Predictions for the proton lifetime in minimal nonsupersymmetric SO(10) models: An update

Dae-Gyu Lee and R. N. Mohapatra
Department of Physics, University of Maryland, College Park, Maryland 20742

M. K. Parida and Merostar Rani
Physics Department, North Eastern Hill University, P.O. Box 21, Laitumkharh, Shillong-793003, India

(Received 8 April 1994)

We present our best estimates of the uncertainties due to heavy particle threshold corrections on the unification scale M_U, intermediate scale M_I, and coupling constant α_U in the minimal nonsupersymmetric SO(10) models. Using recent data from the CERN e^+e^- collider LEP on $\sin^2\theta_W$ and α_{strong} to obtain the two-loop-level predictions for M_U and α_U, we update the predictions for the proton lifetime in minimal nonsupersymmetric SO(10) models.

PACS number(s): 12.10.Dm, 12.60.−i, 13.30.Ce, 14.20.Dh

I. INTRODUCTION

The hypothesis of a single unified gauge symmetry describing all forces and matter at very short distances is a very attractive one from a practical as well as aesthetic point of view. Right now there are several good reasons to think that this gauge symmetry may indeed be SO(10) [1]. The most compelling argument in favor of SO(10) comes from ways to explain [2] the observed deficit [3] of the solar neutrino flux compared to the predictions [4] of the standard solar model in terms of a two-flavor Mikheyev-Smirnov-Wolfenstein (MSW) [5] neutrino oscillation. Consistent understanding of the data from all four experiments using the MSW oscillation hypothesis requires the neutrino masses and mixings to lie in a very narrow range of values. It was shown in a recent paper [6] that the minimal SO(10) theory that implements the seesaw mechanism [8] is a completely predictive theory in the neutrino sector and predicts masses and mixings between ν_e and ν_μ that are in this range. We wish to note however that the MSW explanation is by no means the only way to resolve the solar neutrino puzzle and also the minimal SO(10) model predicts four sets [9] of values for neutrino masses and mixings, only one of which [6] accommodates the MSW solution. Therefore, even though there are good arguments for the SO(10) model, it is by no means proven to be the only possibility. However, our strong belief in SO(10) is due to several other attractive features of the model, such as fermion unification into a single $\{16\}$ representation, a simple picture of baryogenesis [10], asymptotic parity conservation of all interactions, etc. In view of these, we have undertaken a detailed quantitative analysis of the symmetry-breaking scales of this minimal model in order to pinpoint its predictions for the proton lifetime, especially the uncertainties in it arising from unknown Higgs boson masses in the theory.

Since we are going to discuss the minimal SO(10) model, let us explain what we mean by the word "minimal." It stands for the fact that (a) the Higgs sector is chosen to consist of the smallest number of multiplets of SO(10) than is required for symmetry breaking and (b) only those fine-tunings of the parameters needed to achieve the desired gauge hierarchy are imposed. The above fixes the order of magnitude of the Higgs boson masses of the model. Of course, we cannot determine the Higgs boson masses precisely and this will be the source of the uncertainty in our predictions for the proton lifetime. In order to estimate this uncertainty, we will consider the range of Higgs boson masses between $M/10$ to $10M$, where M stands for the relevant gauge symmetry-breaking scale.

Before we proceed further, we wish to make the following important remark about the minimal SO(10) model. For a long time it was thought that this model cannot be realistic since it predicts [11] the relations among fermion masses such as $m_s = m_\mu$ and $m_d = m_e$ at the grand unified theory (GUT) scale M_U, which, after extrapolation to the weak scale, are in complete disagreement with experiment. However, it was shown in Ref. [6] that in the minimal SO(10) models where the small neutrino masses arise from the seesaw mechanism [8], there are additional contributions to charged fermion masses that solve this problem. They arise from the fact that the **(2,2,15)** submultiplet of the $\{126\}$-dimensional Higgs multiplet used in implementing the seesaw mechanism automatically acquires an induced vacuum expectation value (VEV) [7] without additional fine-tuning. These additional contributions correct the above mass relations in such a way as to restore agreement with observations. The same theory, as mentioned above, also predicts interesting values for neutrino masses and mixings making the minimal SO(10) models not only completely realistic but also testable by neutrino oscillation experiments to be carried out soon.

Next, let us mention a word on our choice of the nonsupersymmetric version of the model. While the question of gauge hierarchy certainly prefers a supersymmetric (SUSY) SO(10) model, in the absence of any evidence of supersymmetry at low energies as well as for the sake of simplicity alone, we believe that minimal non-SUSY SO(10) should be thoroughly explored and confronted with experiments.

