Operator product expansion sum rules for heavy flavor transitions and the determination of $|V_{cb}|$

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(Received 29 June 1994)

We derive a model-independent upper bound for the axial-vector form factor of the $B \to D^*$ transition at zero recoil, $F_{B\to D^*}$. The form factor turns out to be noticeably less than unity. The deviation of $F_{B\to D^*}$ from unity is larger than previously anticipated. Using our estimate we extract $|V_{cb}|$ from the measured exclusive rate of $B \to D^* l\nu$ extrapolated to the point of zero recoil. The central " exclusive" value of $|V_{cb}|$ is in agreement with the value obtained from the inclusive semileptonic width $\Gamma(B \to X_c l\nu)$. We argue that the theoretical uncertainty in determining $|V_{cb}|$ from the total inclusive width is significantly reduced if a constraint on the quark mass difference $m_b - m_c$ stemming from the heavy quark expansion is taken into account.

PACS number(s): 12.15.Hh, 12.39.Hg, 13.20.He, 13.25.Hw

I. INTRODUCTION

The precision determination of $|V_{cb}|$, the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, is one of the most important practical applications of the heavy quark expansions in B physics today. Two methods allowing one to extract $|V_{cb}|$ from data are commonly used: the inclusive approach (from the total semileptonic width of the B meson) and the exclusive one (based on the decay amplitude $B \to D^* e\nu$ extrapolated to the point of zero recoil). Both methods have their own advantages and drawbacks, purely experimental and theoretical. In this paper we address the problem of the theoretical uncertainties unavoidable in obtaining $|V_{cb}|$ from experimental data given the status of present-day QCD. The theoretical uncertainty usually quoted for $|V_{cb}|$ dominates all other error bars; see, e.g., [1, 2]. Our task is to show that it can be significantly reduced provided that full information stemming from the fundamental QCD is properly used. Both the "exclusive" and "inclusive" values of $|V_{cb}|$ will be considered. The exclusive method has a larger experimental error due to lower statistics and the need of extrapolation to the point of zero recoil. The measurements of the inclusive semileptonic rates are more accurate.

On the theoretical side, the sources of uncertainty in the two approaches above are different. In the inclusive method the theoretical expression for the width depends on the b and c quark masses which are allowed to vary independently within certain limits. In the exclusive method the amplitude is expressed directly in terms of known masses of B and D^* mesons; however, it is the $B \to D^*$ form factor at zero recoil, $F_{B\to D^*}$, that is not known exactly. According to the theorem of Ref. [3] (see also [4]) in the limit $m_{b,c} \to \infty$ (m_b/m_c fixed) the appropriately normalized form factor $F_{B\to D^*}$ is equal to unity. For the actual values of the quark masses there exist corrections in the inverse powers of the masses. Linear corrections are absent at zero recoil [3, 5], and the leading nonperturbative ones are quadratic in $1/m_{b,c}$. So far, they have not been calculated in a model-independent way, although some estimates exist in the literature.

We go beyond the theorem of Refs. [3, 5] obtaining a model-independent bound on the size of $1/m_{b,c}^2$ corrections in terms of the expectation value of the chromomagnetic operator; see Eq. (5). The value of $F_{B\to D^*}$ is certainly less than unity. The bound we get indicates that the deviation of $F_{B\to D^*}$ from unity [see Eqs. (16a) and (16b) below] is beyond previous estimates.

Moreover, under some very plausible additional assumptions we are able to convert the bound in an estimate of the actual value of $F_{B\to D^*}$; see Eq. (16c). (The size of the $1/m_{b,c}^2$ corrections to $F_{B\to D^*}$ at zero recoil has been previously discussed in [6] with the conclusion that the absolute value of this deviation is very small, less than 0.03.[1] Another analysis of the $1/m_{b,c}^2$ terms published recently [8] also demonstrates that $F_{B\to D^*} < 1$.)

Experimental determination of $|V_{cb}|$ from the measurements of $B \to D^* l\nu$ at maximal q^2 was carried out by ARGUS [9] and CLEO [2]. The results obtained by a *linear* extrapolation of the $B \to D^*$ distribution to the point of zero recoil are

$$F_{B\to D^*}|V_{cb}| = \begin{cases} 0.040 \pm 0.007 \ \text{(Ref. [1])} , \\ 0.038 \pm 0.006 \ \text{(Ref. [2])} . \end{cases} \tag{1}$$

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[1]A. Falk suggested, however [7], that the estimate of Ref. [6], $|1 - F_{B\to D^*}| = \pm 0.03$, should be rather perceived as a "1σ error bar," and doubling the estimate to achieve the "2σ confidence level" is welcome.

