Low energy properties of the heavy vector fermions and electroweak symmetry breaking

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We discuss the properties of the heavy vector fermions with bare Dirac mass terms and an SU(2)×SU(2) global symmetry in the Yukawa interaction Lagrangian. Using the heat kernel expansion method we calculate their contributions to the low energy observables. We argue that these heavy fermions may be responsible for a soft dynamical symmetry breaking through their condensation. We also discuss the possibility of considering ordinary fermions as one part of the vector fermions within our model.

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One of the most profound and important subjects in quantum field theory and particle physics is the spontaneous breaking of continuous symmetries. One well-known example in the real world is the spontaneous breaking of chiral symmetry in hadron interactions, which is, as can be proven modulo some highly plausible assumptions [1], governed by the confining force of the vectorlike gauge interaction, QCD. Despite the fact that too few examples have been given so far by nature of the spontaneously broken symmetry, in the theory aspect, we are also lacking an alternative mechanism in understanding the general features of the issue. In describing electroweak symmetry breaking, the most popular theory is the so-called Higgs mechanism [2] in which the electroweak symmetry is spontaneously broken by the nonvanishing vacuum expectation value (VEV) of an elementary scalar—the Higgs particle. It is also understood that, even if there is no elementary Higgs field, the Higgs mechanism may also be considered as an approximation of some complex dynamical symmetry-breaking mechanism, provided the Higgs field is now only a composite object with a nonzero VEV. It is appealing then to study the varieties of electroweak symmetry-breaking mechanisms. The technicolor model [3] is such an example in which electroweak symmetry breaking is induced by a strong, vectorlike (QCD-type) gauge interaction. However, the technicolor model and its simple extensions are known to encounter some difficulties; for example, the technicolor model leads to a large S parameter [4] compared with experiments. In this situation, it is therefore worthwhile to explore other possibilities for breaking electroweak symmetry spontaneously.

The precision tests at the CERN e^+e^- collider LEP, on the other hand, lead to strong constraints on the possible physics beyond the standard model. Among other things, one of the most severe constraints is from the ρ parameter. The experimental observation of the relation $\rho \simeq 1$ puts an upper bound on the top quark mass. Many discussions have been conducted about this issue. As a general conclusion, within the models studied in the past, it is difficult to weaken the top quark upper bound and, on the contrary, it is easy to lower this bound [5]. There is a recent paper by Caravaglios [6] in which he considered a simple example of a vector family of leptons, that is, a standard fermion family plus a right-handed neutrino and a conjugate family where the role of the left-handed and the right-handed fermions are inverted with respect to the gauge interactions. He concluded that the vector family contributions to the ρ parameter and to the S (or ϵ_{N3} in Ref. [6]) parameter are suppressed by m^2/M^2, where m is generated by the Yukawa interaction and M is the bare mass of the heavy vector fermions. Further, the contribution to the ρ parameter is negative, if one includes the Yukawa interaction with the charge conjugated neutrino fields.

In this paper, we will also discuss the physical effects induced by the heavy vector fermions. The appearance of the vector fermion family, or mirror fermions [7], is a natural consequence of many grand unification models [8], and composite models [9]. In the picture of compositeness, these heavy vector fermions may be related to the parity doubled excitations of the ordinary quarks and leptons. In fact, if the chiral symmetry can be realized in Wigner-Weyl type, which is directly related to the discussion of compositeness of quarks and leptons [10], then the bound state spectrum consists of some chiral bound states and parity doubled massive fermions (their masses are not protected by chiral symmetry).

Although there are various models motivating the possible existence of heavy vector fermions, we will, however, not stick to any concrete model in our discussions. Instead, regardless of the underlying dynamics which controls the behavior of these heavy fermions, we assume, on a rather general ground, that despite the gauge interactions, it is described by an effective Lagrangian of Higgs-Yukawa interactions which has an SU(2) isospin symmetry and another SU(2) global symmetry in the parity

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doublet space.[1] The spontaneous symmetry breaking induced by the Higgs field will break the global symmetry between the parity doubled fermion fields. Especially, it will lead to the mass splitting between the two isospin fermion doublet states which are degenerate in mass before the symmetry breaking.

For each isospin Dirac fermion fields ψ_1, from our assumption, there is another fermion doublet ψ_2 with opposite parity[2]:

$$\mathbf{P}: \Psi(\vec{x}) = \gamma_0 \rho_3 \Psi(-\vec{x}), \quad \Psi = (\psi_1, \psi_2)^T , \tag{1}$$

where the ρ_3 is the third Pauli matrix in the parity doublet space. A fermion and its parity partner appearing in one parity doublet are assumed to be degenerate in mass and all the other quantum numbers when the symmetry are not broken down at high energies. The spontaneous symmetry breaking is responsible for giving the weak gauge bosons' masses as well as the mass splitting between the parity partners.

In Sec. I, we present the construction of the effective Lagrangian, although it is more or less in a standard way. We will only focus on common general features of both quarks and leptons; i.e., we will not discuss those relevant only to the specific property of the neutrino field; this is different from the discussion made in Ref. [6]. In Sec. I, we also investigate the interesting possibility of describing the ordinary quarks and leptons in the parity doublet model. In Sec. II, we calculate the low-energy effects induced by those heavy fermions, by integrating them out using the heat-kernel method in our effective Lagrangian. In agreement with Caravaglios, we find that their contribution to the oblique corrections is suppressed by m^2/M^2. However, their contributions to the decay constant of the Goldstone boson $f_\pi(v)$ can be large. We therefore argue that, if there exists a strong interaction which causes the condensation of the heavy fermion fields, they can be responsible for a soft dynamical symmetry breaking. In other words, the model only leads to a very small contribution to the low-energy observables.

