# Vector and pseudoscalar charm meson radiative decays

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Combining heavy quark effective theory and the chiral Lagrangian approach we investigate radiative decays of pseudoscalar D mesons. We first reanalyze  $D^* \to D\gamma$  decays within the effective Lagrangian approach using the heavy quark spin symmetry, the chiral symmetry Lagrangian, but including also the light vector mesons. We then investigate  $D \to V\gamma$  decays and calculate the  $D^0 \to \bar{K}^{*0}\gamma$  and  $D^{s+} \to \rho^+\gamma$  partial widths and branching ratios.

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### I. INTRODUCTION

The list of D meson decay rates is rather long and further study of their decays would eventually help to better understand their features. There has not yet been any experimental evidence for radiative decays of D mesons, while  $D^*$  radiative decays are known to be important. The  $D^*$  decays [1, 2] can be described in a model-independent framework which incorporates the appropriate constraints on the decay amplitudes. The combination of heavy quark effective theory and chiral Lagrangians have been extensively studied and applied to many D meson decays [3–13]. Wise [14] has proposed an effective Lagrangian to describe, at low momentum, the interactions of a meson containing a heavy quark with the light pseudoscalar mesons  $\pi$ , K,  $\eta$ . Two kinds of symmetries characterize the effective Lagrangian: the heavy quark SU(2) spin symmetry and the nonlinearly realized  $SU(3) \otimes SU(3)$  chiral symmetry in the light sector, corresponding to spontaneous symmetry breaking of the chiral group to the diagonal  $SU(3)_V$ . Because of the rather large masses of the D mesons, the inclusion of resonances with masses below the D mesons seems necessary [11-13].

In this paper, following the requirements of heavy quark and chiral symmetry, we develop a framework for the description of heavy and light pseudoscalar and vector mesons. In Sec. II we write down the most general Lagrangian in the limit of exact heavy quark and chiral symmetries. Section III is devoted to the higher order odd-parity Lagrangian, which also describes the decay  $D^* \rightarrow D\gamma$ , and we reinvestigate this decay in order to learn more about the couplings in the chiral Lagrangian. In Sec. IV we analyze the weak Lagrangian. Finally, as an example of the use of our model, we calculate the  $D \rightarrow V\gamma$  radiative decays in Sec. V.

# II. THE CHIRAL LAGRANGIAN TECHNIQUE AND HEAVY QUARK LIMIT

The strong interaction meson Lagrangian for the light pseudoscalar octet and heavy pseudoscalar and vector triplets in the chiral and heavy quark limits was first written down by Wise [14] (see also [4]). The electromagnetic interactions between these mesons was described in [1, 2, 6]. The octet of light vector mesons was included in the Wise Lagrangian [14] later by Casalbuoni, *et al.* [12] as the gauge particles associated with the hidden symmetry group  $SU(3)_H$  [15]. The next step is to provide a common description of both the light and heavy pseudoscalar mesons, which also includes both light and heavy vector mesons and the electromagnetic interactions. In this section we present the strong and electromagnetic Lagrangian for the description of both light and heavy pseudoscalar and vector mesons.

The light pseudoscalar mesons are described by the  $3 \times 3$  unitary matrix

$$u = \exp\left(\frac{i\Pi}{f}\right) \,, \tag{1}$$

where  $f \simeq 132$  MeV is the pion constant and  $\Pi$  is the pseudoscalar meson unitary matrix defined as

n

$$\Pi = \begin{pmatrix} \frac{\pi^{-}}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^{+} & K^{+} \\ \pi^{-} & \frac{-\pi^{0}}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^{0} \\ K^{-} & \bar{K}^{0} & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} .$$
(2)

The octet of light vector mesons is described by the  $3 \times 3$  unitary matrix

$$\hat{\rho}_{\mu} = i \frac{g_V}{\sqrt{2}} \rho_{\mu} , \qquad (3)$$

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where  $g_V (\simeq 5.8\sqrt{2/a} \text{ with } a = 2 \text{ in the case of exact}$ vector dominance) is the coupling constant of the vector meson self-interaction [15] and  $\rho_{\mu}$  the vector meson unitary matrix

$$\rho_{\mu} = \begin{pmatrix} \frac{\rho_{\mu}^{0} + \omega_{\mu}}{\sqrt{2}} & \rho_{\mu}^{+} & K_{\mu}^{*+} \\ \rho_{\mu}^{-} & \frac{-\rho_{\mu}^{0} + \omega_{\mu}}{\sqrt{2}} & K_{\mu}^{*0} \\ K_{\mu}^{*-} & \bar{K}_{\mu}^{*0} & \Phi_{\mu} \end{pmatrix} .$$
(4)

