Baryon magnetic moments in a simultaneous expansion in $1/N$ and m_s

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We consider the baryon octet and decuplet magnetic moments in a simultaneous expansion in m_s and $1/N$ taking $N_F/N \sim 1$, where N is the number of QCD colors and N_F is the number of light quark flavors. At leading order in this expansion, the magnetic moments obey the nonrelativistic quark-model relations. We compute corrections to these relations using an effective Lagrangian formalism which respects chiral symmetry to all orders in the $1/N$ expansion. Including corrections up to order $m_s^{1/2}$, we find eight relations among the nine measured octet and decuplet magnetic moments; including corrections up to order $1/N$ and m_s, we find four remaining relations. The relations work well, and suggest that the expansion is under control. We give predictions for the unmeasured magnetic moments.

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I. INTRODUCTION

In this paper, we consider the baryon magnetic moments in a simultaneous expansion in m_s and $1/N$, where N is the number of QCD colors. There has been a recent revival of interest in the $1/N$ expansion for baryons, started by the results of Ref. [1]. For example, it was shown that many interesting large-N relations have corrections starting at $\sim 1/N^2$. These results have been extended using a number of different methods [2–5]. We will use the formalism of Ref. [4]. This formalism is based on an exact relativistic treatment of QCD, yet make direct contact with the static quark model. On a more practical level, it allows us to write an explicit effective Lagrangian in which chiral symmetry is kept manifest to all orders in $1/N$.

When the number of light flavors $N_F > 2$, the SU (N_F) flavor representations of baryons grow with N, and so there are ambiguities in how to extrapolate physical baryon states to $N > 3$. We use the approach of Ref. [5], where it is shown that the $1/N$ expansion can be formulated to include all of the states in the SU(N_F) flavor representations for arbitrary N and N_F. The physical results for $N = 3$ can be obtained without having to identify the physical baryon states with particular states for $N > 3$, and SU(N_F) flavor symmetry is kept manifest in this approach. This expansion is well defined even if $N_F/N \sim 1$ [5]; we will work in this limit, since $N_F = N = 3$ in the real world.

In the large-N limit, the magnetic moments obey the nonrelativistic quark-model relations [6]. We find that the leading corrections to these results are suppressed relative to the leading terms by order $1/N$, $m_s^{1/2}$, and m_s. We will assume that $O(m_s)$ and $O(1/N)$ corrections are both $O(\epsilon) \sim 30\%$, and carry out the expansion consistently to $O(\epsilon)$. Including the $O(m_s^{1/2}) = O(\epsilon^{1/2})$ corrections, we find eight relations among the nine measured magnetic moments for the octet and decuplet baryons; including corrections up to $O(\epsilon)$, we find four surviving relations. These relations agree well with data, providing evidence that the combined $1/N$ and chiral expansions work well for baryons. We then predict the unmeasured magnetic moments including all corrections up to $O(\epsilon)$, and compare them to calculations in chiral perturbation theory, lattice calculations, and model-dependent extractions from data.

II. FORMALISM

In this section, we briefly review the formalism we will use to obtain our results. In Ref. [4], it was shown how to write an effective Lagrangian describing the low-energy interactions of baryons with the pseudo Nambu-Goldstone bosons (PNGB's) for large N. The PNGB's are described in the standard way: the field

$$\xi(x) = e^{i\Pi(x)/f} , \tag{1}$$

is taken to transform under $SU(N_F)_L \times SU(N_F)_R$ as

$$\xi \mapsto L\xi U^\dagger = U\xi R^\dagger , \tag{2}$$

where this equation implicitly defines U as a function of L, R, and ξ. For $N_F = 3$, the meson fields are

$$\Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} . \tag{3}$$

Note that the η' is not light if $N_F/N \sim 1$, and is therefore not included. The effective Lagrangian is most conveniently written in terms of the Hermitian fields

$$V_\mu \equiv \frac{i}{2}(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi) \ , \quad A_\mu \equiv \frac{i}{2}(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi) \ , \quad (4)$$

which transform under $\mathrm{SU}(N_F)_L \times \mathrm{SU}(N_F)_R$ as

$$V_\mu \mapsto UV_\mu U^\dagger + iU\partial_\mu U^\dagger \ , \quad A_\mu \mapsto UA_\mu U^\dagger \ . \quad (5)$$