The standard practice in analyzing the exclusive experimental data is as follows: in extracting $|V_{cb}|$ one uses the "reference" value, $F_{B\to D^*} = 1$; then the numbers in Eq. (1) can be read as the results for $|V_{cb}|$. The theoretical uncertainty in $F_{B\to D^*}$ is then added to the experimental error bars quoted for $|V_{cb}|$. If we use the exact bound mentioned above as the actual value of $F_{B\to D^*}$ the result for $|V_{cb}|$ increases by 6%, at least; the actual increase is expected to be even larger, from 8 to 14%.

Next we turn to the inclusive method and analyze the uncertainty in the theoretical expression for the total inclusive decay rate in the $b \to c$ transition. Superficially, the decay rate is proportional to m_b^5 (the quark masses will be denoted by small m with the corresponding subscript), and even a modest error in the b quark mass, say, 100 MeV, is translated in the $\pm 5\%$ error in the numerical value of $|V_{cb}|$, coming on top of other theoretical uncertainties. The total theoretical uncertainty is usually believed to lie in the 10% range. We observe that a large fraction of the events is kinematically close to the small velocity (SV) limit [3]. In the SV limit the inclusive decay rate does not depend on m_b and m_c individually, but rather on the mass difference $m_b - m_c$ that is known to a much better accuracy [see Eq. (24)]. Although in the actual decays not all events occur in the SV regime, the total inclusive probability is only weakly sensitive to $m_b + m_c$. Using the constraint on $m_b - m_c$ we extract the inclusive value of $|V_{bc}|$ with the theoretical uncertainty close to $\pm 5\%$. The dominant part of this uncertainty comes from our rather poor knowledge of μ_π^2, the expectation value of the kinetic energy operator; see Eq. (5). The latter is measurable, in principle, in the very same semileptonic transitions (for more details see Ref. [10]).

II. SUM RULES FOR THE FORM FACTORS AT ZERO RECOIL

The prediction for $F_{B\to D^*}$ stems from the operator product expansion (OPE) and/or heavy quark effective theory (HQET) sum rules for the amplitudes at the point of zero recoil. The main stages of our derivation are outlined below.

The transitions we are interested in are $B \to D^*$ and $B \to$ vector excitations. These transitions are generated by the axial-vector current $A_\mu = \bar{b}\gamma_\mu\gamma_5 c$. If the momentum carried by the lepton pair is denoted by q, the zero recoil point is achieved if $\mathbf{q} = 0$ and $q_0 = \Delta M$, where $\Delta M \equiv M_B - M_{D^*}$. To obtain the sum rule we consider the T product

$$h_{\mu\nu} = i\int d^4x e^{-iqx}\frac{1}{2M_{H_Q}}\langle B|T\{A_\mu^\dagger(x)A_\nu(0)\}|B\rangle \ , \tag{2}$$

assuming that $\mathbf{q} = 0$ and q_0 is close to ΔM. The hadronic tensor $h_{\mu\nu}$ can be systematically expanded in $\Lambda_{\rm QCD}/m_{b,c}$. For our purposes it is sufficient to keep the terms quadratic in this parameter. In general, the hadronic tensor $h_{\mu\nu}$ is decomposed in the sum of five terms [11]; in this way five structure functions h_1 to h_5 are introduced. In the zero recoil point only two independent structures survive. For the spatial components of the axial-vector current we need to consider only h_1 (we will systematically use the notation of Ref. [12]).

The $O(1/m_{b,c}^2)$ terms in $h_{\mu\nu}$ were calculated in Refs. [12, 13]. It is convenient to introduce

$$\epsilon = M_B - M_{D^*} - q_0 = \Delta M - q_0 \ . \tag{3}$$

Using Eq. (A1) of [12] one can write down the result in the form

$$-h_1 = \frac{1}{\epsilon} - \frac{\mu_G^2 - \mu_\pi^2}{2m_b}\left(\frac{1}{3} - \frac{m_c}{m_b}\right)\frac{1}{\epsilon(2m_c+\epsilon)} + \left[\frac{4}{3}\mu_G^2 - (\mu_G^2 - \mu_\pi^2)\frac{q_0}{m_b}\right]\frac{1}{\epsilon^2(2m_c+\epsilon)}, \tag{4}$$

where μ_G^2 and μ_π^2 parametrize the matrix elements of the chromomagnetic and kinetic energy operators:

$$\mu_G^2 = \frac{1}{2M_{H_b}}\left\langle H_b\left|\bar{b}\,\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}\,b\right|H_b\right\rangle \ ,$$
$$\mu_\pi^2 = \frac{1}{2M_{H_b}}\langle H_b|\bar{b}\,(i\mathbf{D})^2\,b|H_b\rangle \ . \tag{5}$$