I. THE PARITY DOUBLET MODEL FOR ELECTROWEAK INTERACTIONS

We begin by introducing a SU(2)×SU(2) global transformation of the Ψ fields defined in Eq. (1):

$$\Psi \to e^{i\vec{\alpha}\cdot\vec{\tau}+i\rho_2\vec{\beta}\cdot\vec{\tau}}\Psi , \tag{2}$$

where $\vec{\tau}$ are the generators of the weak isospin space. We further introduce the notation[3]

$$\Psi^l \equiv \frac{1-\rho_2}{2}\Psi, \quad \Psi^r \equiv \frac{1+\rho_2}{2}\Psi . \tag{3}$$

From Eq. (2), we can further define

$$\begin{aligned} \Psi^l &\to e^{i(\vec{\alpha}-\vec{\beta})\cdot\vec{\tau}}\Psi^l \equiv G_l \Psi^l , \\ \Psi^r &\to e^{i(\vec{\alpha}+\vec{\beta})\cdot\vec{\tau}}\Psi^r \equiv G_r \Psi^r . \end{aligned} \tag{4}$$

There are two invariant fermion bilinears under the rotation Eq. (4), $\bar{\Psi}\Psi$ and $\bar{\Psi}\rho_2\Psi$. We stress again the difference between the conventional chiral symmetry and that in the parity doublet model, that the mass term is allowed by symmetry requirement, only the mass splitting and the mixing between the two parity partners are the right issue of the spontaneous symmetry breaking of the parity doublet model.

We also introduce the collective variable of the Goldstone fields which are other aspects of the spontaneous symmetry breaking[4]

$$\begin{aligned} \mathcal{U} &\equiv e^{i\rho_2\vec{\pi}\cdot\vec{\tau}/f_\pi} = \frac{U+U^\dagger}{2} - \rho_2\frac{U-U^\dagger}{2} , \\ U &= e^{-i\vec{\pi}\cdot\vec{\tau}/f_\pi} . \end{aligned} \tag{5}$$

It can easily be proven that $\bar{\Psi}\rho_3\mathcal{U}\Psi$ is invariant under Eq. (4) if U transforms as

$$U \to G_r U G_l^\dagger . \tag{6}$$

After defining the transformation properties of the fermion and Goldstone fields, the Lagrangian, which is invariant under the global SU(2)×SU(2) transformations, Eqs. (4) and (6), describing the interactions between a parity doublet and the Goldstone excitations of the spontaneous symmetry breaking, with terms of least dimension and bilinear in fermion fields, may be written as

$$\mathcal{L} = \bar{\Psi}\{i\partial\!\!\!/ - M - (m + \gamma_5\rho_2\tilde{m})\rho_3\mathcal{U}\}\Psi . \tag{7}$$

Notice that the three mass parameters in (7) can be different for different parity doublets. The electroweak gauge interactions are embedded in the fermionic La-

[1] Our original motivation comes out from the idea of the parity doublet model in the early discussions of hadron physics [11] which gives an interesting competitive mechanism for spontaneous symmetry breaking. We will use the language of the parity doublet model in the following discussion.

[2] For elementary fermions the definition of relative parity may only be a matter of convention, but when the fermions are composite objects the relative parities can be meaningful. Our convention in the present paper is just borrowed from the parity doublet model. We are not claiming that the fermions under our consideration are necessarily composite objects.

[3] Here we use the small l and r to denote the corresponding decomposition of the fermion fields, so as not to confuse with the conventional left and right notation which we will denote as the capital L and R in this paper, that is $\psi_L = (1-\gamma_5)/2\psi, \psi_R = (1+\gamma_5)/2\psi$.

[4] We chose the nonlinear σ modellike language throughout this paper and do not consider the renormalizability condition, although we can recover the renormalizability whenever it is necessary.

grangian in the following way:

$$i\not{\partial} \to i\not{\Delta} = i\not{\partial} - \not{R}\frac{1+\rho_2}{2} - \not{L}\frac{1-\rho_2}{2} , \quad (8)$$

where L and R are gauge fields coupled to Ψ_l and Ψ_r, respectively. Their concrete expressions depend on the electroweak property of the fermion fields. If they are chosen to be the same as the ordinary fermions, then

$$L_{\text{lepton}} = g_2\frac{\tau^i}{2}W^i - \frac{g_1}{2}B, \quad R_{\text{lepton}} = \left(\frac{\tau^3}{2} - \frac{1}{2}\right)g_1 B , \quad (9)$$

$$L_{\text{quark}} = g_2\frac{\tau^i}{2}W^i + \frac{g_1}{6}B, \quad R_{\text{quark}} = \left(\frac{\tau^3}{2} + \frac{1}{6}\right)g_1 B . \quad (10)$$

Equations (7) and (8) can be rewritten as

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Yukawa}} , \quad (11)$$

where

$$\mathcal{L}_{\text{gauge}} \equiv \bar{\Psi}_l\{i\not{\partial} - M - \not{L}\}\Psi_l + \bar{\Psi}_r\{i\not{\partial} - M - \not{R}\}\Psi_r \quad (12)$$

and

$$\mathcal{L}_{\text{Yukawa}} \equiv -\bar{\Psi}_r(m + \gamma_5\tilde{m})\rho_3 U\Psi_l - \bar{\Psi}_l(m - \gamma_5\tilde{m})\rho_3 U^\dagger\Psi_r . \quad (13)$$