In the following we will also use the gauge field tensor  $F_{\mu\nu}(\hat{\rho})$ , defined as

$$F_{\mu\nu}(\hat{\rho}) = \partial_{\mu}\hat{\rho}_{\nu} - \partial_{\nu}\hat{\rho}_{\mu} + [\hat{\rho}_{\mu}, \hat{\rho}_{\nu}] .$$
(5)

The heavy mesons are  $Q\bar{q}^a$  ground states, where Q is a c (or b) quark and  $q^1 = u$ ,  $q^2 = d$ , and  $q^3 = s$ . In the heavy quark limit they are described by  $4 \times 4$  matrix  $H_a$ (a = 1, 2, 3) [14]:

$$H_{a} = \frac{1}{2} (1 + \not\!\!\!/) (P_{a\mu}^{*} \gamma^{\mu} - P_{a} \gamma_{5}) , \qquad (6)$$

where  $P_{a\mu}^*$  and  $P_a$  annihilate, respectively, a spin-one and spin-zero meson  $Q\bar{q}^a$  of velocity  $v_{\mu}$ . The creation operators  $P_{a\mu}^{*\dagger}$  and  $P_a^{\dagger}$  occur in [14]

$$\bar{H}_{a} = \gamma^{0} H_{a}^{\dagger} \gamma^{0} = (P_{a\mu}^{*\dagger} \gamma^{\mu} + P_{a}^{\dagger} \gamma_{5}) \frac{1}{2} (1 + \not v) .$$
<sup>(7)</sup>

Following the analogy in Refs. [15, 12] we introduce two currents:

$$\mathcal{V}_{\mu} = \frac{1}{2} (u^{\dagger} D_{\mu} u + u D_{\mu} u^{\dagger}) \tag{8}$$

 $\mathbf{and}$ 

$$\mathcal{A}_{\mu} = \frac{1}{2} (u^{\dagger} D_{\mu} u - u D_{\mu} u^{\dagger}) , \qquad (9)$$

where the covariant derivatives of u and  $u^{\dagger}$  are defined as

$$D_{\mu}u = (\partial_{\mu} + \hat{B}_{\mu})u \tag{10}$$

 $\operatorname{and}$ 

$$D_{\mu}u^{\dagger} = (\partial_{\mu} + \hat{B}_{\mu})u^{\dagger} , \qquad (11)$$

while

$$\hat{B}_{\mu} = ieB_{\mu}Q , \qquad (12)$$

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0\\ 0 & -\frac{1}{3} & 0\\ 0 & 0 & -\frac{1}{3} \end{pmatrix} , \qquad (13)$$

and  $B_{\mu}$  is the photon field. To insure that the vertices  $D^{0\dagger}D^{0}\gamma$  and  $D^{*0\dagger}D^{*0}\gamma$ , or those with D replaced by B in the case of the b quark, are absent, we define the covariant derivative for the heavy meson field as

$$D_{\mu}\bar{H}_{a} = (\partial_{\mu} + \mathcal{V}_{\mu} - ieQ'B_{\mu})\bar{H}_{a} \tag{14}$$

with Q' = 2/3 for c quark (-1/3 for b quark). With these definitions we can finally write down the evenparity strong and electromagnetic Lagrangian for heavy and light pseudoscalar and vector mesons:

$$\mathcal{L}_{\text{even}} = \mathcal{L}_{\text{light}} - \frac{1}{4} (\partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu})^{2} + i \text{Tr}(H_{a} v_{\mu} D^{\mu} \bar{H}_{a}) + ig \text{Tr}[H_{b} \gamma_{\mu} \gamma_{5} (\mathcal{A}^{\mu})_{ba} \bar{H}_{a}] + i\beta \text{Tr}[H_{b} v_{\mu} (\mathcal{V}^{\mu} - \hat{\rho}_{\mu})_{ba} \bar{H}_{a}] + \frac{\beta^{2}}{2f^{2}a} \text{Tr}(\bar{H}_{b} H_{a} \bar{H}_{a} H_{b})$$
(15)

 $\operatorname{with}$ 

$$\mathcal{L}_{\text{light}} = -\frac{f^2}{2} \{ \text{tr}(\mathcal{A}_{\mu}\mathcal{A}^{\mu}) + a \text{tr}[(\mathcal{V}_{\mu} - \hat{\rho}_{\mu})^2] \} + \frac{1}{2g_V^2} \text{tr}[F_{\mu\nu}(\hat{\rho})F^{\mu\nu}(\hat{\rho})] .$$
(16)

