly varying electric field, a nonperturbative contribution to particle production exists for the case where the oscillation frequency ω is less than the electron mass m_e ; the result is that the production is exponentially suppressed (the effect $\propto \exp[-\text{const} \times (m_e/\omega)]$) but nonzero. Although we have not found such contributions here, they may exist (see also [3]).

C. Parametric resonance

Recently Kofman, Linde, and Starobinski [7] (see also the work of Shtanov, Traschen, and Brandenberger) [8] noticed that parametric resonance may greatly enhance the production of bosons during reheating in inflation; one can interpret this as the formation of a Bose condensate. In this paper we have been primarily considering the production of fermions, for which there is no parametric resonance. However, we should also consider the production of θ bosons themselves by the classical θ field.

We will consider particle production during the reheating phase of natural inflation. As our potential for the PNGB field we take $U(\theta) = \Lambda^4(1 - \cos\theta)$. Then the mass

of the Lagrangian in Eq. (2.7) we add terms $-m_Q \bar{Q} Q - m_L \bar{L} L$. The equations of motion (2.9) are modified to

$$i\gamma^\mu \partial_\mu Q + \partial_\mu \theta \gamma^\mu Q - m_Q Q + gfL = 0 \quad (4.7a)$$

and

$$i\partial_\mu (\bar{Q} \gamma^\mu) - \partial_\mu \theta \bar{Q} \gamma^\mu + m_Q \bar{Q} - gf\bar{L} = 0. \quad (4.7b)$$

Equations (2.10) and (2.11) are unchanged. Again, we wish to calculate Eq. (2.17), and, again, $\langle \bar{L}(x) G_Q(x, y) L(y) \rangle$ is given by Eq. (A2). This time we keep the masses m_Q and m_L nonzero. After we perform the $\int d^3 y$ integration, there is a term inside the remaining integrals: $(l \cdot p - m_Q m_L) / (l^2 - m_Q^2) = (E_l E_p - l^2 - m_Q m_L) / (E_l^2 - l^2 - m_Q^2)$. There are poles at $E_l = \pm \sqrt{l^2 + m_Q^2}$. Again, only the $E_l < 0$ part gives a nonzero contribution. Our result is

of the PNGB field is $m^2 = \Lambda^4 / f^2$. For natural inflation, $f \sim m_{\text{Pl}}$ and $\Lambda \sim 10^{15}$ GeV, so that $m \sim 10^{12}$ GeV. We will neglect coupling to fermions in our study of the possibility of resonance, and include only coupling of the field to itself. At first, we will neglect expansion of the Universe and see that parametric resonance does indeed exist for a few particular choices of wave number. Then we will include expansion and argue why we believe that the resonance disappears. We have not performed a complete analysis of the equation in the case of expansion, but for the case of natural inflation the arguments are quite robust. We suspect that the disappearance of the resonance in an expanding Universe may happen in other cases as well. We leave investigation of this effect in other cases to future work.

In flat spacetime, the equation of motion for the PNGB is then $\ddot{\theta} + U'(\theta) / f^2 = 0$. For our choice of potential, this becomes $\ddot{\theta} + m^2 \sin\theta = 0$. For small oscillations about $\theta = 0$, we find the solution to this simple equation to be

$$\theta_0(t) = \theta_i \sin mt. \quad (4.9)$$

Here, subscript i refers to initial value of the unperturbed solution. Following the approach of [7,8], we will now add fluctuations $\theta = \theta_0 + \delta\theta$. We will keep terms to first order in $\delta\theta$. Then

$$\begin{aligned} U' &= \Lambda^4 \sin(\theta_0 + \delta\theta) = \Lambda^4 [\sin\theta_0 \cos(\delta\theta) + \sin(\delta\theta) \cos\theta_0] \\ &\approx \Lambda^4 [\sin\theta_0 + \delta\theta \cos\theta_0] \end{aligned}$$

in the small-angle approximation. To first order in $\delta\theta$, after performing a Fourier transform and subtracting the zeroth-order piece, the equation of motion for $\delta\theta$ is then

$$\delta\ddot{\theta} + [k^2 + \cos\theta_0 m^2] \delta\theta = 0. \quad (4.10)$$