The above equation (13) requires some comments. First, the gauge interactions are purely vectorlike since the bare mass term is allowed by gauge symmetries. The Lagrangian $\mathcal{L}_{\text{gauge}}$ is constructed as left-right symmetric. Second, the Yukawa interactions have an SU(2)×SU(2) global symmetry and are parity conserving. But as soon as we introduce the $SU(2)_W \times U(1)_Y$ [which is a subgroup of the global SU(2)×SU(2) field] gauge interactions, parity violation occurs. This is because we have endowed the two fermions with different parities, the parity operation changes the Ψ_l field into the Ψ_r field, and vice versa. If the two fermions have the same parity then the gauge interaction is parity conserving and only the γ_5 term of the Yukawa interaction violates parity explicitly. Also please notice that right-handed neutrinos decouple from other fields in the Lagrangian as guaranteed by Eq. (9).

After the electroweak symmetry breaking, they will, in general, touch other fields again.

We notice that there is a serious flaw in Eq. (12): there is no mass splitting within each weak-isospin doublet, i.e., the mass difference between up- and down-type quarks. In order to fix it we must add a $-M'\bar{\Psi}_r\tau_3\Psi_r$ term, which is isospin violating but (R) gauge invariant, into the Lagrangian Eq. (12). But since this term is parity violating, we can certainly add another parity-violating (but isospin conserving) mass term $-\tilde{M}\bar{\Psi}_r\Psi_r$. The additional Lagrangian then reads

$$\mathcal{L}' = -M'\bar{\Psi}_r\tau_3\Psi_r - \tilde{m}\bar{\Psi}_r\Psi_r . \quad (14)$$

For the convenience of later usage, we introduce the notation

$$v_\mu \equiv -R_\mu/2 - L_\mu/2 \quad \text{and} \quad a_\mu \equiv -R_\mu/2 + L_\mu/2 . \quad (15)$$

Now we discuss the symmetry breaking. To define $\xi = U^{1/2}$ and performing the following rotation of the fermion fields,

$$\Psi = \left(\xi^\dagger\frac{1+\rho_2}{2} + \xi\frac{1-\rho_2}{2}\right)\chi , \quad (16)$$

we can rewrite the above effective Lagrangian in the explicit symmetry-breaking phase (in the following we work in Euclidean convention):

$$\mathcal{L} = \bar{\chi}\left\{\not{\partial} + \gamma_\mu\Gamma_\mu - \frac{i}{2}\rho_2\gamma_\mu\xi_\mu\right\}\chi - \bar{\chi}\left(M + M'\frac{1+\rho_2}{2}\tau_3 + \tilde{M}\frac{1+\rho_2}{2} + i\gamma_5\rho_1\tilde{m} + \rho_3 m\right)\chi , \quad (17)$$

where

$$\Gamma_\mu = \frac{1}{2}\{\xi^\dagger[\partial_\mu - i(v_\mu + a_\mu)]\xi + \xi[\partial_\mu - i(v_\mu - a_\mu)]\xi^\dagger\} , \quad (18)$$

$$\xi_\mu = i\{\xi^\dagger[\partial_\mu - i(v_\mu + a_\mu)]\xi - \xi[\partial_\mu - i(v_\mu - a_\mu)]\xi^\dagger\} . \quad (19)$$

The mass matrix in the above Lagrangian is not diagonal; physical mass eigenstates are obtained by the following rotation of the fermion field:

$$\chi \to \exp\left[i\rho_1\left(\frac{\theta_1+\theta_2}{2} + \frac{\theta_1-\theta_2}{2}\tau_3\right)\right]\exp\left[-i\gamma_5\rho_1\left(\frac{\delta_1+\delta_2}{2} + \frac{\delta_1-\delta_2}{2}\tau_3\right)\right]\chi , \quad (20)$$

where

$$\sin 2\theta_{1,2} = \frac{\frac{1}{2}(\tilde{M} \pm M')}{\sqrt{m^2 + \left(\frac{\tilde{M}\pm M'}{2}\right)^2}} , \quad (21)$$

and

$$\sin 2\delta_{1,2} = \frac{\tilde{m}}{\sqrt{\tilde{m}^2 + \left(M + \frac{\tilde{M}\pm M'}{2}\right)^2}} . \quad (22)$$

The diagonalized fermion mass matrix is then

$$\mathcal{M} = \frac{M_1+M_2}{2} + \frac{M_1-M_2}{2}\tau_3 + \rho_3\left\{\frac{m_1+m_2}{2} + \frac{m_1-m_2}{2}\tau_3\right\} , \quad (23)$$

where

$$M_{1,2} = \sqrt{\tilde{m}^2 + \left(M + \frac{\tilde{M} \pm M'}{2}\right)^2} ,$$
$$m_{1,2} = \sqrt{m^2 + \left(\frac{\tilde{M} \pm M'}{2}\right)^2} . \quad (24)$$

It is noticed that after the rotation Eq. (20) the mass eigenstates are no longer parity eigenstates.[5] The gauge invariant mass terms, denoted by the capital M, may (or may not) be generated dynamically above the electroweak scale but look purely kinematic at the electroweak scale. Another kind of mass term is generated by Yukawa couplings ($\tilde{m}, m$) and is a consequence of electroweak symmetry breaking.