This Lagrangian is invariant under the gauge transformation

$$\begin{aligned} H &\to e^{ieQ'\lambda(x)} H g_0^{\dagger}(x) ,\\ \bar{H} &\to g_0(x) \bar{H} e^{-ieQ'\lambda(x)} ,\\ u &\to g_0(x) u g_0^{\dagger}(x) ,\\ u^{\dagger} &\to g_0(x) u^{\dagger} g_0^{\dagger}(x) ,\\ \mathcal{V}_{\mu} &\to g_0(x) \mathcal{V}_{\mu} g_0^{\dagger}(x) + g_0(x) \partial_{\mu} g_0^{\dagger}(x) ,\\ \mathcal{A}_{\mu} &\to g_0(x) \mathcal{A}_{\mu} g_0^{\dagger}(x) ,\\ \hat{\rho}_{\mu} &\to g_0(x) \hat{\rho}_{\mu} g_0^{\dagger}(x) + g_0(x) \partial_{\mu} g_0^{\dagger}(x) ,\\ \hat{B}_{\mu} &\to g_0(x) \hat{B}_{\mu} g_0^{\dagger}(x) + g_0(x) \partial_{\mu} g_0^{\dagger}(x) , \end{aligned}$$
(17)

where  $g_0(x) = \exp[ieQ\lambda(x)]$ . The last transformation (17) together with (12) and (13) imply, of course, the usual gauge transformation for the photon field:

$$B_{\mu} \to B_{\mu} - \partial_{\mu}\lambda(x)$$
 (18)

In Eq. (15) g and  $\beta$  are constants which should be determined from experimental data [1, 2, 11-13]. The constant a in (15) and (16) is in principle a free parameter, but we shall fix it by assuming exact vector dominance [15], for which a = 2. With exact vector dominance there are no direct vertices between the photon and two pseudoscalar mesons, so that the pseudoscalars interact with the photon only through vector mesons.

The electromagnetic field can couple to the mesons also through the anomalous interaction; i.e., through the oddparity Lagrangian. Even with the  $PP\gamma$  direct vertices absent in  $\mathcal{L}_{\text{light}}$  due to the choice a = 2, direct  $PV\gamma$ vertices are present in the odd-parity Lagrangian. We write down the two contributions which are significant for our calculation:

$$\mathcal{L}_{\text{odd}}^{(1)} = -4 \frac{C_{VV\Pi}}{f} \epsilon^{\mu\nu\alpha\beta} \text{Tr}(\partial_{\mu}\rho_{\nu}\partial_{\alpha}\rho_{\beta}\Pi) , \qquad (19)$$

$$\mathcal{L}_{\text{odd}}^{(2)} = -4e\sqrt{2}\frac{C_{V\pi\gamma}}{f}\epsilon^{\mu\nu\alpha\beta}\text{Tr}(\{\partial_{\mu}\rho_{\nu},\Pi\}Q\partial_{\alpha}B_{\beta}) . \tag{20}$$

Equation (19), together with vector dominance couplings

$$\mathcal{L}_{V-\gamma} = -m_V^2 \frac{e}{g_V} B_\mu \left( \rho^{0\mu} + \frac{1}{3} \omega^\mu - \frac{\sqrt{2}}{3} \Phi^\mu \right) \,, \quad (21)$$

which come from the second term in (16), describe the electromagnetic interaction assuming vector-meson dominance, while the direct photon-light vector mesonpseudoscalar interactions are contained in (20). The contributions to the odd Lagrangian (19) and (20) arise from Lagrangians of the Wess-Zumino-Witten kind [16, 17].

In the  $m_q \rightarrow 0$  and  $m_Q \rightarrow \infty$  limit ( $m_q$  and  $m_Q$  are the masses of the light and heavy quarks, respectively), the strong and electomagnetic interactions of heavy and light pseudoscalar and vector mesons are thus described by the even Lagrangian (15) and (16) and by the odd Lagrangian (19) and (20). However, the  $D^*D\gamma$  vertices are not included in the above Lagrangian since it is of the anomalous type. The terms responsible for it are of higher order  $1/m_Q$  and they will be introduced in the next section.