Another interesting point that needs to be emphasized is that for the minimal set of Higgs multiplets, SO(10) automatically breaks to the standard model via *only* one intermediate stage, that consists of the left-right-symmetric gauge group with or without the parity symmetry [12], depending on the Higgs multiplet chosen to break SO(10). This discrete Z_2 symmetry is denoted by the symbol D in what follows. This leads to the following four possibilities for the intermediate gauge symmetry:

(I) $G_{224D} \equiv SU(2)_L \times SU(2)_R \times SU(4)_C \times D$,

(II) $G_{224} \equiv SU(2)_L \times SU(2)_R \times SU(4)_C$,

(III) $G_{2213D} \equiv SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times D$,

(IV) $G_{2213} \equiv SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$.

Case I arises if the Higgs multiplet used to break is a single {54}-dimensional one [13]. Cases II and III arise if a single {210}-Higgs multiplet is used. Depending on the range of the parameters in the Higgs potential, either case II or case III arises as the intermediate symmetry [14]. Case IV arises when one uses a combination of {45}- and {54}-dimensional Higgs multiplets [15]. Note that both these representations are required to obtain this symmetry-breaking chain and therefore satisfy the criterion of minimality that we adopt. The rest of the symmetry breaking is implemented by a single {126}-dimensional representation to break $SU(2)_R \times U(1)_{B-L}$ as well as to understand neutrino masses and a single complex {10} to break the electroweak $SU(2)_L \times U(1)_Y$ down to $U(1)_{\rm em}$. These four cases therefore represent the four simplest and completely realistic minimal SO(10) models. In the rest of the paper, we present calculations of the predictions for a proton lifetime (τ_p) in these models as well as the uncertainties in these predictions due to unknown Higgs masses and the uncertainties in the low-energy input parameters, in order to see if the next round of proton decay search at Super-Kamiokande (SK) [16] can test this model.

II. COMPUTATION OF THE THRESHOLD UNCERTAINTIES IN M_U AND M_I

The two main equations in our discussion are (i) the two-loop renormalization group equation for the evolution of the gauge couplings, i.e.,

$$\frac{d\alpha_i}{dt} = \frac{a_i}{2\pi}\alpha_i^2 + \sum_j \frac{b_{ij}}{8\pi^2}\alpha_i^2\alpha_j, \tag{1}$$

and (ii) the matching formula at the mass scale where the low-energy symmetry group enlarges [17],

$$\frac{1}{\alpha_i(M_I)} = \frac{1}{\alpha_I(M_I)} - \frac{\lambda_i^I}{12\pi}. \tag{2}$$

In Eqs. (1) and (2), α_i is the "fine-structure" constant corresponding to the gauge group G_i and

$$\lambda_i^I = \mathrm{Tr}\ \theta_i^{V\,2} + \mathrm{Tr}\ \theta_i^{H\,2} \ln\frac{M_H}{M_I}, \tag{3}$$

where θ_i^H is the generator of the gauge group G_i in the representation of the Higgs submultiplet H. The expressions for a_i and b_{ij} for the four cases are given in Table I [18, 19]. In deriving the values of a_i and b_{ij} in various cases as well as to obtain the threshold corrections λ_i, we need to know the order of magnitude of the masses of the various Higgs submultiplets in the models. We obtain these by invoking the survival hypothesis for the Higgs multiplets as dictated by the minimal fine-tuning condition for gauge symmetry breaking [20]. Using this hypothesis, in Tables II(a)–II(d), we list the various Higgs multiplets whose masses are near the relevant symmetry scales along with their contributions to λ_i^I. The omitted fields from each multiplet are the ones that become longitudinal modes of the gauge bosons and therefore do not contribute to the threshold uncertainties discussed here.

We proceed as follows: first, using the two-loop equation, we derive the mean values for the mass scales in various cases. These results already exist in the literature [18, 19, 21, 22] based on the earlier results from the CERN e^+e^- collider LEP. In Table III, we have presented their values from Ref. [22], which uses the inputs $\alpha_1(M_Z) = 0.016\,887 \pm 0.000\,040$; $\alpha_2(M_Z) = 0.033\,22 \pm 0.000\,25$; $\alpha_3(M_Z) = 0.120 \pm 0.007$, for further use in calcuating τ_p. These values of M_U and M_I were obtained using analytic integration of Eq. (1) which has been done exactly for case I. For cases II, III, and IV, we have used the results of Ref. [22], which ignores terms whose effect in the final result of the renormalization group equation is smaller than the error coming from low-energy LEP data by a factor of 10 or more. We have also checked that inputing the most recent LEP data [23] gives results for the mass scales which are within the level of accuracy of our calculations. For instance, for M_U the changes are $10^{0.08}, 10^{0.01}, 10^{0.06}$, and $10^{0.14}$ for cases I, II, III, and IV, respectively. Then, we estimate the uncertainties in M_I and M_U due to both the experimental uncertainties in the low-energy parameters α_s, $\sin^2\theta_W$ and $\alpha_{\rm em}$ as well as the unknown Higgs boson masses. For the cases II and IV, these were discussed in Ref. [24] although we refine these uncertainties somewhat, but the threshold uncertainties for cases I and III are new. These uncertainties

TABLE I. One- and two-loop β-function coefficients for the intermediate symmetries for models I–IV.

