Experimental evidence for simple relations between unpolarized and polarized parton distributions

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The Pauli exclusion principle is advocated for constructing the proton and neutron deep inelastic structure functions in terms of Fermi-Dirac distributions that we parametrize with very few parameters. It allows a fair description of the recent NMC data on $F_2^p(x,Q^2)$ and $F_2^n(x,Q^2)$ at $Q^2=4$ GeV^2, as well as the CCFR neutrino data at $Q^2=3$ and 5 GeV^2. We also make some reasonable and simple assumptions to relate unpolarized and polarized quark parton distributions and we obtain, with no additional free parameters, the spin-dependent structure functions $xg_1^p(x,Q^2)$ and $xg_1^n(x,Q^2)$. Using the correct Q^2 evolution, we have checked that they are in excellent agreement with the very recent SMC proton data at $Q^2=10$ GeV^2 and the SLAC neutron data at $Q^2=2$ GeV^2.

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Many years ago Feynman and Field made the conjecture [1] that the quark sea in the proton may not be flavor symmetric, more precisely $\bar{d}>\bar{u}$, as a consequence of Pauli principle which favors $d\bar{d}$ pairs with respect to $u\bar{u}$ pairs because of the presence of two valence u quarks and only one valence d quark in the proton. This idea was confirmed by the results of the New Muon Collaboration (NMC) experiment [2] on the measurement of proton and neutron unpolarized structure function $F_2(x)$. It yields fair evidence for a defect in the Gottfried sum rule [3] and one finds

$$I_G=\int_0^1\frac{dx}{x}[F_2^p(x)-F_2^n(x)]$$
$$=0.235\pm0.026 \tag{1}$$

instead of the value $\frac{1}{3}$ predicted with a flavor symmetric sea, since we have, in fact,

$$I_G=\tfrac{1}{3}(u+\bar{u}-d-\bar{d})=\tfrac{1}{3}+\tfrac{2}{3}(\bar{u}-\bar{d})\ . \tag{2}$$

A crucial role of the Pauli principle may also be advocated to explain the well-known dominance of u over d quarks at high x [4], which explains the rapid decrease of the ratio $F_2^n(x)/F_2^p(x)$ in this region. Let us denote by $q^{\uparrow}$ ($q^{\downarrow}$), u or d quarks with helicity parallel (antiparallel) to the proton helicity. The double-helicity asymmetry measured in polarized-muon– (electron–) polarized-proton deep-inelastic scattering allows the determination of the quantity $A_1^p(x)$ which increases towards one for high x [5,6], suggesting that in this region $u^{\uparrow}$ dominates over $u^{\downarrow}$, *a fortiori* dominates over $d^{\uparrow}$ and $d^{\downarrow}$, and we will see now how it is possible to make these considerations more quantitative. Indeed at $Q^2=0$ the first moments of the valence quarks are related to the values of the axial vector couplings

$$\begin{aligned}u_{\rm val}^{\uparrow}&=1+F\ ,\\ u_{\rm val}^{\downarrow}&=1-F\ ,\\ d_{\rm val}^{\uparrow}&=\frac{1+F-D}{2}\ ,\\ d_{\rm val}^{\downarrow}&=\frac{1-F+D}{2}\ ,\end{aligned} \tag{3}$$

so by taking $F=\frac{1}{2}$ and $D=\frac{3}{4}$ (rather near to the quoted values [7] 0.461 ± 0.014 and 0.798 ± 0.013) one has $u_{\rm val}^{\uparrow}=\frac{3}{2}$ and $u_{\rm val}^{\downarrow}=\frac{1}{2}$ which is at the center of the rather narrow range $(d_{\rm val}^{\uparrow},d_{\rm val}^{\downarrow})=(\frac{3}{8},\frac{5}{8})$. The abundance of each of these four valence quark species, denoted by $p_{\rm val}$, is given by Eq. (3) and we assume that the distributions at high Q^2 "keep a memory" of the properties of the valence quarks, which is reasonable since for $x>0.2$ the sea is rather small. So we may write, for the parton distributions,