# III. HIGHER ORDER ODD LAGRANGIAN FOR HEAVY MESONS

In our approach vector-meson dominance describes the couplings of light quarks and photons through the higher dimensional invariant operator

$$\mathcal{L}_1 = i\lambda \mathrm{Tr}[H_a \sigma_{\mu\nu} F^{\mu\nu}(\hat{\rho})_{ab} \bar{H}_b] .$$
(22)

In this term the interactions of light-vector mesons with heavy pseudoscalars or heavy vector D mesons are present. The light-vector meson can then couple to the photon by the standard vector-meson dominance prescription (21). These terms effectively describe the light quark-photon interaction inside the charmed (or *b*-quark) mesons.

The coupling  $\lambda$  can be independently determined either from  $D^*$  decays into  $D\pi$  and  $D\gamma$  [1,2,5,6], some ratios of which have been measured [18, 19] or from semileptonic decays of D mesons [11–13].

The heavy quark-photon interaction is generated by the term

$$\mathcal{L}_2 = -\lambda' e \operatorname{Tr}[H_a \sigma_{\mu\nu} F^{\mu\nu}(B) \bar{H}_a] .$$
<sup>(23)</sup>

According to quark models the parameter  $|\lambda'|$  can be approximately related to the charm quark magnetic moment via  $1/(6m_c)$  [1, 2, 5, 6]. In order to reduce the error in determining the couplings we shall reanalyze the decays  $D^* \to D\pi$  and  $D^* \to D\gamma$ . Experimentally one measures [19] the branching fractions  $R_{\gamma}^0 = \Gamma(D^{*0} \to D^0\gamma)/\Gamma(D^{*0} \to D^0\pi^0) = 0.572 \pm 0.057 \pm 0.081$  and  $R_{\gamma}^+ = \Gamma(D^{*+} \to D^+\gamma)/\Gamma(D^{*+} \to D^+\pi^0) = 0.035 \pm 0.047 \pm 0.052$ . Using our Lagrangian these branching ratios are

$$R_{\gamma}^{0} = 64\pi f^{2} \alpha_{\rm EM} \left(\frac{\lambda'}{g} + \frac{2}{3} \frac{\lambda}{g}\right)^{2} \left(\frac{p_{\gamma}^{0}}{p_{\pi}^{0}}\right)^{3}, \qquad (24)$$

$$R_{\gamma}^{+} = 64\pi f^{2} \alpha_{\rm EM} \left(\frac{\lambda'}{g} - \frac{1}{3}\frac{\lambda}{g}\right)^{2} \left(\frac{p_{\gamma}^{+}}{p_{\pi}^{+}}\right)^{3} \,. \tag{25}$$

To determine  $\lambda/g$  and  $\lambda'/g$ , the square roots of the lefthand sides of Eqs. (24) and (25) have to be taken, which introduces an ambiguity in the resulting coupling constants. The experimental errors in the branching fractions are somewhat large but the masses of the particles involved are relatively well known. We have used the standard formulas [18]

$$\widehat{f} = f(\widehat{x}) + \frac{1}{2}\sigma^2 f''(\widehat{x})$$
(26)

$$\sigma_f = \sigma |f'(\widehat{x})| \tag{27}$$

for  $f(x) = \sqrt{x}$   $(x = R_{\gamma}^{0} \text{ or } R_{\gamma}^{+})$ , where  $\hat{x}$ ,  $\hat{f}$  and  $\sigma$ ,  $\sigma_{f}$  are the mean values and standard deviations of the respective distributions. Using expressions (26) and (27) instead of their linearized versions is important due to the large experimental error in  $R_{\gamma}^{+}$ . From that data we then find

$$\left. \frac{\lambda'}{g} + \frac{2}{3} \frac{\lambda}{g} \right| = (0.863 \pm 0.075) \text{ GeV}^{-1}$$
(28)

 $\operatorname{and}$ 

$$\left|\frac{\lambda'}{g} - \frac{1}{3}\frac{\lambda}{g}\right| = (0.089 \pm 0.178) \text{ GeV}^{-1} .$$
<sup>(29)</sup>

The two errors in Eqs. (28) and (29) can in principle be correlated. The main sources of correlations are the experimental efficiencies in the detection of the  $\pi^0$  and  $\gamma$ . Since the contributions of the efficiencies to the net errors are small [19], possible correlations were neglected.

In Fig. 1 the full lines [given from Eq. (28)] and the dashed lines [given from Eq. (29)] enclose the two allowed regions (shaded) for  $\lambda/g$  and  $\lambda'/g$ . The result is compatible with  $|\lambda| = (0.60 \pm 0.11) \text{ GeV}^{-1}$  and  $|g| = 0.57 \pm 0.13$  [13] as determined from different experiments.