Model	a_i	b_{ij}
I	$\{\frac{11}{3}, \frac{11}{3}, -\frac{14}{3}\}$	$\{\{\frac{584}{3}, 3, \frac{765}{2}\}, \{3, \frac{584}{3}, \frac{765}{2}\}, \{\frac{153}{2}, \frac{153}{2}, \frac{1759}{6}\}\}$
II	$\{-3, \frac{11}{3}, -\frac{23}{3}\}$	$\{\{8, 3, \frac{45}{2}\}, \{3, \frac{584}{3}, \frac{765}{2}\}, \{\frac{9}{2}, \frac{153}{2}, \frac{643}{6}\}\}$
III	$\{-\frac{7}{3}, -\frac{7}{3}, 7, -7\}$	$\{\{\frac{80}{3}, 3, \frac{27}{2}, 12\}, \{3, \frac{80}{3}, \frac{27}{2}, 12\}, \{\frac{81}{2}, \frac{81}{2}, \frac{115}{2}, 4\}, \{\frac{9}{2}, \frac{9}{2}, \frac{1}{2}, -26\}\}$
IV	$\{-3, -\frac{7}{3}, \frac{11}{2}, -7\}$	$\{\{8, 3, \frac{3}{2}, 12\}, \{3, \frac{80}{3}, \frac{27}{2}, 12\}, \{\frac{9}{2}, \frac{81}{2}, \frac{61}{2}, 4\}, \{\frac{9}{2}, \frac{9}{2}, \frac{1}{2}, -26\}\}$

TABLE II. (a) The heavy Higgs content of model I. The G_{224} submultiplets in (a)-(1) acquire masses when SO(10) is broken, while the G_{123} submultiplets in (a)-(2) become massive when G_{224D} is broken. The multiplets ϕ, R_i, and L_i in (a)-(2) arise from $\phi(2,2,0)$ in $\{10\}$, from $\Delta_R(1,3,10)$, and $\Delta_L(3,1,\overline{10})$ in $\{126\}$, respectively. Also listed, in the extreme right column of the tables, are the threshold contributions λ_i of the different multiplets. (b) The heavy Higgs particles in model II whose intermediate symmetry is G_{224}. The particles whose masses are on the order of M_U are listed in (b)-(1), and the particles on the order of M_I are listed in (b)-(2). Also listed are their threshold contributions. (c) All the heavy Higgs particles in model III whose intermediate symmetry is G_{2213D}. The submultiplets with masses of order M_U are presented in (c)-(1), and the particles with masses of order M_I are listed in (c)-(2). The entries in the extreme right column denote their threshold contributions. (d) All the heavy Higgs particles in model IV whose intermediate symmetry is G_{2213}. The submultiplets with masses of order M_U are presented in (d)-(1), and the particles with masses of order M_I are listed in (d)-(2). The entries in the far right column give their λ_i contributions.

SO(10) representation	G_{224} submultiplet (a)-(1)	$\{\lambda^U_{2L},\lambda^U_{2R},\lambda^U_{4C}\}$
10	$H(1,1,6)$	$\{0, 0, 2\}$
126	$\zeta_0(2,2,15)$	$\{30,30,32\}$
	$S(1,1,6)$	$\{0, 0, 2\}$
54	$S_\Sigma(3,3,1)$	$\{6, 6, 0\}$
	$S_\zeta(1,1,20')$	$\{0, 0, 8\}$
	$S_+(1,1,1)$	$\{0, 0, 0\}$