To derive the desired sum rule we choose $|\epsilon| \gg \Lambda_{\rm QCD}$ but, at the same time, $|\epsilon| \ll m_{b,c}$. Now we can expand in $\Lambda_{\rm QCD}/\epsilon$ and in $\epsilon/m_{b,c}$, keeping only the term linear in $1/\epsilon$, and compare the theoretical expression obtained in this way with the hadronic saturation of $h_{\mu\nu}$. In the derivation of our sum rule, to the accepted accuracy, one can neglect the difference between $\Delta M = M_B - M_D$ and $\Delta m \equiv m_b - m_c$. The corresponding effect is inversely proportional to the heavy quark mass and affects our result only in a higher order in $\Lambda_{\rm QCD}/m_{b,c}$. We find in this way

$$F_{B\to D^*}^2 + \sum_{i=1,2,\ldots} F_{B\to \rm excit}^2 = 1 - \frac{1}{3}\frac{\mu_G^2}{m_c^2} - \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_cm_b}\right), \tag{6}$$

where the sum on the left-hand side runs over all excited states with the appropriate quantum numbers, and all form factors are taken at the point of zero recoil. The form factor $B \to D^*$ at zero recoil is defined as

$$\langle B|A_\alpha|D^*\rangle = -\sqrt{4M_BM_{D^*}}\ F_{B\to D^*}\,D_\alpha^* \ ,$$

where D_α^* is the polarization vector of D^*.

It is convenient to rewrite Eq. (6) in the form

$$1 - F_{B\to D^*}^2 = \frac{1}{3}\frac{\mu_G^2}{m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_cm_b}\right) + \sum_{i=1,2,\ldots} F_{B\to \rm excit}^2 \ . \tag{7}$$

If terms $O(\Lambda_{\rm QCD}^2/m_Q^2)$ are neglected, then higher states cannot be excited at zero recoil, only the elastic $B \to D^*$ transition survives in this approximation, and we recover the prediction

$$F_{B\to D^*} = 1 \ \ (\text{zero recoil}) \ ,$$

the most well-known consequence of the heavy quark (or the Isgur-Wise [14]) symmetry, a symmetry first observed in the point of zero recoil in Refs. [3, 4].

At the level $O(\Lambda^2_{\rm QCD}/m_Q^2)$ excitation of higher states already takes place; all transition form factors squared are proportional to $\Lambda^2_{\rm QCD}/m_Q^2$. Simultaneously the form factor of the elastic transition shifts from unity by a similar amount. (These facts have a transparent physical interpretation; see [10].)

It is crucial that the contribution of the excited states in the sum rule (7) is strictly positive. Therefore we arrive at the inequality

$$1 - F^2_{B\to D^*} > \frac{1}{3}\frac{\mu_G^2}{m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right) . \quad (8)$$

For completeness, we quote here a similar zero recoil sum rule for the vector form factor of the $B \to D$ transition $F_{B\to D}$:

$$F^2_{B\to D} + \sum_{i=1,2,\dots} F^2_{B\to \rm excit} = 1 - \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c} - \frac{1}{m_b}\right)^2 . \quad (9)$$

This transition is measurable (in principle) at zero recoil in the $B \to D + \tau\nu_\tau$ decays. To derive Eq. (9) one must consider the zero-zero component of the hadronic tensor induced by the vector current $V_\mu = \bar{b}\gamma_\mu c$. Since $\mu_\pi^2 > \mu_G^2$ (see below), Eq. (8) represents a lower bound on the deviation of the form factor from unity at zero recoil.

III. NUMERICAL ESTIMATES AND HIGHER EXCITATIONS

Thus, we found above that the lower bound for the deviation of the elastic form factor $F_{B\to D^*}$ from unity at zero recoil is determined, nonperturbatively, by a local contribution consisting of two terms. (The perturbative correction will be included shortly.) The first parameter μ_G^2 is expressed via the mass difference of B^* and B:

$$\mu_G^2 = \frac{3}{4}(M^2_{B^*} - M_B^2) \approx 0.35 \text{ GeV}^2. \quad (10)$$

As far as μ_π^2 is concerned, it was pointed out recently [15] that this parameter is bounded from below. An improved lower bound

$$\mu_\pi^2 > \mu_G^2 \quad (11)$$

was obtained in Ref. [16], using a quantum-mechanical argument similar to that of Ref. [15] and, later, within the sum rules themselves [10]. Quantum mechanically Eq. (11) stems from the fact that the Pauli Hamiltonian is positive definite.