Expanding around $\theta_0 = 0$ and using Eq. (4.9), we have

$$\delta\ddot{\theta} + \left(k^2 + m^2 - \frac{m^2}{2}\theta_i^2 \sin^2 mt \right) \delta\theta = 0. \quad (4.11)$$

We define $y = 2mt$ and write $\sin^2 mt = (1 - \cos 2mt)/2$. Equation (4.11) can then be written

$$\frac{d^2}{dy^2} \delta\theta + \delta\theta \left[\frac{k^2}{4m^2} + \frac{1}{4} - \frac{1}{16}\theta_i^2 + \frac{1}{16}\theta_i^2 \cos y \right] = 0. \quad (4.12)$$

The standard form of the Mathieu equation is

$$\frac{d^2 z}{dy^2} + (A + 2\epsilon \cos y)z = 0. \quad (4.13)$$

Our flat spacetime equation (4.12) is of the form of the Mathieu equation with $A = \frac{1}{4} - \theta_i^2/16 + k^2/4m^2$ and $2\epsilon = \theta_i^2/16$. Since we made the small-angle approximation earlier on, we have assumed $\theta_i < 1$, i.e., $\epsilon \ll A < 1$. The Mathieu equation has been studied in great depth. It is known that for $A = \frac{1}{4} + A_1\epsilon$ and $\epsilon \ll 1$, there is no instability if $|A_1| > 1$. (In our case $|A_1| = 2$ for $k = 0$.) Therefore, for $k = 0$, there is no instability. However, for particular values of nonzero k (values for which $A \sim n^2/4$ where n is an integer) there are indeed regions of resonance with $\delta\theta$ growing exponentially in time. We refer the reader to literature on the Mathieu equation to see these regions.

However, now let us include the expansion of the Universe. To simplify we will neglect here the interaction with fermions. Without fermions it is convenient to work in terms of the physical time t with the interval $ds^2 = dt^2 - a^2(t)d\mathbf{r}^2$. The relevant change from the nonexpanding case will not only be the additional term $3H\dot{\theta}$ in the equation of motion; rather the important features are (i) the redshifting of the length scales of the perturbations ($k \rightarrow k/a$) and (ii) the fact that the unperturbed solution θ_0 is now different. The equation of motion (again, neglecting the fermion effects) becomes $\ddot{\theta} + 3H\dot{\theta} + U'(\theta)/f^2 = 0$. For our choice of potential, this equation becomes $\ddot{\theta} + 3H\dot{\theta} + m^2 \sin \theta = 0$. Again, we take the small-angle approximation $\sin \theta \sim \theta$. The unperturbed solution for the matter-dominated expansion is

$$\theta_0(t) = \frac{\theta_i \sin(mt + \delta)}{mt}. \quad (4.14)$$

The factor of $1/t$ in the denominator will prove to be an important feature of the expansion. We assumed here that the θ field started to oscillate when the Hubble parameter, $H = 2/3t$, was close to the value of the mass of the field m , as is usually the case for inflation. It fixes the initial value of time, $t_i \sim 1/m$.

This time we take $y = mt$. The equation for the fluctuations becomes

$$\frac{d^2}{dy^2} \delta\theta + \frac{3H}{m} \frac{d}{dy} \delta\theta + \left[\frac{k^2}{m^2 a^2} + 1 - \frac{1}{2}\theta_i^2 \frac{\sin^2 y}{y^2} \right] \delta\theta = 0. \quad (4.15)$$