SO(10) representation	G_{123} submultiplet (a)-(2)	$\{\lambda^I_{1Y},\lambda^I_{2L},\lambda^I_{3C}\}$
10	$\phi(-\frac{1}{2}\sqrt{\frac{3}{5}},2,1)$	$\{\frac{3}{5}, 1, 0\}$
126	$R_1(-2\sqrt{\frac{3}{5}},1,1)$	$\{\frac{24}{5}, 0, 0\}$
	$R_2(+\frac{1}{3}\sqrt{\frac{3}{5}},1,3)$	$\{\frac{2}{5}, 0, 1\}$
	$R_3(-\frac{4}{3}\sqrt{\frac{3}{5}},1,3)$	$\{\frac{32}{5}, 0, 1\}$
	$R_4(-\frac{1}{3}\sqrt{\frac{3}{5}},1,6)$	$\{\frac{4}{5}, 0, 5\}$
	$R_5(+\frac{2}{3}\sqrt{\frac{3}{5}},1,6)$	$\{\frac{16}{5}, 0, 5\}$
	$R_6(-\frac{4}{3}\sqrt{\frac{3}{5}},1,6)$	$\{\frac{64}{5}, 0, 5\}$
	$L_1(+\sqrt{\frac{3}{5}},3,1)$	$\{\frac{18}{5}, 4, 0\}$
	$L_2(+\frac{1}{3}\sqrt{\frac{3}{5}},3,\overline{3})$	$\{\frac{6}{5},12, 3\}$
	$L_3(-\frac{1}{3}\sqrt{\frac{3}{5}},3,\overline{6})$	$\{\frac{12}{5},24, 15\}$

SO(10) representation	G_{224} submultiplet (b)-(1)	$\{\lambda^U_{2L},\lambda^U_{2R},\lambda^U_{4C}\}$
10	$H(1,1,6)$	$\{0, 0, 2\}$
126	$\zeta_0(2,2,15)$	$\{30,30,32\}$
	$S(1,1,6)$	$\{0, 0, 2\}$
	$\Delta_L(3,1,\overline{10})$	$\{40, 0,18\}$
210	$\Sigma_L(3,1,15)$	$\{30, 0,12\}$
	$\Sigma_R(1,3,15)$	$\{0,30,12\}$
	$\zeta_1(2,2,10)$	$\{10,10,12\}$
	$\zeta_2(2,2,\overline{10})$	$\{10,10,12\}$
	$\zeta_3(1,1,15)$	$\{0, 0, 4\}$
	$S'(1,1,1)$	$\{0, 0, 0\}$

G_{123} submultiplet (b)-(2)
ϕ, R_1, R_2, R_3, R_4, R_5, R_6 in Table II(a)-(2)

SO(10) representation	G_{2213} submultiplet (c)-(1)	$\{\lambda^U_{2L},\lambda^U_{2R},\lambda^U_{1X},\lambda^U_{3C}\}$
10	$T_1(1,1,+\frac{1}{3}\sqrt{\frac{3}{2}}, \overline{3})$	$\{0, 0, 1, 1\}$

TABLE II. (*Continued*).