Therefore, neglecting the second (positive) term in Eq. (8) we arrive at a model-independent lower bound

$$1 - F^{\rm nonpert}_{B\to D^*} > \frac{1}{8}\frac{M^2_{B^*} - M_B^2}{m_c^2} \approx 0.035. \quad (12)$$

A stronger result is obtained if one relies on the estimate of μ_π^2 from the QCD sum rules [17]:[2]

$$\mu_\pi^2 = (0.54 \pm 0.12) \text{ GeV}^2 . \quad (13)$$

Then

$$1 - F^{\rm nonpert}_{B\to D^*} > 0.05 - 0.07 . \quad (14)$$

On top of this nonperturbative correction it is necessary to take into account radiative corrections due to the hard gluon exchange. In terms of the hadronic states these corrections correspond to the contribution of sufficiently heavy excited states where the perturbative calculation of inclusive transition rates is justified. The one-loop correction has been found in [3]:

$$\eta_A^{\rm pert} = 1 + \frac{\alpha_s}{\pi}\left(\frac{m_b + m_c}{m_b - m_c}\ln\frac{m_b}{m_c} - \frac{8}{3}\right) \simeq 0.975 , \quad (15)$$

where we used the one-loop value of $\Lambda_{\rm QCD}$ (for consistency it is mandatory to use the one-loop value of $\Lambda_{\rm QCD}$ in the one-loop calculations) and the subscript A marks the axial-vector current.

The renormalization-group improvement (summation of the terms $\alpha_s^n \ln^n m_b/m_c$ and $\alpha_s^n \ln^{(n-1)} m_b/m_c$) has been carried out in [22]; it leads to the result

$$\eta_A^{\rm pert} \approx 0.985 .$$

It is not quite clear, though, whether one can trust this apparent reduction of the perturbative correction. Indeed, for the actual values of the quark masses $\ln m_b/m_c \approx 1.3$ can hardly be considered as a large parameter. Therefore, the exact calculation of the $O(\alpha_s^2)$ term makes much more sense than the summation of the leading and the next-to-leading logs, and will ensure a more reliable and accurate prediction than the above renormalization-group improvement. Note that keeping only the $\ln m_b/m_c$ term in the one-loop calculation results in a dramatically wrong estimate [cf. Eq. (15)]. For practical purposes we will use for the gluon radiative correction to $F_{B\to D^*}$ the factor $\eta_A^{\rm pert} = 0.98$.

Accounting for the hard-gluon radiative corrections makes the matrix elements μ_G^2 and μ_π^2 renormalization-scale dependent; in particular, the first one gets somewhat enhanced due to the anomalous dimension of the chromomagnetic operator [23]. These effects can be readily accounted for and do not produce a noticeable change even if the normalization point is chosen at $\mu \sim 1$ GeV.

To get an idea of the contribution of the inelastic channels one may accept that their joint effect varies between 0 and 100% of the nonperturbative correction in Eq. (7). Then assembling all these numbers together we finally arrive at

[2] Earlier estimates are also available [18, 19]. Note that the author of Ref. [18] essentially revoked his result in a later publication [20], where it is claimed that the value of μ_π^2 is dramatically smaller, even smaller than the lower bound (11) [21].

$$F_{B\to D^*} = \begin{cases} < 0.94 & \text{model-independent bound,} \quad (16a)\\ < 0.92, \ \mu_\pi^2 = 0.54 \text{ GeV}^2 , & \quad (16b)\\ 0.89 \pm 0.03 & \text{educated guess,} \quad (16c)\end{cases}$$

where the last entry assumes our educated guess on the excited state contribution and the central value for the kinetic energy operator, Eq. (13).

To substantiate this estimate of the inelastic contribution let us consider a subclass of possible inelastic contributions in the sum rule (7) due to the final states of the type $D\pi$ with $|\mathbf{p}_\pi| \ll \mu_{\rm hadr}$. The soft-pion corrections in the elastic transition $B \to D^*$ were studied previously in Ref. [24]. We follow the same pattern but calculate, instead, the inelastic $D\pi$ contribution in the sum rule.

