Minimal supersymmetric standard model and large tanβ from SUSY trinification

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We construct a supersymmetric model based on the semisimple gauge group $SU(3)_c \times SU(3)_L \times SU(3)_R$ with the relation $\tan\beta \simeq m_t/m_b$ automatically arising from its structure. The model below a scale $\sim 10^{16}$ GeV naturally gives rise just to the minimal supersymmetric standard model and therefore to the presently favored values for $\sin^2\theta_W$ and α_s without fields in representations higher than the fundamental.

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The remarkable observation [1] that the renormalization group equations of the minimal supersymmetric standard model (MSSM) with a supersymmetry-(SUSY-) breaking scale $M_s \sim 1$ TeV are astonishingly consistent with the observed values of $\sin^2\theta_W$ and α_s and unification of the three gauge couplings at a scale $M_X \sim 10^{16}$ GeV strongly suggests the embedding of the standard gauge group $G_S \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$ in a larger one. It would certainly be very desirable and perhaps a guide to the correct extension if this embedding led to the determination of the important MSSM parameter tanβ. This parameter is defined as the ratio of the vacuum expectation values (VEV's) of the doublet $h^{(1)}$ giving mass to the up-type quarks and the doublet $h^{(2)}$ giving mass to the down-type quarks and charged leptons. The embedding of the MSSM in the minimal SUSY SU(5) model fails to determine tanβ. However, SUSY grand unified theories (GUT's) based on larger groups, such as SO(10), may lead [2] to the asymptotic relation $\tan\beta \simeq m_t/m_b$.

We recently realized [3] that, for this relation to hold, one needs much less than the enormous SO(10) gauge symmetry. It suffices to use the left-right symmetric extension $G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ of the standard gauge group G_S and the further assumption that the Higgs doublets $h^{(1)}, h^{(2)}$ as well as the third generation $SU(2)_L$-singlet antiquarks u^c, d^c form two $SU(2)_R$ doublets h and q^c, respectively. Then, the third generation quark mass terms originate from the single G_{LR}-invariant term fqq^ch [q is the $SU(2)_L$-doublet quark] with the unique Yukawa coupling f and the relation $\tan\beta = m_t/m_b$ follows immediately. Under analogous assumptions for the leptons, we further obtain the relation $\tan\beta = m^D_{\nu_\tau}/m_\tau$ ($m^D_{\nu_\tau}$ being the Dirac mass of the τ neutrino). We then formulated widely applicable sufficient conditions on SUSY GUT's guaranteeing that the situation is as described above. Thus, we opened up the way towards constructing models based on simple as well as on semisimple groups with $\tan\beta \simeq m_t/m_b$.

SUSY GUT's based on semisimple gauge groups are very interesting for various reasons [4] the most important one being related to the proton decay problem. It is well known that in minimal SUSY GUT's there is a close relationship between light quark masses and the proton decay amplitude mediated by color triplet and antitriplet Higgsino exchange. This relationship makes it difficult to forbid proton decay by imposing discrete symmetries without eliminating the light quark mass terms. In contrast, this is easily achieved in SUSY GUT's based on semisimple groups. Another property making semisimple groups attractive is that they offer the possibility to construct viable models using only fundamental representations and singlets. Also, with semisimple groups the gauge hierarchy problem might become easier to solve. To illustrate these nice features of semisimple groups, we recently constructed [5] a SUSY GUT based on the maximal subgroup $G \equiv SU(3)_c \times SU(3)_L \times SU(3)_R$ of E_6 using only fields contained in **27**, $\overline{\mathbf{27}}$ and singlet representations of E_6 in which, below a scale $\sim 10^{16}$ GeV, we naturally recover the MSSM.

The models with $\tan\beta \simeq m_t/m_b$ constructed in [3] either make use of adjoint Higgs fields or do not give rise, below a scale $\sim 10^{16}$ GeV, just to the MSSM with the successful $\sin^2\theta_W$ and α_s predictions following from unification of the three G_S couplings at this scale. The purpose of the present paper is to naturally derive for the first time the MSSM with the usual predictions for $\sin^2\theta_W$ and α_s from a SUSY GUT without adjoint Higgs fields and with the relation $\tan\beta \simeq m_t/m_b$ being automatically guaranteed.

Our model based on the gauge group $G \equiv SU(3)_c \times SU(3)_L \times SU(3)_R$ is a variation of the model of [5] retaining all its good features. We assume that the theory emerges as an effective theory from a more fundamental theory at a scale $M_c = M_P/\sqrt{8\pi}$ close to the Planck mass $M_P \equiv 1.2 \times 10^{19}$ GeV. At the scale M_c the three SU(3) gauge couplings are assumed to be equal and, due to a symmetric spectrum among them, they remain equal (to one loop) until a unification scale $M_X \sim 10^{16}$ GeV where G breaks down to G_S. Higher loop corrections are assumed to be small. Below the scale M_X our model naturally gives rise exactly to the MSSM. Threshold effects at M_X are assumed not to alter the successful MSSM predictions for $\sin^2\theta_W$ and α_s. Using appropriate discrete symmetries we forbid proton decay, we solve the gauge hierarchy problem and we ensure that the relation $\tan\beta \simeq m_t/m_b$ holds. Only fields contained in the 27-dimensional and singlet representations of E_6 are used and generic superpotential couplings are assumed throughout.