SO(10) representation	G_{2213} submultiplet	$\{\lambda^U_{2L},\lambda^U_{2R},\lambda^U_{1X},\lambda^U_{3C}\}$
	$T_2(1,1,-\frac{1}{3}\sqrt{\frac{3}{2}},3)$	$\{0,0,1,1\}$
126	$H_{1R}(1,3,+\frac{1}{3}\sqrt{\frac{3}{2}},6)$	$\{0,24,6,15\}$
	$H_{1L}(3,1,-\frac{1}{3}\sqrt{\frac{3}{2}},\bar{6})$	$\{24,0,6,15\}$
	$H_{2R}(1,3,-\frac{1}{3}\sqrt{\frac{3}{2}},3)$	$\{0,4,3,3\}$
	$H_{2L}(3,1,+\frac{1}{3}\sqrt{\frac{3}{2}},\bar{3})$	$\{4,0,3,3\}$
	$H_3(2,2,+\frac{2}{3}\sqrt{\frac{3}{2}},3)$	$\{6,6,16,4\}$
	$H_4(2,2,-\frac{2}{3}\sqrt{\frac{3}{2}},\bar{3})$	$\{6,6,16,4\}$
	$H_5(2,2,0,8)$	$\{16,16,0,24\}$
	$H_6(2,2,0,1)$	$\{2,2,0,0\}$
	$H_7(1,1,+\frac{1}{3}\sqrt{\frac{3}{2}},\bar{3})$	$\{0,0,1,1\}$
	$H_8(1,1,-\frac{1}{3}\sqrt{\frac{3}{2}},3)$	$\{0,0,1,1\}$
210	$B_{L1}(3,1,0,8)$	$\{16,0,0,9\}$
	$B_{R1}(1,3,0,8)$	$\{0,16,0,9\}$
	$B_{L2}(3,1,-\frac{2}{3}\sqrt{\frac{3}{2}},\bar{3})$	$\{6,0,6,\frac{3}{2}\}$
	$B_{R2}(1,3,-\frac{2}{3}\sqrt{\frac{3}{2}},\bar{3})$	$\{0,6,6,\frac{3}{2}\}$
	$B_{L3}(3,1,+\frac{2}{3}\sqrt{\frac{3}{2}},3)$	$\{6,0,6,\frac{3}{2}\}$
	$B_{R3}(1,3,+\frac{2}{3}\sqrt{\frac{3}{2}},3)$	$\{0,6,6,\frac{3}{2}\}$
	$B_{L4}(3,1,0,1)$	$\{2,0,0,0\}$
	$B_{R4}(1,3,0,1)$	$\{0,2,0,0\}$
	$B_5(2,2,+\frac{1}{3}\sqrt{\frac{3}{2}},6)$	$\{6,6,4,10\}$
	$B_6(2,2,-\frac{1}{3}\sqrt{\frac{3}{2}},\bar{6})$	$\{6,6,4,10\}$
	$B_7(2,2,-\frac{1}{3}\sqrt{\frac{3}{2}},3)$	$\{3,3,2,2\}$
	$B_8(2,2,+\frac{1}{3}\sqrt{\frac{3}{2}},\bar{3})$	$\{3,3,2,2\}$
	$B_9(2,2,-\sqrt{\frac{3}{2}},1)$	$\{1,1,6,0\}$
	$B_{10}(2,2,+\sqrt{\frac{3}{2}},1)$	$\{1,1,6,0\}$
	$B_{11}(1,1,0,8)$	$\{0,0,0,3\}$
	$B_-(1,1,0,1)$	$\{0,0,0,0\}$
	$B_+(1,1,0,1)$	$\{0,0,0,0\}$
	G_{123} submultiplet	
	(c)-(2)	
	ϕ, R_1, L_1 in Table II(a)-(2)	
SO(10) representation	G_{2213} submultiplet	$\{\lambda^U_{2L},\lambda^U_{2R},\lambda^U_{1X},\lambda^U_{3C}\}$
	(d)-(1)	
10	T_1, T_2	
126	All H's in Table II(c)-(1)	
	$H_\Delta(3,1,\sqrt{\frac{3}{2}},1)$	$\{4,0,9,0\}$
45	$S_1(1,1,0,8)$	$\{0,0,0,3\}$
	$S_2(3,1,0,1)$	$\{2,0,0,0\}$
	$S_3(1,3,0,1)$	$\{0,2,0,0\}$
	$S_-(1,1,0,1)$	$\{0,0,0,0\}$
	G_{123} submultiplet	
	(d)-(2)	
	ϕ, R_1 in Table II(a)-(2)	

TABLE III. The values of M_U, M_I, and α_U obtained by solving the two-loop renormalization group equation, Eq. (1) for models I–IV. The results were taken from Ref. [22].

Model	M_I (GeV)	M_U (GeV)	α_U^{-1}
I	$10^{13.64}$	$10^{15.02\pm0.25}$	$40.76\ \pm0.16$
II	$10^{10.70}$	$10^{16.26\pm0.25}$	$46.35\ ^{+0.23}_{-0.22}$
III	$10^{10.16}$	$10^{15.55\pm0.20}$	$43.86\ \pm0.18$
IV	$10^{9.08}$	$10^{16.42^{+0.23}_{-0.22}}$	$46.12\ ^{+0.15}_{-0.16}$

are presented in Table IV. We have allowed the Higgs masses to be between 1/10 and 10 times the scale of the relevant symmetry breaking .

The formulas for the threshold effect on the mass scales M_I and M_U, which were used to obtain Table IV, are given below for each symmetry-breaking chain. We have defined $\eta_i = \ln(M_{H_i}/M)$, where M is the relevant gauge symmetry-breaking scale near M_{H_i}.

Model I:

$$\begin{aligned}\Delta\ln(M_{C_+}/M_Z) &= 0\eta_{10} + 0\eta_{126} + 0\eta_{54}\\ &\quad +0\eta_\phi + 0.431\,818\eta_R - 0.454\,545\eta_L,\end{aligned}$$

$$\begin{aligned}\Delta\ln(M_U/M_Z) &= -0.04\eta_{10} - 0.08\eta_{126} - 0.04\eta_{54}\\ &\quad +0.02\eta_\phi - 0.106\,818\eta_R + 0.194\,545\eta_L.\end{aligned} \tag{4a}$$

Model II:

$$\begin{aligned}\Delta\ln(M_C/M_Z) &= +0.051\,8135\eta_{10} + 0.103\,627\eta_{126}\\ &\quad +0.051\,813\,5\eta_{210} - 0.025\,906\,7\eta_\phi\\ &\quad +1.129\,53\eta_R - 1.295\,34\eta_{\Delta_L},\end{aligned}$$

$$\begin{aligned}\Delta\ln(M_U/M_Z) &= -0.062\,176\,2\eta_{10} - 0.124\,352\eta_{126}\\ &\quad -0.062\,176\,2\eta_{210} + 0.031\,088\,1\eta_\phi\\ &\quad -0.405\,44\eta_R + 0.554\,404\eta_{\Delta_L}.\end{aligned} \tag{4b}$$

