Neutrino electric charge and the possible anisotropy of the solar neutrino flux

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If the electron neutrino had an electric charge, its magnitude should be less than 10^{-13} of the electronic charge. Our question is: Could such a tiny charge affect the solar neutrino flux? While a naive answer would be "no," we present arguments that the opposite may be true. The idea is that a charged neutrino beam is deflected by the solar magnetic field, thus decreasing the observed neutrino flux. We obtain the formula which expresses the solar neutrino flux on the Earth in terms of five parameters: the neutrino charge and energy, the gradient of the toroidal magnetic field in the convective zone, the distance from the center of the Sun to the convective zone, and the width of the convective zone. We show that the neutrino flux deficit can be large enough if the gradient of the magnetic field in the convective zone is of the order of 10^{-5} G/cm. No special structure (such as twisting) of the magnetic field is required. The experimental implications of this scenario are discussed and open problems are identified.

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I. INTRODUCTION

A long-standing puzzle at the crossroads of elementary particle physics and astrophysics is the substantial deficiency of the flux of solar neutrinos registered in four different experiments. The data of the two-decade-long Cl-Ar experiment at Homestake [1] have more recently been complemented by the direct observation of solar neutrinos at the Kamiokande water Cherenkov detector [2] as well as by two different Ga-Ge experiments, SAGE [3] and GALLEX [4]. Being sensitive to different parts of the solar neutrino spectrum, these four experiments together provide data which are difficult to understand with a single set of solar parameters chosen within the standard solar models [5,6]. Chances are that we have to modify some aspects of neutrino behavior in order to explain the observations.

An incomplete list of candidate solutions proposed so far includes neutrino oscillations possibly enhanced by the Mikheev-Smirnov-Wolfenstein mechanism [7,8], neutrino decay [9], neutrino spin precession [10] in the solar magnetic field, and the resonant spin-flavor conversion scenario [11]. Recently, a more sophisticated version of the last scenario has been produced which employs more detailed assumptions about the small scale structure of the solar magnetic field in the convective zone [12].

In this work we would like to analyze a simpler and perhaps more natural hypothesis in connection with the solar neutrino problem: the possible existence of a nonzero electric charge of the neutrino. In fact, the neutrino is the only elementary particle, other than the gauge bosons, whose electric charge is normally assumed to be zero. But if the neutrality of the gauge bosons is deeply rooted in the principle of gauge invariance, there are no compelling reasons whatsoever for the neutrino to have a zero charge.

Of course, the neutrino is assumed to be exactly neutral within the standard model. However, the recently[1] developed approach to the problem of the electric charge quantization has led to the realization of the fact that in a fairly large class of gauge models, including the minimal standard model, the electric charge can be dequantized [15] (see also [16]). This means that the electric charges of elementary particles can take different values from those conventionally assumed: $Q_\nu = 0$, $Q_l = -e, Q_{u,c,b} = 2e/3$, and $Q_{d,s,b} = -e/3$ (e being the modulus of the electronic charge). In particular, the neutrino can acquire nonzero electric charge. (Another interesting aspect of the theories with dequantized electric charges is that one might speculate about the possibility of time dependence of the electric charges within such theories [17].)

To be more specific, consider the standard model with the electric charge dequantized through the formula [15]

$$Q = Q_{\text{standard}} + \epsilon(L_e - L_\mu). \tag{1}$$

Within this model, the electron and muon neutrinos acquire electric charges of the same absolute value but of opposite signs, while the τ neutrino remains neutral:

$$Q(\nu_e) = -Q(\nu_\mu) = \epsilon e, \quad Q(\nu_\tau) = 0. \tag{2}$$

[1]Early attempts to study the possibility and consequences of charge dequantization date back to Einstein [13], who noted that a tiny difference between the electron and proton charges could account for the Earth's and Sun's magnetic fields. Later on, Lyttleton and Bondi [14] proposed the idea that such a tiny difference could cause the expansion of the Universe due to electrostatic repulsion. It took experimentalists several decades to rule out these hypotheses (and some of their modifications).

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Another possibility is to dequantize charge by a similar formula [15]:

$$Q = Q_{\text{standard}} + \epsilon(L_e - L_\tau). \quad (3)$$

In this case, ν_e and ν_τ are (oppositely) charged while ν_μ is neutral. (We would like to emphasize that the above examples by no means exhaust all possible ways of giving the neutrino nonzero charge.)

In other words, in Ref. [15] it has been shown that the standard model contains an additional free parameter ϵ which must be determined experimentally along with the other more familiar parameters such as Higgs boson mass or Yukawa couplings. Of course, if it were found that ϵ is nonzero, it should be very small anyway (see next section) and that would create one more hierarchy problem. Yet taking into account the existence of a few such problems already, the appearance of a new one does not seem a strong enough argument to disregard the possibility of a nonzero ϵ.

Furthermore, one might argue that a nonzero neutrino charge does not follow from any theoretical principle, whether established or hypothetical (with the exception of the well-known rule "all which is not forbidden is allowed"). But now, based on the works [15], we know that the zero neutrino charge does not follow from anywhere, too.

Another possible objection against particles with small fractional charge is that it is difficult to embed them into grand unified theories [18]. Yet theories with a paraphoton provide a viable alternative [19].

So, at present, the cases of zero or nonzero neutrino charges must be considered as two working hypotheses on an equal footing, only experiment being able to provide the ultimate answer. The situation with the neutrino charge is very similar to the situation with the neutrino mass: while zero mass is the prediction of the minimal standard model, most physicists agree that the question of zero or nonzero neutrino mass has much more to do with experimenting than with model building. While it would not be easy to detect the neutrino charge, the consequences of such a discovery should certainly be dramatic, ranging from the prospects of detecting a relic neutrino through its electromagnetic interaction to possible better ways of managing neutrino beams, creation of neutrino optics, etc.

