QCD corrections to Higgs boson self-energies and fermionic decay widths

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(Received 11 July 1994)

We present the QCD corrections to Higgs boson self-energies for an arbitrary momentum transfer and for different internal quark masses to treat the case of CP-even, CP-odd, and charged Higgs bosons which appear in extensions of the standard model scalar sector. Using Ward identities, we then relate these results obtained by directly evaluating the relevant two-loop Feynman diagrams to the known expressions for the electroweak vector boson vacuum polarization functions. Finally, we derive the exact analytical expressions for the QCD corrections to the decays of these Higgs particles into quark pairs in the general case, and reproduce in a completely independent way known results in some special cases.

PACS number(s): 14.80.Cp, 12.15.Lk, 12.38.Bx, 12.60.Cn

I. INTRODUCTION

One of the remaining enigmas in the modern formulation of the theory of strong and electroweak interactions is the mechanism of electroweak symmetry breaking. The existence of at least one scalar particle, the Higgs boson, is required to generate the masses of the other fundamental particles, leptons, quarks, and weak gauge bosons [1]. The discovery of this particle and the study of its fundamental properties will be the most important mission of future high-energy colliders [2].

The phenomenological properties of the unique Higgs particle of the standard model (SM) have been studied in great detail in the literature [1,2]. In fact, because the precise knowledge of the Higgs boson decay widths, branching fractions, and production cross sections is mandatory, quantum corrections must be included and this subject has received much attention recently [3]. In particular, QCD corrections to Higgs boson decay and production processes are of the utmost importance. For instance, the Higgs boson decays into quark pairs and gluons, which together with the $H \to \tau^+\tau^-$ decays, are the most important decay modes in the intermediate mass range $M_W < M_H < 140$ GeV, receive very large QCD corrections. In the case of $H \to q\bar{q}$, they are known exactly to $O(\alpha_s)$ [4–6] and up to $O(\alpha_s^2)$ [7] in the approximation $m_q \ll M_H$; in the case of the gluonic decays, the QCD corrections are known up to next to leading order [8].

The electroweak radiative corrections to Higgs boson decays [10] are also of significance, since the leading contribution is quadratically proportional to the mass of the heavy top quark [11]. In fact, a fourth generation of heavy fermions, the existence of which is still allowed by present experimental data with the proviso that the associated neutrino is heavy enough [9], would have a dramatic effect on the Higgs boson decay widths. Its contribution is universal in the sense that it does not depend on the final state particle, and will also increase quadratically with the heavy fermion masses. The universal part of the two-loop mixed $O(\alpha_s G_F m_Q^2)$ corrections, which have been calculated very recently [12,13], will screen the leading one-loop contributions by a non-negligible amount.

This is how it is, so far, for the SM Higgs particle. However, many extensions of the standard model predict the existence of a larger Higgs sector. For instance supersymmetric (SUSY) theories, which are very attractive since at low energies they provide a theoretical framework in which the problem of naturalness and hierarchy in the Higgs sector is solved while retaining Higgs bosons with moderate masses as elementary particles, require the existence of at least two isodoublet scalar fields Φ_1 and Φ_2 to give masses separately to isospin up and down particles, thus extending the physical spectrum of scalar particles to five [1]. The physical Higgs bosons introduced by the minimal supersymmetric extension of the standard model (MSSM) are of the following type: two CP-even neutral bosons h and H (where h will be the lightest particle), a CP-odd neutral boson A (usually called pseudoscalar), and two charged Higgs bosons $H^{\pm}$. In addition to the four masses M_h, M_H, M_A, and $M_{H^\pm}$, two additional parameters define the properties of the scalar particles and their interactions with gauge bosons and fermions: the ratio of the two vacuum expectation values $\tan\beta = v_2/v_1$ and a mixing angle α in the neutral CP-even sector. Note that, contrary to a general two-Higgs doublet model where the six parameters are free, supersymmetry leads to several relations among these parameters and, in fact, only two of them are independent.[1]

[1]Note that there are also large radiative corrections to the supersymmetric Higgs boson masses and couplings [14]; these corrections are beyond the scope of this paper and will not be discussed here.

0556-2821/95/51(1)/218(11)/$06.00

In this paper we calculate, by directly evaluating the relevant Feynam diagrams using dimensional regularization, the fermionic contributions to the Higgs boson self-energies at $O(\alpha\alpha_s)$. To treat the case of scalar, pseudoscalar and charged Higgs particles on the same footing, we have considered the most general case where, in addition to leaving the momentum transfer arbitrary, we allow the internal quarks to be of different flavors $U \neq D$ and therefore to have different masses $m_U \neq m_D$. Our motivations for performing such a calculation are threefold.

(i) As in the case of the SM Higgs boson, the strong [4–6,15] and some of the electroweak [16] radiative corrections to SUSY Higgs boson decays are known at the one-loop level. In some limiting cases, as for the gluonic corrections for nearly massless quarks, SM two-loop results can be adapted to the SUSY case [17]. Here, we provide the necessary material which allows us to derive the universal part of the mixed $O(\alpha_s G_F)$ radiative corrections to these Higgs decays. This is a generalization to a multi-Higgs-doublet model of the recent SM calculation [12], which is just a special case [in the limit $m_U = m_D$ and when the Higgs boson is CP even] of the present results.

(ii) The imaginary parts of the Higgs boson self-energies are related, through the optical theorem, to the hadronic[2] partial decay widths of the Higgs bosons. We give here the exact analytical expression of the QCD corrections to the Higgs decay widths in the most general case $m_U \neq m_D$ which is not available in the literature.[3] In the special cases $m_U = m_D$, we recover the known results for the QCD corrections to CP-even and CP-odd [4–6] neutral Higgs bosons which have been obtained by directly evaluating the relevant Feynman amplitudes. Since our expressions have been obtained with a completely different method, this serves as an independent check of these results.

(iii) The fermionic contributions to the transverse and longitudinal components of the electroweak vector boson self-energies at $O(\alpha\alpha_s)$ have been evaluated recently in the general case [18]. It is known that the longitudinal parts of the latter self-energies are directly related to the corresponding Goldstone boson self-energies through a Ward identity. Here, we will show explicitly that this is indeed the case also for the Higgs bosons and therefore provide a very powerful check of both calculations.

The paper is organized as follows. In the next section we will first set the notation and summarize the one-loop results which will be relevant to our discussion. In Sec. III we will give a few details on the calculation of the two-loop Higgs boson self-energies and discuss the renormalization procedure. The complete results for the Higgs boson two-point functions will be given in Sec. IV. In Sec. V we will derive the imaginary parts of the two-loop self-energies which correspond to the QCD corrections to the decays of the Higgs bosons into quark-antiquark pairs. Section VI will summarize our results. Finally, in the Appendix, we discuss the Ward identity which allows us to relate our results to the ones derived for the longitudinal components of the electroweak vector bosons.

II. NOTATION AND ONE-LOOP RESULTS

In this section we will first set the notation and for the sake of completeness, we rederive some of the one-loop results which will be relevant to our next discussion. We will closely follow the notation of Refs. [18,19].