Our approach is different from [1], [2], and [6], since we



FIG. 1. The experimentally allowed values for  $\lambda/g$  and  $\lambda'/g$  (shaded regions) are given by the conditions from the measured branching fractions  $R_{\gamma}^{0}$  (full line) and  $R_{\gamma}^{+}$  (dashed line).

do not use any quark model prediction for the parameter  $\lambda'$  but treat it on an equal footing with the parameter  $\lambda$ , so that both are considered as purely phenomenological. Nevertheless we were able to obtain reasonably good precision in the determination of model parameters. We have to point out that our method of not fixing  $\lambda'$  has both a distinct advantage and a distinct drawback. The advantage is related to the fact that we explicitly retain the leading piece (the  $\lambda$  terms which are of order  $\frac{1}{m}$  where *m* is a light quark mass) in the contribution of the light-quark current in  $D^* \to D\gamma$ . At the same time the corrections of the order  $\frac{1}{m_c}$  are effectively absorbed into the parameter  $\lambda'$ . This is a drawback, since such a new  $\lambda'$  is now no more equal to the one appearing in Eq. (23).

### IV. WEAK LAGRANGIAN FOR LIGHT AND HEAVY MESONS

In addition to strong and electromagnetic interactions, we must also address the weak decays within these schemes. We will follow the approach of [12, 14] and use an effective current between the heavy mesons and the light mesons. The weak current is  $L_{Q_a}^{\mu} = \bar{q}_a \gamma^{\mu} (1 - \gamma_5) Q$ and it transforms as  $(\bar{3}_L, 1_R)$ . The lowest dimension operator with the same transformation properties but with meson degrees of freedom is

$$J_{Q_{a}}^{\mu} = \frac{1}{2} i \alpha \operatorname{Tr}[\gamma^{\mu} (1 - \gamma_{5}) H_{b} u_{ba}^{\dagger}] + \alpha_{1} \operatorname{Tr}[\gamma_{5} H_{b} (\hat{\rho}^{\mu} - \mathcal{V}^{\mu})_{bc} u_{ca}^{\dagger}] + \cdots, \qquad (30)$$

where the ellipsis denotes terms vanishing in the limit  $m_q \to 0, m_Q \to \infty$ , or terms with derivatives.

The light meson decay constants  $f_{P,V}$  are defined by the usual relations

$$\langle 0|J_{q\mu}^{\ ab}(0)|P_i(p)\rangle = if_P \frac{\lambda_i^{ab}}{\sqrt{2}} p_\mu ,$$
  
$$\langle 0|J_{q\mu}^{\ ab}(0)|V_i(\epsilon,p)\rangle = f_V \frac{\lambda_i^{ab}}{\sqrt{2}} m_V \epsilon_\mu .$$
(31)

Here  $\lambda_i^{ab}$ ,  $i = 1, \ldots, 8$ , a, b = 1, 2, 3 are the usual eight Gell-Mann  $3 \times 3$  matrices, normalized as  $\text{Tr}(\lambda_i \lambda_j) = 2\delta_{ij}$ . In the chiral limit  $m_q \to 0$  the above decay constants are related through  $f_P = f_V / \sqrt{a} = f$  for all P, V. Similarly we can define the heavy meson decay constants by [14]

$$\langle 0|J_{Q_{\mu}^{a}}(0)|D^{b}(p)\rangle = -if_{D}\delta^{ab}m_{D}v_{\mu} ,$$
  
$$\langle 0|J_{Q_{\mu}^{a}}(0)|D^{*b}(\epsilon,p)\rangle = if_{D^{*}}\delta^{ab}m_{D^{*}}\epsilon_{\mu} .$$
(32)

In the heavy quark limit  $m_Q \to \infty$  we have  $m_D = m_{D^*} \to \infty$  and  $f_D = f_{D^*}$ . The constant  $\alpha$  in Eq. (30) can be fixed in this limit by taking the matrix elements of  $J^{\mu}_a$  between the heavy meson state and the vacuum, with the result

$$\alpha = f_P \sqrt{m_P} \ . \tag{33}$$

Unfortunately, up to now, neither a theoretical prediction nor experimental data exist for the other parameter,  $\alpha_1$ , in the current (30). The situation is different and better in the light sector, where a well-known prescription exists, for deriving the weak current directly from the strong Lagrangian [20]. Since the quark weak current  $L_{q_{ab}}^{\mu} = \bar{q}_b \gamma^{\mu} (1-\gamma_5) q_a$  must transform as  $(\bar{3}_L, 3_R)$  the light meson weak current with these transformation properties can be obtained from the Lagrangian (16). Of course, one has to properly define the covariant derivative for pseudoscalars, enlarging the gauge group to include the W-boson contributions [21]. The resulting light meson part of the weak current is

$$J_q^{\ \mu} = i f^2 u [\mathcal{A}^{\mu} + a (\mathcal{V}^{\mu} - \hat{\rho}^{\mu})] u^{\dagger} .$$
 (34)

