β decay of hyperons in a relativistic quark model

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A relativistic constituent quark model is used to calculate the semileptonic β decay of nucleons and hyperons. The parameters of the model, namely, the constituent quark mass and the confinement scale, are fixed by a previous calculation of the magnetic moments of the baryon octet within the same model. We discuss the momentum dependence of the form factors, possible configuration mixing, and SU(3) symmetry breaking. We conclude that the relativistic constituent quark model is a good framework to analyze electroweak properties of the baryons.

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I. INTRODUCTION

In this paper we consider the application of the relativistic constituent quark model to semileptonic hyperon decay. We compare our result with the new data from the Particle Data Group (PDG) [1]. The predictive power of a relativistic constituent quark model formulated on the light front was recently investigated in Ref. [2]. It provides a simple model wherein we have overall an excellent and consistent picture of the magnetic moments and of the semileptonic decays of the baryon octet. This paper extends the analysis of the semileptonic β decays and addresses specific questions for the hyperon β decay.

The effect of configuration mixing has recently been studied [3] in the context of deep inelastic scattering. We show below that such configuration mixing is not favored for hyperon decays.

Our quark model provides a unique scheme for calculating the momentum dependence of the form factors. Although its effect is generally small, a change of the dipole masses M_V or M_A by 0.15 GeV in the decay $\Sigma^- \to ne\nu$ causes a relative change in g_1/f_1 of 2%. Ignoring the momentum dependence altogether would shift g_1/f_1 by 17%.

SU(3) symmetry breaking can also be studied in our model. It plays a major role in the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{us} from baryon decay.

The parameters of the model are the constituent quark mass m and the scale parameter β, which is a measure of the size of the baryon. Parameter set 2 of Ref. [2] is chosen for the present work. The results reported in this paper are independent of the wave function assumed in the calculation. It has been shown in Ref. [4] that relations between observables at zero momentum transfer are independent of the wave function, and Ref. [5] shows that this independence holds up to 1 GeV^2 for the baryons.

This article is organized as follows. Section II describes the basics of hyperon semileptonic decay. In Sec. III we give a brief summary of our model as described in Ref. [2] with the explicit expressions for the β decay. The numerical results are presented in Sec. IV, and are compared with experiment, other calculations, and some extensions of the model. We summarize our investigation in Sec. V.

II. HYPERON SEMILEPTONIC DECAY

In the low energy limit the standard model for semileptonic weak decays reduces to an effective current-current interaction Hamiltonian

$$H_{\rm int} = \frac{G}{\sqrt{2}} J_\mu L^\mu + \text{H.c.} \ , \qquad (2.1)$$

where $G \simeq 10^{-5}/M_p^2$ is the weak coupling constant,

$$L^\mu = \bar{\psi}_e \gamma^\mu (1-\gamma_5)\psi_\nu + \bar{\psi}_\mu \gamma^\mu (1-\gamma_5)\psi_\nu \qquad (2.2)$$

is the lepton current, and

$$\begin{aligned} J_\mu &= V_\mu - A_\mu \ , \\ V_\mu &= V_{ud}\bar{u}\gamma_\mu d + V_{us}\bar{u}\gamma_\mu s \ , \\ A_\mu &= V_{ud}\bar{u}\gamma_\mu\gamma_5 d + V_{us}\bar{u}\gamma_\mu\gamma_5 s \end{aligned} \qquad (2.3)$$

is the hadronic current; V_{ud}, V_{us} are the elements of the CKM mixing matrix. The τ-lepton current cannot contribute since m_τ is much too large.

The matrix elements of the hadronic current between spin-$\frac{1}{2}$ states are

$$\langle B',p'\,|V^\mu|\,B,p\rangle = V_{qq'}\bar{u}(p')\left[f_1(K^2)\gamma^\mu - \frac{f_2(K^2)}{M_i} i\sigma^{\mu\nu}K_\nu + \frac{f_3(K^2)}{M_i}K^\mu\right]u(p) \ , \qquad (2.4)$$

$$\langle B',p'\,|A^\mu|\,B,p\rangle = V_{qq'}\bar{u}(p')\left[g_1(K^2)\gamma^\mu - \frac{g_2(K^2)}{M_i} i\sigma^{\mu\nu}K_\nu + \frac{g_3(K^2)}{M_i}K^\mu\right]\gamma_5 u(p) \ , \qquad (2.5)$$