$$p(x)=F(x,p_{\rm val})\ , \tag{4}$$

where F is an increasing function of $p_{\rm val}$. The fact that the dominant distribution at high x is just the one corresponding to the highest value of $p_{\rm val}$ gives the correlation *abundance shape* suggested by the Pauli principle, so we expect broader shapes for more abundant partons. If $F(x,p_{\rm val})$ is a smooth function of $p_{\rm val}$, its value at the center of a narrow range is given, to a good approximation, by half the sum of the values at the extrema, which then implies [8]

$$u_{\rm val}^{\downarrow}(x)=\tfrac{1}{2}d_{\rm val}(x)\ . \tag{5}$$

This leads to

$$\begin{aligned}\Delta u_{\rm val}(x)&\equiv u_{\rm val}^{\uparrow}(x)-u_{\rm val}^{\downarrow}(x)\\ &=u_{\rm val}(x)-d_{\rm val}(x)\end{aligned} \tag{6}$$

and, in order to generalize this relation to the whole u quark distribution, we assume that Eq. (6) should also hold for quark sea and antiquark distributions, so we have

$$\Delta u_{\rm sea}(x) = \Delta \bar{u}(x) = \bar{u}(x) - \bar{d}(x) \ . \tag{7}$$

Moreover as a natural consequence of Eq. (3), we will assume[1]

$$\Delta d_{\rm val}(x) = (F-D)d_{\rm val}(x) \ . \tag{8}$$

Finally we will suppose that the d sea quarks (and antiquarks) are not polarized, i.e.,

$$\Delta d_{\rm sea}(x) = \Delta \bar{d}(x) = 0 \tag{9}$$

and similarly for the strange quarks,

$$\Delta s(x) = \Delta \bar{s}(x) = 0 \ . \tag{10}$$

Clearly the above simple relations (6)–(10) are enough for fixing the determination of the spin-dependent structure functions $xg_1^{p,n}(x,Q^2)$, in terms of the spin-average quark-parton distributions. We now proceed to present our approach for constructing the nucleon structure functions $F_2^{p,n}(x,Q^2)$, $xF_3^{\nu N}(x,Q^2)$, etc., in terms of Fermi-Dirac distributions which is motivated by the importance of the Pauli exclusion principle, as we stressed above. A first attempt for such a construction was made in [10], but here, as we shall see, our method is slightly different and leads to significant improvements. Let us consider u quarks and antiquarks only, and let us assume that at fixed Q^2, $u_{\rm val}^{\uparrow}(x)$, $u_{\rm val}^{\downarrow}(x)$, $\bar{u}\uparrow(x)$, and $\bar{u}^{\downarrow}(x)$ are expressed in terms of Fermi-Dirac distributions, in the scaling variable x, of the form

$$xp(x) = a_p x^{b_p}/(\exp\{[x-\tilde{x}(p)]/\bar{x}\}+1) \ . \tag{11}$$

Here $\tilde{x}(p)$ plays the role of the "thermodynamical potential" for the fermionic parton p and $\bar{x}$ is the "temperature" which is a universal constant. Since valence quarks and sea quarks have very different x dependences, we expect $0 < b_p < 1$ for $u_{\rm val}^{\uparrow,\downarrow}(x)$ and $b_p < 0$ for $\bar{u}^{\uparrow,\downarrow}(x)$. Moreover $\tilde{x}(p)$ is a constant for $u_{\rm val}^{\uparrow,\downarrow}(x)$, whereas for $\bar{u}^{\uparrow,\downarrow}(x)$, it has a smooth x dependence. This might reflect the fact that parton distributions contain two phases: a gas contributing to the nonsinglet part with a constant potential and a liquid which prevails at low x, contributing to the singlet part with a potential slowly varying in x, that we take linear in $\sqrt{x}$. In addition, in a statistical model of the nucleon [11], we expect quarks and antiquarks to have opposite potentials, consequently the gluon, which produces $q\bar{q}$ pairs, will have a zero potential. Moreover since in the process $G \to q_{\rm sea} + \bar{q}$, $q_{\rm sea}$ and $\bar{q}$ have opposite helicities, we expect the potentials for $u_{\rm sea}^{\uparrow}$ (or $\bar{u}^{\uparrow}$) and $\bar{u}^{\downarrow}$ (or $u_{\rm sea}^{\downarrow}$) to be opposite. So we take