The contribution of a quark loop to the self-energy of a scalar Higgs boson Φ will be denoted by $\Pi^{\Phi}(s = q^2)$, where q is the four-momentum transfer. To treat the cases of neutral CP-even, neutral CP-odd, and charged Higgs bosons on the same footing, it is convenient to work in the general situation where the internal quarks in the loop are of different flavor, and thus have different masses. This will correspond to the case of a charged Higgs boson which couples to an up-type and down-type quark, with masses $m_U \neq m_D \neq 0$; the self-energies of neutral scalar and pseudoscalar Higgs bosons will simply be special cases of the previous one.

The coupling of charged Higgs bosons to fermions is a P-violating mixture of scalar and pseudoscalar couplings,

$$g(H^+U\bar{D}) = i(G_F/\sqrt{2})^{1/2}[h_U(1-\gamma_5) + h_D(1+\gamma_5)] \ , \tag{2.1}$$

where in a (type II) two-Higgs doublet model [1] such as in the MSSM,

$$h_U = m_U/\tan\beta, \quad h_D = m_D \tan\beta \ . \tag{2.2}$$

It is often convenient to use the scalar and pseudoscalar components of this coupling:

$$v = h_D + h_U, \quad a = h_D - h_U \ . \tag{2.3}$$

The couplings of scalar, that we will denote by S, and pseudoscalar A Higgs bosons take the general form

$$\begin{aligned} g(SQ\bar{Q}) &= -i(G_F\sqrt{2})^{1/2}h_Q, \\ g(AQ\bar{Q}) &= -(G_F\sqrt{2})^{1/2}\gamma_5 h_Q \ . \end{aligned} \tag{2.4}$$

In the SM, the reduced couplings h_Q are just the quark masses; in the MSSM these couplings for $\Phi = S, A$ are given in Table I for U and D quarks.

In the one-loop approximation, the contribution of a quark loop to the vacuum polarization amplitude of a charged Higgs boson, $\Pi^C(q^2)$, will correspond to $-i$ times the standard Feynman amplitude of the diagram Fig. 1(a). For arbitrary fermion masses $m_U \neq m_D \neq 0$ and momentum transfer q^2, this amplitude reads

[2]By hadronic Higgs boson decays, we mean here the decays into quark pairs; the QCD corrections to the gluonic neutral Higgs boson decays have been evaluated in Ref. [8].

[3]QCD corrections to the charged Higgs boson decay for $m_U \neq m_D$ have been calculated in Ref. [15], but not in a full analytical form since the integrals for the real corrections have been performed numerically.

$$\Pi^C(q^2) = \frac{N_C G_F}{\sqrt{2}} \left(\frac{\mu^2 e^\gamma}{4\pi}\right)^\epsilon i \int \frac{d^n k}{(2\pi)^n} \mathrm{Tr} \frac{(\not k + m_U)(v - a\gamma_5)(\not k - \not q + m_D)(v + a\gamma_5)}{(k^2 - m_U^2)[(k-q)^2 - m_D^2]} , \tag{2.5}$$

where $N_C = 3$ is a color factor, μ is the 't Hooft renormalization mass scale introduced to make the coupling constant dimensionless in $n = 4 - 2\epsilon$ dimensions; we have also introduced an extra term $(e^\gamma/4\pi)^\epsilon$ [γ is the Euler constant] to prevent uninteresting combinations of $\ln 4\pi, \gamma, \ldots$, in the final result. After calculating the trace and integrating over the loop momentum, one obtains

$$\Pi^C(s) = \frac{3G_F}{2\sqrt{2}\pi^2} s \left[h_U^2 \Pi_U^+(s) + h_D^2 \Pi_D^+(s) + 2h_U h_D \frac{m_U m_D}{s} \Pi^-(s)\right] , \tag{2.6}$$

where, in the general case $m_U \neq m_D \neq 0, \Pi^\pm(s)$ take the form

$$\begin{aligned} \Pi_{U,D}^+(s) = {}& \frac{1}{2\epsilon}(1 + 2\alpha + 2\beta) - \frac{1}{4}(\rho_a + \rho_b)(1 + \alpha + \beta) - \frac{1}{2}\alpha\rho_a - \frac{1}{2}\beta\rho_b + 1 + \frac{3}{2}\alpha + \frac{3}{2}\beta \\ & + \frac{1}{4}(1 + \alpha + \beta)[(\alpha - \beta + \lambda^{1/2}) \ln x_a + (\beta - \alpha + \lambda^{1/2}) \ln x_b] , \\ \Pi^-(s) = {}& -\frac{1}{\epsilon} - 2 + \frac{1}{2}(\rho_a + \rho_b) - \frac{1}{2}[(\alpha - \beta + \lambda^{1/2}) \ln x_a + (\beta - \alpha + \lambda^{1/2}) \ln x_b] , \end{aligned} \tag{2.7}$$

and where we use the variables [$\beta = -m_D^2/s, \rho_b$ and x_b are defined in a similar way]

$$\alpha = -\frac{m_U^2}{s}, \quad \rho_a = \ln \frac{m_U^2}{\mu^2}, \quad x_a = \frac{2\alpha}{1 + \alpha + \beta + \lambda^{1/2}} ,$$

$$\lambda = 1 + 2\alpha + 2\beta + (\alpha - \beta)^2 . \tag{2.8}$$

In the limit where one of the internal quarks is nearly massless compared to its partner, as is the case for the top-bottom isodoublet, the coefficients in front of $\Pi_D^+(s)$ and $\Pi^-(s)$ vanish and the expression of $\Pi_U^+(s)$ simplifies to

$$\begin{aligned} \Pi_U^+(s) = {}& \frac{1}{2}\left(\frac{1}{\epsilon} - \rho_a\right)(1 + 2\alpha) + 1 + \frac{3}{2}\alpha \\ & + \frac{1}{2}(1 + \alpha)^2 \ln \frac{\alpha}{1 + \alpha} . \end{aligned} \tag{2.9}$$

The expressions of the self-energies for neutral scalar and pseudoscalar Higgs bosons can simply be obtained by setting $m_U = m_D = m_Q$ in Eqs. (2.6), (2.7) and by using the relevant couplings which are given in Table I. With the help of the variable $x = 4\alpha/(1 + \sqrt{1 + 4\alpha})^2$ with $\alpha = -m_Q^2/s$, one has

$$\Pi^{S,A}(s) = \frac{3G_F}{2\sqrt{2}\pi^2} s h_Q^2 [\Pi_Q^+(s) \pm (m_Q^2/s)\Pi^-(s)] , \tag{2.10}$$

leading to

$$\begin{aligned} \Pi^S(s) = {}& \frac{3G_F}{2\sqrt{2}\pi^2} s h_Q^2 \\ & \times \left[\frac{1}{2}\left(\frac{1}{\epsilon} - \rho_a\right)(1 + 6\alpha) + 1 + 5\alpha \right. \\ & \left. + \frac{1}{2}(1 + 4\alpha)^{3/2} \ln x\right] , \end{aligned}$$

$$\begin{aligned} \Pi^A(s) = {}& \frac{3G_F}{2\sqrt{2}\pi^2} s h_Q^2 \\ & \times \left[\frac{1}{2}\left(\frac{1}{\epsilon} - \rho_a\right)(1 + 2\alpha) + 1 + \alpha \right. \\ & \left. + \frac{1}{2}(1 + 4\alpha)^{1/2} \ln x\right] . \end{aligned} \tag{2.11}$$

Note that Eq. (2.6) exhibits the fact that $\Pi^{A,S}(s)$ can be obtained from $\Pi^{S,A}(s)$ by simply making the substitution $m_U(m_D) \to -m_U(-m_D)$ in $\Pi^C(s)$ as expected from γ_5 reflection symmetry.