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where $K = p - p'$ and M_i is the mass of the initial baryon. The quantities f_1 and g_1 are the vector and axial-vector form factors, f_2 and g_2 are the weak magnetism and electric form factors, and f_3 and g_3 are the induced scalar and pseudoscalar form factors, respectively. Time invariance implies real form factors. We do not calculate f_3 and g_3 since we put $K^+ = 0$ and their dependence on the decay spectra is of the order

$$\left(\frac{m_l}{M_i}\right)^2 \ll 1 , \tag{2.6}$$

where m_l is the mass of the final charged lepton. The other form factors are

$$\begin{aligned} f_1 &= \langle B', \uparrow |V^+| B, \uparrow\rangle , \\ K_\perp f_2 &= M_i \langle B', \uparrow |V^+| B, \downarrow\rangle , \\ g_1 &= \langle B', \uparrow |A^+| B, \uparrow\rangle , \\ K_\perp g_2 &= -M_i \langle B', \uparrow |A^+| B, \downarrow\rangle . \end{aligned} \tag{2.7}$$

What is usually measured is the total decay rate Γ, the electron-neutrino correlation $\alpha_{e\nu}$, and the electron α_e, neutrino α_ν, and final baryon α_B asymmetries. The e-ν correlation is defined as

$$\alpha_{e\nu} = 2\frac{N(\Theta_{e\nu} < \frac{1}{2}\pi) - N(\Theta_{e\nu} > \frac{1}{2}\pi)}{N(\Theta_{e\nu} < \frac{1}{2}\pi) + N(\Theta_{e\nu} > \frac{1}{2}\pi)} , \tag{2.8}$$

where $N(\Theta_{e\nu} < \frac{1}{2}\pi)$ is the number of e-ν pairs that form an angle $\Theta_{e\nu}$ smaller than 90°. The correlations α_e, α_ν, and α_B are defined analogously by Θ_e, Θ_ν, and Θ_B now being the angles between the e, ν, B directions and the polarization of the initial baryon.

Ignoring the lepton-mass one can calculate expressions for the measured quantities. Expressions for Γ, $\alpha_{e\nu}$, α_e, α_ν, and α_B are given in Ref. [6]. For the decay rate Γ we have, for instance,

$$\Gamma = G^2\frac{\Delta M^5|V|^2}{60\pi^3}\Bigg[\left(1 - \frac{3}{2}\beta + \frac{6}{7}\beta^2\right)f_1^2 + \frac{4}{7}\beta^2 f_2^2 + \left(3 - \frac{9}{2}\beta + \frac{12}{7}\beta^2\right)g_1^2 + \frac{12}{7}\beta^2 g_2^2 + \frac{6}{7}\beta^2 f_1 f_2 + (-4\beta + 6\beta^2)g_1 g_2 + \frac{4}{7}\beta^2(f_1\lambda_f + 5g_1\lambda_g)\Bigg], \tag{2.9}$$

where β is defined as $\beta = (M_i - M_f)/M_i$, and $\Delta M = M_i - M_f$, M_i and M_f being the masses of the initial and final baryons, respectively. The K^2 dependence of f_2 and g_2 is ignored and f_1 and g_1 are expanded as

$$f_1(K^2) = f_1(0) + \frac{K^2}{M_i^2}\lambda_f , \quad g_1(K^2) = g_1(0) + \frac{K^2}{M_i^2}\lambda_g . \tag{2.10}$$

We get the corresponding expression for the dipole parametrization $f(K^2) = (1 - K^2/M^2)^{-2}$ by putting

$$\lambda_f = 2M_i^2 f_1/M_V^2 , \quad \lambda_g = 2M_i^2 g_1/M_A^2 . \tag{2.11}$$

These quantities are corrected by the nonvanishing lepton mass and radiative corrections [6–8].

III. FORM FACTORS IN A RELATIVISTIC CONSTITUENT QUARK MODEL

The constituent quark model described in Ref. [2] provides a framework for representing the general structure of the three-quark wave function for baryons. The model is formulated on the light front, which is specified by the invariant hypersurface $x^+ = x^0 + x^3 = 0$. The wave function is constructed as the product of a momentum wave function, which is spherically symmetric and invariant under permutations, and a spin-isospin wave function, which is uniquely determined by SU(6) symmetry requirements. A Wigner (Melosh) rotation [9] is applied to the spinors, so that the wave function of the proton is an eigenfunction of J^2 and J_z in its rest frame [10]. To represent the range of uncertainty in the possible form of the momentum wave function, a harmonic oscillator and a pole-type wave function have been chosen in Refs. [2,4,5]. Surprisingly, it has been found that observables at zero momentum transfer are independent of the wave function chosen [4], and form factors do not differ up to 1 GeV^2 [5] for a wide range of wave functions. Since the momentum transfer involved in hyperon β decays is much smaller than 1 GeV^2, it is representative to use one special wave function. The form factors in Eq. (2.7) are calculated as shown in Ref. [2]. Parameter set 2 of Ref. [2] does not assume additional structure of the constituent quarks, and uses symmetric wave functions. The parameters are the two masses $(m_{u/d}, m_s)$ and three scale parameters $(\beta_N, \beta_{\Sigma/\Lambda}, \beta_\Xi)$.