$$\tilde{x}(\bar{u}^{\uparrow}) = -\tilde{x}(\bar{u}^{\downarrow}) = x_0 + x_1\sqrt{x} \ . \tag{12}$$

The d quarks and antiquarks are obtained by using Eqs. (5) and (7) and concerning the strange quarks, we take, in accordance with the data [12],

$$s(x) = \bar{s}(x) = \frac{\bar{u}(x) + \bar{d}(x)}{4} \ . \tag{13}$$

Finally for the gluon distribution, for the sake of consistency, we take a Bose-Einstein expression given by

$$xG(x) = \frac{a_G x^{b_G}}{e^{x/\bar{x}} - 1} \tag{14}$$

with the same temperature $\bar{x}$ and a vanishing potential, as discussed above. Since it is reasonable to assume that for very small x, $xG(x)$ has the same dependence as $x\bar{q}(x)$, we will take $b_G = 1 + \bar{b}$, where $\bar{b}$ is b_p for the antiquarks. So, except for the overall normalization a_G, $xG(x)$ has no free parameter.

To determine our parameters we have used the most recent NMC data [2] on $F_2^p(x)$ and $F_2^n(x)$ at $Q^2 = 4$ GeV^2 together with the most accurate neutrino data from the Chicago-Columbia-Fermilab-Rochester (CCRF) Collaboration [12,13] on $xF_3^{\nu N}(x)$ and the antiquark distribution $x\bar{q}(x)$ [12].

The universal temperature is found to be

$$\bar{x} = 0.120 \tag{15}$$

and for valence quarks we get the *three* free parameters

$$\begin{aligned} b(u_{\rm val}^{\uparrow}) &= \tfrac{1}{2} b(u_{\rm val}^{\downarrow}) = 0.417 \ , \\ \tilde{x}(u_{\rm val}^{\uparrow}) &= 0.442 \ , \\ \tilde{x}(u_{\rm val}^{\downarrow}) &= 0.128 \ . \end{aligned} \tag{16}$$

This relation between the b's is imposed by the small-x behavior of $xF_3^{\nu N}(x)$, $a^{\uparrow}$ and $a^{\downarrow}$ are not free parameters, but two normalization constants which are fixed from the obvious requirements to have the correct number of valence quarks in the proton. As we noticed before $u_{\rm val}^{\uparrow}(x)$ dominates, so it is not surprising to find that it has a larger potential than[2] $u_{\rm val}^{\downarrow}(x)$.

For antiquarks we have *four* additional free parameters

$$\begin{aligned} \bar{b} &= -0.358 \ , \\ \bar{a}^{\uparrow} &= 0.024 \ , \\ x_0 &= 0.215 \ , \end{aligned}$$

and

$$x_1 = -0.388 \quad \text{for } \bar{u}^{\uparrow} \ . \tag{17}$$

[1]It is amusing to remark that with the values of F and D quoted above, we have in fact $\Delta d_{\rm val}(x) = -\frac{1}{3} d_{\rm val}(x)$ which coincides with the so-called conservative SU(6) model [9].