Finally, in the limit where the momentum squared is much larger or much smaller than the quark masses squared, the self-energies read

$$\begin{aligned} s\Pi^+(s \ll m_{U,D}^2) = {}& -(m_U^2 + m_D^2)\left(\frac{1}{\epsilon} - \ln \frac{m_U m_D}{\mu^2} + 1\right) \\ & + \frac{1}{2} \frac{m_U^4 + m_D^4}{m_U^2 - m_D^2} \ln \frac{m_U^2}{m_D^2} , \\ \Pi^-(s \ll m_{U,D}^2) = {}& -\frac{1}{\epsilon} + \ln \frac{m_U m_D}{\mu^2} - 1 \\ & + \frac{1}{2} \frac{m_U^2 + m_D^2}{m_U^2 - m_D^2} \ln \frac{m_U^2}{m_D^2} , \end{aligned} \tag{2.12}$$

$$s\Pi^+(s \gg m_{U,D}^2) = \frac{1}{2\epsilon} - \frac{1}{2} \ln \frac{-s}{\mu^2} + 1 . \tag{2.13}$$

In all the previous expressions the momentum transfer

TABLE I. Neutral Higgs couplings to up-type and down-type fermions in the SM and MSSM.

Φ	h_U/m_U	h_D/m_D
$H_{\rm SM}$	1	1
h	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$
H	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$
A	$1/\tan\beta$	$\tan\beta$

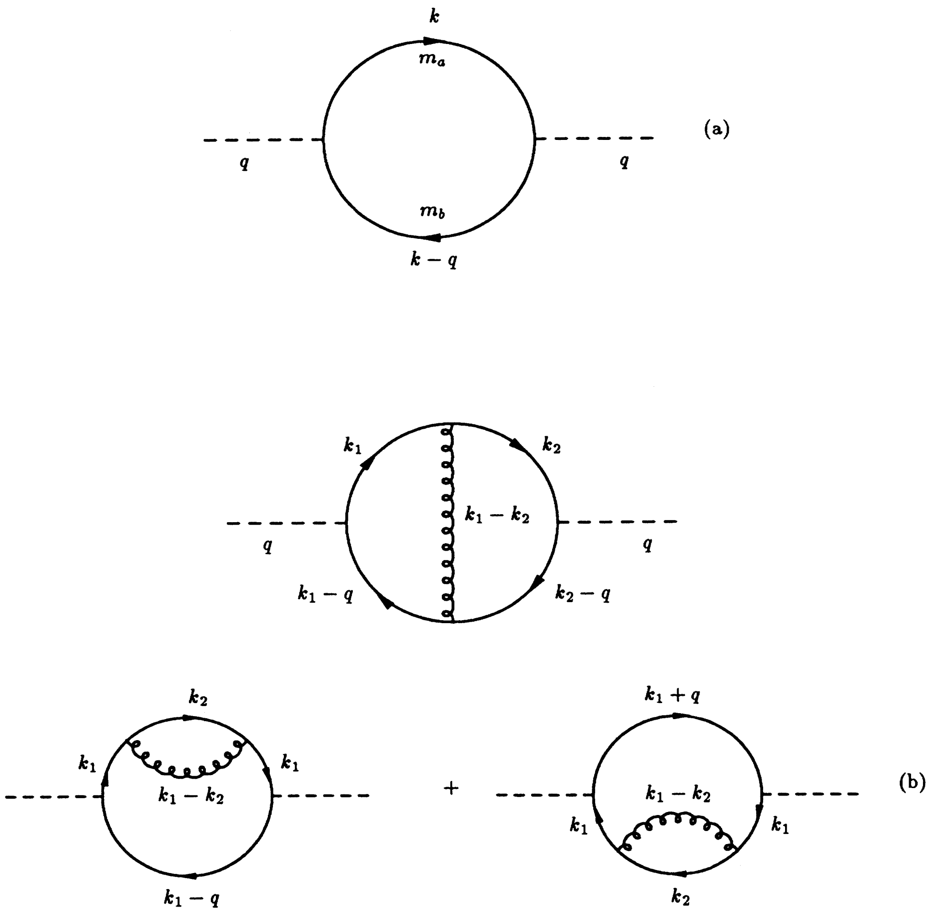

FIG. 1. Feynman diagrams for the contribution of quark pairs to the self-energy of a Higgs boson at one-loop (a) and two-loop (b).

has been defined to be in the spacelike region, i.e., $s < 0$. When continued to the physical region above the threshold for the production of two fermions, $s \geq (m_U + m_D)^2$, the Higgs boson self-energies acquire imaginary parts. These imaginary parts are related to the decay widths of the Higgs particles into quark-antiquark pairs. Adding a small imaginary part $-i\epsilon$ to the fermion masses squared, the analytical continuation is consistently defined.

From expressions (2.7), the imaginary parts can be straightforwardly obtained by making the substitution

$$\ln x_{a,b} \to \ln|x_{a,b}| + i\pi . \qquad (2.14)$$

One then has for the partial decay widths of a charged Higgs boson H^+ into $U\bar{D}$ quark pairs [$s = M_{H^+}^2$]

$$\Gamma(H^+ \to U\bar{D}) = \frac{N_C G_F M_{H^+}}{2\sqrt{2}\pi^2}\left[h_U^2 \mathrm{Im}\Pi_U^+(s) + h_D^2 \mathrm{Im}\Pi_D^+(s) + 2h_U h_D \frac{m_U m_D}{s}\mathrm{Im}\Pi^-(s)\right] \qquad (2.15)$$

with

$$\mathrm{Im}\Pi_{U,D}^+(s) = \frac{\pi}{2}\lambda^{1/2}(1+\alpha+\beta), \quad \mathrm{Im}\Pi^-(s) = -\pi\lambda^{1/2} . \qquad (2.16)$$

In the limit $m_D = 0$ this partial decay width reduces to the more familiar form

$$\Gamma(H^+ \to U\bar{D}) = \frac{N_C G_F}{4\sqrt{2}\pi} M_{H^+} h_U^2 \left(1 - \frac{m_U^2}{s}\right)^2 . \qquad (2.17)$$

In a similar manner, one also obtains the familiar expressions of the partial decay widths of neutral scalar and pseudoscalar Higgs bosons into quark-antiquark pairs:

$$\Gamma(S \to Q\bar{Q}) = \frac{N_C G_F}{4\sqrt{2}\pi} M_S h_Q^2 \left(1 - 4\frac{m_Q^2}{s}\right)^{3/2} ,$$

$$\Gamma(A \to Q\bar{Q}) = \frac{N_C G_F}{4\sqrt{2}\pi} M_A h_Q^2 \left(1 - 4\frac{m_Q^2}{s}\right)^{1/2} . \qquad (2.18)$$