Finally, let us note that in the present work we are not concerned if the neutrino mass is zero or not. Certainly, there exist well-known difficulties associated with charged massless particles [20]. However, one can take a pragmatic point of view [21] and keep developing a theory until one runs into any inconsistency. No such inconsistency seems to show up in our treatment. An alternative point of view is to give the neutrino a Dirac mass by introducing additional Higgs multiplets.

Note also that even if the neutrino is massless in vacuum, it cannot be considered massless inside a plasma. This is because the vacuum dispersion relation $E = |\mathbf{p}|$ is changed by the weak interaction of the neutrino with the plasma [8]. In other words, there arises a refraction index for the neutrino propagating through a plasma. Thus the situation with infrared divergencies might be better for a neutrino in a plasma than in a vacuum.

To conclude, it seems that assuming nonzero neutrino charge is certainly not more heretical than assuming nonzero neutrino magnetic moment, or mass and mixing angles.

In addition, there exists a quite independent motivation to study the behavior of a charged neutrino inside the Sun. The point is that the neutrino electromagnetic properties are modified by plasma effects and under certain conditions these modifications result in *an induced electric charge of the neutrino* [22]. We stress that it happens in the minimal standard model where the neutrino has zero *intrinsic* electric charge.

The purpose of this paper is to draw attention to the fact that the possible existence of a very small electric charge of the neutrino may be a clue to the solar neutrino problem.[2] The idea is that the charged neutrinos are deflected by the solar magnetic field while passing inside the Sun. Thus the resulting neutrino flux is made anisotropic, which leads to the solar neutrino deficit registered on the Earth. (Within our scenario, this deficit is not real but only apparent in the sense that *the total* 4π solar neutrino flux is *not changed* as compared with the standard solar models.)

Section II summarizes the existing bounds on the electron neutrino charge. In Sec. III we give an order-of-magnitude estimate of the effect. Section IV is devoted to a more detailed calculation of the solar neutrino deficit in the present scenario. In Sec. V some experimental implications are discussed and one of them, the prediction of a second neutrino flux, is briefly outlined in Sec. VI. Our main results are summarized in Sec. VII.

II. PHENOMENOLOGICAL LIMITS ON THE ELECTRIC CHARGE OF THE ELECTRON NEUTRINO

For a recent detailed analysis of various bounds on the neutrino charges, see [23].

The strongest model-independent[3] constraints on the electron neutrino charge, $Q(\nu_e) = \epsilon e$, come from three sources.

(1) Analysis of the data on $\nu_e e$ elastic scattering [21]:

$$\epsilon \lesssim 3 \times 10^{-10}. \quad (4)$$

(2) Study of the electromagnetic neutrino production in the core of the Sun and its influence on the solar energetics [21]:

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[2]The possible role of the charged neutrino interaction with the terrestrial electric field, in connection with the solar neutrino problem, was discussed previously by Joshi and Volkas (unpublished).

[3]By model-independent we mean the constraints that do not rely on additional assumptions such as charge conservation or the equality $Q(\nu_e) = Q(\bar{\nu}_e)$.

$$\epsilon \lesssim 10^{-13}. \tag{5}$$

This estimate follows from the fact that if the neutrino is charged then neutrino-antineutrino pairs will be produced in the decays of plasmons (which can be viewed as massive photons) in the core of the Sun. The neutrino will then escape freely from the Sun, thus taking away a lot of energy which would be inadmissible in terms of the solar energy balance.

(3) Analysis of the data on SN 1987A supernova explosion [24]:

$$\epsilon \lesssim 10^{-15} \text{ to } 10^{-17}. \tag{6}$$

These upper limits on the electron neutrino charge ϵ were based on the experimental detection [25] of the antineutrino signal from the supernova 1987A explosion. It was claimed that if ϵ were larger than (6) then the intergalactic and galactic magnetic fields would lengthen the neutrino paths, and neutrinos of different energy could not arrive on the Earth within a few seconds of each other, even if emitted simultaneously by the supernova.

Yet it is generally believed (see, e.g., [26]) that the constraint based on SN 1987A arguments, although stronger, is less reliable than the previous ones because it involves details of the galactic magnetic field which are not very well known.

There exist even more severe, but less direct constraints. They are based on the experimental data on the neutrality of atoms and neutrality of the neutron. These data give limits on the sum of the proton and electron charges [27]: $Q(p) + Q(e) = (0.8 \pm 0.8) \times 10^{-21}e$, and the neutron charge [28]: $Q(n) = (-0.4 \pm 1.1) \times 10^{-21}e$. Then, assuming charge conservation in the neutron β-decay $n \rightarrow p + e^- + \bar{\nu}_e$ we can obtain the bound on the electron *antineutrino* charge: $Q(\bar{\nu}_e) < 3 \times 10^{-21}e$. Finally, assuming validity of CPT symmetry[4] with respect to ν_e and $\bar{\nu}_e$ charges, one can claim that

$$Q(\nu_e) < 3 \times 10^{-21}e. \tag{7}$$

In particular, this constraint is valid within the model with charge dequantization through Eq. (1) and Eq. (3). Yet the requirements of the electric charge conservation and CPT symmetry, although very general and perfectly valid up to now, are themselves a subject of experimental testing at present.[5] Furthermore, there exist several models in which a charged neutrino arises as a natural consequence of the electric charge violation [31] (in those specific models the constraint (7) is still not invalidated).

