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Transverse polarization of top quarks produced at a photon-photon collider

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At future $\gamma\gamma$ colliders copious production of $t\bar{t}$ pairs is possible. This would allow for a detailed investigation of the interactions involving the top quark. We propose some correlations which are sensitive to $t\bar{t}$ final state interactions and we compute the QCD and standard model Higgs boson contributions to these correlations. A correlation resulting from the QCD induced transverse polarization of top quarks is found to be sizable and measurable at a high energy e^+e^- collider, which is operated as a photon collider through backscattering of laser photons, at an integrated luminosity of 10 fb^{-1}.

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I. INTRODUCTION

One of the attractive possible uses of a future high energy linear e^+e^- collider [1] is to convert it via backscattering of laser photons off the initial lepton beams into a high energy $\gamma\gamma$ collider [2]. It is expected that such a facility would provide a tool for a number of novel precision studies of strong and electroweak interactions [1,3–8]. For instance, copious production of top quarks is feasible. This would allow for detailed studies of top quark physics [5–8] which would be to some extent complementary[1] to studies of $e^+e^- \to t\bar{t}$. One important aspect of top quark physics is the quasifree behavior of the top quark due to its heavy mass ($m_t \approx 174$ GeV [11]). On average the top quark will decay before forming hadrons. This property makes the spin polarization of the top quark a good observable as it can be traced in the angular distributions of the t and /or $\bar{t}$ decay products.

In this Brief Report we exploit this property to investigate the transverse polarization (i.e., the polarization transverse to the production plane) of the t and/or $\bar{t}$ produced in unpolarized $\gamma\gamma$ collisions. In this case the transverse polarization is due to $t\bar{t}$ final state interactions and it may serve as a probe of (non) standard model (SM) interactions [12–15] in the $t\bar{t}$ system. We propose observables constructed from the momenta of the t and $\bar{t}$ decay products, which are sensitive to the transverse polarization, and compute the dominant SM contributions to their expectation values. This work is based on [14], to which the reader is referred for further details of the definitions and conventions which we use in this report.

We consider $t\bar{t}$ production via photon fusion:

$$\gamma(p_1) + \gamma(p_2) \to t(k_1, s_1) + \bar{t}(k_2, s_2), \qquad (1)$$

where the momenta are defined in the photon-photon c.m. frame and s_1 and s_2 label the spins of t and $\bar{t}$. In the following we consider only unpolarized photon beams. The process may then be described by the density matrix

$$R_{\alpha\alpha',\beta\beta'} = \sum{}' \langle t(k_1,\alpha')\bar{t}(k_2,\beta')|T|\gamma\gamma\rangle^* \times \langle t(k_1,\alpha)\bar{t}(k_2,\beta)|T|\gamma\gamma\rangle, \qquad (2)$$

where the $\sum'$ denotes averaging over the $\gamma\gamma$ polarizations, and $\alpha, \alpha', \beta, \beta'$ are polarization indices. Note that R is an even function of the three-momentum $\mathbf{p}_1$ due to Bose symmetry of the two-photon state. In the $t\bar{t}$ spin space it has the matrix structure

$$R = AI \otimes I + \mathbf{B}\cdot\sigma \otimes I + I \otimes \mathbf{C}\cdot\sigma + D_{ij}\sigma^i \otimes \sigma^j, \qquad (3)$$

where σ^i denote the Pauli matrices, I is the 2×2 unit matrix, and the first (second) factor in the tensor products refers to the t ($\bar{t}$) spin space. Because of rotation invariance the structure functions $\mathbf{B}$, $\mathbf{C}$, and D_{ij} can be expressed in terms of the unit vectors $\hat{\mathbf{p}}_1$, $\hat{\mathbf{k}}_1$, and $\hat{\mathbf{n}} = \hat{\mathbf{p}}_1 \times \hat{\mathbf{k}}_1/|\hat{\mathbf{p}}_1 \times \hat{\mathbf{k}}_1|$, which is orthogonal to the production plane. For $\mathbf{B}$ and $\mathbf{C}$ one can write