III. TWO-LOOP CALCULATION

At $O(\alpha\alpha_s)$, the two-loop diagrams contributing to the Higgs boson self-energies $\Pi^\Phi(q^2)$ [up to a factor $-i$] are shown in Fig. 1(b). In the 't Hooft–Feynman gauge, using the routing of momenta shown in the figure and following the notation introduced in the preceding section, one can write the bare amplitude as

$$\Pi^{\Phi}(q^2)|_{\rm bare} = \frac{16\pi G_F}{3\sqrt{2}} N_C \alpha_S \left(\frac{\mu^2 e^{\gamma}}{4\pi}\right)^{2\epsilon} \int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} [\mathcal{A} + \mathcal{B}] \,, \tag{3.1}$$

where, for $m_U \neq m_D \neq 0$, are given by

$$\mathcal{A} = \mathrm{Tr} \frac{(\not k_1 + m_U)(v - a\gamma_5)(\not k_1 - \not q + m_D)\gamma_\lambda(\not k_2 - \not q + m_D)(v + a\gamma_5)(\not k_2 + m_U)\gamma^\lambda}{(k_1 - k_2)^2 (k_1^2 - m_U^2)(k_2^2 - m_U^2)[(k_1 - q)^2 - m_D^2][(k_2 - q)^2 - m_D^2]} \,,$$

$$\mathcal{B} = \mathrm{Tr} \frac{(\not k_1 + m_U)(v - a\gamma_5)(\not k_1 - \not q + m_D)(v + a\gamma_5)(\not k_1 + m_U)\gamma_\lambda(\not k_2 + m_U)\gamma^\lambda}{(k_1 - k_2)^2 (k_1^2 - m_U^2)^2 (k_2^2 - m_U^2)[(k_1 - q)^2 - m_D^2]} + m_U \leftrightarrow m_D \,. \tag{3.2}$$

This bare amplitude has to be supplemented by counterterms; these include the quark wave function and mass counterterms as well as the Higgs-boson–quark vertex counterterm. However, the renormalization of the Higgs-boson–quark vertex is connected with the renormalization of the quark masses and wave functions; because the latter counterterms cancel, one only needs to include quark mass renormalization and considers the diagrams where the quark mass counterterms are inserted into the one-loop Higgs boson self-energy [Figs. 2(b) and 2(c)]. The mass counterterm is obtained by evaluating the amplitude of the diagram shown in Fig. 2(a), which reads in dimensional regularization

$$-i\Sigma(\not p) = -\alpha_s \frac{16\pi}{3} \left(\frac{e^{\gamma}\mu^2}{4\pi}\right)^{\epsilon} \times \int \frac{d^n k}{(2\pi)^n} \frac{\gamma^\lambda(\not p - \not k + m_Q)\gamma_\lambda}{[(p-k)^2 - m_Q^2]k^2} \,, \tag{3.3}$$

where p is the four-momentum of the quark and m_Q its bare mass. This expression can be decomposed into a piece proportional to $(\not p - m_Q)$ which will enter the wave function renormalization and another piece proportional to m_Q which will give the mass counterterm. After integration over the loop momentum, the latter is given by

$$\Sigma(p^2) = \frac{\alpha_s}{\pi} \frac{m_Q}{\epsilon} \left(\frac{e^{\gamma}\mu^2}{m_Q^2}\right)^{\epsilon} \frac{1 - 2/3\epsilon}{1 - 2\epsilon} \Gamma(1+\epsilon) \times [1 + O(p^2/m_Q^2 - 1)] \,. \tag{3.4}$$

The mass counterterm will now depend on the renormalization procedure, i.e., on the way the quark mass is defined. In the on-shell scheme which is usually used to calculate radiative corrections in the electroweak theory [20], the fermion masses are defined at $p^2 = m_Q^2$ and correspond to the position of the pole of the fermion propagators; they are referred to as the on-shell masses and the counterterm reads in this case[4]

$$\delta m_Q \equiv m_Q(m_Q^2) - m_Q = \frac{\alpha_s}{\pi} \frac{m_Q}{\epsilon} \left(\frac{\mu^2}{m_Q^2}\right)^{\epsilon} \left(1 + \frac{\pi^2}{12}\epsilon^2\right) \frac{1 - 2\epsilon/3}{1 - 2\epsilon} \,. \tag{3.5}$$

One then inserts this mass counterterm in the one-loop self-energies, as depicted in the diagrams of Fig. 2(a), which is equivalent to calculate

$$\Pi^{\Phi}(q^2)|_{\rm CT} = -\delta m_U \left[1 + \frac{\partial}{\partial m_U}\right] \Pi^{\Phi}(q^2)|_{1\ \rm loop} - \delta m_D \left[1 + \frac{\partial}{\partial m_D}\right] \Pi^{\Phi}(q^2)|_{1\ \rm loop} \,, \tag{3.6}$$

where the one-loop vacuum polarization function is given by Eqs. (2.6) and (2.7), up to $O(\epsilon)$ terms which have to be included. The renormalized two-loop self-energies will then read

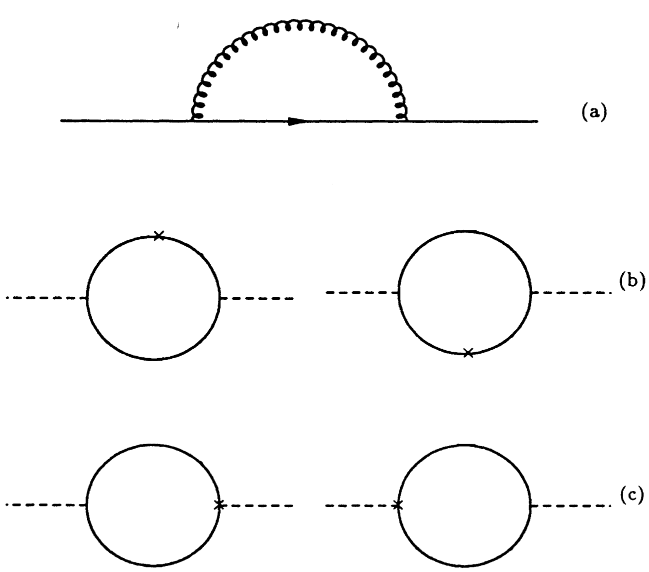

FIG. 2. Feynman diagrams for the (a) one-loop quark self-energy and for the (b) mass and (c) vertex counterterms contribution to the self-energies at the two-loop level.

[4]Note that one can also employ a different definition of the quark masses; for instance one can use the modified minimal subtraction scheme ($\overline{\rm MS}$) mass which is defined by just picking the divergent term in the expression of $\Sigma(p^2)$ in Eq. (3.4), or the running mass where one evaluates $\Sigma(p^2)$ at a scale $p^2 = M_\Phi^2$. Having at hand the expressions of the two-loop self-energies in the on-shell scheme, the procedure for obtaining the corresponding result in any other renormalization scheme is straightforward and can be found, e.g., in Ref. [18]. In practice, however, it is sufficient to replace in the one-loop result, the on-shell mass by the $\overline{\rm MS}$ or the running mass: the difference between this result and the one obtained using the procedure discussed in Ref. [18] is of higher order in α_s.

$$\Pi^{\Phi}(q^2) = \Pi^{\Phi}(q^2)|_{\text{bare}} + \Pi^{\Phi}(q^2)|_{\text{CT}} \; . \tag{3.7}$$

Similarly to the one-loop case, after calculating the trace in Eqs. (3.2) and expressing the scalar products of the momenta appearing in the numerators in terms of combinations of the polynomials in the denominators, one is led to the calculation of a set of scalar two-loop integrals [21] which can also be found in Ref. [18]. In the following we will not give any more details on the cumbersome calculation: we will simply list our main results in the next two sections.