To eliminate the $\delta\dot{\theta}$ term we define $\theta(t) = \bar{\theta}(t)/a^{3/2}$. During the reheating portion of inflation, the Universe

is matter dominated and we take $a \propto t^{2/3}$ (of course after reheating the Universe is radiation dominated). With this matter-dominated expansion, Eq. (4.15) becomes

$$\frac{d^2}{dy^2} \delta\bar{\theta} + \left[\frac{k^2}{m^2 a^2} + 1 - \frac{\theta_i^2 \sin^2 y}{2 y^2} \right] \delta\bar{\theta} = 0. \quad (4.16)$$

As before, we use $\sin^2 y = [1 - \cos(2y)]/2$ to write the equation in the form closest to the Mathieu equation. This time it is not exactly the Mathieu equation because of the time dependence in the denominator of k^2/a^2 and because of the factor of $1/y^2$ that came from Eq. (4.14).

We have numerically integrated Eq. (4.12) without Universe expansion and Eq. (4.15) with the expansion to see what happens to the resonance. As we expected the solutions of Eq. (4.12) show the resonance behavior for a particular region of parameters while solutions of Eq. (4.16) do not resonate. The resonance might be excited if the oscillations of θ began when $H \sim 1/t_i \ll m$ (not what usually happens in inflation). We did not perform a numerical study of the entire range of parameter space, and in principle could have missed the particular choices of k/a that do resonate. Hence we proceed here with a simple analytic discussion.

One can see from simple analytic arguments that Eq. (4.16) is unlikely to lead to resonance. When one includes expansion, there are two effects that reduce the instability. First, the redshift of the wave number, $k \rightarrow k/a$, quickly moves any wave number that happens to be in a resonance band, out of the resonance region. In other words, if at one time there is an instability on some length scale, shortly afterwards this length scale has redshifted to a value for which there is no instability. Thus it is difficult to see how there could be exponential increase in particle production (numerically we could get factors of a few, not of 10^5). Second, in the long-time limit, $y \gg 1$, both the first and third terms inside the brackets become small. Then Eq. (4.16) is simply a harmonic-oscillator equation with oscillating rather than unstable solutions. The last term, whose negative sign could make it responsible for resonance, becomes unimportant for times $t > \theta_i/2m$. Since the frequency of oscillations m is usually faster than the frequencies corresponding to other relevant time scales, such as the time scale of reheating in inflation, the last term quickly becomes unimportant. There may be enhancement in the first oscillation or two, but it is probably not very large (as above, this assumes that the oscillations began when $H \sim m$, i.e., $y \sim 1$). Instead the solutions quickly become oscillatory, with amplitudes at most slightly larger than those in the nonexpanding case. Again, we have not performed a complete analysis of Eq. (4.16), but we have argued why we believe the resonance effects are not very strong here.

V. BARYOGENESIS IN NATURAL INFLATION

The inflationary Universe model [1] provides an elegant means of solving several cosmological problems, including the horizon problem, the flatness problem, and the

monopole problem. In addition, quantum fluctuations produced during the inflationary epoch may provide the initial conditions required for the formation of structure in the Universe. During the inflationary epoch, the energy density of the Universe is dominated by a (nearly constant) vacuum energy term $\rho \simeq \rho_{\text{vac}}$, and the scale factor R of the Universe expands superluminally (i.e., $\ddot{R} > 0$). If the time interval of accelerated expansion satisfies $\Delta t \geq 60R/\dot{R}$, a small causally connected region of the Universe grows sufficiently to explain the observed homogeneity and isotropy of the Universe, to dilute any overdensity of magnetic monopoles, and to flatten the spatial hypersurfaces (i.e., $\Omega \rightarrow 1$).

The model of natural inflation [2] was proposed to provide a natural explanation of the required flatness of the potential in inflation. The flatness is achieved by mimicking the axion physics described earlier. The inflaton is a PNCB field and two different mass scales describe the height and width of the potential. The model has several nice features, including the possibility of extra large-scale power in the density fluctuations, negligible production of gravitational waves, and possible tie-ins to particle physics models under consideration.