We can incorporate $\mathrm{SU}(N_F)$ breaking due to $m_s \neq 0$ by including the quark mass spurion (for arbitrary N_F)

$$m_q \equiv m_s S \ , \quad S \equiv \begin{pmatrix} 0 & & & \\ & \ddots & & \\ & & 0 & \\ & & & 1 \end{pmatrix} \ , \quad (6)$$

transforming under $\mathrm{SU}(N_F)_L \times \mathrm{SU}(N_F)_R$ as $m_Q \mapsto Lm_q R^\dagger$. We find it convenient to define the even-parity field

$$m \equiv \tfrac{1}{2}(\xi^\dagger m_q \xi + \text{h.c.}) \mapsto UmU^\dagger \ . \quad (7)$$

In this notation, the leading terms giving rise to meson interactions are

$$\mathcal{L} = f^2\,\mathrm{tr}(A^\mu A_\mu) + \beta f^3 \mathrm{tr}(m) + \cdots \ , \quad (8)$$

where β is a coupling and $f = f_\pi \simeq 93$ MeV; $f \sim \sqrt{N}$ in the large-N limit [7].

We now discuss the baryon fields. Because the baryon mass is of order $N\Lambda_{\mathrm{QCD}}$, we can describe the baryons using a heavy-particle effective field theory [8]. We write the baryon momentum as $P = M_0 v + k$, where $M_0 \sim N$ is a baryon mass and v is a four-velocity ($v^2 = 1$) which defines the baryon rest frame. We then write an effective field theory in terms of baryon fields whose momentum modes are the residual momenta k. This effective field theory gives an expansion in $1/M_0$ around the static limit.

For N large, the baryon $\mathrm{SU}(N_F)$ representations are large, and it is convenient to use a compact notation to keep track of baryon flavor quantum numbers. We use a Fock-space notation in which the baryons fields are written

$$|\mathcal{B}(x)) \equiv \mathcal{B}^{a_1\alpha_1\cdots a_N\alpha_N}(x)\alpha^\dagger_{a_1\alpha_1}\cdots\alpha^\dagger_{a_N\alpha_N}|0) \ . \quad (9)$$

The $\alpha^\dagger$'s are *bosonic* creation operators which create a "quark" with definite flavor and spin, and $|0)$ is the Fock "vacuum" state; $a_1,\ldots,a_N$ are $\mathrm{SU}(N_F)$ flavor indices and $\alpha_1,\ldots,\alpha_N = \uparrow,\downarrow$ are spin indices in the rest frame defined by v.

Under $\mathrm{SU}(N_F)_L \times \mathrm{SU}(N_F)_R$, the baryon fields transform as

$$\mathcal{B}^{a_1\alpha_1\cdots a_N\alpha_N} \mapsto U^{a_1}{}_{b_N}\cdots U^{a_N}{}_{b_N}\mathcal{B}^{b_1\alpha_1\cdots b_N\alpha_N} \ , \quad (10)$$

where U is defined in Eq. (2). $\mathcal{B}^{\cdots}$ transforms under a highly reducible representation of $\mathrm{SU}(N_F)$, but we will have to carry out calculations explicitly only for $N = N_F = 3$.