$$\mathbf{B} = b_1\hat{\mathbf{p}}_1 + b_2\hat{\mathbf{k}}_1 + b_3\hat{\mathbf{n}}, \quad \mathbf{C} = c_1\hat{\mathbf{p}}_1 + c_2\hat{\mathbf{k}}_1 + c_3\hat{\mathbf{n}}, \qquad (4)$$

while D_{ij} which characterizes the correlation between the t and $\bar{t}$ spins can be written in terms of eight independent scalar functions. At the tree level in the standard model $\mathbf{B} = \mathbf{C} = \mathbf{0}$, i.e., the top quarks are not polarized. If time reversal (T) invariance holds the structure functions b_3 and c_3, which describe t and $\bar{t}$ polarization transverse to the production plane, become nonzero only due to the interference between the dispersive part and the absorptive part of the scattering amplitude for the process (1). It is easy to show from the Bose symmetry of the $\gamma\gamma$ state that

$$b_3(x) = -b_3(-x) \text{ and } c_3(x) = -c_3(-x), \qquad (5)$$

[1]For $t\bar{t}$ production by bremsstrahlung photons at a linear e^+e^- collider, see [9,10].

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where $x = \hat{\mathbf{p}}_1 \cdot \hat{\mathbf{k}}_1$ is the cosine of the scattering angle. This implies that the spin projection $s_1 \cdot \hat{\mathbf{n}}$, where $s_1 = \frac{1}{2}\sigma \otimes I$, must be weighted with x in order to have a nonzero expectation value, as must the corresponding $\bar{t}$ spin projection. To construct an observable sensitive to final-state interaction effects but not to CP-violating phenomena, which may also occur in the $t\bar{t}$ system, it is useful to employ CP-even correlations which are, however, odd under reflection of momenta and spins[2] [14]. For studying final state interactions which are induced by parity- and CP-invariant interactions in the process (1) the appropriate observable would be

$$x(\mathbf{s}_1 + \mathbf{s}_2) \cdot \hat{\mathbf{n}} = \frac{x}{2}(\sigma \otimes I + I \otimes \sigma) \cdot \hat{\mathbf{n}}, \qquad (6)$$

which, however, cannot be used in an experiment because measurements of the t and $\bar{t}$ spin cannot be made on an event-by-event basis.

It is well known that the charged lepton from W decay subsequent to the parity-violating weak decay $t \to W + b$ is an efficient analyzer of the top quark spin [16]. We use this to construct realistic observables. Consider first the case where both t and $\bar{t}$ decay semileptonically: i.e.,

$$\begin{aligned} t &\to W^+ + b \to l^+ + \nu_l + b, \\ \bar{t} &\to W^- + \bar{b} \to l^- + \bar{\nu}_l + \bar{b}. \end{aligned} \qquad (7)$$

A CP-even but $\tilde{T}$-odd observable can then be formed using the momenta $\mathbf{q}_\pm$ of the charged leptons $l^\pm$ measured in the e^+e^- laboratory frame and the direction $\hat{\mathbf{p}}$ of the electron beam:

$$O_L = \frac{1}{m_t^3}\{\hat{\mathbf{p}} \cdot (\mathbf{q}_+ + \mathbf{q}_-)\}\hat{\mathbf{p}} \cdot (\mathbf{q}_+ \times \mathbf{q}_-), \qquad (8)$$

where the top quark mass m_t is used to make O_L dimensionless.

Events with semileptonic t and nonleptonic $\bar{t}$ decay (and vice versa) also provide opportunities for analyzing the polarizations. For these events the momentum direction $\hat{\mathbf{k}}_-$ ($\hat{\mathbf{k}}_+$) of the $\bar{t}$ (t) can be reconstructed. (Again we refer to the laboratory frame.) For the respective channels we can then use the $\tilde{T}$-odd observables:

$$\begin{aligned} O_{B_1} &= \frac{1}{m_t}(\hat{\mathbf{p}} \cdot \hat{\mathbf{k}}_-)\hat{\mathbf{p}} \cdot (\hat{\mathbf{k}}_- \times \hat{\mathbf{q}}_+), \\ O_{B_2} &= \frac{1}{m_t}(\hat{\mathbf{p}} \cdot \hat{\mathbf{k}}_+)\hat{\mathbf{p}} \cdot (\hat{\mathbf{k}}_+ \times \mathbf{q}_-). \end{aligned} \qquad (9)$$

As these observables are intimately related to the t and $\bar{t}$ spin-momentum projections $x\mathbf{s}_{1,2} \cdot \hat{\mathbf{n}}$, they are more sensitive to $t\bar{t}$ final-state interaction effects than (8). The observable $O_{B_1} - O_{B_2}$, which is CP even, measures the spin-momentum correlation (6). Below we shall evaluate

$$B_t = \langle O_{B_1} \rangle - \langle O_{B_2} \rangle. \qquad (10)$$

In this quantity contributions from CP-invariant absorptive parts add constructively.