In order to fix the notation we repeat here the essential formalism in Ref. [2]. The four-vector is given by $x = (x^+, x^-, x_\perp)$, where $x^\pm = x^0 \pm x^3$ and $x_\perp = (x^1, x^2)$. Light-front vectors are denoted by boldface $\mathbf{x} = (x^+, x_\perp)$, and they are covariant under kinematic Lorentz transformations. The three-momenta $\mathbf{p}_i$ of the quarks can be transformed to the total and relative momenta to facilitate the separation of the center of mass motion:

$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3, \quad \xi = \frac{p_1^+}{p_1^+ + p_2^+}, \quad \eta = \frac{p_1^+ + p_2^+}{P^+},$$
$$q_\perp = (1-\xi)p_{1\perp} - \xi p_{2\perp}, \quad Q_\perp = (1-\eta)(p_{1\perp} + p_{2\perp}) - \eta p_{3\perp}. \tag{3.1}$$

In the light-front dynamics the Hamiltonian takes the form

$$H = \frac{P_\perp^2 + M^2}{P^+}, \tag{3.2}$$

where M is the mass operator with the interaction term W:

$$M = M_0 + W, \quad M_0^2 = \frac{Q_\perp^2}{\eta(1-\eta)} + \frac{M_3^2}{\eta} + \frac{m_3^2}{1-\eta}, \quad M_3^2 = \frac{q_\perp^2}{\xi(1-\xi)} + \frac{m_1^2}{\xi} + \frac{m_2^2}{1-\xi}, \tag{3.3}$$

with m_i being the masses of the constituent quarks. To get a clearer picture of M_0 we transform to q_3 and Q_3 by

$$\xi = \frac{E_1 + q_3}{E_1 + E_2}, \quad \eta = \frac{E_{12} + Q_3}{E_{12} + E_3},$$
$$E_{1/2} = (\mathbf{q}^2 + m_{1/2}^2)^{1/2}, \quad E_3 = (\mathbf{Q}^2 + m_3^2)^{1/2}, \quad E_{12} = (\mathbf{Q}^2 + M_3^2)^{1/2}, \tag{3.4}$$

where $\mathbf{q} = (q_1, q_2, q_3)$, and $\mathbf{Q} = (Q_1, Q_2, Q_3)$. The expression for the mass operator is now simply

$$M_0 = E_{12} + E_3, \quad M_3 = E_1 + E_2. \tag{3.5}$$

For $K^2 = 0$ we have for $\Delta S = 0$ transitions

$$f_1 = A(f_1), \quad f_2 = \frac{N_c}{(2\pi)^6} \int d^3q d^3Q |\Phi|^2 A(f_2),$$
$$g_1 = A(g_1) \frac{N_c}{(2\pi)^6} \int d^3q d^3Q |\Phi|^2 \frac{b^2 - Q_\perp^2}{b^2 + Q_\perp^2}, \quad g_2 \simeq 0, \tag{3.6}$$

with As given in Table I. The values $A(f_1)$ and $A(g_1)$ are the values in the nonrelativistic quark model. The factors A_1, A_2, and A_3 are given by

$$A_1 = \frac{\eta\left(a - \frac{Q_\perp^2}{2(1-\eta)M}\right)}{a^2 + Q_\perp^2} \frac{c^2}{c^2 + q_\perp^2}, \quad A_2 = \frac{\eta\left(a - \frac{Q_\perp^2}{2(1-\eta)M}\right)}{a^2 + Q_\perp^2} \frac{d^2}{d^2 + q_\perp^2}, \quad A_3 = \frac{\frac{Q_\perp^2}{2M} - \eta b}{b^2 + Q_\perp^2}, \tag{3.7}$$

where we used the notation

$$a = M_3 + \eta M_0, \quad b = m_3 + (1-\eta)M_0, \quad c = m_1 + \xi M_3, \quad d = m_2 + (1-\xi)M_3.$$

Note that for equal u and d quark masses there is an equality $A_1 = A_2$ under the integral.

The $\Delta S = 1$ transitions for $K^2 = 0$ are

TABLE I. Parameters in Eq. (3.6).