After the rotation, the gauge interaction form in the Lagrangian Eq. (17) will change its form except for the electromagnetic interaction. For example, if defining the $Z\bar{\Psi}\Psi$ term by the $(g_V - g_A\gamma_5)$ form and the $W\bar{\Psi}\Psi$ coupling the $g_V^W - g_A^W\gamma_5$ term (our notation, in the standard model, is $g_V^b = 1 - \frac{4}{3}s_W^2$, $g_V^t = 1 - \frac{8}{3}s_W^2$, $g_A^b = g_A^t = 1$, and $g_V^W = g_A^W = 1$), then we have the following results (terms with the nondiagonal coupling in the parity doublet space can be found in Appendix A):

$$\delta g_V^b = -\rho_3 \sin 2\theta_2 \cos 2\delta_2, \quad \delta g_V^t = -\rho_3 \sin 2\theta_1 \cos 2\delta_1 , \quad (25)$$

$$g_A^b = -\rho_3 \cos 2\theta_2 \sin 2\delta_2, \quad g_A^t = -\rho_3 \cos 2\theta_1 \sin 2\delta_1 , \quad (26)$$

$$g_V^W = \cos(\theta_1-\theta_2)\cos(\delta_1-\delta_2) - \rho_3 \sin(\theta_1+\theta_2)\cos(\delta_1+\delta_2) ,$$
$$g_A^W = -\sin(\theta_1-\theta_2)\sin(\delta_1-\delta_2) - \rho_3 \cos(\theta_1+\theta_2)\sin(\delta_1+\delta_2) .$$

We observe from the above relations that these parameters g_V and g_A for the heavy fermions are very different from the standard model results because of the large bare mass terms M, M', and $\tilde{M}$ in Eq. (21). It is interesting to notice that in the above framework we can also consider that the ordinary fermions appear in parity doublets. This is possible when the bare mass parameters are much smaller than the masses generated by the Yukawa interaction. After the symmetry breaking, while the known quarks and leptons are "left" handed, their cousins with odd parity are "right" handed[6] and massive ($\simeq 2m$ or $2\tilde{m}$) (notice that the neutrino partners are also massive). The "deviation" of an ordinary fermion field in our model to how it behaves in the standard model may, roughly speaking, be characterized by the axial-vector coupling g_A (proportional to the Z coupling to fermions) ranging from 0 to 1. In the $g_A = 1$ ($M \to 0$) limit, the standard model result is recovered, i.e., light fermions are of purely $V - A$ type. But fermion masses and the gauge couplings in our model are correlated with each other; the value of g_A cannot be exactly unity. Within the experimental constraints on the deviation (typically less than 10^{-2}–10^{-3}) of the gauge couplings to their standard model values, we find no difficulty in reproducing the light fermion spectrum (even when assuming universal Yukawa coupling for different families) by varying the bare mass parameters. The most severe difference in our model compared with the standard one occurs on the top quark. In order to explain its large mass (with the constraints on δg_V^b and g_A^b from experiments) we find significant difference for the top couplings, within reasonable range of the scale parameter, m of $\tilde{m}$. As can be seen from Figs. 1 and 2, m and $\tilde{m}$ are necessarily large in order to explain the large top quark mass. In fact a lower bound on m can be obtained: $m \geq M_t/\sqrt{2(1-g_A^b)}$ (the equality holds when $M' \gg m$). Taking for example $g_A^b > 0.99$ and $M_t > 140$ GeV one obtains $m > 1$ TeV. This means that in the model the Yukawa couplings are unavoidably nonperturbative in the top quark sector, which unfortunately causes our model to be less predictive beyond the tree level. The Yukawa couplings, on the other hand, are known to suffer from the triviality problem which leads to an upper bound on the magnitude. It is expected that m and $\tilde{m}$ should be no greater than a few TeV.[7] The constraints on m, for fixed M_t, can be translated to the constraints on g_A^t as can be seen from Fig. 1. If we neglect the unpleasant fact of the large Yukawa couplings, we may estimate the one loop contribution to the S and T parameter (the Yukawa couplings only enter in the two loop contribution). Numerical results show that the S parameter is acceptable but show a large positive T parameter. This may be considered the main shortcoming of the model when trying to explain the ordinary fermions as part of the parity doublets. The nondecoupling effects of the heavy chiral fermion to low-energy physics is a difficult task. Efforts have been made in the literature using various nonperturbative methods, and it

[5]A subsequent unitary rotation on fermion fields (after the parity operation) transforms the Lagrangian back to the mass eigenstates. Physics such as strong or electromagnetic interaction will not be influenced.

[6]It is exactly this symmetry that guarantees the anomaly cancellation within each parity doublet, without the necessity to rescue the standard model anomaly cancellation condition $\sum_{q,l} Q = 0$.

[7]We expect that we are fortunate to have room to put this m parameter. The upper bound on the conventional Higgs-Yukawa coupling obtained in [12] by using the large N_F analysis is $M_Y < 10\pi v$.

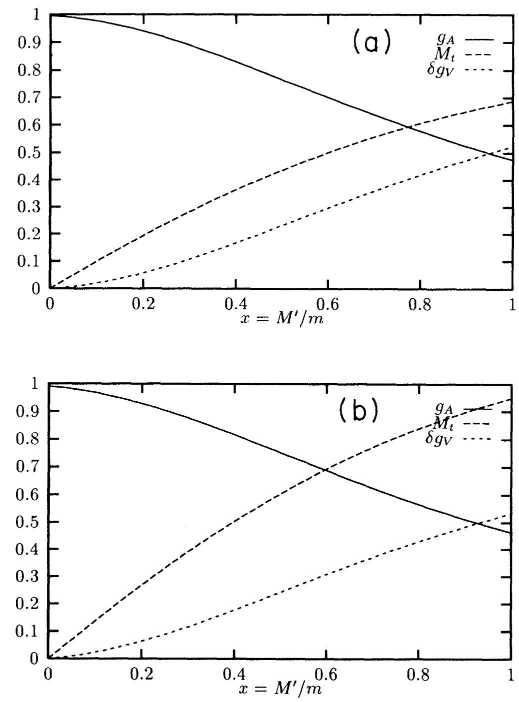

FIG. 1. (a) The $g^t_A, \delta g^t_V$, and the top quark mass (scaled as $m/10$) as a function of M'/m, at $\epsilon = M/\tilde{m} = 0.1$ ($g^b_A \sim 1 - \epsilon^2/2$). (b) The same as (a) but $\epsilon = 0.14$.