Model III:

$$\begin{aligned}\Delta\ln(M_{R_+}/M_Z) &= +0.095\,238\,1\eta_{10} - 0.095\,238\,1\eta_{126}\\ &\quad +0\eta_{210} - 0.047\,619\eta_\phi + 0.190\,476\eta_{R1}\\ &\quad -0.142\,857\eta_{L1},\end{aligned}$$

$$\begin{aligned}\Delta\ln(M_U/M_Z) &= -0.054\,421\,8\eta_{10} - 0.159\,864\eta_{126}\\ &\quad -0.035\,714\,3\eta_{210} + 0.027\,210\,9\eta_\phi\\ &\quad +0.034\,013\,6\eta_{R1} + 0.117\,347\eta_{L1}.\end{aligned} \tag{4c}$$

Model IV:

$$\begin{aligned}\Delta\ln(M_R/M_Z) &= +0.124\,138\eta_{10} - 0.082\,758\,6\eta_{126}\\ &\quad +0.006\,896\,55\eta_{45} - 0.062\,069\eta_\phi\\ &\quad +0.220\,69\eta_{R1} - 0.193\,103\eta_{H_\Delta},\end{aligned}$$

$$\begin{aligned}\Delta\ln(M_U/M_Z) &= -0.078\,160\,9\eta_{10} - 0.170\,115\eta_{126}\\ &\quad -0.041\,379\,3\eta_{45} + 0.039\,080\,5\eta_\phi\\ &\quad +0.009\,195\,4\eta_{R1} + 0.158\,621\eta_{H_\Delta}.\end{aligned} \tag{4d}$$

In obtaining the above equations, we have assumed that the particles from a single SO(10) representation which have masses of the same order are degenerate. This is the same assumption as in Ref. [24]. Since we are interested in estimating the maximum and minimum values of the uncertainties, we believe that this is not too unreasonable an assumption to make. Before proceeding to give our predictions for the proton lifetime, a few comments are in order.

(a) We want to clarify how we get the uncertainties presented in Table IV. First, as already mentioned, we chose M_H/M_I or M_H/M_U to vary between 10^{-1} and 10^{+1}. The maximum values of the uncertainties are obtained by allowing the different η's to vary independently to their extreme values that lead to the largest positive or negative uncertainty. The only exception to this are the two parameters η_{Δ_L} and η_{H_Δ}, which are always kept negative. (See below.) Second, in chains I and III we do not assume that the left- and right-handed Higgs submultiplets have the same mass (that would lead to $\eta_L = \eta_R$). The reason is that since the masses are close to the intermediate scale, where left-right symmetry is broken, the multiplets need not necessarily be degenerate. If we assumed the degeneracy, there would be a cancellation between the η_L and η_R terms reducing the threshold uncertainties [25]; the uncertainties we present in Table IV are, therefore, rather conservative.

(b) In cases II and IV, since D parity is broken at the GUT scale, the masses of Δ_L in Table II(b)-(1) and H_Δ in Table II(d)-(1) are always above the scale M_I, but below M_U [24]. Although *a priori* M_{Δ_L} could be bigger than M_U, we have kept it smaller in presenting the uncertainty in τ_p. Therefore, η_{Δ_L} and η_{H_Δ} in Eqs. (4b) and (4d) are always negative, since we use $M = M_U$ to define them. In any case, from an experimental point of view, the upper value of the uncertainty is not too relevant and making M_{Δ_L} bigger than M_U simply adds to the upper value of the uncertainty.

TABLE IV. The threshold uncertainties due to the difference between the symmetry-breaking scale and masses of Higgs bosons on the order of that scale. For the cases where threshold effects in M_I and M_U are maximized, corresponding threshold uncertainties are given in the first two lines and the last two lines, respectively.

Threshold uncertainty	Model I	Model II	Model III	Model IV
M_I/M_I^0	$10^{\pm0.886}$	$10^{+2.658}_{-0.067}$	$10^{\pm0.571}$	$10^{+0.690}_{-0.303}$
M_U/M_U^0	$10^{\pm0.481}$	$10^{+0.131}_{-1.240}$	$10^{\pm0.031}$	$10^{-0.138}_{-0.179}$
M_I/M_I^0	$10^{\pm0.886}$	$10^{+2.658}_{-0.067}$	$10^{\pm0.000}$	$10^{+0.303}_{+0.083}$
M_U/M_U^0	$10^{\pm0.481}$	$10^{+0.131}_{-1.240}$	$10^{\pm0.429}$	$10^{+0.179}_{-0.497}$

(c) The first set of entries in Table IV is obtained by maximizing the uncertainty in M_I whereas the second set is obtained by doing the same for M_U.