Reaction	$A(f_1)$	$A(f_2)$	$A(g_1)$
np	1	$(2A_2 - 5A_3)/3$	$\frac{5}{3}$
$\Sigma^+\Lambda$	0	$(A_2 + A_1 - 2A_3)/\sqrt{6}$	$\sqrt{\frac{2}{3}}$
$\Sigma^-\Lambda$	0	$(A_2 + A_1 - 2A_3)/\sqrt{6}$	$\sqrt{\frac{2}{3}}$
$\Sigma^-\Sigma^0$	$\sqrt{2}$	$-(4A_3 + A_2 + A_1)/(3\sqrt{2})$	$\frac{2\sqrt{2}}{3}$
$\Sigma^0\Sigma^+$	$-\sqrt{2}$	$(4A_3 + A_2 + A_1)/(3\sqrt{2})$	$-\frac{2\sqrt{2}}{3}$
$\Xi^-\Xi^0$	-1	$(2A_2 + 2A_1 - A_3)/3$	$\frac{1}{3}$

TABLE II. Parameters in Eq. (3.9).

Reaction	B_1	B_2	B_3	B_4	B_5	B_6
Λp	$-\sqrt{\frac{3}{2}}$	$-\sqrt{\frac{3}{2}}$	0	$-\sqrt{\frac{3}{2}}$	0	0
$\Sigma^0 p$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{3\sqrt{2}}$	$\frac{\sqrt{2}}{3}$	$\frac{1}{3\sqrt{2}}$	$\frac{4\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$
$\Sigma^- n$	-1	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{8}{3}$	$\frac{2}{3}$
$\Xi^-\Lambda$	$\sqrt{\frac{3}{2}}$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$-2\sqrt{\frac{2}{3}}$	$-\frac{1}{6}$
$\Xi^-\Sigma^0$	$\frac{1}{\sqrt{2}}$	$\frac{5}{3\sqrt{2}}$	$\frac{1}{3\sqrt{2}}$	$\frac{5}{3\sqrt{2}}$	$\frac{4}{3\sqrt{2}}$	$\frac{\sqrt{2}}{6}$
$\Xi^0\Sigma^+$	1	$\frac{5}{3}$	$\frac{1}{3}$	$\frac{5}{3}$	$\frac{4}{3}$	$\frac{1}{3}$

$$f_1 = \frac{N_c}{(2\pi)^6}\int d^3q d^3Q \left(\frac{E_3' E_{12}' M_0}{E_3 E_{12} M_0'}\right)^{1/2} \frac{\Phi^\dagger(M_0')\Phi(M_0)B(f_1)}{(a'^2+Q_\perp^2)(a^2+Q_\perp^2)\sqrt{b'^2+Q_\perp^2}\sqrt{b^2+Q_\perp^2}} ,$$

$$g_1 = \frac{N_c}{(2\pi)^6}\int d^3q d^3Q \left(\frac{E_3' E_{12}' M_0}{E_3 E_{12} M_0'}\right)^{1/2} \frac{\Phi^\dagger(M_0')\Phi(M_0)B(g_1)}{(a'^2+Q_\perp^2)(a^2+Q_\perp^2)\sqrt{b'^2+Q_\perp^2}\sqrt{b^2+Q_\perp^2}} , \tag{3.8}$$

$$B(f_1) = B_1(a'a+Q_\perp^2)^2(b'b+Q_\perp^2) + B_2(a'-a)^2Q_\perp^2(b'b+Q_\perp^2)\frac{(cd-q_\perp^2)^2}{(c^2+q_\perp^2)(d^2+q_\perp^2)}$$
$$+B_3(a'-a)(b'-b)Q_\perp^2(a'a+Q_\perp^2)\left(\frac{c^2}{c^2+q_\perp^2}+\frac{d^2}{d^2+q_\perp^2}\right) ,$$

$$B(g_1) = B_4(b'b-Q_\perp^2)\left[(a'a+Q_\perp^2)^2+(a'-a)^2Q_\perp^2\frac{(cd-q_\perp^2)^2}{(c^2+q_\perp^2)(d^2+q_\perp^2)}\right]$$
$$+B_5(a'-a)^2Q_\perp^2(b'b-Q_\perp^2)\frac{cdq_\perp^2}{(c^2+q_\perp^2)(d^2+q_\perp^2)}$$
$$+B_6(a'-a)Q_\perp^2(b'+b)(a'a+Q_\perp^2)\left(\frac{c^2}{c^2+q_\perp^2}+\frac{d^2}{d^2+q_\perp^2}\right) . \tag{3.9}$$

The B_i for the different decays are given in Table II.

Equations (3.8) and (3.9) confirm the Ademollo-Gatto theorem [11]. Since $(a'-a) \sim \Delta m$, and $(b'-b) \sim \Delta m$, the symmetry breaking for f_1 is of the order $(\Delta m)^2$ whereas it is of the order Δm for g_1 owing to the term containing B_6. In addition to Ademollo-Gatto we see that the symmetry breaking for $g_1(\Lambda \to p)$ is also of second order.