Meson-baryon interactions are written in terms of operators constructed from the creation and annihilation operators. For example, the leading terms involving baryons can be written

$$\mathcal{L} = (\mathcal{B}|iv^\mu\nabla_\mu|\mathcal{B}) + g(\mathcal{B}|\{A^\mu\sigma_\mu\}|\mathcal{B}) + \cdots \ , \quad (11)$$

where $\sigma_\mu \equiv (\not{v}\gamma_\mu - v_\mu)\gamma_5$ is the spin matrix and we define

$$\{A^\mu\sigma_\mu\} \equiv \alpha^\dagger_{a\alpha}(A^\mu)^a{}_b(\sigma_\mu)^\alpha{}_\beta\alpha^{b\beta} \ , \quad (12)$$

etc. The chiral covariant derivative acting on the baryon fields is defined by

$$\nabla_\mu|\mathcal{B}) \equiv (\partial_\mu - i\{V_\mu\})|\mathcal{B}) \ . \quad (13)$$

The coupling g can be determined for matrix elements of the $\Delta S = 1$ axial current measured in semileptonic hyperon decays. We obtain $g = 0.83 \pm 0.08$, where the error is obtained by assigning a 30% uncertainty to the higher-order corrections. This value should be used with caution, since the $\mathrm{SU}(N_F)$-breaking corrections are known to be large [8,9].

In this notation, the leading N dependence of an arbitrary term in the effective Lagrangian is given by associating a factor of $1/N^{r-1}$ with every r-body operator (that is, an operator constructed from r creation and r annihilation operators), and a factor of $1/N$ for every explicit flavor trace [4]. The reason for these rules is that an r-body operator can arise only from quark-level diagrams involving at least $r-1$ gluon exchanges, and flavor traces arise from quark loops. Each gluon exchange or quark loop gives rise to a suppression of $1/N$, yielding the rules given above. For more details, see Ref. [4]. According to these rules, the coupling g in Eq. (11) is order 1 in the large-N limit.

III. MAGNETIC MOMENTS

We now apply the formalism discussed in the previous section to the magnetic moments.

A. Leading order

At leading order in the large-N and chiral limits, the baryon magnetic moments are described by a single term in the effective Lagrangian:

$$\delta\mathcal{L} = \frac{a_0 e}{\Lambda}v_\mu\tilde{F}^{\mu\nu}(\mathcal{B}|\{\mathcal{Q}\sigma_\nu\}|\mathcal{B}) \ , \quad (14)$$

where $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}$, $\epsilon_{0123} = +1$, and

$$\mathcal{Q} \equiv \tfrac{1}{2}(\xi^\dagger\mathcal{Q}_L\xi^\dagger + \xi\mathcal{Q}_R\xi) \mapsto U\mathcal{Q}U^\dagger \ . \quad (15)$$

Here (at $N_F = 3$)

$$\mathcal{Q}_L = \mathcal{Q}_R \equiv \begin{pmatrix} \frac{2}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3} \end{pmatrix} \quad (16)$$

are the left- and right-handed quark charge spurions. The parameter $\Lambda \sim 1$ GeV is the chiral expansion scale; naive dimensional analysis leads us to expect that $a_0 \sim 1$. The magnetic moments arising from Eq. (14) are given by

$$\mu_{B'B}\sigma^j_{B'B} = -\frac{a_0 e}{\Lambda}(B'|\{\mathcal{Q}\sigma^j\}|B) \ , \tag{17}$$

where $\sigma^j_{B'B}$ is the matrix element of the spin matrix between the states $|B)$ and $|B')$ normalized so that its maximal value is $+1$. (We use nonscript capital letters to refer to specific baryon states.) The operator $\{\mathcal{Q}\sigma^3\}$ has matrix elements $O(N)$, so that the leading contributions to the magnetic moments are $O(N)$.[1]