IV. TWO-LOOP SELF-ENERGIES

We begin by giving the expression of the contribution of a (U, D) isodoublet to the charged Higgs boson two-point function at order $O(\alpha\alpha_s)$ in the general case $m_U \neq m_D \neq 0$. The result will be given in the on-shell mass scheme; i.e., $m_{U,D}$ will stand for the on-shell masses. Using the same notations as in the one loop case [confusion should be rare], the charged Higgs boson self-energy $\Pi^C(q^2)$ at the two-loop level is given by

$$\Pi^C(s) = \frac{G_F}{2\sqrt{2}\pi^2}\frac{\alpha_s}{\pi} s \left[h_U^2 \Pi_U^+(s) + h_D^2 \Pi_D^+(s) + 2h_U h_D \right.$$
$$\left. \times \frac{m_U m_D}{s} \Pi^-(s) \right] \tag{4.1}$$

with $\Pi^{\pm}(s)$ given by the relatively simple and compact expressions

$$\begin{aligned}
\Pi_U^+ = & -\frac{3}{2\epsilon^2}(1+4\alpha+4\beta) - \frac{1}{\epsilon}\left[\frac{11}{4} + 14\alpha + 14\beta - 3\rho_a - 12\alpha\rho_a - 6\beta\rho_a - 6\beta\rho_b\right] \\
& + (\rho_a+\rho_b)\left[\frac{11}{4} + 14\alpha + 14\beta - 3(\rho_a+\rho_b)\left(\frac{1}{4}+\alpha+\beta\right) - 3(\alpha-\beta)(\rho_a-\rho_b)\right] \\
& + (\rho_a-\rho_b)\left[5 + \frac{17}{2}\alpha + \frac{17}{2}\beta - \frac{3}{2}(1+2\alpha+2\beta)(\rho_a+\rho_b) + \frac{3}{4}(\alpha^2-\beta^2)(\rho_a-\rho_b)\right] \\
& + \frac{3}{8} - \frac{53}{2}(\alpha+\beta) - \frac{\pi^2}{4}(1+4\alpha+4\beta) + \frac{3}{4}\lambda^{1/2}(1+\alpha+\beta)(\ln x_a + \ln x_b)(\rho_a-\rho_b) \\
& + \frac{1}{4}\ln x_a[31(\alpha-\beta) + 9(1+\alpha+\beta)(\alpha-\beta+\lambda^{1/2})] + \frac{1}{4}\ln^2 x_a\{(1-2\alpha-2\beta)[-\lambda+1+\alpha+\beta-(\alpha-\beta)\lambda^{1/2}] \\
& -3(\alpha+\beta) + 3(\alpha-\beta)(\alpha-\beta+\lambda^{1/2})\} + \frac{1}{4}\ln x_b[31(\beta-\alpha) + 9(1+\alpha+\beta)(\beta-\alpha+\lambda^{1/2})] \\
& + \frac{1}{4}\ln^2 x_b\{(1-2\alpha-2\beta)[-\lambda+1+\alpha+\beta+(\alpha-\beta)\lambda^{1/2}] - 3(\alpha+\beta) + 3(\alpha-\beta)(\alpha-\beta-\lambda^{1/2})\} \\
& + \frac{3}{2}\ln x_a \ln x_b(\lambda + 2\alpha + 2\beta + 6\alpha\beta) - (1+\alpha+\beta)^2\mathcal{I} - 2(1+\alpha+\beta)\mathcal{I}' \; ,
\end{aligned} \tag{4.2}$$

$$\Pi_D^+ = \Pi_U^+[m_U \leftrightarrow m_D] \; , \tag{4.3}$$

$$\begin{aligned}
\Pi^- = & \frac{6}{\epsilon^2} + \frac{1}{\epsilon}(14 - 6\rho_a - 6\rho_b) - 14\rho_a - 14\rho_b + 3(\rho_a+\rho_b)^2 + 20 + \pi^2 \\
& -6\ln x_a(\alpha-\beta+\lambda^{1/2}) - \ln^2 x_a[\lambda - 1 - \alpha - \beta + (\alpha-\beta)\lambda^{1/2}] \\
& -6\ln x_b(\beta-\alpha+\lambda^{1/2}) - \ln^2 x_b[\lambda - 1 - \alpha - \beta + (\beta-\alpha)\lambda^{1/2}] \\
& -6\ln x_a \ln x_b(1+\alpha+\beta) + 2(1+\alpha+\beta)\mathcal{I} + 4\mathcal{I}' \; .
\end{aligned} \tag{4.4}$$

In these expressions, $\mathcal{I}$ and $\mathcal{I}'$ are given by

$$\mathcal{I} = F(1) + F(x_a x_b) - F(x_a) - F(x_b) \; ,$$

$$\mathcal{I}' = \lambda^{1/2}G(x_a x_b) - \frac{1}{2}(\beta-\alpha+\lambda^{1/2})G(x_a) - \frac{1}{2}(\alpha-\beta+\lambda^{1/2})G(x_b) \; , \tag{4.5}$$

where in terms of the polylogarithmic functions [22] $\mathrm{Li}_2(x) = -\int_0^1 y^{-1}\ln(1-xy)dy$ and $\mathrm{Li}_3(x) = -\int_0^1 y^{-1}\ln y \ln(1-xy)dy$, the functions F and G are given by

$$F(x) = 6\mathrm{Li}_3(x) - 4\mathrm{Li}_2(x)\ln x - \ln^2 x \ln(1-x) \; ,$$

$$G(x) = 2\mathrm{Li}_2(x) + 2\ln x \ln(1-x) + \frac{x}{1-x}\ln^2 x \; . \tag{4.6}$$

In the limit where one of the quarks is massless, $m_D = 0$, the coefficients of Π_D^+ and Π^- vanish while Π_U^+ takes the much simpler form [$x = \alpha/(1+\alpha)$ with $\alpha = -m_U^2/s$]

$$\Pi_U^+(s) = -\frac{3}{2\epsilon^2}(1+4\alpha) - \frac{1}{\epsilon}\left[\frac{11}{4} + 14\alpha - 3\rho_a - 12\alpha\rho_a\right] + \frac{1}{2}\rho_a(11+56\alpha)$$
$$-3\rho_a^2(1+4\alpha) + \frac{3}{8} - \frac{53}{2}\alpha - \frac{\pi^2}{4}(1+4\alpha) + \frac{9}{2}(1-\alpha)^2\ln x$$
$$+\frac{1}{2}(1+\alpha)^2(3+2\alpha)\ln^2 x + (1+\alpha)^2[F(x)-F(1)] + (1+\alpha)G(x)\ . \qquad (4.7)$$

Note that in this limit the expression of $\Pi_U^+(s)$ is free of mass singularities as it should be.