The gauge nonsinglet left-handed lepton, quark, and antiquark superfields transform under G as follows:

$$\lambda = (1,\bar{3},3) = \begin{pmatrix} H^{(1)} & H^{(2)} & L \\ E^c & \nu^c & N \end{pmatrix},$$
$$Q = (3,3,1) = \begin{pmatrix} q \\ g \end{pmatrix},$$
$$Q^c = (\bar{3},1,\bar{3}) = (\, u^c, d^c, g^c \,).$$

Here N and ν^c denote G_S-singlet superfields while $g(g^c)$ is an additional down-type quark (antiquark). We will be working with eight fields of each type (λ, Q, Q^c) and five corresponding mirror fields $(\bar{\lambda}, \bar{Q}, \bar{Q}^c)$. Notice that the field content ensures identical running of the three SU(3) gauge couplings to the one-loop approximation in the G-symmetric phase.

We impose invariance under three Z_2 symmetries P, C, and S_1 and a Z_3 symmetry S_2. Under P, all $\lambda, \bar{\lambda}$ fields remain invariant while all $Q, \bar{Q}, Q^c, \bar{Q}^c$ fields change sign. Under C, all fields remain invariant except $\lambda_6, \lambda_8, \bar{\lambda}_3$, and $\bar{\lambda}_5$ which change sign. Under S_1, all fields remain invariant except $\lambda_3, \lambda_6, \lambda_7, \bar{\lambda}_3, \bar{\lambda}_4$, Q_1, Q_2, and Q_3^c which change sign. Finally, under the generator of S_2, all fields remain invariant except $\lambda_1, \lambda_2, Q_1, Q_2$, Q_3^c which are multiplied by α and λ_3, Q_3, Q_1^c, Q_2^c which are multiplied by $\alpha^2 (\alpha = e^{2\pi i/3})$. Out of these discrete symmetries only C does not carry $\mathrm{SU}(3)^3$ anomalies and, therefore, could be possibly gauged. Discrete symmetries which are not gauged might be broken by gravitational effects. We assume that here such effects can be neglected.

The symmetry breaking of G down to G_S is obtained through appropriate superpotential couplings of the fields acquiring a superlarge VEV. These are the fields $\lambda_7, \bar{\lambda}_4, \lambda_8, \bar{\lambda}_5$. The superlarge VEV's are $\langle N_7 \rangle = \langle \bar{N}_4 \rangle^* = \langle \nu_8^c \rangle = \langle \bar{\nu}_5^c \rangle^*$. We also introduce in the superpotential explicit mass terms of order M_X (wherever allowed by the discrete symmetries) for $Q, \bar{Q}$ and $Q^c, \bar{Q}^c$ pairs as well as the $\lambda, \bar{\lambda}$ pairs involving fields which do not acquire superlarge VEV's.

The above VEV's leave the discrete symmetries P and S_2 unbroken. The discrete symmetry C combined with the diagonal Z_2 subgroup of the center of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ gives a Z_2 group C' which remains unbroken by all VEV's and acts as "matter parity." The discrete symmetry S_1 combined with the Z_2 subgroup of $\mathrm{SU}(3)_R$ generated by the element diag$(-1,+1,-1)$ gives a Z_2 group S_1' which remains unbroken by the superlarge VEV's. The symmetry P leads to a practically stable proton. The "matter parity" C' suppresses some lepton-number-violating couplings at the level of the MSSM (which is important for keeping the already generated baryon asymmetry of the Universe) and stabilizes the lightest supersymmetric particle. The combined effect of C' and S_1' solves in a natural manner the gauge hierarchy problem. Finally, the symmetry S_2 forces the relation $\tan\beta \simeq m_t/m_b \simeq m_{\nu_\tau}^D/m_\tau$.