Within the SM the dominant contributions to the absorptive part of the scattering amplitude (1) arise from QCD corrections, and also from Higgs boson (H) exchange because of the sizable Yukawa coupling of the top quark. As to QCD corrections, only gluon exchange between the final t and $\bar{t}$ quarks leads to an absorptive part of the one-loop amplitude. An absorptive part from Higgs boson interactions arises from H exchange between the final t and $\bar{t}$ and from s-channel H exchange with a $\gamma\gamma H$ vertex being induced by W boson and t quark loops. However, interference of the s-channel diagrams with the Born amplitude does not induce a transverse polarization of the top quark.

In general, the interference between the Born amplitude and the absorptive part will give contributions not only to b_3 and c_3 but also to some terms in D_{ij}. However, in the case at hand, only b_3 and c_3 are nonzero. Gluon exchange gives

$$\begin{aligned} b_3^{\text{gluon}} = c_3^{\text{gluon}} &= \alpha_s e^4 Q_t^4 |\hat{\mathbf{p}}_1 \times \hat{\mathbf{k}}_1|(m_t/\beta E_1)(1-\beta^2x^2)^{-1}(x^2-1)^{-1} \\ &\times \Big[4\beta^2 x(1-x^2) + 2\beta x(-5\beta^2 - 6\beta - 1)\ln(\beta+\beta^2) + 2\beta x(5\beta^2 - 6\beta + 1)\ln(\beta - \beta^2) \\ &+2\beta(-2\beta^2x^2 - 3\beta^2 + 6\beta x - x^2)\ln(\beta - \beta^2 x) + 2\beta(2\beta^2x^2 + 3\beta^2 + 6\beta x + x^2)\ln(\beta + \beta^2 x)\Big], \end{aligned} \qquad (11)$$

where Q_t is the charge of the top quark in units of e, E_1 is the photon energy, $\beta = (1 - m_t^2/E_1^2)^{1/2}$ is the velocity of the top quark, and x is the cosine of the scattering angle defined above. Higgs boson exchange induces

$$\begin{aligned} b_3^{\text{Higgs}} = c_3^{\text{Higgs}} &= \frac{3e^4 Q_t^4}{32\pi}\frac{m_t^2}{v^2}|\hat{\mathbf{p}}_1 \times \hat{\mathbf{k}}_1|(m_t/E_1)(x^2-1)^{-1}(1-\beta^2x^2)^{-1} \\ &\times \Big\{2\beta^2 x(x^2 + 2d_H - 3)\ln\frac{1+\beta}{1-\beta} + 2\beta x(x^2 d_H - 3d_H + 2)\ln\frac{d_H+1}{d_H-1} \\ &+\beta x(-\beta^2x^2 d_H - \beta^2 x^2 - 2\beta^2 d_H^2 + 3\beta^2 d_H + \beta^2 + 2d_H - 3)[F(\beta) + F(-\beta)] \\ &+(\beta^2x^4 + 3\beta^2x^2 d_H - 2\beta^2x^2 - 2\beta^2 d_H^2 + \beta^2 d_H - \beta^2 - x^2 + 1)[F(\beta) - F(-\beta)] + 4\beta x(1-x^2)\Big\}, \end{aligned} \qquad (12)$$

[2]We will refer to correlations which are odd under reflection of momenta and spins as $\tilde{T}$-odd. Time reversal (T) symmetry also involves interchanging initial and final states.