In this work we are not concerned with the problem of constructing a viable model which would allow one to avoid the constraint (7). Instead, in the main part of the paper we do not use any model-dependent arguments and we treat the neutrino charge ϵ as a free parameter; also we consider some interesting consequences of the choice $\epsilon \simeq 10^{-13}$. Of course, should any of the above constraints change in the future, our results would be easily reformulated by simply rescaling the magnitude of ϵ.

A massive Dirac ν_e with an electric charge of $10^{-13}e$ and a mass of < 7–9 eV [32] would have a Dirac magnetic moment $\mu > 5 \times 10^{-9}\mu_B$, where μ_B is the Bohr magneton. Therefore one might suspect a contradiction with the existing upper bounds on the neutrino magnetic moment obtained recently by a number of authors (see, e.g., [33] and references therein): $\mu(\nu_e) < 10^{-10}\mu_B$ to $10^{-12}\mu_B$. However, these limits apply only to the *anomalous* magnetic moment, but not to the Dirac magnetic moment. In our case, the neutrino anomalous magnetic moment due to the electromagnetic radiative correction is equal to $\epsilon^2\alpha/2\pi$ and therefore negligible. Still, some of the constraints on $\mu(\nu_e)$ can be translated into limits on $Q(\nu_e)$. For example, the analyses of plasmon decay into neutrino-antineutrino pairs due to $\mu(\nu_e)$ in various astrophysical contexts can be carried out also for the case of a charged neutrino. Yet the resulting limits are not significantly different from the solar constraint (5).

Note also that recently there has been considerable interest in discussing the possible existence of new particles carrying very small electric charge ("millicharged particles") [16]. These works contain detailed discussion of many phenomenological constraints on such particles obtained from a variety of sources (including astrophysics, cosmology, geophysics, and macroscopic electrodynamics). Many of those constraints apply to the case of the electron neutrino, too; we shall not repeat that material here.

III. SIMPLE ESTIMATE

Since the neutrino charge, if any, must be so tiny, $\lesssim 10^{-13}e$, at first sight it seems unlikely that it would play any role at all. To make a rough check whether this is true or not, let us start by a dimensional order-of-magnitude estimate of the effect of the solar magnetic

[4]In addition to charge conservation and CPT, a number of usually unspoken but very important assumptions underlying the constraint (7) are made. For instance, one has to assume that the electric charges of *free* electrons and protons are exactly the same as those of *atom-bound* electrons and protons. Another fundamental assumption, as noted in Ref. [21], is that the electric charge, as measured by interaction with an electromagnetic field, coincides with the electric charge assigned by the charge-conservation law (see also a discussion of that point in [29]). According to Ref. [21], it is possible to construct models in which it is not the case. Under ordinary circumstances there is no doubt of the correctness of the above axioms, but when it comes to such fantastic accuracies as 10^{-21}, it does not seem unreasonable to question those axioms, too.

[5]Note that it is possible to constrain the ν_e charge assuming only charge conservation in the decay $n \rightarrow p+e^+ +\nu_e$, but not the equality $Q(\nu_e) = Q(\bar{\nu}_e)$. Naturally, this constraint turns out to be much weaker than (7), namely, $Q(\nu_e) < 4 \times 10^{-8}e$ [30].

field[6] on the the charge neutrino motion inside the Sun. As a dimensionless characteristic of the effect it is natural to choose the neutrino deflection angle. The deflection angle γ may be written as the product of the deflection rate $d\gamma/dt$ and the time of flight through the convective zone τ:

$$\gamma \sim \frac{d\gamma}{dt}\tau, \; \tau \simeq \frac{D}{c}, \tag{8}$$

where $D \simeq 2 \times 10^{10}$ cm is the thickness of the convective zone and c is the speed of light. Now, in a crude approximation, the value of the deflection rate $d\gamma/dt$ will depend only on the neutrino electric charge, ϵe, the neutrino energy $E \simeq 1$ MeV, and some characteristic value of the solar magnetic field, H_c. The only combination of dimension time^{-1} which can be made out of these quantities is the Larmour frequency:

$$\omega_L = \frac{\epsilon e c H_c}{E}. \tag{9}$$

(We use the Gauss system of units throughout the paper, i.e., $e = 4.8 \times 10^{-10}$ esu, 1 MeV $= 1.6 \times 10^{-6}$ erg.) Putting all together and adopting $\epsilon \simeq 10^{-13}$ for the neutrino charge and $H_c \simeq 10^4$–10^5 G for the solar magnetic field[7] we finally obtain, for the deflection angle,

$$\gamma \sim \frac{\epsilon e H_c D}{E} \simeq 0.01\text{–}0.1. \tag{10}$$

Thus we see that, contrary to a naive expectation, the possible tiny charge of the neutrino may indeed play some role in the neutrino propagation through the solar magnetic field. Yet from our estimate it is hard to see the relation between the neutrino deflection angle and the neutrino deficit observed on the Earth. So it is worthwhile to study the problem more carefully.

IV. CALCULATION OF THE NEUTRINO FLUX ON THE EARTH

We have seen, by an order-of-magnitude estimate, that the suggested value of the neutrino charge, $\epsilon \simeq 10^{-13}$, may be relevant to the solar neutrino problem. Now we can start a more exact calculation of the reduction of the solar neutrino flux on the Earth due to the bending of neutrino trajectories by the solar magnetic field. The key quantity we need to find is the angular distribution of the neutrino flux coming out from the surface of the Sun, assuming that the initial flux coming out from the central region of the Sun is isotropic.