At this order, there are nine experimentally measured octet and decuplet baryon magnetic moments determined by a single unknown constant a_0/Λ. There are therefore eight relations among the magnetic moments. Of these, six are the Coleman-Glashow relations[2] [10]

$$\begin{aligned} \mu_p - \mu_{\Sigma^+} &= 0 \ \ (15\%) \ , \\ \mu_n + \mu_p + \mu_{\Sigma^-} &= 0 \ \ (10\%) \ , \\ \mu_n - 2\mu_\Lambda &= 0 \ \ (40\%) \ , \\ \mu_{\Sigma^-} - \mu_{\Xi^-} &= 0 \ \ (55\%) \ , \\ \mu_n - \mu_{\Xi^0} &= 0 \ \ (40\%) \ , \\ \sqrt{3}\mu_n + 2\mu_{\Sigma^0\Lambda} &= 0 \ \ (5\%) \ , \end{aligned} \tag{18}$$

which hold in the limit of exact SU(3) to all orders in $1/N$; the remaining relations can be taken to be the quark-model relations

$$3\mu_n + 2\mu_p = 0 \ \ (3\%) \ , \tag{19}$$

$$\mu_{\Omega^-} + \mu_p = 0 \ \ (35\%) \ , \tag{20}$$

which are the consequences of the large-N limit. The numerical accuracy indicated is defined by dividing the numerical value by the average of the positive and negative terms on the left-hand side. If we perform a best fit, the average deviation is $0.3\mu_N$.

B. $1/N$ corrections

The $O(1/N)$ corrections to the magnetic moments arise from the term

$$\delta\mathcal{L} = \frac{a_1 e}{N\Lambda} v_\mu \tilde{F}^{\mu\nu}(\mathcal{B}|\{\mathcal{Q}\}\{\sigma_\nu\}|\mathcal{B}) \ . \tag{21}$$

Because matrix elements of the operator $\{\mathcal{Q}\}\{\sigma^\mu\}$ can be $O(N)$, the contributions to the magnetic moments arising from this term are $O(1)$ in the large-N limit. Including this term, the Coleman-Glashow relations in Eq. (18) still hold, but the quark-model relations Eqs. (19) and (20) no longer hold. Note that the relation Eq. (19) involves states with zero strangeness, and therefore receives further corrections only from isospin breaking, which are expected to be about 5%. The experimental deviation of this relation is therefore a direct measure of the $1/N$ corrections, and we have no understanding of why these corrections are so small in this case.

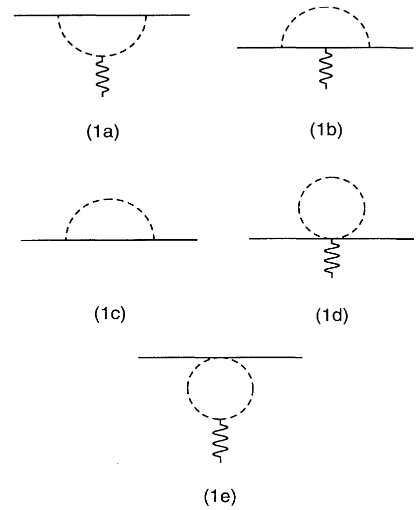

FIG. 1. Feynman graphs giving rise to nonanalytic corrections to the baryon magnetic moments. Figure 1(c) is wave function renormalization; Fig. 1(e) does not contribute to the magnetic moments.

C. SU(N_F)-breaking corrections

In chiral perturbation theory, the leading SU(N_F)-breaking corrections generally arise from loop graphs with PNGB intermediate states. Such graphs can give nonanalytic dependence on the quark masses. In the case of the magnetic moments, the leading dependence on m_s is $\sim m_s^{1/2}$ and $\sim m_s \ln m_s$ [11]. The diagrams which give rise to these corrections are shown in Fig. 1.