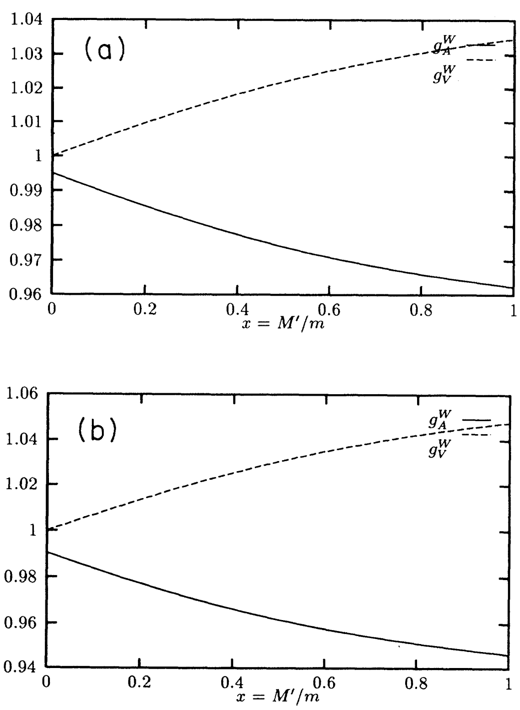

FIG. 2. (a) The coupling g^W_V and g^W_A as a function of M'/m with $\epsilon = 0.1$. These couplings only depend weakly on the "top" quark mass; especially when $\epsilon = 0$ they are degenerate to the constant unity, i.e., the standard model value. (b) Same as (a), but $\epsilon = 0.14$.

is agreed that there may exist nondecoupling effects, but the T parameter will not blow up when the fermion mass gets large [13], in contrast to the results indicated by the one loop perturbative calculation.

II. ELECTROWEAK SYMMETRY BREAKING AND PROPERTIES OF THE "VECTORLIKE" FERMIONS

The W and Z bosons acquire their masses by absorbing the corresponding Goldstone excitations. This is achievable because of the existence of the kinematic term of the Goldstone fields in the Lagrangian:

$$\mathcal{L}_G = \frac{f_\pi^2}{4} \mathrm{tr} D_\mu U D^\mu U^\dagger \ , \qquad (27)$$

where

$$D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu) \qquad (28)$$

and $f_\pi = (1/\sqrt{2}G_F)^{-1/2} = 246$ GeV. The Goldstone field Lagrangian (27) can be either fundamental like that in the standard model or an induced one from dynamical symmetry breaking. In the latter case, these Goldstone excitations are composite objects, and f_π is induced by the nonvanishing vacuum expectation values of certain kind of composite field operators. In our case, this should be $\langle \bar{\Psi} \rho_3 \Psi \rangle$ or so. The dynamically broken symmetry is our main concern, although we may build a renormalizable Lagrangian from Eqs. (7) and (8) by invoking the Higgs mechanism. Now the standard procedure can be performed to give the desired weak gauge boson masses and the Weinberg angle. In the unitary gauge where $U = 1$,

$$\mathcal{L}_G \to M_W^2 W^+ W^- + \frac{1}{2} M_Z^2 Z^2 \ , \qquad (29)$$

where

$$M_W^2 = \frac{f_\pi^2}{4} g_2^2, \quad M_Z^2 = \frac{M_W^2}{\cos^2\theta_W}, \quad \cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} \ ,$$
$$e = \frac{g_2 g_1}{\sqrt{g_2^2 + g_1^2}} \ . \qquad (30)$$

We will calculate the effects of the vectorlike heavy fermions on low energy physics in the limit $M \gg m$.[8] These fermions are decoupling at low energies and hence can be integrated out safely, because the mass parameter M comes out from a bare mass term and the decoupling theorem [14] applies. These vectorlike fermions influence low-energy physics by modifying these coupling

[8] It is contrary to the situation in the above section, when discussing the possibility to consider the ordinary fermions as appearing in parity doublets, where another limit $m, \tilde{m} \gg M$ should be taken.

constants of the low-energy effective Lagrangian. These modifications are calculable using the heat kernel expansion method [15].

To proceed we assume the Lagrangian equation (17) is an effective Lagrangian below a cutoff Λ. This means that once we run down from the energy scale much higher than Λ, more and more high frequency modes including those very massive vectorlike fermions are integrated out until the scale Λ is reached where the symmetry breaking is assumed to occur. The effective Lagrangian describing the interaction between the parity doublet fermions and the Goldstone bosons therefore only contains particles below the scale Λ. We further assume that Λ is much larger than the decay constant of the Goldstone bosons, $f_\pi = v = 246$ GeV. This assumption is quite nontrivial; please compare with the so-called walking technicolor model in which the similar requirement is tried to be adjusted dynamically [16].