The full formulas for $K^2 \leq 0$ are longer than the ones for $K^2 = 0$; they are given in Ref. [12].

IV. NUMERICAL RESULTS

The form factors can be determined by the generalization of Eqs. (3.6) and (3.8). With the parametrization of the form factor $f(K^2)$,

$$f(K^2) \simeq \frac{f(0)}{1-K^2/\Lambda_1^2+K^4/\Lambda_2^4} , \tag{4.1}$$

we get the result shown in Tables III and IV together with the rates, angular correlation, and asymmetries. The parameters Λ_n are determined by the calculation of the appropriate derivatives of $f(K^2)$ at $K^2 = 0$. The rates have been corrected taking into account the nonvanishing lepton mass and radiative corrections.

In this paper, we use parameter set 2 of Ref. [2]. The values for the constituent quark masses and the confinement scales are

$$m_u = m_d = 0.267 \text{ GeV} ,$$
$$m_s = 0.40 \text{ GeV} ,$$
$$\beta_N = 0.56 \text{ GeV} ,$$
$$\beta_\Sigma = \beta_\Lambda = 0.60 \text{ GeV} ,$$
$$\beta_\Xi = 0.62 \text{ GeV} .$$

These parameters also give good results for the magnetic moments of the baryon octet [2].

A. Rates, $f_1(0)$ and $g_1(0)$

The largest discrepancy between theory and experiments comes from the rates and g_1/f_1 for the processes $\Lambda \to pe^-\bar{\nu}_e$ and $\Sigma^- \to ne^-\bar{\nu}_e$. By changing the axial couplings of the quarks, i.e., $g_{1us} \simeq 0.9$, we could improve the rates of both reactions, but the ratios g_1/f_1 clearly force us to use $g_{1us} = 1$. Another modification could be the Λ-Σ^0 mixing, which was considered in Ref. [13]. Let us write

$$\Lambda_{\rm phys} = \Lambda \cos\phi + \Sigma^0 \sin\phi \,, \qquad \Sigma^0_{\rm phys} = -\Lambda \sin\phi + \Sigma^0 \cos\phi \,. \tag{4.2}$$

A reasonable value for the mixing angle is $\phi = -0.015$ [13] which lies within one standard deviation of experiment [14]. The decay rate and the ratio g_1/f_1 are only modified by some percent with this mixing angle, not helping the disagreement between theory and experiment.

This inconsistency of our values is a general feature of quark models with a SU(6) flavor-spin symmetry [15]. The ratio g_1/f_1 can generally be written as

$$\frac{g_1}{f_1} = \rho\eta \left(\frac{g_1}{f_1}\right)_{\rm nonrel} , \tag{4.3}$$

where $(g_1/f_1)_{\rm nonrel}$ is the nonrelativistic value. The quantity ρ is a relativistic suppression factor due to the "small" components in the quark spinors (in the bag model) or due to the Melosh transformation (in our model). The quantity η is an enhancing factor due to SU(3) symmetry breaking in $\Delta S = 1$ transitions. From Tables III and IV we see that $\rho \simeq 0.73$–0.76 [4] depending on the strangeness content of the wave functions and $\eta \simeq 1.11$. This simple estimate shows that every quark model is *a priori* constrained to

TABLE III. Results for $\Delta S = 0$ weak β decay. Experimental data are from PDG [1].

		np	$\Sigma^+\Lambda$	$\Sigma^-\Lambda$	$\Sigma^-\Sigma^0$	$\Sigma^0\Sigma^+$	$\Xi^-\Xi^0$
f_1	$f_1(0)$	1.00	0	0	1.41	−1.41	−1.00
	Λ_1 (GeV)	0.69	-0.32^{a}	-0.32^{a}	0.60	0.60	0.56
	Λ_2 (GeV)	0.96	-1.72^{a}	-1.72^{a}	0.81	0.81	0.71
g_1	$g_1(0)$	1.25	0.60	0.60	0.69	−0.69	0.24
	Λ_1 (GeV)	0.76	0.77	0.77	0.77	0.77	0.76
	Λ_2 (GeV)	1.04	1.05	1.05	1.04	1.04	1.04
g_1/f_1	Theor.	1.252	0.736^{b}	0.736^{b}	0.491	0.491	−0.244
	Expt.	1.2573	0.742^{b}	–	–	–	$< 2\times 10^{3}$
		±0.0028	±0.018				
$\frac{f_2}{M}$ (GeV^{-1})	Theor.	1.81	1.04	1.04	0.76	−0.76	0.73
	CVC	1.85	1.17	1.17	0.60	−0.60	1.00
$\frac{g_2}{M}$ (GeV^{-1})		0	0	0	0	0	0
Rate (10^6 s^{-1})	Theor.	1.152×10^{-9}	0.24	0.389	1.47^{c}	3.65^{d}	1.55^{c}
e mode	Expt.	1.127×10^{-9}	0.25	0.387	–	–	–
		±0.003	±0.06	±0.018			
$\alpha_{e\nu}$	Theor.	−0.101	−0.404	−0.412	0.436	0.438	0.793
	Expt.	−0.102	−0.35	−0.404			
		±0.005	±0.15	±0.044			
α_e	Theor.	−0.112	−0.701	−0.704	0.287	0.288	−0.514
	Expt.	−0.1127					
		±0.0011					
α_ν	Theor.	0.989	0.647	0.645	0.850	0.850	−0.314
	Expt.	0.997					
		±0.028					
α_B	Theor.	−0.548	0.070	0.077	−0.710	−0.711	0.518
	Expt.						