For soft pions the amplitude $\langle D\pi|A_i|B\rangle$ is given by the diagrams of Fig. 1 and is reliably calculable:

$$\langle D^-\pi^+|\mathbf{A}|B^+\rangle = -\lambda\sqrt{4M_BM_D}\,\vec{p}_\pi\left(\frac{1}{\epsilon}-\frac{1}{\epsilon+\Delta}\right), \quad (17)$$

where ϵ is defined in Eq. (3), $\mathbf{p}_\pi$ is the pion momentum,

$$\Delta = M_{B^*} - M_B + M_{D^*} - M_D,$$

and λ is the heavy-meson-pion constant,

$$\mathcal{L}_{\rm int} = 2M_D\,\lambda D^*_\mu D\partial^\mu\pi + 2M_B\,\lambda B^*_\mu B\partial^\mu\pi$$

(for $D^{*0}D^-\pi^+$). The $D^*D\pi$ and $B^*B\pi$ constants are related by the heavy quark plus chiral symmetry [25]; $1/m_{b,c}$ deviations from the heavy quark symmetry in the vertices are inessential for the infrared part we are interested in. In Eq. (17) we also neglected some other irrelevant terms of higher order in $1/m_{b,c}$. Equation (17) explicitly shows cancellation of two graphs of Fig. 1 in the limit of the heavy quark symmetry, when $\Delta \to 0$, $\epsilon \gg \Delta$. This cancellation is just a manifestation of the fact that $F_{B\to D^*}$ at zero recoil must be equal to unity up to $1/m_{b,c}^2$ corrections [3, 5]. At $\epsilon < \Delta$ no trace of the heavy quark symmetry is left and no cancellation occurs; diagram 1(a) is much smaller than that of Fig. 1(b). The absence of the heavy quark symmetry in similar kinematical conditions has been noted previously in Ref. [26].

The $D\pi$ production threshold is situated at

$$\epsilon_0 = -(M_{D^*} - M_D - M_\pi),$$

i.e., at a small negative value of ϵ. The pole at $\epsilon = 0$ in Eq. (17) evidently corresponds to the actual production of D^* with the subsequent decay into $D\pi$. The second singularity, the B^* meson pole $(\epsilon+\Delta)^{-1}$ [see Fig. 1(a)], lies outside the physical domain $\epsilon > \epsilon_0$.

The amplitude (17) implies the hadronic tensor

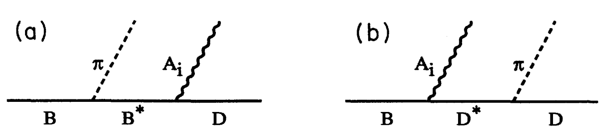

FIG. 1. The graphs determining the amplitude $\langle D\pi|A_i|B\rangle$ in the soft-pion limit.

$$\frac{1}{\pi}{\rm Im}\,h_1|_{D\pi} = \frac{\lambda^2}{8\pi^2}|\mathbf{p}_\pi|^3\frac{\Delta^2}{\epsilon^2(\epsilon+\Delta^2)^2}. \quad (18)$$

Here we also added the channel $\bar{D}^0\pi^0$. Integrating this expression over ϵ we get the contribution to the sum rule (6) sought for; cf. Eq. (4).

To make the integral

$$\frac{1}{\pi}\int d\epsilon\,{\rm Im}\,h_1 \quad (19)$$

well defined, the factor ϵ^{-2} has to be replaced by $[\epsilon^2 + (\Gamma^2/4)]^{-1}$, where Γ is the pion width of D^*. Then integration near $\epsilon = 0$ yields unity — this is nothing else than the leading elastic contribution to the sum rule (6), which has to be unity in the calculation at hand. It must be removed from our inelastic part. To this end the lower limit of integration in Eq. (19) must be chosen at some $\epsilon_{\rm min} \gg \Gamma$. The upper limit of integration is also needed, since the integral (19) is logarithmically divergent at large ϵ. This is a standard situation with the soft-pion amplitudes containing chiral logarithms. We will cut off the integral at $\epsilon = \mu_{\rm hadr} \sim 1$ GeV, so that the expression (17) for the amplitude stays valid inside the integration range. Notice that not only is the coefficient in front of the chiral logarithm reliably calculable, but the constant term from the domain $\epsilon \sim \Delta$ also comes out correctly.

Doing the integration, we find that

$$\sum F^2_{B\to{\rm excit}} \to \frac{1}{\pi}\int d\epsilon\,{\rm Im}\,h_1|_{D\pi} = \frac{\lambda^2\Delta^2}{8\pi^2}\left(\ln\frac{\mu_{\rm hadr}}{\Delta} + C\right), \quad (20)$$

$$C \approx 3 \text{ at } \Delta/M_\pi \approx 1.4.$$

Parametrically the right-hand side of Eq. (20) is proportional to $1/m_{b,c}^2$ (through Δ^2), as it should be, of course. Moreover, the $D\pi$ inelastic contribution is additionally suppressed by $1/N_c$ (through λ^2), where N_c is the number of colors.