We proceed further by integrating out those heavy fermions (where the bare mass parameter M lies in the region $\Lambda > M \gg m$; this is only possible because of the assumption we made above) in the effective Lagrangian. The resulting effective Lagrangian is of Gasser-Leutwyler type [up to $O(p^4)$ terms in the derivative expansions] [17] with external gauge field coupling to the Goldstone excitations. To illustrate the main idea we consider the simplified case $\tilde{m} = M' = \tilde{M} = 0$. We conclude from our calculation that the heavy parity doublets contribute to the vacuum expectation value of electroweak symmetry breaking as

$$\delta(f_\pi^2) = \delta v^2 \simeq \frac{m^2 N_d}{2\pi^2}\left(\ln\frac{\Lambda^2}{M^2}\right) , \tag{31}$$

where N_d is the number of parity doublets. This is to be compared with the pion decay constant obtained in the QCD effective action approach [18], $f_\pi^2 = (N_c/4\pi^2)M_Q^2\ln(\Lambda_{\rm QCD}^2/M_Q^2)$, where M_Q is the constitute quark mass which is similar to m in our present discussion. We notice that if in the above equation (31) $m = O(v)$ then several of these heavy fermions would be enough to take charge of the electroweak symmetry breaking. Therefore if there are strong attractive forces in the appropriate channel to cause the heavy fermion condensation, then they may play the role similar to techniquarks in the technicolor model. Each parity doublet contributes to the coupling of the $O(p^4)$ term as the following:

$$H_1 = -\frac{1}{48\pi^2}\ln\frac{\Lambda^2}{M^2} , \tag{32}$$

$$L_1 = L_2 = 0 , \tag{33}$$

$$L_3 = \frac{1}{48\pi^2}\frac{m^2}{M^2} , \tag{34}$$

$$L_9 = \frac{1}{24\pi^2}\frac{m^2}{M^2} , \tag{35}$$

$$L_{10} = -\frac{1}{48\pi^2}\frac{m^2}{M^2} . \tag{36}$$

The remarkable property of the heavy parity doublet fermions is that their contribution to the oblique corrections of the standard processes is suppressed by m^2/M^2 which can be understood in the point of view of the decoupling theorem. Especially, its contribution to the S parameter is $(1/3\pi)(m^2/M^2)$, which is still positive definite but much smaller in magnitude than the QCD-like technicolor model result. We have not calculated the isospin breaking effects in the effective Lagrangian approach. However, we estimate the heavy vector fermion contribution to the ρ parameter in one loop perturbative calculation. We find that the correction is vanishing in the large M limit (keeping M' fixed). In this sense, the dynamics responsible for the symmetry breaking at the electroweak scale is rather weak, but still strong enough to give the masses of the weak gauge bosons. This way of dynamical electroweak symmetry breaking, if possible, may also be distinguishable from SM at low energies because of the difference between the L_1 terms [17].

We have not been able to discuss what kind of underlying dynamics can be responsible for the above scenario. One difficulty we may encounter when discussing the possibility of dynamical symmetry breaking is from the Vafa-Witten theorem [19] which states that a parity conserving vectorlike gage interaction cannot be responsible for the spontaneous breaking of vectorlike symmetry. If this theorem applies to our case, then we may lack the underlying dynamics of the effective Yukawa interaction although it is possible to break vector symmetries through the four-fermion interactions [20]. However, when there are parity violation interactions such as in our case it is in general possible to violate vector symmetry spontaneously, and massless bound states can be composed of massive constituents [21].

III. CONCLUSIONS

In this paper we have discussed the physical consequences of assuming interactions between vector fermions. When the assumption is applied to the ordinary fermion sector, the model predicts a sizable deviation of the axial vector coupling g_A of the top quark to its SM value. This phenomenological consideration is not successful, partly due to the difficulty in understanding the behavior of large Yukawa couplings. However, it may still be interesting. For example we point out that it is very difficult (if not impossible) to understand the large fermion mass hierarchy problem within the context of SM, i.e., how to understand the very different fermion masses generated at the electroweak scale. In our model, this is translated to mean that different fermions have different kinematic mass terms. The Yukawa interaction itself is isospin invariant and may even be universal for different families. The role of those heavy fermions with their masses mainly coming from bare mass terms is discussed. We calculated the low-energy effects induced by these heavy fermions using the heat-kernel expansion method and argue that they may be responsible for a soft breaking of the electroweak symmetry which leads to a safely small S parameter.

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APPENDIX A

We list in the following all the vector and axial-vector couplings of the W gauge boson to fermions,

$$g^V_{Wtb} = \cos(\theta_1 - \theta_2)\cos(\delta_1 - \delta_2) + \sin(\theta_1 + \theta_2)\cos(\delta_1 + \delta_2) , \tag{A1}$$

$$g^A_{Wtb} = \cos(\theta_1 + \theta_2)\sin(\delta_1 + \delta_2) - \sin(\theta_1 - \theta_2)\sin(\delta_1 - \delta_2) , \tag{A2}$$

$$g^V_{W\tilde{t}\tilde{b}} = \cos(\theta_1 - \theta_2)\cos(\delta_1 - \delta_2) - \sin(\theta_1 + \theta_2)\cos(\delta_1 + \delta_2) , \tag{A3}$$

$$g^A_{W\tilde{t}\tilde{b}} = -\cos(\theta_1 + \theta_2)\sin(\delta_1 + \delta_2) - \sin(\theta_1 - \theta_2)\sin(\delta_1 - \delta_2) , \tag{A4}$$

$$g^V_{W\tilde{t}b} = -\cos(\theta_1 + \theta_2)\cos(\delta_1 + \delta_2) + \sin(\theta_1 - \theta_2)\cos(\delta_1 - \delta_2) , \tag{A5}$$