Therefore the procedure has three steps. First, we start with the isotropic angular distribution. Second, we find out how the neutrino trajectories are bent by the solar magnetic field. Mathematically, this bending of trajectories can be viewed as a change of variables in the distribution function. The last step, then, is to make explicitly such a change of angular variables as would lead us to the final angular distribution. From this anisotropic distribution we shall easily obtain our key result, Eq. (29): the reduction of the neutrino flux predicted by the present hypothesis (and then compare it with the reduction of the flux observed experimentally).

For definiteness and simplicity of the calculation, we make the following assumptions. First, we assume that all the neutrinos are emitted from a point source located in the center of the Sun. In other words we neglect the radius of the neutrino-emitting zone, R_ν, as compared to the solar radius $R_\odot$ (their ratio is $R_\nu/R_\odot \simeq 0.1$). Next, we suppose that the neutrino deflection angle is small (the precise meaning of that assumption will be discussed below). Furthermore, what we are interested in is not the complete 4π angular distribution of the neutrino flux but only a small part of it within the angular interval swept by the line of vision Earth-Sun. This is a narrow interval close to the solar equatorial plane: $-7° < \theta < 7°$. This fact will be used throughout the calculation to simplify it.

Finally, a few remarks about the solar magnetic field. The structure of the solar magnetic field is very complicated and is probably one of the worst known aspects of the Sun. Yet some important information about it can be obtained from the observations of solar activity such as sunspots which strongly depend on the behavior of the large scale magnetic field inside the Sun. For instance, observing the Zeeman effect for the light emitted from the sunspots, one can measure the strength of the magnetic field near the surface of the Sun which can then be translated into estimates of the azimuthal magnetic field in the convective zone of the Sun. In what follows we will rely only on relatively well understood features of the solar magnetic fields which include the following (see, e.g., [34]).

(1) The existence of the large scale toroidal magnetic field in the convective zone of the Sun (toroidal means the magnetic lines of force are closed circles parallel to the Sun's equatorial plane).

(2) The directions of the toroidal fields in the northern and southern hemispheres of the Sun are opposite. In addition, these directions reverse themselves every 11 years.

(3) The strength of the toroidal magnetic field is nearly zero in the equatorial plane and it grows with the separation from the equatorial plane within the narrow transition zone, between about $+10°$ and $-10°$ latitudes. Therefore, in this transition zone, there exists a gradient of the toroidal magnetic field in the direction parallel to the rotation axis of the Sun. Unfortunately, the magnitude of the gradient is not very well known and we will consider it as a parameter varying in a certain range [see below, between Eqs. (30) and (31)]. It is this gradient rather than magnetic field itself that plays the key role

[6]Here, by "the solar magnetic field" we mean the large scale toroidal magnetic field in the convective zone. For the moment, we ignore the magnetic fields in the core and the radiative zone, not because they are irrelevant but because we do not know much about them.

[7]This choice is discussed in more detail below, between Eqs. (30) and (31).

in reducing the neutrino flux on the Earth within the present scenario.

According to our plan, we start with the isotropic neutrino flux from the center of the Sun described by the angular distribution

$$\frac{dN}{d\Omega} = N_0 = \text{const}, \tag{11}$$

or equivalently

$$F_0(\theta, \phi) \equiv \frac{dN}{d\theta d\phi} = \sin\theta, \tag{12}$$

where θ and ϕ are the usual angles in the spherical coordinate frame whose origin is placed in the center of the Sun, the y axis points to the Earth, and the z axis is the rotation axis of the Sun. After passing the convective zone filled with magnetic field the neutrino is deflected from the direction characterized by the pair of spherical angles θ and ϕ to the new direction characterized by the spherical angles α and β. Using the equation of motion, we can explicitly write down the angles α and β as functions of θ and ϕ. Conversely, we can express α and β as functions of θ and ϕ:

$$\theta = T(\alpha, \beta), \phi = P(\alpha, \beta). \tag{13}$$

Now, all we need to get the new distribution function from the old one is to change the variables from θ and ϕ to α and β. Then the new distribution function which we denote by $F_1(\alpha, \beta)$ will look like

$$F_1(\alpha, \beta) = F_0(T(\alpha, \beta), P(\alpha, \beta))J(\alpha, \beta), \tag{14}$$

where $J(\alpha, \beta)$ is the Jacobian

$$J(\alpha, \beta) = \begin{vmatrix} \frac{\partial T}{\partial \alpha} & \frac{\partial T}{\partial \beta} \\ \frac{\partial P}{\partial \alpha} & \frac{\partial P}{\partial \beta} \end{vmatrix}. \tag{15}$$

Finally, within our model, the neutrino flux observed on the Earth (Φ_1) and the flux predicted by the standard solar model (Φ_0) are related simply by

$$\frac{\Phi_1}{\Phi_0} = \frac{F_1(\pi/2, \pi/2)}{F_0(\pi/2, \pi/2)}. \tag{16}$$

Now, we proceed to the calculation of the angular distribution function F_1. To do that, we need to find the deflection angles, that is, the functions $T(\alpha, \beta)$ and $P(\alpha, \beta)$. For this purpose, let us consider the equations of motion for a charged neutrino in the magnetic field:

$$\frac{d\mathbf{p}}{dt} = \frac{\epsilon e}{c}[\mathbf{v} \times \mathbf{H}], \; \frac{dE}{dt} = 0, \tag{17}$$

where $\mathbf{p}$ and $\mathbf{v}$ are the neutrino's momentum and velocity, respectively, and ϵe is the neutrino electric charge. Since the electron neutrino mass is experimentally known to be less than 7–9 eV [32], to a very high accuracy the neutrino velocity equals the velocity of light, $v \approx c$. As for the neutrino energy, E, its magnitude depends on the reaction in which the neutrino is born. On the other hand, it must be greater than the experimental threshold, which is different for different experiments, ranging from about 0.2 MeV to 7 MeV.