[a]Instead of Λ_i we list $f_1^{(i)}$.
[b]Instead of g_1/f_1 we list $\sqrt{3/2}g_1$.
[c]$\times 10^{-6}$.
[d]$\times 10^{-8}$.

$$\frac{g_1/f_1(\Lambda \to pe^-\bar{\nu}_e)}{g_1/f_1(\Sigma^- \to ne^-\bar{\nu}_e)} = -3, \quad (4.4)$$

in contrast with the experimental value -2.11 ± 0.15 for $g_2 = 0$. This puzzle was pointed out independently by Lipkin [16] and the author [12]. For $g_2 \neq 0$ it is measured that [17]

$$\left|\frac{g_1}{f_1}\right|_{\Lambda p} = 0.715 + 0.28\frac{g_2}{f_1} \quad (4.5)$$

and [18]

$$\left|\frac{g_1}{f_1} - 0.237\frac{g_2}{f_1}\right|_{\Sigma^- n} = 0.34 \pm 0.017\ , \quad (4.6)$$

which will bring the data closer to -3, but in our model $g_2/g_1 \simeq 0.062$, which is much too small to remove the discrepancy.

B. Configuration mixing

In this section we investigate the effect caused by configuration mixing suggested by spectroscopy. The analysis of the Δ-nucleon mass splitting suggests [19,20]

$$|\text{baryon}\rangle = A\,[56, 0^+] + B\,[56, 0^+]^* + C\,[70, 0^+], \quad (4.7)$$

TABLE IV. Results for $\Delta S = 1$ weak β decay. Experimental data are from PDG [1].

		Λp	$\Sigma^0 p$	$\Sigma^- n$	$\Xi^-\Lambda$	$\Xi^-\Sigma^0$	$\Xi^0\Sigma^+$
f_1	$f_1(0)$	−1.19	−0.69	−0.97	1.19	0.69	0.98
	Λ_1 (GeV)	0.71	0.64	0.64	0.68	0.75	0.75
	Λ_2(GeV)	0.98	0.84	0.90	0.89	1.05	1.05
g_1	$g_1(0)$	−0.99	0.19	0.27	0.33	0.94	1.33
	Λ_1 (GeV)	0.81	0.83	0.83	0.81	0.81	0.81
	Λ_2 (GeV)	1.12	1.16	1.16	1.10	1.12	1.12
g_1/f_1	Theor.	0.826	−0.275	−0.275	0.272	1.362	1.362
	Expt.	0.718	–	−0.340	0.25	1.287	< 2.93
		±0.015		±0.017	±0.05	±0.158	
$\frac{f_2}{M}$ (GeV^{-1})	Theor.	−0.85	0.44	0.62	0.070	0.98	1.38
	CVC	−1.19	–	1.12	−0.080	1.38	1.95
$\frac{g_2}{M}$ (GeV^{-1})		−0.062	0.011	0.015	–[a]	–[a]	–[a]
Rate (10^6 s^{-1})	Theor.	3.51	2.72	5.74	2.96	0.549	0.942
e mode	Expt.	3.170	–	6.88	3.36	0.53	–
		±0.058		±0.26	±0.19	±0.10	
Rate (10^6 s^{-1})	Theor.	0.58	1.18	2.54	0.80	7.47×10^{-3}	7.74×10^{-3}
μ mode	Expt.	0.60	–	3.04	2.1	–	–
		±0.13		±0.27	±2.1		
$\alpha_{e\nu}$	Theor.	−0.100	0.443	0.437	0.531	−0.252	−0.248
	Expt.	−0.019		0.279	0.53		
		±0.013		±0.026	±0.1		
α_e	Theor.	−0.021	−0.536	−0.537	0.236	−0.226	−0.223
	Expt.	0.125		−0.519[b]			
		±0.066		±0.104			
α_ν	Theor.	0.992	−0.318	−0.318	0.592	0.973	0.973
	Expt.	0.821		−0.230[b]			
		±0.066		±0.061			
α_B	Theor.	−0.582	0.568	0.569	−0.519	−0.437	−0.439
	Expt.	−0.508		0.509[b]			
		±0.065		±0.102			