Here we want to consider an idea for baryogenesis during natural inflation. More standard ideas such as (a) reheating to above the baryogenesis temperature (e.g., electroweak) or (b) baryon-violating decays have already been considered [2]. Instead, here we consider a model of baryogenesis in which the baryon number is produced as the inflaton is rolling down its potential. In this paper we will merely suggest the idea, and leave study of the implementation of the idea for future work.

For this particular idea to work, the inflaton would have to carry baryon number. If the inflaton rolls clockwise down the hump in the Mexican hat, then baryons are produced; if the inflaton rolls counterclockwise down the hump, then antibaryons are produced. In different regions of the Universe, there will be these two different kinds of behavior. Any one region will be blown up to become very large by the inflation. Thus our observable Universe, which lies inside one of these regions, had a 50-50 chance of being made primarily of baryons or of antibaryons. CP violation is not explicitly required in the Lagrangian, as the sign of the baryon number is determined by the initial conditions, namely the direction of the roll of the field. These ideas are very similar to those of the authors of [5,6].

The reheating temperature in this scenario could be very low. In particular, if $T_{\text{reheat}} < T_{\text{electroweak}}$, sphalerons do not erase any baryon asymmetry generated during inflation. Of course, we also need to return to the standard evolution of the Universe at a high enough temperature for nucleosynthesis; i.e., $T_{\text{reheat}} > T_{\text{nucleosynthesis}}$. It may be a nice feature to have a very low reheating temperature, as many inflationary models are very constrained by the requirement of a high reheat temperature.

In order for this to work, the Φ field must carry a baryon number. The current that is explicitly broken by instantons or by whatever else provides the tilt (the cosine potential) cannot be orthogonal to baryon number.

In that case the baryon number of the Φ field will be proportional to the angular momentum as the inflaton rolls down: one direction of roll will correspond to baryon production and the other to antibaryon production. The baryon current carried by the Φ field is $J^\mu = i[\Phi \partial^\mu \Phi^* - \Phi^* \partial^\mu \Phi]$. Since $\langle \Phi \rangle = f e^{i\theta}$, the baryon number density $\langle n_B \rangle \equiv \langle J_0 \rangle = f^2 \dot{\theta}$, namely the angular momentum of the two-dimensional mechanical motion of the Φ field in the plane ($\text{Re}\Phi, \text{Im}\Phi$). As an example, in the Lagrangian in Eq. (2.7), we have considered a symmetry whereby Φ and Q transform whereas L does not. Thus Φ and Q could carry baryon number while L does not. Q and L would not be ordinary quarks and leptons; rather they would be hidden sector particles that could be made to couple to quarks and leptons in such a way that Q carries baryon number while L does not.

There are many constraints that such a model must satisfy. One must be careful about the quantum numbers carried by the various fields: namely $SU(3)$ color, the gauge group that became strong at scale Λ and produced the cosine potential in the first place, and baryon number. One must also ensure that the present day violation of baryon number predicted for ordinary matter is not in excess of observations. Also, we do not want the only decay mode to be to baryonic matter of our Universe. Somehow there must be nonbaryonic decay modes or decays to baryons that remain in the hidden sector. Simultaneously satisfying all these constraints is difficult. However, we have by no means exhausted all the possibilities, and leave this investigation to future work should the idea prove promising enough. As indicated near the end of Sec. III above, the calculation of the baryon number produced in such a model will be performed in future work.

VI. CONCLUSIONS

As a Nambu-Goldstone boson θ moves in a potential, it can produce fermions that it couples to. A semiclassical calculation of particle production was performed. The back reaction of quantum fermions on the evolution of a classical θ was calculated for a specific simple model, to provide a general framework within which one can calculate production of other particles as well. The primary results of our calculations in flat spacetime are Eqs. (2.21) and (2.22). Generalization to curved spacetime with massive fermions was discussed. We argued that enhanced production of bosons due to parametric resonance is probably not important here in an expanding universe; a more general investigation of the effects of expansion on resonance is warranted in the future. We are especially interested in the model of natural inflation, in which the inflaton is a pseudo-Nambu-Goldstone boson. It may be possible for the inflaton to create baryon asymmetry at the exit from inflation simultaneously with the Universe reheating.