To simplify the equation of motion, recall that we are only interested in what happens in the close vicinity of the direction Sun-Earth, that is, near the direction $\theta = \pi/2, \phi = \pi/2$. Therefore we can neglect the curvature of the magnetic lines of force and of the convective zone containing these lines of force and approximate the convective zone by a flat slab of the same thickness, $D \simeq 2 \times 10^{10}$ cm, containing *straight* lines of force. This slab is placed perpendicular to the axis Earth-Sun. The distance between the center of the Sun and the middle of the slab is taken to be $d \simeq 6 \times 10^{10}$ cm, which is the distance between the center of the Sun and the middle of the convective zone. Furthermore, we assume that the magnetic field is zero outside this slab whereas inside the slab the only nonzero component is H_x (i.e., we ignore the poloidal magnetic field altogether). In the absence of a detailed model for z dependence of the magnetic field, we adopt the simplest form, i.e., a linear variation with z:

$$H_y = H_z = 0, \; H_x = \left\langle \frac{\partial H_x}{\partial z} \right\rangle z, \tag{18}$$

where $\langle \frac{\partial H_x}{\partial z} \rangle$ is the average vertical gradient of the large scale magnetic field.

Since the deflection angles are assumed to be small, we can neglect the variation of magnetic field over the trajectory of any single neutrino flying at some θ. In other words, we assume that all the neutrinos flying at the *same* angle θ will experience *the same* strength of the magnetic field, while the neutrinos flying at *different* angles θ will experience *different* strengths of the magnetic field. To paraphrase it once more, we assume that a neutrino flying at an angle θ upon entering the convective zone will move in a *uniform* magnetic field all the way through until it leaves the Sun; the strength of that uniform magnetic field is taken to be the same as the strength of the magnetic field at the "entrance point:"

$$H_y = H_z = 0, \; H_x = \left\langle \frac{\partial H_x}{\partial z} \right\rangle d \cot\theta. \tag{19}$$

Now, the motion of a charged particle in a uniform magnetic field is well known: along the magnetic field the motion is free, while in the perpendicular plane the trajectories, both in coordinate and momentum spaces, are circles orbited with Larmor frequency. Thus, if a neutrino enters the convection zone with the speed components v_{0x}, v_{0y}, v_{0z}, it will leave the Sun with the speed components

$$v_x = v_{0x}, \tag{20}$$

$$v_y = v_{0y} \cos\omega\tau + v_{0z} \sin\omega\tau \approx v_{0y} + v_{0z}\delta, \tag{21}$$

$$v_z = -v_{0y} \sin\omega\tau + v_{0z} \cos\omega\tau \approx -v_{0y}\delta + v_{0z}. \tag{22}$$

Here, $\tau = D/c$ is the time of flight through the convection zone,[8] $\delta = \omega\tau$ is the angle by which the magnetic field rotates the neutrino speed vector projected to the yz plane, and ω is the Larmor frequency:

$$\omega = \omega_0 \cot\alpha, \; \omega_0 = \frac{\epsilon e c}{E}\left\langle \frac{\partial H_x}{\partial z} \right\rangle d. \tag{23}$$

From these equations we see that the condition of validity of our small-angle approximation can be formulated as $\sin\delta \approx \delta$.

Having obtained the solution of the equations of motion, we are now able to find the relation between the initial angles θ, ϕ at which a neutrino enters the convective zone and the final angles α, β at which the neutrino leaves the Sun. For this purpose, let us express the initial speed components v_{0x}, v_{0y}, v_{0z} through the initial angles θ, ϕ, and the final speed components v_x, v_y, v_z through the final angles α, β:

$$v_{0x} = c\sin\theta\cos\phi, \; v_{0y} = c\sin\theta\sin\phi, \; v_{0z} = c\cos\theta, \tag{24}$$

$$v_x = c\sin\alpha\cos\beta, \; v_y = c\sin\alpha\sin\beta, \; v_z = c\cos\alpha. \tag{25}$$

Assuming as usual that the deflection angles are small, we can write

$$\theta = \alpha + \Delta\alpha, \; \phi = \beta + \Delta\beta, \tag{26}$$

where $\Delta\alpha, \Delta\beta$ are small. Now, solving the system of equations, Eqs. (20)–(26) with respect to $\Delta\alpha, \Delta\beta$, we find

$$\theta = \alpha - \delta_0 \sin\beta\cot\alpha \equiv T(\alpha, \beta), \tag{27}$$

$$\phi = \beta - \delta_0 \cot^2\alpha\cos\beta \equiv P(\alpha, \beta), \tag{28}$$

where $\delta_0 = \omega_0\tau$. Having expressed initial angles in terms of final angles, that is, having obtained the functions $T(\alpha, \beta)$ and $P(\alpha, \beta)$, we can now calculate the Jacobian in Eq. (15) and then insert the result, together with Eqs. (27) and (28), into our basic formula, Eq. (14). Lastly, we recall Eq. (16) giving the neutrino flux observed on the Earth within our model.

Thus, finally, the neutrino flux observed on the Earth within our model is given by

$$\frac{\Phi_1}{\Phi_0} = 1 + \delta_0 = 1 + \frac{\epsilon e \langle \frac{\partial H_x}{\partial z} \rangle dD}{E}. \tag{29}$$

An important thing about this result is that it is *the field gradient* rather than *magnetic field itself* that causes the effect. Indeed, using our method, one can show that, if the toroidal magnetic field in the convective zone were *uniform*, it would not lead to any substantial anisotropy of the neutrino flux at all.