$$g^A_{W\tilde{t}b} = \cos(\theta_1 - \theta_2)\sin(\delta_1 - \delta_2) + \sin(\theta_1 + \theta_2)\sin(\delta_1 + \delta_2) , \tag{A6}$$

$$g^V_{Wt\tilde{b}} = \cos(\theta_1 + \theta_2)\cos(\delta_1 + \delta_2) + \sin(\theta_1 - \theta_2)\cos(\delta_1 - \delta_2) , \tag{A7}$$

$$g^A_{Wt\tilde{b}} = \cos(\theta_1 - \theta_2)\sin(\delta_1 - \delta_2) - \sin(\theta_1 + \theta_2)\sin(\delta_1 + \delta_2) , \tag{A8}$$

and Z to fermions,

$$g^V_{Ztt} = 1 + \sin(2\theta_1)\cos(2\delta_1) , \tag{A9}$$
$$g^A_{Ztt} = \cos(2\theta_1)\sin(2\delta_1) , \tag{A10}$$
$$g^V_{Zbb} = 1 + \sin(2\theta_2)\cos(2\delta_2) , \tag{A11}$$
$$g^A_{Zbb} = \cos(2\theta_2)\sin(2\delta_2) , \tag{A12}$$
$$g^V_{Z\tilde{t}t} = \cos(2\delta_1)\cos(2\theta_1) , \tag{A13}$$
$$g^A_{Z\tilde{t}t} = -\sin(2\theta_1)\sin(2\delta_1) , \tag{A14}$$
$$g^V_{Z\tilde{b}b} = \cos(2\theta_2)\cos(2\delta_2) , \tag{A15}$$
$$g^A_{Z\tilde{b}b} = -\sin(2\theta_2)\sin(2\delta_2) , \tag{A16}$$
$$g^V_{Z\tilde{t}\tilde{t}} = 1 - \sin(2\theta_1)\cos(2\delta_1) , \tag{A17}$$
$$g^A_{Z\tilde{t}\tilde{t}} = -g^A_{Ztt} , \tag{A18}$$
$$g^V_{Z\tilde{b}\tilde{b}} = 1 - \sin(2\theta_2)\cos(2\delta_2) , \tag{A19}$$
$$g^A_{Z\tilde{b}\tilde{b}} = -g^A_{Zbb} . \tag{A20}$$

From these formulas we read off useful relations obeyed by these couplings:

$$\sum\{(g^V_W)^2 + (g^A_W)^2\} = 4 , \tag{A21}$$

$$\sum_{t,\tilde{t}}\{(g^V_Z)^2 + (g^A_Z)^2\} = 4 , \tag{A22}$$

$$\sum_{b,\tilde{b}}\{(g^V_Z)^2 + (g^A_Z)^2\} = 4 . \tag{A23}$$

Especially, we have relations which guarantee the cancellation of the triangle anomaly within each parity doublet, which are

$$\sum g^V_W g^A_W = 0 \tag{A24}$$

for the γW^+W^- triangle diagram,

$$g^A_{Ztt} + g^A_{Z\tilde{t}\tilde{t}} = g^A_{Zbb} + g^A_{Z\tilde{b}\tilde{b}} = 0 \tag{A25}$$

for the $Z\gamma\gamma$ diagram, and

$$\sum g^V_Z g^A_Z = 0 \tag{A26}$$

for the γZZ diagram. Relations required for anomaly cancellations in other diagrams are not independent.

APPENDIX B

In this section we present our calculations on the low-energy effects of heavy vectorlike fermions, using the heat-kernel expansion in the proper-time regularization scheme. For simplicity, we only consider the $M' = \tilde{M} = \tilde{m} = 0$ and the $M \gg m$ limit to illustrate our idea. The effective Lagrangian Eq. (17) is now simplified:

$$\mathcal{L} = \bar{\chi}\left\{ \not{\partial} + \gamma_\mu \Gamma_\mu - \frac{i}{2}\rho_2\gamma_\mu\xi_\mu \right\}\chi - \bar{\chi}(M + \rho_3 m)\chi \equiv \bar{\chi}\not{D}_E\chi . \tag{B1}$$

The expressions for Γ_μ and ξ_μ are already given in Eqs. (18) and (19). Now defining

$$D_\mu \equiv \partial_\mu + X_\mu \equiv \partial_\mu + \Gamma_\mu - \frac{i}{2}\rho_2\xi_\mu , \tag{B2}$$

we obtain

$$\not{D}^\dagger_E \not{D}_E = -D_\mu D_\mu - \frac{i}{2}\sigma_{\mu\nu}R_{\mu\nu} - m\gamma_\mu\rho_1\xi_\mu + 2Mm\rho_3 + M^2 + m^2 \equiv -D^2 + Y + (M^2 + m^2) , \tag{B3}$$

where

$$\sigma_{\mu\nu} \equiv -\frac{i}{2}[\gamma_\mu, \gamma_\nu] , \tag{B4}$$

and

$$R_{\mu\nu} \equiv [D_\mu, D_\nu] = \Gamma_{\mu\nu} - \frac{1}{4}[\xi_\mu, \xi_\nu] + \frac{i}{2}\rho_2\xi_{\mu\nu} \tag{B5}$$

in which

$$\Gamma_{\mu\nu} = \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + [\Gamma_\mu, \Gamma_\nu] (\Gamma^\dagger_{\mu\nu} = -\Gamma_{\mu\nu} = \Gamma_{\nu\mu}) , \tag{B6}$$