ACKNOWLEDGMENTS

K.F. would like to thank Liz Jenkins, Salman Habib, Glennys Farrar, and especially Steve Selipsky and Fred

Adams for useful conversations. K.F. gratefully acknowledges support from the NSF Presidential Young Investigator Program and from NSF Grant No. INT91-16745. A.D. is grateful for the hospitality of the Department of

Physics of the University of Michigan where this work was started. K.F. thanks the Aspen Center for Physics for hospitality during her stay.

APPENDIX A: CALCULATION OF SEMICLASSICAL EQUATION FOR θ FIELD

We wish to calculate the right-hand side of Eq. (2.17). Using the quantization in Eq. (2.19), we find that

$$\begin{aligned} \langle \bar{L}(x)G_Q(x,y)L(y) \rangle &= \sum_s \sum_{s'} \int \frac{d^3 p d^3 p'}{(2\pi)^3} \frac{m_L}{\sqrt{E_p E_{p'}}} \bar{v}_{p'}^{s'} G_Q(x,y) v_p^s e^{-ip' \cdot x} e^{ip \cdot y} \langle d_{p'}^{s'} d_p^{s\dagger} \rangle \\ &= \sum_s \int \frac{d^3 p}{(2\pi)^3} \frac{m_L}{E_p} \bar{v}_p^s G_Q(x,y) v_p^s e^{-ip \cdot (x-y)}, \end{aligned} \quad (\text{A1})$$

where the second equality follows since $\langle d_{p'}^{s'} d_p^{s\dagger} \rangle = \delta_{ss'} \delta^3(\mathbf{p} - \mathbf{p}')$. Using

$$G_Q(x,y) = i \int \frac{d^4 l}{(2\pi)^4} \frac{\exp[-il \cdot (x-y)]}{l_\mu \gamma^\mu - m_Q + i\epsilon} \quad (\text{A1a})$$

and $\sum_s v_p^s \bar{v}_p^s = (p_\mu \gamma^\mu - m_L)/2m_L$, we find

$$\langle \bar{L}(x)G_Q(x,y)L(y) \rangle = \int \frac{d^3 p}{(2\pi)^7} \int \frac{d^4 l}{2E_p} \left[\frac{i}{l^2 + m_Q^2} \right] e^{[i(p+l) \cdot (y-x)]} \text{Tr}[(l_\mu \gamma^\mu + m_Q)(p_\mu \gamma^\mu - m_L)]. \quad (\text{A2})$$

In the massless limit considered in Sec. II, the trace becomes $\text{Tr}[l_\mu p_\nu \gamma^\mu \gamma^\nu] = 4l \cdot p$ and we have Eq. (2.20). The calculation of $\langle \bar{Q}(x)G_L(x,y)Q(y) \rangle$ is similar.

Using Eq. (2.20), we now have terms on the right-hand side of Eq. (2.17) such as

$$I = -ig^2 \int d^4 y e^{[i\theta(x) - i\theta(y)]} \int \frac{d^3 p}{(2\pi)^7} \frac{d^4 l}{E_p l^2} e^{[i(p-l) \cdot (y-x)]} l \cdot p + \text{H.c.}, \quad (\text{A3})$$

where we have taken $l \rightarrow -l$ compared to previous expressions. We now write $l^\mu = (E_l, \mathbf{l})$ and $p^\mu = (E_p, \mathbf{p})$ so that $l \cdot p = E_l E_p - \mathbf{l} \cdot \mathbf{p}$. Henceforth we will assume no spatial gradients in the θ field; i.e., $\theta = \theta(t)$ only. We will now use $\int d^3 y e^{[i(\mathbf{p}-\mathbf{l}) \cdot \mathbf{y}]} = (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{l})$ to write

$$I = -ig^2 \int dt_y e^{[i\theta(t_x) - i\theta(t_y)]} \int \frac{d^4 l}{(2\pi)^4} \frac{d^3 p}{E_p} e^{[i(E_p - E_l)(t_y - t_x)]} \delta^3(\mathbf{p} - \mathbf{l}) \frac{E_l E_p - \mathbf{l}^2}{E_l^2 - \mathbf{l}^2} + \text{H.c.} \quad (\text{A4})$$