(d) Note that, in case I, the intermediate mass scale M_I and the unification scale M_U are so close that one might think of this as an almost single step breaking. This is similar to the D-parity broken scenario (case II) recently discussed in Ref. [26]. For the proton lifetime estimate, this is inconsequential.

III. PREDICTIONS FOR THE PROTON LIFETIME

Now, we present our predictions for the proton lifetime in the four SO(10) models I–IV. For this purpose, we need the values of M_U and α_U and remember that in SO(10) there are extra gauge bosons contributing to proton decay compared to the SU(5) model. We use the following formula from the review by Langacker [27], where the original literature can be found. We write

$$\tau_p = \tau_p^{(0)} F_p,$$

where F_p denotes the uncertainty arising from threshold corrections as well as the experimental errors in α_s, $\alpha_{\rm em}$, and $\sin^2\theta_W$. From Ref. [27], we get, for $\tau_p^{(0)}$,

$$\tau^{(0)}_{p\to e^+\pi^0} = \frac{5}{8}\left(\frac{\alpha_U^{\rm SU(5)}}{\alpha_U^{\rm SO(10)}}\right)^2 \times 4.5\times 10^{29\pm 0.7}\left(\frac{M_U}{2.1\times 10^{14}\ {\rm GeV}}\right)^4 \ {\rm yr}. \tag{5}$$

Including the F_p factors, we present below the predictions for the proton lifetime in SO(10) (noting that $\alpha_U^{\rm SU(5)} \approx \alpha_U^{\rm SO(10)}$). The first uncertainty in the predictions below arises from the proton decay matrix element evaluation which is included in $\tau_p^{(0)}$, whereas the second and the third ones come from LEP data and threshold corrections, respectively [28].

Model I:

$$\tau_{p\to e^+\pi^0} = 1.44\times 10^{32.1\pm 0.7\pm 1.0\pm 1.9}\ {\rm yr}.$$

Model II:

$$\tau_{p\to e^+\pi^0} = 1.44\times 10^{37.4\pm 0.7\pm 1.0^{+0.5}_{-5.0}}\ {\rm yr}.$$

Model III:

$$\tau_{p\to e^+\pi^0} = 1.44\times 10^{34.2\pm 0.7\pm 0.8\pm 1.7}\ {\rm yr}.$$

Model IV:

$$\tau_{p\to e^+\pi^0} = 1.44\times 10^{37.7\pm 0.7\pm 0.9^{+0.7}_{-2.0}}\ {\rm yr}.$$

IV. CONCLUSION

We have computed the threshold uncertainties in both the intermediate and the unification scales for all four possible minimal nonsupersymmetric SO(10) models I–IV. We then update the predictions for the proton lifetime in all these cases including the most conservative estimates for the threshold uncertainties in them. We see that for case I, τ_p is very much within the range of the Super-Kamiokande search even without threshold corrections. This is the most useful new result of the paper not discussed in the paper [24] on the subject. For cases II and III, the threshold uncertainties have the effect of bringing the proton lifetime within the range of the SK search. In our discussions, we have worked within the framework of a renormalizable field theory and have therefore not included the effects of any higher-dimensional nonrenormalizable terms. Their presence will of course add uncertainties by changing the intermediate scales and GUT scales, etc.; but, our philosophy in this paper is to stay strictly within a renormalizable model as was for instance done in the case of the conventional SU(5) model.

ACKNOWLEDGMENTS

This work was supported by a grant from the National Science Foundation. The work of M.K.P. was supported by the Project No. SP/S2/K-09/91 from the Dept. of Science and Technology, Govt. of India.

[1] H. Georgi, in *Particles and Fields*, edited by C.E. Carlson (American Institute of Physics, New York, 1975); H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) **93**, 193 (1975).

[2] N. Hata and P. Langacker, Phys. Rev. D **50**, 632 (1994); P.I. Krastev and S.T. Petcov, Phys. Rev. Lett. **72**, 1960 (1994); G. Fogli, E. Lisi, and D. Montanino, Phys. Rev. D **50**, 3626 (1994).