As in the one-loop case, one can derive the expressions of the self-energies for neutral scalar and pseudoscalar Higgs bosons from Eqs. (4.1)–(4.4) by setting $m_U = m_D = m_Q$ and using the proper coupling. One would have $[x = 4\alpha/(1+\sqrt{1+4\alpha})^2$ with $\alpha = -m_Q^2/s]$

$$\Pi_Q^{S(A)}(s) = \frac{G_F}{2\sqrt{2}\pi^2}\frac{\alpha_s}{\pi} s h_Q^2 S_Q(A_Q) \qquad (4.8)$$

with $S_Q = \Pi_Q^+(s) - \alpha\Pi^-(s)$ and $A_Q = \Pi_Q^+(s) + \alpha\Pi^-(s)$ given by

$$\begin{aligned}
S_Q =& -\frac{3}{2\epsilon^2}(1+12\alpha) - \frac{1}{\epsilon}\left(\frac{11}{4} - 3\rho_a + 42\alpha - 36\alpha\rho_a\right) + \frac{11}{2}\rho_a - 3\rho_a^2 + 84\alpha\rho_a - 36\alpha\rho_a^2\\
&+\frac{3}{8} - 73\alpha - \frac{\pi^2}{4}(1+12\alpha) + \frac{3}{2}(1+4\alpha)^{1/2}(14\alpha+3)\ln x + \left(\frac{3}{2} + 14\alpha + 29\alpha^2\right)\ln^2 x\\
&-(1+2\alpha)(1+4\alpha)[F(1)+F(x^2)-2F(x)] - 2(1+4\alpha)^{3/2}[G(x^2)-G(x)]\ ,\\
A_Q =& -\frac{3}{2\epsilon^2}(1+4\alpha) - \frac{1}{\epsilon}\left(\frac{11}{4} - 3\rho_a + 14\alpha - 12\alpha\rho_a\right) + \frac{11}{2}\rho_a - 3\rho_a^2 + 28\alpha\rho_a - 12\alpha\rho_a^2\\
&+\frac{3}{8} - 33\alpha - \frac{\pi^2}{4}(1+4\alpha) + \frac{3}{2}(1+4\alpha)^{1/2}(3-2\alpha)\ln x + \left(\frac{3}{2} + 2\alpha - 3\alpha^2\right)\ln^2 x\\
&-(1+2\alpha)[F(1)+F(x^2)-2F(x)] - 2(1+4\alpha)^{1/2}[G(x^2)-G(x)]\ . \qquad (4.9)
\end{aligned}$$

The expressions of Π_Q^S has been derived very recently [12] in the case of the standard model Higgs boson; we have verified that both results are in agreement with each other.[5]

Finally, in the limit where the momentum squared is much smaller than the internal quark masses squared, the components Π_Q^+ and Π^- read

$$\begin{aligned}
s\Pi_Q^+ =& \frac{6}{\epsilon^2}m_+ + \frac{1}{\epsilon}(14m_+ - 3m_+\rho_- - 3m_-\rho_- - 6m_+\rho_+) - \frac{3}{4}\frac{m_+^3}{m_-^2}\rho_-^2 + \frac{3}{4}\frac{m_+^2}{m_-}\rho_-^2 + \pi^2 m_+\\
&+m_+\left(3\rho_+^2 - 14\rho_+ - 7p_- + 3\rho_+\rho_- + \frac{9}{4}\rho_-^2 + 30\right) + m_-\left(\frac{3}{4}\rho_-^2 - 7\rho_- + 3\rho_+\rho_-\right)\ ,\\
\Pi^- =& \frac{6}{\epsilon^2} + \frac{1}{\epsilon}(14-6\rho_+) - \frac{3}{2}\frac{m_+^2}{m_-^2}\rho_-^2 + 30 - 14\rho_+ + 3\rho_+^2 + \pi^2 + \frac{3}{2}\rho_-^2 \qquad (4.10)
\end{aligned}$$

with $\rho_\pm = \ln m_U^2/\mu^2 \pm \ln m_D^2/\mu^2$ and $m_\pm = m_U^2 \pm m_D^2$. In the opposite limit, i.e., when the masses are very small compared to the momentum transfer squared, the coefficient of Π^- vanishes and Π_Q^+ will read $[\zeta(3) = F(1)/6 = 1.202]$

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[5] We have also found that the leading $O(G_F\alpha_s m_t^2)$ universal radiative correction factor to the Higgs boson fermionic decay widths, for $m_t \gg m_b$, is indeed $[1 - \frac{6}{7}(1+\pi^2/9)\frac{\alpha_s}{\pi}]$, in accord with Ref. [12]. In the case of the pseudoscalar Higgs boson with $\tan\beta = 1$, the correction factor in the limit $m_t \gg m_b$ is found to be $[1 - \frac{2}{9}(3+\pi^2)\frac{\alpha_s}{\pi}]$; this is just the well-known QCD correction to the ρ parameter [19]. This result is not surprising if one recalls that the pseudoscalar Higgs boson couples like the longitudinal component of the Z boson [see Appendix] which, in turn, is the same as the transverse component for $q^2 = 0$. Note, however, that this contribution is the full $O(G_F\alpha_s m_t^2)$ radiative correction only for the leptonic Higgs boson decays: for decays into quark pairs [and not only in the case of the b quark] additional diagrams have to be considered; see also Ref. [13].

$$s\Pi_Q^+ = -\frac{3}{2\epsilon^2} + \frac{1}{\epsilon}\left(3\ln\frac{m_Q^2}{\mu^2} - \frac{11}{4}\right) + \frac{3}{8} - \frac{\pi^2}{4} - 6\zeta(3) + 10\ln\frac{m_Q^2}{\mu^2} - 3\ln^2\frac{m_Q^2}{\mu^2} - \frac{9}{2}\ln\frac{-s}{\mu^2} + \frac{3}{2}\ln^2\frac{m_Q^2}{-s}\ . \tag{4.11}$$

V. HADRONIC DECAY WIDTHS

We now turn to the discussion of the partial decay widths of these various Higgs bosons in quark-antiquark pairs. At $O(\alpha_s)$, the partial decay width of a charged Higgs boson into $U\bar{D}$ pairs is given by $[s = M_{H^+}^2]$

$$\Gamma(H^+ \to U\bar{D}) = \frac{G_F \alpha_s M_{H^+}}{2\sqrt{2}\pi^3}\left[h_U^2 \mathrm{Im}\Pi_U^+(s) + h_D^2 \mathrm{Im}\Pi_D^+(s) + 2h_U h_D \frac{m_U m_D}{s}\mathrm{Im}\Pi^-(s)\right]\ . \tag{5.1}$$