We now do $\int dE_l$ and find a nonzero contribution from the pole at $E_l = -|\mathbf{l}|$. Note that we take the retarded Green's function, which gives nonzero result only for $t_y < t_x$. We have

$$I = \frac{g^2}{2\pi^2} \int_0^\infty E_l^2 dE_l \int_{-\infty}^{t_x} dt_y e^{[2iE_l(t_y - t_x)]} e^{[i\theta(t_x) - i\theta(t_y)]} + \text{H.c.} \quad (\text{A5})$$

Adding all terms of this form that contribute to the right-hand side of Eq. (2.17), relabeling E_l as ω , and defining $t' = t_y - t_x$, we find

$$\langle \mathcal{D}^2 \theta + U'(\theta)/f^2 \rangle = -\frac{4g^2}{\pi^2} \int_0^\infty d\omega \omega^2 \int_{-\infty}^0 dt' \sin(2\omega t') \sin[\theta(t+t') - \theta(t)]. \quad (\text{A6})$$

This is the result quoted in Eq. (2.21).

We can rewrite this in the form given in Eq. (2.22) if we now perform the ω integration:

$$\begin{aligned} \int_0^\infty d\omega \omega^2 \sin(2\omega t') &= -\frac{1}{4} \frac{\partial^2}{\partial t'^2} \left[\int_0^\infty d\omega \sin 2\omega t' \right] \\ &= \lim_{\omega \rightarrow \infty} \frac{\partial^2}{\partial t'^2} \left[\frac{1}{2t'} (\cos 2\omega t' - 1) \right]. \end{aligned} \quad (\text{A7})$$

Next we do the t' integral by parts. We will use the notation $\Delta\theta = [\theta(t+t') - \theta(t)]$. The nonvanishing contribution is

$$\begin{aligned} \frac{1}{8} \lim_{w \rightarrow \infty} \int_{-\infty}^0 dt' \frac{\partial^2}{\partial t'^2} \left[\frac{1}{t'} (\cos 2wt' - 1) \right] \sin \Delta\theta &= -\frac{1}{8} \lim_{w \rightarrow \infty} \int_{-\infty}^0 dt' \frac{\partial}{\partial t'} \left[\frac{1}{t'} (\cos 2wt' - 1) \right] \frac{\partial}{\partial t'} \sin \Delta\theta \\ &= \frac{1}{8} \lim_{w \rightarrow \infty} \int_{-\infty}^0 dt' \left[\frac{1}{t'} (\cos 2wt' - 1) \right] \frac{\partial^2}{\partial t'^2} \sin \Delta\theta, \end{aligned} \quad (\text{A8})$$

where surface terms have all vanished. We now perform the derivative on $\sin \Delta\theta$. Then Eq. (A6) becomes

$$\langle \mathcal{D}^2\theta + U'(\theta)/f^2 \rangle = -\frac{g^2}{2\pi^2} \lim_{w \rightarrow \infty} \int_{-\infty}^0 dt' \left[\frac{\cos 2wt' - 1}{t'} \right] \left[\ddot{\theta}(t+t') \cos \Delta\theta - \dot{\theta}^2(t+t') \sin \Delta\theta \right]. \quad (\text{A9})$$

This is the result quoted in Eq. (2.22)