where $v = 246$ GeV,

$$d_H = 1 + \frac{M_H^2}{2E_1^2\beta^2},$$

$$F(\beta) = \frac{1}{r}\ln\frac{d_H - \beta x + r}{d_H - \beta x - r},$$

and

$$r = \sqrt{(1 - \beta x d_H)^2 + \beta^2(d_H^2 - 1)(1 - x^2)}.$$

The transverse polarization P of the top quark is defined by

$$P = b_3/A, \tag{13}$$

where to lowest order in the SM couplings the structure function A is given by

$$A = \frac{3Q_f^4 e^4}{(1-\beta^2x^2)^2} \times(1 + 2\beta^2 - 2\beta^4 - 2\beta^2x^2 + 2\beta^4x^2 - \beta^4x^4). \tag{14}$$

Transverse polarization effects due to QCD and QED interactions were previously studied in [17]. In particular, in this work the transverse polarization $P_{\rm QED}$ of charged leptons l produced in $\gamma\gamma \to l^+l^-$ was given to order α. Defining $P_{\rm QCD} = b_3^{\rm gluon}/A$ the relation $P_{\rm QCD} = (4\alpha_s/3\alpha)P_{\rm QED}$ holds. We have compared our result for $P_{\rm QCD}$ with $P_{\rm QED}$ of [17] and find agreement.

Finally, we compute the expectation values of (8) and (9). In addition to the formulas above we need the Born amplitude contribution to D_{ij} which is easily calculated, the distributions of polarized t and $\bar{t}$ decay into the respective channels which we take from the SM Born decay amplitudes, and the $\gamma\gamma$ luminosity spectrum which we take from [2]. We assume the energy of the laser photon to be 1.26 eV and the e-γ conversion factor to be unity. Assuming for definiteness a top mass of $m_t = 170$ GeV we obtain for a e^+e^- collider at center-of-mass energy $\sqrt{s} = 500$ GeV

$$\langle O_L\rangle = 3.0\times10^{-4}\alpha_s, \quad B_t = 0.047\alpha_s. \tag{15}$$

For $\sqrt{s} = 1$ TeV we get

$$\langle O_L\rangle = 0.016\alpha_s, \quad B_t = 0.18\alpha_s. \tag{16}$$

The Higgs boson contributions are of the order 10^{-5}–10^{-8} and are therefore omitted from Eqs. (15) and (16). To estimate the sensitivity of the correlations to the QCD induced transverse polarization we need the effective $t\bar{t}$ cross sections [i.e., the cross sections for the process (1) folded with the $\gamma\gamma$ luminosity spectrum] and the width of the distribution of the observables $\Delta O = (\langle O^2\rangle - \langle O\rangle^2)^{1/2} \simeq (\langle O^2\rangle)^{1/2}$. For $m_t = 170$ GeV,

$$\sigma_{\rm tot} = 0.09 \text{ pb}, \quad \langle O_L^2\rangle = 0.000\,26, \quad \langle O_{B_1}^2\rangle = 0.0056 \quad \text{for } \sqrt{s} = 500 \text{ GeV} \tag{17}$$

$$\sigma_{\rm tot} = 0.60 \text{ pb}, \quad \langle O_L^2\rangle = 0.0092, \quad \langle O_{B_1}^2\rangle = 0.0044 \quad \text{for } \sqrt{s} = 1000 \text{ GeV}.$$

From Eqs. (15)–(17) we find that the sensitivity, signified by the signal-to-noise ratios $\langle O\rangle/\Delta O$, of the observable (10) is much higher than that of the observable (8). One gets $B_t/\Delta O_{B_1} \simeq 0.044$ (0.20) at $\sqrt{s} = 500(1000)$ GeV. With the $t\bar{t}$ event sample produced at a e^+e^- collider with an integrated luminosity of 10 fb^{-1} the correlation B_t would be measurable as a 2σ effect at 500 GeV, and as a 10σ effect at 1 TeV.

In conclusion, the transverse polarization of the top quark produced with unpolarized photon-photon collisions is due to radiative corrections and results within the SM primarily from QCD final state interactions. We have shown that the triple product correlations (9) and (10), respectively, are sensitive tools for detecting transverse polarization of t and $\bar{t}$ produced in this way. As the couplings of the top quark have yet to be measured, the observables above may be useful for a detailed investigation of the forces in the $t\bar{t}$ system.

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