The imaginary parts of $\Pi_{U,D}^+$ and Π^- can be derived along the same lines as discussed previously in the one-loop case. Using the fact that

$$\mathrm{Im}\,\ln x_{a,b} = \pi,\ \ \mathrm{Im}\,\ln^2 x_{a,b} = 2\pi\,\ln|x_{a,b}|\ ,$$

$$\begin{aligned}\mathrm{Im}\mathcal{I}' = \pi\mathcal{J}' = \pi\Bigg\{&4\lambda^{1/2}\left[\ln(1-x_a x_b) + \frac{x_b x_b}{1-x_a x_b}\ln|x_a x_b|\right] - (\beta-\alpha+\lambda^{1/2})\Bigg[\ln(1-x_a)\\ &+\frac{x_a}{1-x_a}\ln|x_a|\Bigg] - (\alpha-\beta+\lambda^{1/2})\left[\ln(1-x_b) + \frac{x_b}{1-x_b}\ln|x_b|\right]\Bigg\}\ ,\\ \mathrm{Im}\mathcal{I} = \pi\mathcal{J} = &-2\pi[4\mathrm{Li}_2(x_a x_b) - 2\mathrm{Li}_2(x_a) - 2\mathrm{Li}_2(x_b) + 2|x_a x_b|\\ &\times\ln(1-x_a x_b) - \ln|x_a|(1-x_a) - \ln|x_b|\ln(1-x_b)]\ ,\end{aligned} \tag{5.2}$$

one obtains, for $\mathrm{Im}\Pi^\pm$ in the general case $m_U \neq m_D \neq 0$,

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\Pi_U^+(s) = &\left[(1+\alpha+\beta)\left(\alpha-\beta+\frac{3}{2}\right)\lambda^{1/2} + \left(\frac{3}{2}+\alpha+\beta\right)\lambda + 5\alpha\beta\right]\ln|x_a|\\ &+\left[(1+\alpha+\beta)\left(\beta-\alpha-\frac{3}{2}\right)\lambda^{1/2} + \left(\frac{3}{2}+\alpha+\beta\right)\lambda + 5\alpha\beta\right]\ln|x_b|\\ &+\frac{9}{2}(1+\alpha+\beta)\lambda^{1/2} - (1+\alpha+\beta)^2\mathcal{J} - 2(1+\alpha+\beta)\mathcal{J}'\ ,\end{aligned} \tag{5.3}$$

$$\mathrm{Im}\Pi_D^+(s) = \mathrm{Im}\Pi_U^+(s)[m_U \leftrightarrow m_D]\ , \tag{5.4}$$

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\Pi^-(s) = &-2[\lambda + 2(1+\alpha+\beta) + (\alpha-\beta)\lambda^{1/2}]\ln|x_a| - 2[\lambda + 2(1+\alpha+\beta) + (\beta-\alpha)\lambda^{1/2}]\ln|x_b|\\ &-12\lambda^{1/2} + 2(1+\alpha+\beta)\mathcal{J} + 4\mathcal{J}'\ .\end{aligned} \tag{5.5}$$

From these formulas one can derive again the expressions of the hadronic decay widths in the previous special situations of physical relevance. In the limit where one of the quark is nearly massless, $m_D \to 0$, one has, for $\mathrm{Im}\Pi_U^+(s)$,

$$\begin{aligned}C_Q = \frac{1}{\pi}\mathrm{Im}\Pi_U^+(s) = &\frac{9}{2}(1+\alpha)^2 + (1+\alpha)(3+7\alpha+2\alpha^2)\ln\frac{-\alpha}{1+\alpha} - 2(1+\alpha)^2\\ &\times\left[\frac{\ln(1+\alpha)}{1+\alpha} + 2\mathrm{Li}_2\left(\frac{\alpha}{\alpha+1}\right) - \ln(1+\alpha)\ln\frac{-\alpha}{1+\alpha}\right]\ .\end{aligned} \tag{5.6}$$

In the case of scalar and pseudoscalar Higgs bosons, the partial decay widths $\Gamma(S, A \to Q\bar{Q})$ will be given by

$$\Gamma[S(A) \to Q\bar{Q}] = \frac{G_F}{2\sqrt{2}\pi}\frac{\alpha_s}{\pi}h_Q^2 M_{S(A)} S_Q(A_Q)\ , \tag{5.7}$$

where $S_Q/A_Q = \mathrm{Im}\Pi_Q^+(s, \beta=\alpha)/\pi \mp \alpha\,\mathrm{Im}\Pi^-(s, \beta=\alpha)/\pi$ are given by

$$\begin{aligned}S_Q &= \frac{3}{2}(1+4\alpha)^{1/2}(14\alpha+3) + (58\alpha^2+28\alpha+3)\ln|x| - 4(1+4\alpha)J\ ,\\ A_Q &= \frac{3}{2}(1+4\alpha)^{1/2}(3-2\alpha) + (3+4\alpha-6\alpha^2)\ln|x| - 4J\ ,\end{aligned} \tag{5.8}$$

where

$$J = -(1+2\alpha)[2\mathrm{Li}_2(x^2) - 2\mathrm{Li}_2(x) + 2\ln|x|\ln(1-x^2) - \ln|x|\ln(1-x)] \\ +2\sqrt{1+4\alpha}\left[2\ln(1-x^2) - \ln(1-x) + \frac{x(3x-1)}{1-x^2}\ln|x|\right] . \tag{5.9}$$

Finally, let us note that in the limit where the quark masses are much smaller than the Higgs boson masses, the QCD corrections to the decay widths will exhibit the well known logarithmic behavior [4,5] which, because of chiral symmetry, is the same for the scalar, pseudoscalar and charged Higgs boson:

$$S_Q, A_Q, C_Q \to \frac{9}{2} + 3\ln\frac{m_Q^2}{M_\Phi^2} . \tag{5.10}$$

One has therefore to sum these potentially large logarithmic terms; this is equivalent to replace the on-shell quark masses by the running masses defined at $p^2 = M_\Phi^2$ when renormalizing the $\Phi Q\bar{Q}$ vertex.

Analytical results for $\mathrm{Im}\Pi^S(s)$ [4–6] and $\mathrm{Im}\Pi^A(s)$ [5,6] have been obtained in the past by a number of authors by directly calculating the QCD corrections to the decay of a scalar Higgs boson into quark pairs. The results that we obtain here using a completely different method agree with the previous ones; this serves as check of our full calculation in the general case. Note also that for the value $\tan\beta = 1$, we recover the expression of the imaginary part for the longitudinal component of the electroweak vector bosons in the general case, which is given in Refs. [6,18,23]. Indeed, because of a Ward identity which will be discussed in the Appendix, the imaginary part of the longitudinal component of the vector boson self-energy is the same as the one for the Higgs boson self-energy for this value of $\tan\beta$. This feature provides also a very powerful check of the calculation presented here.

VI. SUMMARY

In this paper, the contribution of heavy quarks to the Higgs boson self-energies was calculated at first order in the strong interaction. We have considered the most general case: finite momentum transfer and arbitrary masses for the internal quarks to treat on the same footing the case of scalar, pseudoscalar and charged Higgs bosons; these particles appear in many extensions of the standard model scalar sector such as two-Higgs doublet models and in particular, supersymmetric theories. Full analytical formulas for the real parts of the self-energies at $O(\alpha_s)$ were presented in the on-shell quark mass scheme.

We have also given the expressions of the self-energies in some situations of physical interest: the case where the two quarks have equal masses which corresponds to neutral scalar and pseudoscalar Higgs bosons, the case where one of the quarks has a negligible mass with respect to the other which would correspond to the approximate contribution of the top-bottom isodoublet to the charged Higgs boson self-energy, and finally the case where the momentum transfer squared is much larger or much smaller than the quark masses squared.

By analytical continuation, we have then derived the imaginary part of the Higgs boson self-energies in the general case $m_U \neq m_D \neq 0$; this imaginary part corresponds to the QCD correction to the partial decay width of the Higgs bosons into quark-antiquark pairs. In the special case $m_U = m_D$ these corrections have been obtained by several authors in a full analytical form. In these limits, the results that we obtain here using a completely different method provide independent checks of these calculations.