To compare Eq. (29) with the experimental data, we need to know the value of the mean gradient (along the solar rotation axis) of the large scale toroidal magnetic field in the convective zone, $\langle \frac{\partial H_x}{\partial z} \rangle$. Unfortunately, the magnitude of this gradient is not very well known, so let us try to reverse our problem and ask: what value of the gradient will be needed for our mechanism to explain the solar neutrino deficit? Assuming the neutrino charge to be $\epsilon = 10^{-13}$, its energy $E = 0.8$ MeV, and requiring the flux deficit to be $\delta_0 = -0.5$, from Eq. (29) we find that the gradient must be

$$\left\langle \frac{\partial H_x}{\partial z} \right\rangle \approx -1.1 \times 10^{-5} \text{ G/cm}. \tag{30}$$

Is it a reasonable figure or not? A crude estimate of the gradient can be obtained by dividing H by h where H is the maximum value of the magnetic field reached at the latitudes of about $\pm 10°$ [34] and h is the distance from that latitude to the solar equatorial plane, $h = d\sin 10° \approx 10^{10}$ cm.

As for the possible value of H, it is a subject of a debated controversy. On the one hand, it is claimed [35] that values of H greater than 10^4 G are ruled out by nonlinear growth-limiting effects; on the other hand, there are arguments based on helioseismology data that it can reach as large values as a few million gauss [36]. Anyway, magnetic fields up to 10^4 G (or even 10^5 G [37]) are widely used by many authors trying to explain the solar neutrino puzzle. So we leave it to the reader to make his/her own judgement on that point. Note also that it is the magnetic field close to the surface of the Sun which reaches its maximum at 10° latitude, and this latitude may be higher (or lower) for magnetic fields located at larger depths. That brings an additional uncertainty to the estimate of the gradient. If we do admit that the magnetic field in the convective zone may vary in the range $H = 10^3$–10^6 G then the value of the gradient may vary in the range

$$\left\langle \frac{\partial H_x}{\partial z} \right\rangle \simeq \frac{H}{h} \approx (10^{-7}\text{–}10^{-4}) \text{ G/cm}. \tag{31}$$

Hence we see that the value of gradient needed to explain the neutrino deficit, Eq. (30), *may indeed exist in the convective zone of the Sun.*

A few remarks are now in order.

(1) We have just considered the case of $\alpha = \pi/2$ corresponding to the case when the Earth traverses the plane of the solar equator. But that happens only twice a year: in June and December, the days of summer and winter solstice. During the rest of the time the direction Sun-Earth is at an angle to the solar equatorial plane. Fortunately, this angle is rather small: $|\pi/2 - \alpha| \lesssim 7°$, so that our small-angle approximation still applies. (The boundaries are reached when the Sun-Earth axis lies in the northern solar hemisphere at an angle of 7° in September, the autumn equinox, and through the southern solar hemisphere, at the same angle, in March, the spring equinox.) If we keep α dependence in our calculation

[8]Note that the actual times of flight, besides being different for different neutrinos, are in fact longer than D/c because the neutrino paths are not straight, but this difference is negligible (and even if it were not, assuming $\tau = D/c$ for all neutrinos would only decrease the effect rather than increase it).

then we obtain that the deficit of the neutrino flux varies according to

$$\frac{\Phi_1}{\Phi_0} = 1 + \frac{\delta_0}{\sin^2 \alpha}. \tag{32}$$

Thus for extreme values of α we have[9]

$$\frac{\Phi_1}{\Phi_0} = 1 + \frac{\delta_0}{\sin^2 97^\circ} = 1 + \frac{\delta_0}{\sin^2 83^\circ} \approx 1 + 1.01 \times \delta_0. \tag{33}$$

Recall that this result is valid within our system of approximations: we neglected the curvature of the solar convective zone and substituted the true value of the magnetic field gradient by its average value. But even if these assumptions are dropped, there are hardly any reasons to expect that this result would change dramatically.

So we can conclude that the seasonal variations of the neutrino flux predicted by the present model are rather small, which makes them very hard to observe in the experiment.

(2) All the above derivation was of course completely classical. Indeed, one can reasonably expect that quantum corrections (e.g., due to neutrino diffraction) must be vanishingly small.

(3) One might wonder about the possible influence of other magnetic fields encountered by the neutrinos on their long way from the Sun to the Earth. Specifically, there are the solar magnetic field outside the Sun, the interplanetary magnetic field, and, finally, the terrestrial magnetic field.

First of all, the terrestrial magnetic field is completely transparent for solar neutrinos because the penetration ability of a particle of charge q and momentum p is controlled by the factor pc/q which is huge for a neutrino with the charge 10^{-13} — it is by many orders of magnitude greater than this factor for the penetrating particles of cosmic rays. The neutrino deflection angle due to Earth's magnetism is also negligibly small since both the terrestrial magnetic field and the time of neutrino flight through the terrestrial magnetosphere are much less than those for the convective zone of the Sun. Similarly, the influence of the outer magnetic field of the Sun and the interplanetary magnetic field can be neglected: although the time of flight outside the Sun is about 10^3 longer than inside, the magnetic field outside is 10^{-4}–10^{-5} G, i.e., many orders of magnitude weaker than inside, so that the product of these two factors outside the Sun is much smaller than inside. In addition to that, let us stress once again that we need large *field gradients* rather than large *magnetic fields*, which are unlikely to emerge in the interplanetary space.