[3] R. Davis *et al.*, in *Proceedings of the 21st International Cosmic Ray Conference*, Adelaide, Australia, 1989, edited by R.J. Protheroe (University of Adelaide Press, Adelaide, 1990), Vol. 12, p. 143; K.S. Hirata *et al.*, Phys. Rev. D **44**, 2241 (1991); A.I. Abazov *et al.*, Phys. Rev. Lett. **67**, 3332 (1991); T. Bowles (unpublished); P. Anselmann *et al.*, Phys. Lett. B **285**, 376 (1992); **314**, 445 (1993); T. Kirsten (unpublished).

[4] For a theoretical discussion, see J.N. Bahcall and M.H. Pinsonneault, Rev. Mod. Phys. **64**, 885 (1992); S. Turck-chieze *et al.*, Phys. Rep. **230**, 57 (1993).

[5] S.P. Mikheyev and A.Yu. Smirnov, Yad. Fiz. **42**, 1441 (1975) [Sov. J. Nucl. Phys.]; L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).

[6] K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993).

[7] The question of getting induced VEV's for (2, 2, 15) of 126 in the context of nonminimal SO(10) models has been discussed by Q. Shafi and C. Wetterich, Nucl. Phys. **B289**, 787 (1988); Z. Berezhiani and A. Asatryan (unpublished).

[8] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, Proceedings of the Workshop, Stony Brook, New York, 1979, edited by D. Freedman and P. van Nieuwenhuizen (North-Holland, Amsterdam, 1980); T. Yanagida, *Proceedings of the Workshop on Unified Theories and*

Baryon Number in the Universe, Tsukuba, Japan, 1979, edited by A. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, 1979); R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).
[9] L. Lavoura, Phys. Rev. D **48**, 5440 (1993).
[10] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986); P. Langacker, R.D. Peccei, and T. Yanagida, Mod. Phys. Lett. **A 1**, 541 (1986); M. Luty, Phys. Rev. D **45**, 455 (1992).
[11] M. Chanowitz, J. Ellis, and M.K. Gaillard, Nucl. Phys. **B135**, 66 (1978).
[12] D. Chang, R.N. Mohapatra, and M.K. Parida, Phys. Rev. Lett. **52**, 1072 (1984).
[13] G. Lazaridis, M. Magg, and Q. Shafi, Phys. Lett. **97B**, 87 (1980); F. Buccella, L. Cocco, and C. Wetterich, Nucl. Phys. **B248**, 273 (1984).
[14] D. Chang and A. Kumar, Phys. Rev. D **33**, 2695 (1986); J. Basaq, S. Meljanac, and L. O'Raifeartaigh, *ibid.* **39**, 3110 (1989); X.-G. He and S. Meljanac, *ibid.* **40**, 2098 (1989).
[15] Ö. Kaymakcalan, L. Michel, K.C. Wali, W.D. McGlinn, and L. O'Raifeartaigh, Nucl. Phys. **B267**, 203 (1986); R. Thornburg and W.D. McGlinn, Phys. Rev. D **33**, 2991 (1986); R. Kuchimanchi, *ibid.* **47**, 685 (1993).
[16] A. Suzuki, in *Proceedings of the ICEPP Symposium 'From LEP to the Planck World,'* Tokyo, Japan, 1992, edited by K. Kawagoe and T. Kobayashi (Tokyo University, Japan, 1993).
[17] S. Weinberg, Phys. Lett. **91B**, 51 (1980); L. Hall, Nucl. Phys. **B178**, 75 (1981).
[18] D. Chang, R.N. Mohapatra, J.M. Gipson, R.E. Marshak, and M.K. Parida, Phys. Rev. D **31**, 1718 (1985).
[19] N.G. Deshpande, E. Keith, and P.B. Pal, Phys. Rev. D **46**, 2261 (1992).
[20] F. del Aguila and L. Ibáñez, Nucl. Phys. **B177**, 60 (1981); R.N. Mohapatra and G. Senjanović, Phys. Rev. D **27**, 1601 (1983).
[21] F. Buccella, G. Miele, L. Rosa, P. Santorelli, and T. Tuzi, Phys. Lett. B **233**, 178 (1989).
[22] D.-G. Lee, Phys. Rev. D **50**, 2071 (1994).
[23] The L3 Collaboration, Phys. Rep. **236**, 1 (1993).
[24] R.N. Mohapatra and M.K. Parida, Phys. Rev. D **47**, 264 (1993).
[25] M.K. Parida and P.K. Patra, Phys. Rev. Lett. **66**, 858 (1991); **68**, 754 (1992).
[26] L. Lavoura and L. Wolfenstein, Phys. Rev. D **48**, 264 (1993).
[27] P. Langacker, in *Inner Space and Outer Space*, edited by E. Kolb *et al.* (University of Chicago Press, Chicago, 1986), p. 1.
[28] The predictions for cases II and IV are different from those given in Ref. [24], because of different input LEP data.