The graph which gives rise to the $\sim m_s^{1/2}$ corrections is easily evaluated using the meson-baryon coupling from Eq. (11):

$$\begin{aligned} \text{Fig. 1(a)} = &-\frac{g^2 e}{2f^2}(B'|\{T_A\sigma^\nu\}\{T_B\sigma^\lambda\}|B)\,\mathrm{tr}(T_A[T_B,\mathcal{Q}]) \\ &\times \int \frac{d^4k}{(2\pi)^4}\frac{(2k+q)_\mu k_\nu (k+q)_\lambda}{(k^2 - M_A^2 + i0^+)[(k+q)^2 - M_B^2 + i0^+](k\cdot v + i0^+)} \ , \end{aligned} \tag{22}$$

[1]We could normalize the electric charge of the quarks to be of order $1/N$ so that the baryons have electric charge of order 1 in the large-N limit. If we did this, the magnetic moments would be $O(1)$ in the large-N limit. Such a rescaling would modify the formulas which follow in a trivial way, and would not affect our results for $N = 3$.

[2]Note that $M_{\Sigma^0} - M_\Lambda = O(m_s/N)$, so that we can treat the Σ^0 and Λ as degenerate to the order we are working.

where the T_A are SU(N_F) generators normalized so that $\mathrm{tr}(T_A T_B) = \delta_{AB}$, and

$$M^2_{AB} = \frac{\beta f}{2}\,\mathrm{tr}(\{T_A, T_B\}m_q) \tag{23}$$

is the mass-squared matrix of the PNGB's [see Eq. (8)]. We have neglected the $O(m_s)$ mass differences between baryons in the same SU(N_F) multiplet, as well as the $O(1/N)$ differences between octet and decuplet baryons. (Including these effects gives corrections suppressed by m_s and/or $1/N$.) The sum over all intermediate spin states is then included in Eq. (22), with the large-N relations properly taken into account. Evaluating Eq. (22) gives rise to a contribution to the magnetic moments

$$\delta\mu_{B'B}\sigma^j_{B'B} = \frac{g^2 e M_K}{16\pi f^2}(B'|\mathcal{O}^j_K|B) , \tag{24}$$

where

$$\mathcal{O}^j_K = (N + N_F)\,\mathrm{tr}[\mathcal{Q}(1 - \mathcal{S})]\{\mathcal{S}\sigma^j\}$$

$$-\{\mathcal{S}\}\{\mathcal{Q}\sigma^j\} + \{\mathcal{Q}\}\{\mathcal{S}\sigma^j\} . \tag{25}$$

This gives contributions to the magnetic moments which are $O(Nm_s^{1/2}) = O(N\epsilon^{1/2})$.

Including these contributions along with the leading term in Eq. (14), we obtain seven relations valid to $O(N\epsilon^{1/2})$ which are independent of g. One of these is the quark-model relation Eq. (19). The remaining relations can be written

$$\begin{aligned}
\mu_{\Sigma^-} + \mu_n + \mu_p &= 0 \quad (10\%) ,\\
\mu_{\Xi^0} - 2\mu_{\Xi^-} &= 0 \quad (4\%) ,\\
\mu_{\Sigma^+} - 2\sqrt{3}\mu_{\Sigma^0\Lambda} + \mu_p &= 0 \quad (7\%) ,\\
\mu_{\Sigma^+} + \frac{1}{\sqrt{3}}\mu_{\Sigma^0\Lambda} + \mu_\Lambda + \mu_p &= 0 \quad (2\%) ,\\
\mu_{\Xi^-} + \mu_{\Sigma^+} - \mu_{\Sigma^-} - \mu_p &= 0 \quad (4\%) ,\\
\mu_{\Omega^-} - \mu_{\Xi^0} - \mu_{\Xi^-} &= 0 \quad (2\%) .
\end{aligned} \tag{26}$$

In addition, there is one relation which depends on g,

$$\mu_{\Sigma^-} - \mu_{\Xi^-} = \frac{g^2 e M_K}{8\pi f^2} \quad (-0.51 = -2.0) . \tag{27}$$

(We use $f = f_K \simeq 114$ MeV in the evaluation.) The relations which are independent of g work much better than the g-dependent relation: a fit including the $O(Nm_s^{1/2})$ corrections and treating g as a free parameter has an average deviation of 0.08 μ_N. The nonanalytic corrections have the right sign, but their predicted magnitude for the lowest-order value $g \simeq 0.8$ is too large. However, we expect that including the SU(N_F)-breaking corrections in the fit to the semileptonic decays will substantially *decrease* g [8,9], and it is not clear to us that the large discrepancy in Eq. (27) indicates a breakdown of the expansion.