The part of the weak Lagrangian for the pseudoscalar and vector, light and heavy mesons, which we will use, can be written as [22-24]

$$\mathcal{L}_{W}^{\text{eff}}(\Delta c = \Delta s = 1) = -\frac{G}{\sqrt{2}} V_{ud} V_{cs}^{*}[a_{1}(\bar{u}d)_{V-A}^{\mu}(\bar{s}c)_{V-A,\mu} + a_{2}(\bar{s}d)_{V-A}^{\mu}(\bar{u}c)_{V-A,\mu}], \quad (35)$$

where  $V_{ud}$ , etc., are the relevant Cabibbo-Kobayashi-Maskawa (CKM) mixing parameters, while  $a_1$  and  $a_2$  are the QCD Wilson coefficients, which depend on a scale  $\mu$ . One expects the scale to be the heavy quark mass and we take  $\mu \simeq 1.5$  GeV which gives  $a_1 = 1.2$  and  $a_2 = -0.5$ , with an approximate 20% error. In the factorization model the quark currents are approximated by the corresponding meson currents defined in Eqs. (30) and (34):

$$(\bar{q}_a Q)^{\mu}_{V-A} \equiv \bar{q}_a \gamma^{\mu} (1 - \gamma^5) Q \simeq J_{Q_a}^{\mu} ,$$
 (36)

$$(\bar{q}_b q_a)_{V-A}^{\mu} \equiv \bar{q}_b \gamma^{\mu} (1 - \gamma^5) q_a \simeq J_{q_{ab}}^{\mu}$$
 (37)

Many heavy meson weak nonleptonic amplitudes [22-25] have been calculated using the factorization approximation. It has been shown in [22], however, that for some of the *D* meson decays there are rather important final state interactions and the factorization approximation can be improved by the inclusion of the SU(3) symmetrybreaking effects [26].

The authors of Ref. [23] have classified the weak nonleptonic decays into three classes: decays determined by  $a_1$  only (class I), decays determined by  $a_2$  only (class II), and decays where  $a_1$  and  $a_2$  amplitudes interfere (class III). Factorization can therefore be tested in several ways. There are the following two categories of decays: "quark decays," in which the heavy quark decays while the remaining antiquark acts as a spectator, and "annihilation processes," in which heavy and light quarks annihilate and two new quarks are created. For the annihilation processes the factorization approximation is usually only a small contribution [23, 24, 22].

We are forced to use this approximation in our calculations, since there are no better approaches developed so far for nonleptonic weak decays.

### V. $D \rightarrow V\gamma$ DECAYS

The simplest radiative decays of D mesons are into a light meson and a photon. Since the process  $D \to P\gamma$  (P

is a light pseudoscalar) is forbidden due to the requirement of gauge invariance and chiral symmetry [27], as well as angular momentum conservation, we will concentrate on the  $D \to V\gamma$  (V is a light vector meson) decays. Although there are not yet any data on such processes we can predict the partial widths and branching ratios. We consider the only two processes which are possible at the tree level and are not Cabibbo suppressed, namely,  $D^0 \to \bar{K}^{*0}\gamma$  and  $D^{s+} \to \rho^+\gamma$ . Both processes have contributions from the odd-parity interaction Lagrangian. The second one has, in addition, a direct emission term, due to the charged initial and final mesons.

Regarding the anomalous term, there are two contributions which are important. The photon can first be emitted from the  $D^0$   $(D^{s+})$  meson, which becomes a

 $D^{*0}$   $(D^{*s+})$  and then  $D^{*0}$   $(D^{*s+})$  decays weakly into  $\bar{K}^{*0}$   $(\rho^+)$ . The other contribution comes from the weak decay of  $D^0$   $(D^{s+})$  first into an off-shell  $\bar{K}^0$   $(\pi^+)$ , which then decays into  $\bar{K}^{*0}\gamma$   $(\rho^+\gamma)$ . Both contributions are proportional to  $a_2$   $(a_1)$  (35). For the description of this amplitude we need the  $D^*D\gamma$  and  $K^*K\gamma$   $(\rho\pi\gamma)$  couplings and these couplings were obtained in the previous section. The amplitude for the  $D^0 \to \bar{K}^{*0}\gamma$  is