$$d_\mu \xi_\nu = \partial_\mu \xi_\nu + [\Gamma_\mu, \xi_\nu] \tag{B7}$$

and

$$\xi_{\mu\nu} = d_\mu \xi_\nu - d_\nu \xi_\mu . \tag{B8}$$

The effective action obtained by integrating out the fermion fields is

$$\Gamma_{\text{eff}} = -\frac{1}{2} \int d^4x \text{tr} \int_0^\infty \frac{d\tau}{\tau} \rho(\epsilon, \tau) \frac{e^{-\tau(M^2+m^2)}}{(4\pi\tau)^2} \sum_{n=0}^\infty a_n \tau^n , \tag{B9}$$

where a_n are the Seely-deWitt coefficients in terms of D_μ and Y. The function $\rho(\epsilon, \tau)$ is the cutoff regulator:

$$\rho(\epsilon, \tau) \to 1, \epsilon \to 0; \quad \rho(\epsilon, \tau) \to 0, \tau \to 0 .$$

We choose the proper time regularization:

$$\rho(\epsilon, \tau) = \theta(\tau - \epsilon) = \theta\left(\tau - \frac{1}{\Lambda^2}\right) . \tag{B10}$$

In this regularization scheme Eq. (B9) is rewritten as

$$\Gamma_{\text{eff}} = -\frac{1}{32\pi^2} \int d^4x \sum_{n=0}^\infty \Gamma\left(n-1, \frac{M_Q^2}{\Lambda^2}\right) \frac{\text{tr}(a_{n+1})}{(M_Q^2)^{n-1}} , \tag{B11}$$

where $M_Q^2 = M^2 + m^2$ and $\Gamma(n-1, x)$ is the incomplete Γ function:

$$\Gamma(n-1, x) = \int_x^\infty \frac{dz}{z} e^{-z} z^{n-1} . \tag{B12}$$

We also list the expressions of a few leading Seely-deWitt coefficients:

$$a_1 = -Y , \tag{B13}$$

$$a_2 = \frac{1}{2}Y^2 - \frac{1}{6}D^2Y + \frac{1}{12}R_{\mu\nu}R_{\mu\nu} , \tag{B14}$$

$$a_3 = -\frac{1}{6}Y^3 + \frac{1}{12}[YD^2Y + (D^2Y)Y + D_\mu Y D_\mu Y] - \frac{1}{60}D^2D^2Y + \cdots , \tag{B15}$$

$$a_4 = \frac{1}{24}Y^4 \cdots . \tag{B16}$$

Terms which are neglected above are irrelevant to our calculation.

The expansion in the effective Lagrangian is in inverse power of M_Q^2. In the $M \gg m$ limit, the expansion series converges. Up to fourth order terms, the results are

$$\begin{aligned}
16\pi^2\Gamma_{(2)} &= \{-2m^2\text{tr}(D_\mu U^\dagger D_\mu U) - \tfrac{1}{3}\text{tr}(F^L_{\mu\nu}F^L_{\mu\nu}) \\
&\quad - \tfrac{1}{3}\text{tr}(F^R_{\mu\nu}F^R_{\mu\nu})\}\Gamma_0 , \\
16\pi^2\Gamma_{(3)} &= -i\frac{m^2}{M_Q^2}\text{tr}\{F^R_{\mu\nu}D_\mu U D_\nu U^\dagger + F^L_{\mu\nu}D_\mu U^\dagger D_\nu U\}\Gamma_1 \\
&\quad + \frac{m^2}{3M_Q^2}\text{tr}\{(d_\mu\xi_\nu)^2 + \tfrac{1}{2}(\xi_\mu\xi_\mu\xi_\nu\xi_\nu + \xi_\mu\xi_\nu\xi_\mu\xi_\nu) \\
&\quad + 4M^2\xi_\mu\xi_\mu\}\Gamma_1 , \\
16\pi^2\Gamma_{(4)} &= -\frac{M^2m^2}{(M^2+m^2)^2}(F^L_{\mu\nu}F^L_{\mu\nu} + F^R_{\mu\nu}F^R_{\mu\nu})\Gamma_2 \\
&\quad - \frac{4}{3}\frac{M^2m^4}{(M^2+m^2)^2}\text{tr}(D_\mu U^\dagger D_\mu U)\Gamma_2 ,
\end{aligned} \tag{B17}$$

which can also be summarized using the parameters as defined in Ref. [17] (the $\Gamma_{(4)}$ contribution can already be neglected):

$$\frac{f_\pi^2}{4} = \left(2m^2\Gamma_0 - \frac{4M^2m^2}{3(M^2+m^2)}\Gamma_1\right)\frac{1}{16\pi^2} , \tag{B18}$$

$$H_1 = \left(-\frac{1}{3}\Gamma_0 + \frac{m^2}{6(M^2+m^2)}\Gamma_1\right)\frac{1}{16\pi^2} , \tag{B19}$$

$$L_1 = L_2 = 0 , \tag{B20}$$

$$L_3 = \frac{m^2}{3(M^2+m^2)}\frac{\Gamma_1}{16\pi^2} , \tag{B21}$$

$$L_9 = \frac{2m^2}{3(M^2+m^2)}\frac{\Gamma_1}{16\pi^2} , \tag{B22}$$

$$L_{10} = -\frac{m^2}{3(M^2+m^2)}\frac{\Gamma_1}{16\pi^2} . \tag{B23}$$

Taking the leading term in m^2/M^2 expansions we easily obtain the result given in Sec. II.

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