[a] $\frac{g_2}{g_1 M} \simeq 0.057$ since $\frac{g_2}{g_1} \simeq$ const.
[b] From Ref. [18].

TABLE V. Parameters for the configuration mixing of the baryon octet given in Eq. (4.7) for two different references.

	A	B	C
Ref. [19]	0.93	−0.29	−0.23
Ref. [20]	0.90	−0.34	−0.27

in the notation [SU(6),L^p], where $A^2 + B^2 + C^2 = 1$, L denotes the angular momentum, and p is the parity of the nucleon. The values for A, B, C are listed in Table V for different references.

Unfortunately, the mixing configuration does not improve the fit; it is even worse for the crucial ratio in Eq. (4.4). A rough estimate gives

$$\frac{g_1/f_1(\Lambda \to pe^-\bar{\nu}_e)}{g_1/f_1(\Sigma^- \to ne^-\bar{\nu}_e)} \simeq -3\left(1+\frac{8}{3}C^2\right) = -3.5 \pm 0.1 , \tag{4.8}$$

to be compared with the value −3 for no mixing, and the experimental data −2.11 ± 0.15. Other values such as the ratio $\mu(p)/\mu(n)$ also get worse with the configuration mixing suggested in Eq. (4.7). A configuration mixing has recently been suggested in the context of deep inelastic scattering [3]. Equation (4.8) shows that such a possibility is not favored for hyperon decays.

C. Form factors $f_2(0)$ and $g_2(0)$

Our model agrees with the conserved vector current (CVC) hypothesis. The deviations have the same origin as the too small neutron magnetic moment [2] since f_2 and the magnetic moments have similar analytic forms. The experimental situation is not yet clear; some experiments favor [18] and some disfavor [21] the CVC hypothesis.

For $\Delta S = 1$ transitions the prediction of g_2/g_1 for nonrelativistic quark models is ~ 0.37 and for the bag model ~ 0.15 [22]. Our model gives also a constant value

$$\left(\frac{g_2}{g_1}\right)_{\Delta S=1} \simeq 0.062 . \tag{4.9}$$

While the sign of the ratio g_2/g_1 is quite clear, the magnitude is more model dependent as already mentioned in Ref. [22]. For $\Delta S = 0$ transitions we get

$$\left(\frac{g_2}{g_1}\right)_{\Delta S=0} \simeq 0.0033 , \tag{4.10}$$

if we put $m_d - m_u = 7$ MeV. This confirms the viewpoint of the PDG [1] which fixes $g_2 = 0$. Experiments also find a vanishing or small g_2 [6].

With the CVC hypothesis and the absence of g_2 we reach the same conclusion that was reached in nuclear physics.

D. K^2 dependence of the form factors

Tables III and IV suggest that the form factor of Eq. (4.1) can be approximated by the dipole form

$$f(K^2) \simeq \frac{f(0)}{(1 - K^2/\Lambda_2^2)^2} . \tag{4.11}$$

The axial-vector form factor g_1 for the neutron decay gives a value $M_A = \Lambda_2 = 1.04$ GeV compared to the experimental value $M_A = (1.00 \pm 0.04)$ GeV [23,24].

Table VI compares our values for M_V and M_A with the results of other work.

The contribution of M_V and M_A to the rate and to $x = g_1/f_1$ to first order is

TABLE VI. The parameters M_V and M_A for various models in units of GeV.

	This work		Gaillard and Sauvage [8]		Garcia and Kielanowski [6]		Gensini [27]	
	M_V	M_A	M_V	M_A	M_V	M_A	M_V	M_A
np	0.96	1.04	0.84	1.08	0.84	0.96	0.84	1.08
$\Sigma\Lambda$	-	1.05	-	1.08	-	0.96	-	1.08
$\Sigma\Sigma$	0.81	1.04	0.84	1.08	0.84	0.96	0.84	1.08
$\Xi\Xi$	0.71	1.04	0.84	1.08	0.84	0.96	0.84	1.08
Λp	0.98	1.12	0.98	1.25	0.97	1.11	0.94	1.16
Σp	0.84	1.16	0.98	1.25	0.97	1.11	0.94	1.16
Σn	0.90	1.16	0.98	1.25	0.97	1.11	0.94	1.16
$\Xi\Lambda$	0.89	1.10	0.98	1.25	0.97	1.11	0.94	1.16
$\Xi\Sigma$	1.05	1.12	0.98	1.25	0.97	1.11	0.94	1.16

TABLE VII. Symmetry breaking for f_1. The ratio $f_1/f_1^{\mathrm{SU}(3)}$ is shown.