 $A(D^{0}(p_{D^{0}}) \to \bar{K}^{*0}(p_{K^{*0}})\gamma(q))$   $= e \frac{G}{\sqrt{2}} V_{ud} V_{cs}^{*} a_{2} [C_{D^{0}K^{*0}\gamma}^{(1)} \epsilon_{\mu\nu\alpha\beta}$   $\times q^{\mu} \epsilon_{\nu}^{\nu*} v^{\alpha} \epsilon_{\kappa^{*0}}^{\beta*}], \qquad (38)$ 

$$C_{D^{0}K^{\ast 0}\gamma}^{(1)} = \left(C_{VV\Pi}\frac{1}{g_{V}} + C_{V\Pi\gamma}\right) \frac{f_{D^{0}}f_{K^{0}}m_{D^{0}}^{2}}{(m_{D^{0}}^{2} - m_{K^{0}}^{2})} \frac{8\sqrt{2}m_{D^{0}}}{3f} + 4(\lambda' + \frac{2}{3}\lambda)f_{D^{\ast 0}}f_{K^{\ast 0}}\frac{m_{D^{\ast 0}}m_{K^{\ast 0}}}{(m_{D^{\ast 0}}^{2} - m_{K^{\ast 0}}^{2})}\sqrt{m_{D^{0}}m_{D^{\ast 0}}} \right)$$
(39)

Similarly, the amplitude for  $D^{s+} \to \rho^+ \gamma$  is

$$A(D^{s+}(p_{D^s}) \to \rho^+(p_{\rho})\gamma(q)) = e \frac{G}{\sqrt{2}} V_{ud} V_{cs}^* a_1 \left[ C_{D^s \rho \gamma}^{(1)} \epsilon_{\mu\nu\alpha\beta} q^{\mu} \epsilon_{\gamma}^{\nu*} v^{\alpha} \epsilon_{\rho}^{\beta*} + i C_{D^s \rho \gamma}^{(2)} m_{\rho} \left( \epsilon_{\gamma}^* \cdot \epsilon_{\rho}^* - \frac{\epsilon_{\gamma}^* \cdot p_{\rho} \epsilon_{\rho}^* \cdot q}{p_{\rho} \cdot q} \right) \right],$$
(40)

where

$$C_{D^{*}\rho\gamma}^{(1)} = -\left(C_{VV\Pi}\frac{1}{g_{V}} + C_{V\Pi\gamma}\right) \frac{f_{D^{*}}f_{\pi}m_{D^{*}}^{2}}{(m_{D^{*}}^{2} - m_{\pi}^{2})} \frac{4\sqrt{2}m_{D^{*}}}{3f} + 4(\lambda' - \frac{1}{3}\lambda)f_{D^{**}}f_{\rho}\frac{m_{D^{**}}m_{\rho}}{(m_{D^{**}}^{2} - m_{\rho}^{2})}\sqrt{m_{D^{*}}m_{D^{**}}}$$
(41)

and

$$C_{D^{s}\rho\gamma}^{(2)} = f_{D^{s}}f_{\rho} \ . \tag{42}$$

In our numerical calculations we used the following numerical values  $C_{VV\Pi} = 0.423$ ,  $C_{V\Pi\gamma} = -3.26 \times 10^{-2}$  [28, 29],  $g_V = 5.8$  [12],  $f \simeq f_{\pi} = 132$  MeV, and the other decay constants  $f_{P,V}$  were taken from [25]. It is straightforward to calculate the decay widths. The result, of course, depends on which numerical values we take for  $(\lambda' + 2\lambda/3)$  and  $(\lambda' - \lambda/3)$ . Computing these decay widths using  $\lambda$  and  $\lambda'$  as derived in a previous paragraph, we have to point out that the  $\frac{1}{m_c}$  corrections, coming from light-quark current, effectively included into the  $\lambda'$  parameter, are not necessarily the same as in the case  $D^* \to D\gamma$  decay. Of course, this uncertainty unfortunately increases the theoretical and experimental uncertainties already present in the calculation of the  $D^* \to V\gamma$ . We will assume that the differences are negligible.