Substituting the existing upper bound for λ^2 [25] ($\lambda^2 < 0.5f_\pi^{-2}$) as the actual value of λ^2 and $\mu_{\rm hadr} \sim 1$ GeV, we find that the inelastic contribution in the sum rule (6) due to the channel D plus the soft pion is close to 7%. Approximately one-third comes from the logarithmic term and two-thirds from the constant.

Thus, if λ^2 is close to its upper bound, this inelastic channel alone produces the same effect on $F_{B\to D^*}$ as the μ_G^2 term in Eq. (7), i.e., decreases $F_{B\to D^*}$ by 0.035. Thus, we conclude that it is perfectly reasonable and safe to assume the inelastic effect to vary between zero and 100% of the nonperturbative terms in Eq. (7), and our educated guess is fully substantiated.

Comparing our result with the renormalization of $F_{B\to D^*}$ due to the soft-pion exchange considered in [24] we observe, with satisfaction, that the elastic renormalization of Ref. [24] is exactly the same (parametrically) and has the opposite sign, so that the combined effect of the soft pions in the integral (19) vanishes, and this integral remains equal to unity, as it should if we limit ourselves to the infrared contributions and thus neglect the nonperturbative corrections in Eq. (6). Numerically our estimate of Eq. (20) is higher, by a factor of ~ 2, since in Ref. [24] $1/M_B$ was set equal to zero and Δ was approximated by $M_{D^*} - M_D$.

IV. DETERMINATION OF $|V_{cb}|$ FROM THE INCLUSIVE RATE

The CKM matrix element $|V_{cb}|$ can be alternatively determined from the inclusive semileptonic width $\Gamma B \to X_c l\nu$. The theoretical expression for the widths is known in the literature including the α_s and the leading nonperturbative correction,

$$\Gamma(B \to l\nu X_c) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \times \left\{ \left(z_0(x) - \frac{2\alpha_s}{3\pi}(\pi^2 - 25/4) z_0^{(1)}(x) \right) \times \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} \right) - z_1(x) \frac{\mu_G^2}{m_b^2} + O(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3) \right\} , \tag{21}$$

where the phase space factors z account for the mass of the final quark:

$$z_0(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, z_1(x) = (1-x)^4 ,$$
$$z_0^{(1)}(0) = 1, z_0^{(1)}(1) = 3/(2\pi^2 - 25/2) \approx 0.41 ,$$
$$x = (m_c/m_b)^2. \tag{22}$$

The function $z_0^{(1)}$ can be found in Ref. [27]. The nonperturbative terms proportional to μ_G^2 and μ_π^2 were first found in Ref. [28]; all corrections together are compiled in Refs. [29, 30].

The explicit form of the perturbative correction here refers to the one-loop value of the so-called pole mass; see, e.g., [31]. Theoretically this object is ill defined [32]. Generally speaking, if the result for the decay rate is expressed in terms of the pole mass, one should expect large (factorially divergent) coefficients in the α_s expansion. If nonperturbative terms are included, it is necessary to express the result in terms of a Euclidean mass normalized appropriately.[3] Transition from the pole mass to the Euclidean mass might change the coefficient of the α_s correction. We do not need to do that, however. Indeed, our point is that the decay rate (21) depends essentially on the difference of the quark masses, $m_b - m_c$, and in this difference all uncertainties and definition dependence cancel. The residual weak dependence on the individual masses is reflected in the error bars we will ascribe to our result.

For the pole mass of the b quark (or for the mass normalized not far from the would-be mass shell) it is reasonable to accept

$$m_b = 4.8 \pm 0.1 \text{ GeV}. \tag{23}$$

The central value, 4.8 GeV, follows from the QCD sum rule analysis of the Υ system [33]. To be on the safe side, we multiplied the original error bars by a factor of 4. It is worth noting that it is very difficult, practically impossible, to go outside the indicated limits given the constraint on $m_b - m_c$ to be discussed below. Indeed the central value of m_b above implies $m_c \approx 1.30$ GeV [provided we accept the estimate (13) for μ_π^2], which matches very well with an independent determination of the c quark pole mass [31]. Plus or minus 100 MeV in m_b is translated in ± 100 MeV in m_c. It seems perfectly safe to say that m_c lies between 1.20 and 1.40 GeV; one can hardly imagine that the one-loop pole c quark mass is less than 1.20 or larger than 1.40. Thus, we believe that allowing m_b to vary in the interval (23) we fully cover the existing uncertainty in this parameter. We will *not* allow m_c to change independently, however; this parameter will be tied to m_b. This simple step dramatically reduces the uncertainty in the theoretical prediction for $\Gamma(B \to X_c l\nu)$.