Finally, in the Appendix, we relate the results for the two-loop Higgs boson self-energies that we obtained here by directly evaluating the relevant Feynman amplitudes, to the results for the longitudinal components of the electroweak vector boson vacuum polarization functions which are available in the literature. This provides a consistency check of both calculations.

ACKNOWLEDGMENTS

Discussions with Alberto Sirlin and technical help from J. Papavassiliou are gratefully acknowledged. One of us (P.G.) would like to thank the Groupe de Physique des Particules of the Université de Montréal for the hospitality extended to him in the final stage of this work. This work was partially supported by the National Sciences and Engineering Research Council of Canada and by the National Science Foundation under Grant No. PHY-9313781.

APPENDIX: WARD IDENTITIES AND GOLDSTONE BOSON SELF-ENERGIES

As is well known, there are Ward identities relating the longitudinal components of the electroweak vector bosons and the corresponding Goldstone bosons [24]. In this Appendix, we use the current algebra of the standard model to derive these Ward identities; we will then briefly show how to relate the Higgs boson self-energies calculated in the preceding sections to the longitudinal parts of the electroweak vector boson self-energies, the expressions of which have been derived up to $O(\alpha\alpha_s)$ in the general case in Ref. [18].

Define the fermionic contribution to the vacuum polarization function of the W boson and of the corresponding Goldstone boson Φ as [g is the $\mathrm{SU}(2)_L$ coupling constant]

$$\Pi_{WW}^{\mu\nu}(q^2) = -i\frac{g^2}{2}\int d^n x e^{-iq\cdot x}\langle 0|T^* J_W^{\dagger\mu}(x) J_W^\nu(0)|0\rangle , \tag{A1}$$

$$\Pi_{\Phi\Phi}(q^2) = +i\frac{g^2}{2M_W^2}\int d^n x e^{-iq\cdot x}\langle 0|T^* S^\dagger(x) S(0)|0\rangle , \tag{A2}$$

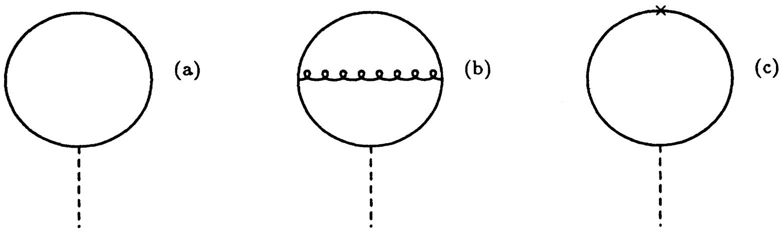

FIG. 3. Tadpole diagrams relating the Higgs boson self-energies to the vector boson self-energies (a) at one-loop, (b) at two-loop, and (c) mass counterterm contributing at the two-loop level.

where $J_W^\mu(w)$ and $S(x)$ are the charged fermionic currents coupled to the W and to the Φ bosons and T^* denotes the covariant time ordering product; for the notation and normalization of the currents, we will follow Ref. [25]. Contracting $\Pi_{WW}^{\mu\nu}$ with the tensor $q^\mu q^\nu$, one obtains

$$q^\mu q^\nu \int d^n x e^{-iq\cdot x}\langle 0|T^* J_W^{\dagger\mu}(x)J_W^\nu(0)|0\rangle = \int d^n x e^{-iq\cdot x}\langle 0|T^* S^\dagger(x)S(0)|0\rangle - \frac{i}{2}\langle 0|S_1(0)|0\rangle \quad \text{(A3)}$$

with S_1 the current coupled to the standard model Higgs boson and where we have used

$$\partial_\mu J_W^\mu(x) = iS(x) \quad \text{(A4)}$$

and

$$[J_W^0(x), S^\dagger(y)]_{x^0=y^0} = +\frac{1}{2}\delta^3(\mathbf{x}-\mathbf{y})[S_1(x) - iS_2(x)] \ , \quad \text{(A5)}$$

where S_2 is the current coupled to the neutral Goldstone boson. This, in turn, can be written as

$$q^2\Pi_{WW}^L(q^2) = -M_W^2\Pi_{\Phi\Phi}(q^2) - \frac{g^2}{4}\langle 0|S_1(0)|0\rangle \ . \quad \text{(A6)}$$

This equation relates the longitudinal part of the vacuum polarization function of the W boson to the self-energy of the corresponding unphysical charged boson. One can see that the subtraction term $\langle 0|S_1(0)|0\rangle$ [a tadpole] is needed to cancel a spurious quartic dependence on the mass of the fermions.

Even though the previous derivation was at the one-loop level in the electroweak interactions, it is valid at any order in the strong interactions as the QCD generators commute with the ones of the electroweak group. To derive the self-energy of the charged Goldstone boson at $O(\alpha)$ and $O(\alpha\alpha_s)$, we therefore need only the expressions of the electroweak vector boson self-energies given in Ref. [18] in the general case and the one of the tadpole diagrams of Fig. 3 where both the two quarks of the same weak isodoublet are running in the loop. Using the same notation as in the main text, and for a single quark of mass m_Q [which is renormalized "on-shell"], we obtain, for the tadpole amplitude up to order α_s,

$$\langle 0|S_1(0)|0\rangle = \frac{3m_Q^4}{4\pi^2}\left\{\frac{1}{\epsilon} + 1 - \ln\frac{m_Q^2}{\mu^2} + \frac{\alpha_s}{3\pi}\left[-\frac{6}{\epsilon^2} - \frac{1}{\epsilon}\left(14 - 12\ln\frac{m_Q^2}{\mu^2}\right) - 30 - \pi^2 + 28\ln\frac{m_Q^2}{\mu^2} - 12\ln^2\frac{m_Q^2}{\mu^2}\right]\right\} . \quad \text{(A7)}$$

This equation, added to the one- and two-loop expressions for the longitudinal part of the W boson self-energies in the general case $m_U \neq m_D \neq 0$ given in Ref. [18] [Eqs. (2.5), (2.8) and (4.1), (4.2) of that paper, respectively] leads to the one- and two-loop expressions of the charged Higgs boson self-energy given in Eqs. (2.6) and (2.7) and (4.1)–(4.4) of the present paper. This is just because for tan$\beta = 1$, the charged Higgs boson couples to fermions exactly like the charged Goldstone of the standard model, up to a relative minus sign for up-type and down-type quarks.

In a completely analogous manner, one can derive the Ward identity in the case of the neutral Goldstone boson Φ_2, which writes

$$q^2\Pi_{ZZ}^L(q^2) = -M_Z^2\Pi_{\Phi_2\Phi_2}(q^2) - \frac{g^2}{4\cos^2\theta_W}\langle 0|S_1(0)|0\rangle \ , \quad \text{(A8)}$$

which allows us to check the expressions of the self-energies of the pseudoscalar Higgs boson which, again for tan$\beta = 1$, has exactly the same couplings as the neutral standard model Goldstone up to, again, a relative minus sign for isospin up and down quarks.

This provides a powerful consistency check of both the calculation of the electroweak vector boson self-energies performed in Ref. [18] and the one of the neutral pseudoscalar and charged Higgs boson self-energies presented here.

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