The vertex graph [Fig. 1(b)] and wave function graphs [Fig. 1(c)] combine to give the contribution

$$\text{Figs. 1(b,c)} = \frac{ieg^2}{4f^2}\epsilon_{\mu\nu\lambda\rho}v^\nu q^\lambda (B'|[\{T_A\sigma^\alpha\},[\{T_A\sigma^\beta\},\{\mathcal{Q}\sigma^\rho\}]]|B)$$

$$\times \int \frac{d^4k}{(2\pi)^4}\frac{k_\alpha k_\beta}{(k^2 - M_A^2 + i0^+)(k\cdot v + i0^+)[(k-q)\cdot v + i0^+)]} . \tag{28}$$

The double commutator can be written as a one-body operator; this contribution is therefore $\sim 1/N$ times a one-body operator, and can be at most $O(m_s \ln m_s) = O(N\epsilon^2 \ln\epsilon)$. This is negligible compared to the $O(N\epsilon)$ counterterms which we will discuss below. The graph in Fig. 1(d) gives a contribution which is $1/N$ times a one-body operator, which is negligible for the same reason as for Eq. (28); the graph in Fig. 1(e) gives no contribution to the magnetic moments.

The leading SU(N_F)-violating counterterms are

$$\delta\mathcal{L} = ev^\mu \tilde{F}^{\mu\nu}(\mathcal{B}|\left[\frac{b_1}{\Lambda^2}\{(\mathcal{Q}m)\sigma_\nu\} + \frac{b_2}{N\Lambda^2}\{\mathcal{Q}\sigma_\nu\}\{m\} + \frac{b_3}{N\Lambda^2}\{\mathcal{Q}\}\{m\sigma_\nu\}\right]|\mathcal{B}) . \tag{29}$$

These counterterms give $O(Nm_s) = O(N\epsilon)$ contributions to the magnetic moments. There are also $O(Nm_s)$ contributions from the loop graph in Fig. 1(a), but these have the same spin-flavor dependence as the counterterms considered above, so we will not need to evaluate them explicitly. Including the counterterms in Eq. (29) and the nonanalytic corrections computed above, we obtain the relations

$$\begin{aligned}
\mu_{\Xi^0} + 2\mu_{\Sigma^+} + 2\mu_{\Sigma^-} + \mu_n &= 0 \quad (10\%) ,\\
\mu_{\Xi^-} + 4\mu_{\Sigma^-} + 2\sqrt{3}\mu_{\Sigma^0\Lambda} + 5\mu_p + 8\mu_n &= 0 \quad (2\%) ,\\
\mu_{\Omega^-} + 4\mu_{\Xi^0} - 3\mu_{\Xi^-} + 8\mu_{\Sigma^+} + 5\mu_{\Sigma^-} - 3\mu_p + \mu_n &= 0 \quad (8\%) ,\\
4\mu_{\Xi^0} - \mu_{\Sigma^+} - \mu_{\Sigma^-} + 4\sqrt{3}\mu_{\Sigma^0\Lambda} - 6\mu_\Lambda + 4\mu_n &= 0 \quad (6\%) .
\end{aligned} \tag{30}$$

The last to these relations was noted in Ref. [12], and is valid to $O(m_x)$ independent of the $1/N$ expansion. The average deviation of a fit which treats all of the counterterm couplings as free parameters is 0.08 μ_N, the same as the fit including only the $O(Nm_s^{1/2})$ terms.