At first sight it might seem that the fifth power of the b quark mass in Eq. (21) strongly magnifies the uncertainty in m_b. It is possible to check, however, that the perturbative expression for the width, $m_b^5 z_0(m_c^2/m_b^2)$, being a rather sophisticated function of both masses m_b and m_c is sensitive mostly to the quark mass difference. Moreover, even though each mass individually has a noticeable scatter around the canonical central values ($\pm 2\%$ for b and $\pm 6\%$ for c), the quark mass difference is known to a much better accuracy within HQET [34, 35]:

$$m_b - m_c = \frac{M_B + 3M_{B^*}}{4} - \frac{M_D + 3M_{D^*}}{4} + \mu_\pi^2 \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) + O(1/m_c^3, 1/m_b^3) . \tag{24}$$

This relation holds for the masses normalized at the scale below the mass of the c quark. However, the infrared renormalon singularities cancel in this difference, and it is free of intrinsic theoretical uncertainties at any normalization point.

The expression for the width has such a structure that for the given fixed value of $m_b - m_c$ the variation of the individual masses inside the allowed interval affects the theoretical prediction at a much weaker level than one gets in the standard approach, with both masses changing independently. This observation noted long ago in phenomenological studies has a reason: in the semileptonic decays in a large part of the phase space, effectively we are not far from the SV limit, i.e., $|\mathbf{q}| < M_D$. Exactly in this limit, at $|\mathbf{q}|/M_D \to 0$, the semileptonic transition would depend *only* on the quark mass difference. The most clear-cut manifestation of the proximity to the SV limit is the fact that about 60% of the semileptonic decay rate is due to the "elastic" channels, $B \to Dl\nu$

[3]It was shown in Ref. [32] that the b quark mass that enters the expressions for inclusive widths at the level of nonperturbative corrections is a well-defined running mass normalized at a sufficiently high momentum scale that *per se* does not have any intrinsic theoretical uncertainty. It can be determined to a very good accuracy from the spectra of quarkonia.

and $B \to D^* l\nu$; in the SV limit these two channels completely saturate the probability (up to nonperturbative corrections of order $\Lambda^2_{\rm QCD}/m^2_{b,c}$). This reason explains why we expect the higher-order perturbative corrections to behave in the same manner.

In this way we get numerically

$$|V_{cb}| = 0.0415 \left(\frac{1.49 \text{ ps}}{\tau_B}\right)^{1/2} \left(\frac{BR_{\rm sl}(B)}{0.106}\right)^{1/2} , \quad (25)$$

where we used the central value 4.80 GeV (Ref. [33]) for the one-loop pole mass of the b quark, and the value of the strong coupling $\alpha_s = 0.22$. The expectation value of the kinetic energy is also set equal to its central value, $\mu_\pi^2 = 0.54 \text{ GeV}^2$.

Let us discuss which error bars in Eq. (25) theory is responsible for. First, the variation of m_b (or, alternatively, m_c) in the range ± 100 MeV results only in a $\mp 1.6\%$ relative variation of $|V_{cb}|$ if other parameters are kept fixed. The most sizable uncertainty arises in this approach due to dependence of $m_b - m_c$ on the value of μ_π^2. Again, to be on the safe side, we double the original theoretical error bars [17] in this parameter and allow the value to vary within the limits

$$0.35 \text{ GeV}^2 < \mu_\pi^2 < 0.8 \text{ GeV}^2. \quad (26)$$

This uncertainty leads to the change in $|V_{cb}|$ of $\mp 2.8\%$, with practically linear dependence. It seems obvious that the interval (26) overestimates the existing uncertainty in μ_π^2. It is worth noting that the value of μ_π^2 can, and will, be measured soon via the shape of the lepton spectrum in $b \to c l \nu$ inclusive decays [10] with theoretical accuracy of at least 0.1 GeV^2.

Finally there is some dependence on the value of the strong coupling in Eq. (21). Numerically the uncertainty constitutes about $\pm 1\%$ when α_s is varied between 0.2 and 0.25. This must and will be reduced by explicit calculation of the next loop correction, which is straightforward (though somewhat tedious in practice). All perturbative QCD corrections are well behaved here: no logarithms of the mass ratio can appear, at least if the result is expressed in terms of the quark masses and α_s normalized at a proper Euclidean scale.