(4) Also, the role of various *electric* fields, both inside and outside the Sun, seems to be negligible for the present problem. Inside the Sun, as is well known for any plasma, any electric field dissipates very quickly due to the high conductivity of the plasma.[10] Outside the Sun, an electric field could exist, for instance, due to nonzero total electric charge of the Sun. Although there is not much information available about the strength of such an electric field, we can be quite confident that it cannot be strong enough to capture neutrinos with the charge $10^{-13}e$ and the energy $E \sim 1$ MeV before they reach the Earth (or even decrease their energy to any extent). If that were possible, then the solar wind could also never reach close to the Earth. The terrestrial electric field, about 1 V/cm in strength, is also helpless to stop the neutrinos.

(5) One might wonder if the nonzero neutrino charge would affect the solar neutrino detection at Kamiokande. The point is that there arises an electromagnetic contribution to the cross section of the neutrino-electron scattering which might modify the Kamiokande results. However, if $Q_\nu = 10^{-13}e$ then the cross section of the electromagnetic $\nu_e e$ scattering is about ten orders of magnitude smaller than the cross section of weak $\nu_e e$ scattering and therefore can be completely neglected.

(6) At this stage, we have not considered the effect of solar matter on our results.

(7) Now we come to the discussion of the most serious flaw of the suggested scenario in its present form. The point is that up to now we tacitly assumed that the direction of the magnetic field is such that the value of the deficit, $(-\delta_0)$, is positive. But recall that the large scale magnetic field in the convective zone *reverses* every 11 years, which means that the gradient, too, changes its sign every 11 years. That means that taken as it is, Eq. (29) would predict that each 11 year period of neutrino flux *deficiency* must be followed by an 11 year period of neutrino flux *excess of the same magnitude* so that the flux averaged over the 22 year cycle would be the same as predicted by the standard solar model.

One can think of several possible ways out of that difficulty.

The most natural one is to recall that our previous calculation was based on the approximation of small deflection angles, or, more precisely, the smallness of the parameter δ_0. We can expect that this small-angle approximation is valid as long as $\sin \delta_0 \approx \delta_0$ or $|\delta_0| \lesssim 0.7$. But if the gradient of the magnetic field is greater than 2×10^{-5} G/cm then our approximation does not work anymore and a more exact calculation is needed.

Naively, one might expect that the neutrino deficiency must alternate with the neutrino excess at 11 year intervals independently of the magnitude of the gradient: just note that when the magnetic field configuration is defocusing, one would expect the neutrino deficiency, and when it is focusing, the neutrino excess. Each reversal of the magnetic field means a switch between focusing and

[9]Note that Eq. (32) cannot be extrapolated to large angles so it does not mean that the neutrino flux is zero at $\alpha = \pi$ and $\alpha = 0$.

[10]In fact, there may exist exceptions to this rule. Under certain circumstances (for example, in the conditions of solar flares) the electrical resistivity may be greatly increased so that a local electric field may arise [34]. This point is probably worth further study.

defocusing modes so that any 11 year "deficiency" cycle would be followed by the 11 year "excess" cycle, however great is the gradient of the magnetic field. Nevertheless, there are arguments based on simple geometrical optics considerations which show that this is not the case and if the gradient is large enough then the neutrino deficiency can occur both for the defocusing *and the focusing* configuration.

Another option is to try to relax the solar upper bound on the possible electric charge of the electron neutrino obtained in [21], since the neutrino charge and the magnetic field gradient come always as the product $\epsilon\langle\frac{\partial H_x}{\partial z}\rangle$ rather than separately. Note that if we used only the most reliable bound (i.e., the direct experimental bound extracted from the elastic $\nu_e e$ scattering data) on the electron neutrino charge, $\epsilon \lesssim 3\times10^{-10}$ [21], the required value of the gradient would be less than the above estimate by a factor of 300.

Let us also mention briefly that at present we cannot rule out the possible existence of a primordial magnetic field of as much as 10^6 G inside the core of the Sun [34]. Within the present context, it would be very interesting if any evidence could be obtained concerning the existence of significant gradients of that field near the plane of the solar equator.

Finally, although it is not as appealing, we should not discard the possibility that our mechanism is effective only during alternate 11 year cycles or even only during the periods of active sun within the alternate 11 year cycles while some other mechanism is responsible for neutrino depletion during the rest of the time. This possibility will have to be considered much more seriously if the anticorrelation of the neutrino deficiency with solar activity is established firmly by future experiments.

V. DISCUSSION

As we stressed in the preceding sections, our present knowledge of the structure of the solar magnetic field is rather limited. Thus we cannot rule out such values of the magnetic field as would lead to *large* deflections of the neutrinos traveling through the convective zone of the Sun. The quantitative theory for this case seems more difficult to construct and we do not attempt it here. Yet it is instructive to discuss here some qualitative features of such a theory based on simple physical considerations, keeping in mind the results of the four different solar neutrino experiments available by now.

Before doing that, let us very briefly summarize the solar neutrino data.

(1) Anticorrelation of the neutrino flux with solar activity is probably observed in the Homestake data [38,39].

(2) No such anticorrelation is observed in the Kamiokande data [2].

(3) Higher neutrino flux (i.e., less neutrino deficit) is observed in the Kamiokande experiment than in the Homestake experiment.

(4) Higher neutrino flux is observed in the SAGE [3] and GALLEX [4] experiments than in the Homestake experiment.

It is very important for us to note that the experimental thresholds of neutrino energy are rather different in those experiments: $E_{\rm Home} = 0.816$ MeV, $E_{\rm Kam} \sim 7.5$ MeV, and $E_{\rm GALLEX} = 0.233$ MeV.