IV. PREDICTIONS

We can use the results obtained above to predict the unmeasured octet and decuplet magnetic moments including all contributions up to $O(N\epsilon)$. We first give these predictions in the form of relations, and then give numerical predictions.

The predictions include the isospin relations

$$\begin{aligned} 2\mu_{\Sigma^0} &= \mu_{\Sigma^+} + \mu_{\Sigma^-} \ , \\ \mu_{\Delta^{++}} - \mu_{\Delta^+} &= \mu_{\Delta^0} - \mu_{\Delta^-} \ , \\ \mu_{\Delta^{++}} - \mu_{\Delta^-} &= 3(\mu_{\Delta^+} - \mu_{\Delta^0}) \ , \\ 2\mu_{\Sigma^{*0}} &= \mu_{\Sigma^{*+}} + \mu_{\Sigma^{*-}} \ , \end{aligned} \tag{31}$$

which are valid to all orders in m_s and $1/N$; we also have the SU(3) relations

$$\begin{aligned} (\mu_{\Delta^{++}} - \mu_{\Delta^+}) - (\mu_{\Sigma^+} - \mu_{\Sigma^-}) + (\mu_{\Xi^0} - \mu_{\Xi^-}) &= 0 \ , \\ \mu_{\Sigma^{*-}} - 2\mu_{\Xi^{*-}} + \mu_{\Omega^-} &= 0 \ , \\ \mu_{\Delta^0} - 2\mu_{\Sigma^{*0}} + \mu_{\Xi^{*0}} &= 0 \ , \end{aligned} \tag{32}$$

which are valid to $O(m_s)$ to all orders in $1/N$. The new relations which are consequences of the $1/N$ expansion may be written

$$\begin{aligned} \mu_{\Delta^0} &= 0 \ , \\ \mu_{\Delta^+} &= 3(\mu_p + \mu_n) \ , \\ \mu_{\Sigma^{*+}} - \mu_{\Sigma^{*-}} &= 3(-\mu_p + \mu_{\Sigma^-} + 3\mu_{\Sigma^+} - \mu_{\Xi^-} + 2\mu_{\Xi^0}) \ . \end{aligned} \tag{33}$$

There are two more relations than one would expect from naive parameter counting, because $\{\mathcal{Q}\sigma_\mu\} \propto \{\mathcal{Q}\}\{\sigma_\mu\}$ and $\{\mathcal{Q}\sigma_\mu\}\{m\} \propto \{Q\}\{m\sigma_\mu\}$ on decuplet states. The leading contribution to the Δ^0 magnetic moment comes from terms such as

$$\delta\mathcal{L} = \frac{eb_4}{N\Lambda^2}\,\mathrm{tr}(m\mathcal{Q})v^\mu\epsilon_{\mu\nu\lambda\rho}F^{\nu\lambda}(\mathcal{B}|\{\sigma^\rho\}|\mathcal{B}) \ , \tag{34}$$

where $b_4 \sim 1$. (The reason for the factor of $1/N$ in the coefficient is that the QCD diagrams which contribute to this term contain a quark loop; see Refs. [4,5].) This gives a contribution to the magnetic moments $O(1/N) = O(N\epsilon^2)$, which is higher order than the terms we are keeping.

We can also give numerical predictions for the unmeasured magnetic moments by fitting to the measured moment. Up until now, the fits were used only to give a rough idea of how well the expansion works. We now give some details on the fit and the treatment of errors to help the reader understand the numerical predictions. We add a theoretical uncertainty of 0.1 μ_N in quadrature with the experimental error to obtain the error on the individual magnetic moments, and then perform a χ^2 fit. This theoretical uncertainty is approximately the average deviation of the fit when corrections up to $O(N\epsilon)$ are included, and is consistent with the expectations of dimensional analysis: the largest contributions not included are $m_s^{3/2}$ nonanalytic terms

$$\delta\mu \sim \frac{M_K^3 M_N}{16\pi f^2\Lambda^2}\mu_N \sim 0.2\mu_N \ . \tag{35}$$

The predictions we obtain are given in Table I, along with a comparison to predictions from chiral perturbation theory [13] and lattice data [14]. Note that in the limit of exact SU(3) flavor symmetry, the decuplet magnetic moments are proportional to their charges. Table I therefore should be viewed as giving predictions for the pattern of SU(3) violation.