Therefore, the above numerical estimates imply that already at present the theoretical uncertainty in the inclusive value of $|V_{cb}|$ does not exceed $\sim \pm 5\%$ and is quite competitive with the existing experimental uncertainties in this quantity. It seems possible to further reduce this error to 4% or even 3% by measuring μ_π^2 and calculating the two-loop perturbative correction to the width. The inclusive method, thus, is not only more accurate experimentally (due to higher statistics than in the exclusive case) but it also seems more promising from the side of both the theoretical uncertainties as they exist now and their future possible reduction.

V. CONCLUSIONS

Let us summarize our findings. From the OPE sum rules, we obtained a lower bound on the deviation of $F_{B\to D^*}$ from unity from from fundamental QCD. Supplementing this result by a plausible estimate of the contribution due to higher excitations we are able to estimate the actual value of $F_{B\to D^*}$, which turns out substantially lower than unity, in drastic contrast to previous expectations. Then we argued that the proper use of the heavy quark expansion allows one to extract $|V_{cb}|$ from the total inclusive rate with much higher accuracy than is usually stated.

The value of $|V_{cb}|$ determined from the inclusive semileptonic width combined with our estimates of the $F_{B\to D^*}$, Eq. (16a), implies a prediction for the extrapolation of the exclusive decay rate to zero recoil. Alternatively, one can express the same result by explicitly introducing the correction due to the fact that $F_{B\to D^*} \neq 1$ in Eq. (1). Assuming $F_{B\to D^*} = 0.89$ and using the value from Eq. (1) one obtains, e.g., from the CLEO data

$$|V_{cb}| = 0.043 \pm 0.007 . \quad (27)$$

The agreement with the inclusive result (25) seems to be even too good, taking into consideration the experimental uncertainties and the theoretical assumptions involved in the estimates of $F_{B\to D^*}$.

It is instructive to make a brief comparative analysis of the theoretical uncertainties in the two methods: inclusive versus exclusive. In the both cases there are no nonperturbative corrections at the level $1/m_{b,c}$. The main difference, however, is that the leading $1/m^2_{b,c}$ nonperturbative corrections *can* be, and have been fully calculated for the inclusive widths, whereas they *cannot* be determined in a model-independent way for the exclusive $B \to D^*$ form factor even at zero recoil (see, e.g., [8]). Either we have a contamination due to the contribution from higher states (in our approach), or nonlocal contributions within the more traditional approach of Refs. [6, 8]. The latter are poorly controllable. It is fortunate that in our approach the μ_G^2 and μ_π^2 terms make $F_{B\to D^*}$ smaller than unity; the contamination due to the higher states, being positive-definite, works in the same direction.

The exclusive approach has certain conceptual advantages: apart from the form factor itself, the measured rate is given in terms of the masses of real B and D^* mesons, whereas in calculating the inclusive widths one uses, though well defined theoretically, but still uncertain at some level, quark masses. On the other hand, the total semileptonic width, in turn, has a theoretical advantage over the exclusive predictions: in the inclusive approach, determination of the CKM matrix elements is meaningful even in the limit of the light final quark (i.e., for the $b \to u$ transition), whereas the exclusive approach becomes useless due to our ignorance of the heavy-to-light form factors even at zero recoil. (In the opposite limit when m_c increases approaching m_b, the gold-plated situation for exclusive form factors, the OPE-based analysis of the inclusive transitions automatically reproduces all results of HQET [10].)

The perturbative corrections have been calculated in both cases to one loop. The one-loop gluon correction turns out to be somewhat smaller for the exclusive transition due to numerical cancellations. Although some higher-order logarithmic summation has been performed in the exclusive case, it does not seem to be very useful for $m_b/m_c \sim 3$. Logarithmic terms of this type do not appear at all in the inclusive widths due to the infrared stability of the latter. The real improvement of the perturbative estimates can be achieved only by the exact two-loop calculation of the α_s^2 terms in both cases. Summarizing, numerically and statistically the inclusive approach seems best.

ACKNOWLEDGMENTS

This work was done during our stay at the Institute of Theoretical Physics, University of California (Santa Barbara). We are grateful to L. Randall and D. Kaplan for inviting us to participate in the program on heavy quarks at ITP. We would like to thank our colleagues and the staff of ITP for kind hospitality. Useful discussions with I. Bigi, A. Falk, Y. Kubota, R. Poling, L. Randall, M. Voloshin, and M. Wise are acknowledged. We are grateful to L. Randall for pointing out Ref. [24] to us. This work was supported in part by the U.S. DOE under Grant No. DE-FG02-94ER40823 and by the NSF under Grant No. NSF-PHY89-04035.

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