In Figs. 2 and 3 the decay widths for  $D^0 \to \bar{K}^{*0}\gamma$  and  $D^{s+} \to \rho^+\gamma$  are shown as functions of the combinations  $\lambda' + 2\lambda/3$  and  $\lambda' - \lambda/3$ , respectively. The full (dashed) lines denote the values for these combinations, which are allowed (forbidden) by experimental constraints [Eqs. (28) and (29)], together with  $|g| = 0.57 \pm 0.13$  [13].

An interesting feature can be seen from Fig. 2: a not very precise measurement of the  $D^0 \to \bar{K}^{*0}\gamma$  decay width is sufficient to differentiate between positive or negative solutions for  $\lambda' + 2\lambda/3$ , which are predicted to be of one order of magnitude different. This is because the first and second terms in Eq. (39) have equal (opposite) sign for positive (negative) values of  $\lambda' + 2\lambda/3$ , and thus interfere constructively (destructively). Given the total  $D^0$  decay



FIG. 2. The decay width for  $D^0 \to \overline{K}^{*0}\gamma$  as function of the combination  $\lambda' + 2\lambda/3$ . The full (dashed) lines denote the experimentally allowed (forbidden) values for this combination.



FIG. 3. The decay width for  $D^s \to \rho \gamma$  as function of the combination  $\lambda' - \lambda/3$ . The full (dashed) line denotes the experimentally allowed (forbidden) values for this combination.

width  $\Gamma_{\rm tot}(D^0) \approx 0.0016$  eV [18], the branching ratio is constrained by  $2.1 \times 10^{-4} < B < 4.3 \times 10^{-4}$  for positive and  $B < 0.27 \times 10^{-4}$  for negative values of  $\lambda' + 2\lambda/3$ .

A less dramatic situation is obtained for the decay  $D^{s+} \rightarrow \rho^+ \gamma$  in Fig. 3, where a not precise determination of the partial width is not enough to further constrain the combination  $\lambda' - \lambda/3$ . From  $\Gamma_{\rm tot}(D^{s+}) \approx 0.0014$  eV [18] we see that the branching ratio for this decay is in the range  $2.2 \times 10^{-4} < B < 7.2 \times 10^{-4}$ .

Unfortunately, due to the much larger branching ratio for the weak decay  $D^0 \to K^{*0}\pi^0$  and the difficulty in differentiating the photon from the  $\pi^0$  in this energy range, the decay  $D^0 \to \overline{K}^*\gamma$  has not been seen by the ARGUS Collaboration [30]. The situation is similar for the detection of  $D^{s+} \to \rho^+\gamma$  by the ARGUS Collaboration: because of the very small branching ratio, low detector's acceptance ( $3\gamma$  events have to be measured) and poor mass resolution, the ARGUS data are not likely to find this decay. But, hopefully, some experimental signals for these radiative decays will come from the CLEO data. The experimental measurement of this branching ratio will determine the relative sign between the first and the

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second contributions, giving in such a way new information on the parameters  $\lambda$ ,  $\lambda'$ , and g.

In conclusion, we used both chiral symmetry and heavy quark symmetry to obtain an effective strong, EM and weak Lagrangian for the description of both light and heavy pseudoscalar and vector mesons. In this framework we reanalyzed the  $D^*$  strong and radiative decays, obtaining without any reference to quark models, a good determination of some of the parameters in the effective Lagrangian. Within the same framework and with these values for the parameters we calculated the  $D \to V \gamma$  decay widths, providing numerical predictions. These results can be used to test the validity of the approximations that were made in the context of the heavy quark effective theory. At the least, our numerical results are reasonable estimates and provide some guidance. In the framework developed here, other D meson radiative nonleptonic decays  $(D \rightarrow VP\gamma \text{ or } PP\gamma)$  can also be calculated [31], giving estimates for future experiments and further tests of the applicability of heavy quark effective theory (HQET).

Note added. After completion of this work the paper by P. Jain, A. Momen, and J. Schechter (hep-ph/9406338), where some of these results have been independently obtained, came to our attention. Their approach is quite similar to ours in Sec. III, but they fix the parameter  $\lambda'$  to be proportional to  $1/m_c$ , while we leave it free. Also we noticed a paper by H.Y. Cheng, C.Y. Cheung, G.L. Lin, Y.C. Lin, T.M. Yan, and H.L. Yu (hep-ph/9407303). In this paper, the  $D \to K^*\gamma$  decay rate was estimated using an effective electromagnetic and weak Lagrangian developed from quark diagrams for  $b\bar{d} \to c\bar{u}\gamma$ . They then made the replacement  $b \to c$  and  $c \to s$ . Their result is comparable with ours in Fig. 2 for negative values of  $\lambda' + 2\lambda/3$ .

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