APPENDIX B: CALCULATION OF MASS RENORMALIZATION TERM FOR THE CASE OF SMALL OSCILLATIONS AROUND THE MINIMUM

To obtain Eq. (3.4), we must calculate the term

$$\begin{aligned} m_R^2 \theta_0 \cos m_R t \int_{-\infty}^0 \frac{dt'}{t'} [1 - \cos 2wt'] \cos m_R t' &= m_R^2 \theta_0 \cos m_R t \int_{-\infty}^0 \frac{dt'}{2t'} [\cos m_R t' - \cos \alpha t' + \cos m_R t' - \cos \beta t'] \\ &= m_R^2 \theta_0 \cos m_R t \int_{-\infty}^0 \frac{dt'}{t'} [\sin(m_R + w)t' \sin wt' + \sin wt' \sin(w - m_R)t'], \end{aligned} \quad (\text{B1})$$

where $\alpha = 2w + m_R$ and $\beta = 2w - m_R$. After doing the integration, we find that this term is

$$-\frac{m_R^2 \theta_0}{2} \cos m_R t \left[\ln \left(\frac{2w + m_R}{m_R} \right) + \ln \left(\frac{2w - m_R}{m_R} \right) \right]. \quad (\text{B2})$$

In the limit $w \rightarrow \infty$, this becomes

$$-m_R^2 \theta_0 \cos m_R t \ln(2w/m_R), \quad (\text{B3})$$

a logarithmically divergent term that renormalizes the mass.

-
- [1] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[2] K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990); F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. D **47**, 426 (1993).
[3] A. Dolgov and Kirilova, Yad. Fiz. **51**, 273 (1990) [Sov. J. Nucl. Phys. **51**, 172 (1990)].
[4] L. F. Abbott, E. Farhi, and M. B. Wise, Phys. Lett. **117B**, 29 (1982); A. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, Phys. Rev. Lett. **48**, 1437 (1982); A. D. Dolgov and A. D. Linde, Phys. Lett. **116B**, 329 (1982); J. Traschen and R. Brandenberger, Phys. Rev. D **42**, 2491 (1990); J. P. Paz, *ibid.* **42**, 530 (1990); D. Boyanovsky, H. J. de Vega, R. Holman, D. S. Lee, and A. Singh, Carnegie-Mellon University Report No. PITT-94-07, 1994 (unpublished).
[5] I. Affleck and M. Dine, Nucl. Phys. B **249**, 361 (1985).
[6] A. G. Cohen and D. B. Kaplan, Phys. Lett. B **199**, 251 (1987).
[7] L. Kofman, A. Linde, and A. Starobinski, Phys. Rev. Lett. **73**, 3195 (1994).
[8] Y. Shtanov, J. Traschen, and R. Brandenberger, "Universe Reheating after Inflation," Brown University Report No. BROWN-HET-957 (unpublished).
[9] H. Quinn and R. Peccei, Phys. Rev. Lett. **38**, 1440 (1977).
[10] As written, the Yukawa coupling $g\Phi\bar{Q}L+\text{H.c.}$, would generate masses for the fermions $m_\psi \sim gf$. These masses would be extremely large, in fact so large that the Nambu-Goldstone boson would never be able to produce them. However, this Yukawa coupling is just a schematic for the real coupling. As in the case of the axion models, the real Yukawa coupling that gives a fermion its mass is a coupling of the fermion to a different scalar, not directly the PQ scalar. For example, for the case of the Dine, Fischler, Srednicki axion [11], the Yukawa couplings are with two scalar doublets ϕ_u and ϕ_d , which transform under the chiral symmetry in the same way as the PQ field Φ . The vacuum expectation values (VEV's) of these other scalars are much lower in energy scale than the VEV of the Φ field. Thus the fermions retain smaller masses. The axion is then a linear combination of all these scalars, but predominantly made of the Φ component. To mimic this physics without including all the complications, we keep the simple Yukawa coupling of Eq. (2.1), but take $g \ll 1$ to ensure proper fermion masses.
[11] M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981).
[12] C. T. Hill and G. G. Ross, Phys. Lett. B **203**, 125 (1988); C. T. Hill and G. G. Ross, Nucl. Phys. **B311**, 253 (1979).
[13] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, *ibid.* **40**, 279 (1978).