	This work	Donoghue *et al.* [22]	Krause [32]	Anderson and Luty [33]
$\Delta S=0$	1.000	1.000	1.000	1.000
Λp	0.976	0.987	0.943	1.024
Σp	0.975	0.987	-	-
Σn	0.975	0.987	0.987	1.100
$\Xi\Lambda$	0.976	0.987	0.957	1.059
$\Xi\Sigma$	0.976	0.987	0.943	1.011

$$\frac{\Delta\Gamma}{\Gamma}=\frac{8}{7}\frac{\beta^2M^2}{(1+3x^2)}\left(\frac{1}{M_V^2}+\frac{5x^2}{M_A^2}\right),$$
$$\frac{\Delta x^2}{x^2}=-\frac{8}{7}\beta^2M^2\left[\frac{(1-\alpha_{e\nu})\alpha_{e\nu}}{M_V^2}+\frac{6+5\alpha_{e\nu}}{M_A^2}\right], \tag{4.12}$$

which shows that our parameters give for the decay $\Sigma^-\to ne^-\bar{\nu}_e$ a 0.3% larger rate and a 4% smaller g_1/f_1 than with the parameters of Gaillard and Sauvage [8] that are often used for the experimental analysis. Although this does not explain the inconsistency of the data with our calculation, it shows that future high-statistics experiments should pay more attention to M_V and M_A in analyzing g_1/f_1.

E. SU(3) symmetry breaking

There are some questions concerning flavor SU(3) breaking in semileptonic weak hyperon decays [25–27]. In a recent, careful analysis Ref. [28] shows that there is both consistency and evidence for SU(3) breaking. The SU(3) symmetry breaking for f_1 and g_1 within our model is given in Tables VII and VIII, respectively. It originates from the mass difference $\Delta m = m_s - m_{u/d}$, and it is included to all orders of Δm in our approach. The values in the present model are similar to the bag model calculation of Ref. [22]. Note that the center of mass corrections are already included in our formalism. Reference [29] suggests that $f_1/f_1^{\mathrm{SU}(3)}>1$ to reconcile the value for V_{us} for both the K_{l3} and hyperon decays. In our approach we find $f_1/f_1^{\mathrm{SU}(3)}<1$ since the wave function overlap is smaller for $\Delta m\neq 0$.

TABLE VIII. Symmetry breaking for g_1. The ratio $g_1/g_1^{\mathrm{SU}(3)}$ is shown.

	This work	Donoghue *et al.* [22]
np	1.000	1.000
$\Sigma\Lambda$	0.981	0.9383/0.9390
$\Sigma\Sigma$	0.982	-
$\Xi\Xi$	0.977	-
Λp	1.072	1.050
Σp	1.051	-
Σn	1.056	1.040
$\Xi\Lambda$	1.072	1.003
$\Xi\Sigma$	1.061	0.9954

In order to determine the CKM matrix element V_{us} we can fit the hyperon decay rate and asymmetries within the Cabibbo model using the f_1 and g_1 from Tables VII and VIII, and using the dipole masses from Table VI. We get a value similar to Ref. [29]:

$$V_{us}=0.225\pm 0.003\ [12]. \tag{4.13}$$

This has to be compared to the value from K_{e3} which is 0.2196 ± 0.0023 [30]. A discussion about this discrepancy can be found in Ref. [29]. Note that the matrix element V_{us} is a crucial input for the determination of all parameters of the CKM matrix in the framework proposed in Ref. [31].

V. CONCLUSIONS

In this paper we have analyzed in detail the semileptonic β decay of the nucleons and hyperons within a relativistic constituent quark model. All parameters of the model have previously been determined by a fit to the magnetic moments of the baryon octet. We see no evidence for configuration mixing. The momentum dependence of the form factors has been calculated and we find some deviation from popular parametrizations. The SU(3) symmetry breaking for the vector and axial form factors is determined. We find that the symmetry breaking for $g_1(\Lambda\to p)$ is of second order. Our value for V_{us} is somehow larger than the K_{e3} one in agreement with other studies [28,29]. We conclude that our relativistic constituent quark model does a good job in analyzing the electroweak properties of the baryon octet.

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