Now let us discuss qualitatively some effects that are consequences of our hypothesis. These effects are controlled by the neutrino electric charge ϵ, the gradient of the solar magnetic field $\langle\frac{\partial H_x}{\partial z}\rangle$, and the neutrino energy E (for the moment, we forget about other relevant parameters). However, these quantities enter not separately but only through the ratio $\epsilon\langle\frac{\partial H_x}{\partial z}\rangle/E$. Therefore changing the magnetic field will be equivalent to changing the neutrino energy correspondingly. Furthermore, it is natural to assume that both neutrino flux deficit and anticorrelations grow with increase of the gradient. Hence we obtain the result that the anticorrelations have to be *smaller* for *more energetic* neutrinos. And this is exactly what is needed to qualitatively explain the difference between the Homestake and Kamiokande data [see (1) and (2) above]. Also, by the same reasoning, within our scenario one can expect less deficit in the Kamiokande than in the Homestake experiment. Thus we can summarize that it is rather plausible that the present scenario can, in principle, account for three out of the four main experimental features; see (1)–(3) above.

Now, as for the fourth feature, i.e., the results of the gallium experiments, our hypothesis seems to predict *greater* deficit than Homestake and thus looks disfavored by the gallium results. However, one must remember the following. (1) The difference between the gallium and Homestake results, from the viewpoint of our hypothesis, must be *less* pronounced than the difference between the Homestake and Kamiokande data. This follows from the fact that the ratio of the characteristic neutrino momenta for Homestake-gallium data are *smaller* than for Kamiokande-Homestake data; (2) The errors of the gallium data are still larger than those of the Homestake data.

Now we would like to draw attention to a curious coincidence in the solar neutrino data. The Kamiokande experiment does not detect anticorrelations during the whole period of its operation, i.e., 1987–1993 (part of solar cycle No. 22). And according to [39] there is no anticorrelation in the Homestake data during the years 1970–1977 (part of solar cycle No. 20). Also, the latest data do not confirm the anticorrelation: the large number of sunspots in 1991–1992 was accompanied by a high counting rate [40]. Therefore one is tempted to speculate that, due to some reason, the anticorrelations are much more prominent in the *odd-numbered* solar cycles while being suppressed in the *even-numbered* cycles. If we take this conjecture seriously, it would be easy to conclude that the neutrino-depleting mechanism must somehow be correlated not only with *the strength* of the solar magnetic field but also with *the direction* of the toroidal solar magnetic field which reverses every 11 years. Obviously, this feature would be difficult to accommodate within any of the existing scenarios except the present one.

VI. A SECOND NEUTRINO FLUX

Apart from the reduction of the conventional (i.e., thermonuclear) neutrino flux, a spectacular feature of our scenario is the prediction of a "second flux" of electron neutrinos and antineutrinos from the Sun. While thermonuclear neutrinos are produced due to the *weak* interactions of the neutrino, the second flux arises due to the *electromagnetic* production of neutrino-antineutrino pairs. The most important process would be that of plasmon decay into a neutrino-antineutrino pair. Thus the second flux would consist of low-energy (about 200 eV) neutrinos produced in plasmon decays in the core of the Sun, the number of such neutrinos being much greater than that of the thermonuclear neutrinos. It would be very interesting to consider the possibility of detecting this second neutrino flux, about 10^{16} s^{-1} cm^{-2} in magnitude, on the Earth.

VII. CONCLUSION

To conclude, in the context of the solar neutrino problem we studied the consequences of the hypothesis that the electron neutrino has a small but nonvanishing electric charge. The main general consequence is that the solar neutrino flux can be anisotropic. The cause of that anisotropy is the antisymmetry of the large scale toroidal solar magnetic field about the solar equatorial plane, which leads to a large gradient of the magnetic field along the direction normal to the solar equatorial plane, denoted by $\langle\frac{\partial H_x}{\partial z}\rangle$. It is this gradient that results in anisotropic deflection of the neutrino flux. The magnitude of the anisotropy is controlled by the product of the gradient $\langle\frac{\partial H_x}{\partial z}\rangle$ and the neutrino charge ϵe. Arguments based on the energetics of the Sun show that the neutrino charge must be less than $\epsilon \lesssim 10^{-13}$ [21]. Unfortunately, the value of the gradient $\langle\frac{\partial H_x}{\partial z}\rangle$ is not very well known and, according to a rough estimate, may vary from 10^{-7} G/cm to 10^{-5} G/cm (or even perhaps up to 10^{-4} G/cm). We calculated, in the linear approximation, the deficit of the solar neutrino flux observed on the Earth caused by the anisotropy of the neutrino flux at the surface of the Sun. Assuming that the neutrino charge is equal to $10^{-13}e$ and its energy $E = 0.8$ MeV, we found the value of the gradient which is needed to obtain a 50% deficit by our mechanism: about 10^{-5} G/cm. If the neutrino charge is much less than $10^{-13}e$, it is unlikely to produce any observable effect under the action of the magnetic field *of the convective zone*. (If one considers the magnetic field of *the core and radiative zone*, it is not ruled out that much smaller charges are still interesting, but any quantitative conclusion on that point is difficult to reach in view of our poor knowledge of those magnetic fields.)

We then discussed some attractive experimental implications of this scenario as well as the problems which have to be solved so that this scenario could be considered as a full-fledged solution to the solar neutrino puzzle.

Independently of whether this scenario survives or not in its present form, our arguments show that the more general problem of the possible anisotropy of the neutrino flux due to the interactions of the neutrino with the solar matter and electromagnetic fields is certainly worth pursuing further.

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