To see how these properties are realized, we now turn to a brief discussion of the mass spectrum. All G_S-nonsinglet states which remain invariant under S_2 acquire masses $\sim M_X$. The remaining G_S-nonsinglets are all contained in $\lambda_1, \lambda_2, \lambda_3, Q_1, Q_2, Q_3, Q_1^c, Q_2^c, Q_3^c$ and transform nontrivially under S_2. These fields, because of the unbroken S_2 symmetry, can only form mass terms by pairing up among themselves. Also, because they all remain invariant under C, the only λ with a large VEV that can be used in the renormalizable superpotential to generate these mass terms is the C-invariant λ_7 which acquires a VEV in the N direction and therefore leaves the $\mathrm{SU}(2)_R$ symmetry unbroken. We conclude that all G_S-nonsinglet states which remain massless after taking into account the above mass terms are necessarily either $\mathrm{SU}(2)_R$ singlets or partners in $\mathrm{SU}(2)_R$ doublets. It is straightforward to see that these states are the q_i, u_i^c, d_i^c, L_i, E_i^c with i=1,2,3, one linear combination of $H_1^{(1)}$ and $H_2^{(1)}$, which we call $h^{(1)}$, and its partner $h^{(2)}$ in the $\mathrm{SU}(2)_R$-doublet h to which it belongs. These are exactly the states of the MSSM and can, of course, be classified by their charges under the unbroken discrete symmetries P, C', S_1', and S_2. From the G_S-singlet states, we are only interested in the $C' = -1$ sector because it contains the right-handed neutrinos. In this sector all states acquire masses $\sim M_X$ or $\sim M_X^2/M_c$ except the ν_i^c which are the partners of E_i^c in their $\mathrm{SU}(2)_R$-doublets ℓ_i^c and remain massless as expected.

From the above discussion we see that the electroweak Higgs doublets $h^{(1)}, h^{(2)}$ form a single $\mathrm{SU}(2)_R$-doublet h, the light $\mathrm{SU}(2)_L$-singlet antiquarks u_i^c, d_i^c form three $\mathrm{SU}(2)_R$-doublets q_i^c and the light $\mathrm{SU}(2)_L$-doublet quarks q_i are $\mathrm{SU}(2)_R$-singlets. In the lepton sector, the $\mathrm{SU}(2)_L$-singlet charged antileptons belong to three $\mathrm{SU}(2)_R$-doublets ℓ_i^c and the $\mathrm{SU}(2)_L$-doublet leptons L_i are $\mathrm{SU}(2)_R$-singlets. Furthermore, the unbroken discrete symmetries allow tree-level masses at least for the third generation. In the quark sector there is only one such mass term allowed, namely, $q' q_3^c h$ (where q' is one linear combination of q_1, q_2). In the lepton sector the allowed mass terms are $L_k \ell_m^c h$ with $k, m = 1, 2$. The dominant among these mass terms corresponds to the third generation and contains the only allowed τ-neutrino Dirac mass term. This suffices to prove the relation $\tan\beta = m_t/m_b = m_{\nu_\tau}^D/m_\tau$. Analogous relations for the lighter quarks are not expected to hold.

With the symmetry S_2 unbroken, Majorana masses for the states ν_i^c are forbidden. Therefore S_2 has to break at a superheavy scale. To this end, we introduce two gauge singlets T_1 and T_2 which remain invariant under P, C, and S_1 but get multiplied by α under the generator of S_2. Let T' be the linear combination of T_1 and T_2 which has a coupling of the type $T' \lambda_3 \bar{\lambda}_4$ and T the orthogonal linear combination. We assume that T acquires a VEV of the order of M_X thereby breaking the S_2 symmetry. One can check that this breaking generates mixings among states with different S_2 charges which are, however, at most $\sim M_X/M_c$. Therefore our relations for $\tan\beta$ become now only very good approximations. This breaking leads to generation of Majorana masses which are $\sim M_X^3/M_c^2$ for ν_k^c $(k = 1, 2)$ and $\sim M_X^4/M_c^3$ for ν_3^c. This, in turn, means that the τ-neutrino mass is close to 10 eV and therefore contributes significantly to the hot dark matter of the universe. The other two neutrinos could be light enough to allow for a solution of the solar neutrino puzzle through the Mikheyev-Smirnov-Wolfenstein (MSW) effect. The ν^c Majorana masses are also of the correct order of magnitude to allow for the generation of lepton number asymmetry [4] in the Universe which at temperatures close to M_W will be transformed into the observed baryon asymmetry.

Finally, we point out that neither the S_1' symmetry can be left unbroken because it forbids a Higgsino mass term of the type $h^{(1)}h^{(2)}$. The breaking of S_1' can be done using superheavy VEV's acquired by gauge singlets $\tilde{T}$ which change sign under S_1'. If such singlets transform as $\beta = e^{2\pi i/N}$ or β^{-1} under an additional Z_N symmetry with $N =$ odd then superpotential terms such as $h^{(1)}h^{(2)}N_7\frac{T}{M_c}(\frac{\tilde{T}}{M_c})^N$ naturally give rise to a sufficiently small Higgsino mass.

We constructed a SUSY GUT based on the semisimple group $SU(3)^3$ which below a scale $M_X \sim 10^{16}$ GeV gives naturally rise to the MSSM and therefore to the presently favored values of $\sin^2\theta_W$ and α_s. In addition, the relations $\tan\beta \simeq m_t/m_b \simeq m_{\nu_\tau}^D/m_\tau$ hold as automatic consequences of the structure of the theory. This is the first example of a SUSY GUT where these two features coexist and no use of fields in representations higher than the fundamental is made.

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