The chiral expansion of Ref. [13] includes the $O(m_s^{1/2})$ and $O(m_s \ln m_s)$ contributions to the decuplet magnetic moments without making use of the $1/N$ expansion. Their predictions differ considerably from ours: for example, their predicted values for $\mu_{\Sigma^{*0}}$ and $\mu_{\Xi^{*0}}$ are significantly smaller than ours. It is worth noting that they do not include $O(m_s)$ counterterm contributions, which are not expected to be significantly smaller than the $O(m_s \ln m_s)$ which they compute, whereas the largest terms which are omitted in our analysis are suppressed by a *power* of the expansion parameter. Our predictions agree well with the lattice computation of Ref. [14], which also disagrees with the chiral perturbation theory predictions of Ref. [13]. We do not regard the lattice calculation of Ref. [14] as definitive, and we hope that results from experiment or improved lattice calculations will be able to decide between our predictions and those of Ref. [13] in the future.

There is also an extraction of the Δ^{++} magnetic moment from the reaction $\pi p \to \pi p\gamma$, which yields $\mu_{\Delta^{++}} = 4.52 \pm 0.32\ \mu_N$ [15]. This agrees better with the chiral perturbation theory prediction than with our results or the lattice calculation, but the extraction relies on hadronic models.

The octet-decuplet transition magnetic moments are also predicted in the $1/N$ expansion considered here. However, the momentum transfer for the process $T \to B\gamma$ is

TABLE I. Decuplet magnetic moments predicted by the $1/N$ expansion of this paper, compared with the chiral perturbation theory (χPT) results of Ref. [13] and the lattice results of Ref. [14]. The errors quoted in our predictions are the formal fit errors; for a discussion of the errors in the other predictions, see Refs. [13,14].

μ	$1/N$	χPT	Lattice
Δ^{++}	5.9 ± 0.4	4.0 ± 0.4	6.09 ± 0.88
Δ^+	2.9 ± 0.2	2.1 ± 0.2	3.05 ± 0.44
Δ^-	-2.9 ± 0.2	-2.25 ± 0.25	-3.05 ± 0.44
Σ^{*+}	3.3 ± 0.2	2.0 ± 0.2	3.16 ± 0.40
Σ^{*0}	0.3 ± 0.1	-0.07 ± 0.02	0.329 ± 0.067
Σ^{*-}	-2.8 ± 0.3	-2.2 ± 0.2	-2.50 ± 0.29
Ξ^{*0}	0.65 ± 0.2	0.10 ± 0.04	0.58 ± 0.010
Ξ^{*-}	-2.3 ± 0.15	2.0 ± 0.2	-2.08 ± 0.24

$$\mathbf{q} \sim M_T - M_B \sim \frac{\Lambda}{N}\,, \tag{36}$$

and the loop graph in Fig. 1(a) has nontrivial dependence on $\mathbf{q}/(M_T - M_B) \sim 1$. This makes the coefficients of the effective operators in the expansion different from the ones appearing in the expansion of the magnetic moments. In principle, this difference is computable, but it depends on the value of the axial coupling g, which is not well determined. We will return to these issues in a future publication.

After this paper was completed, we received Ref. [16], which also analyzes baryon magnetic moments using a $1/N$ expansion. Their expansion differs from ours in that physical baryon states are identified with particular large-N states. The relationship between their expansion and the one carried out in this paper is discussed in Ref. [5].

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