[2]In a statistical model of the nucleon [11], the potentials associated with u and d quarks are taken in the ratio $2^{1/3}$ which is much smaller than the value of $\tilde{x}(u_{\rm val}^{\uparrow})/\tilde{x}(u_{\rm val}^{\downarrow}) \sim 3$ we have found.

$\bar{b}$ is the same for $\bar{u}^{\uparrow}$ and $\bar{u}^{\downarrow}$.

When $x \to 0$, from Pomeron universality, one expects $x\bar{u}(x) = x\bar{d}(x) \neq 0$, so $\bar{a}^{\downarrow}$ is determined by this constraint.

We show the results of our fit for $F_2^p(x) - F_2^n(x)$ and $F_2^n(x)/F_2^p(x)$ by the solid lines in Figs. 1(a) and 1(b) and for $xF_3^{\nu N}(x)$ and $x\bar{q}(x)$ in Figs. 2(a) and 2(b). The accuracy of these neutrino data gives strong constraints on both valence and sea quark distributions. The description of the data is very satisfactory, taking into account the fact that we only have *eight* free parameters and this certainly speaks for Fermi-Dirac distributions. Note that

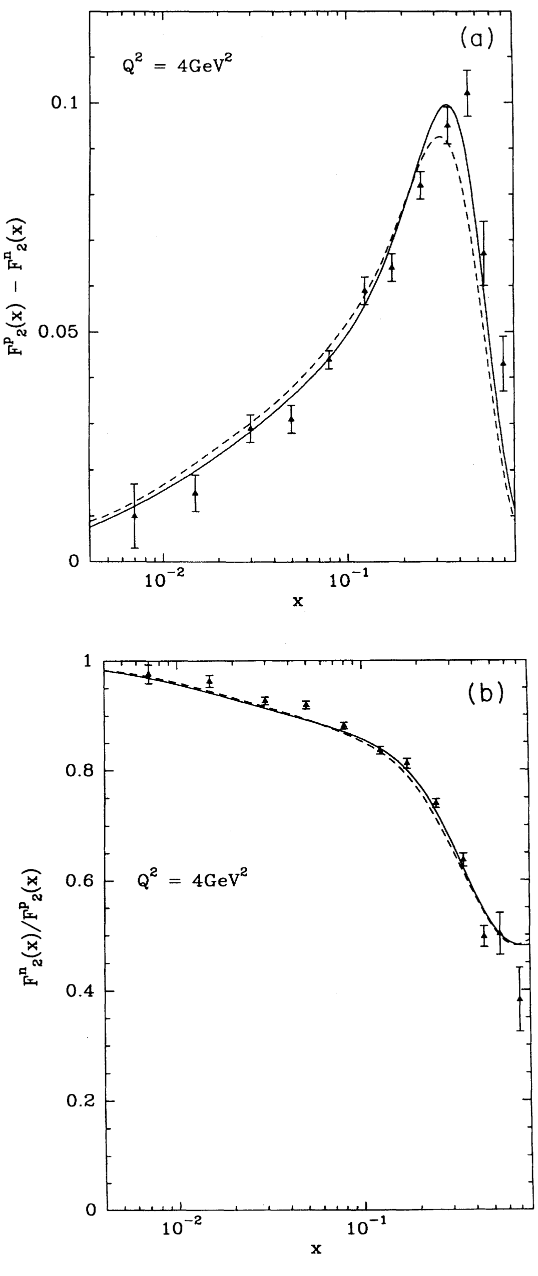

FIG. 1. (a) The difference $F_2^p(x) - F_2^n(x)$ at $Q^2 = 4$ GeV^2 vs x. Data are from [2] and the solid line is the result of our fit. The dashed line is the theoretical result after evolution at $Q^2 = 10$ GeV^2. (b) The ratio $F_2^n(x)/F_2^p(x)$ at $Q^2 = 4$ GeV^2 vs x. Data are from [2] and the solid line is the result of our fit. The dashed line is the theoretical result after evolution at $Q^2 = 10$ GeV^2.

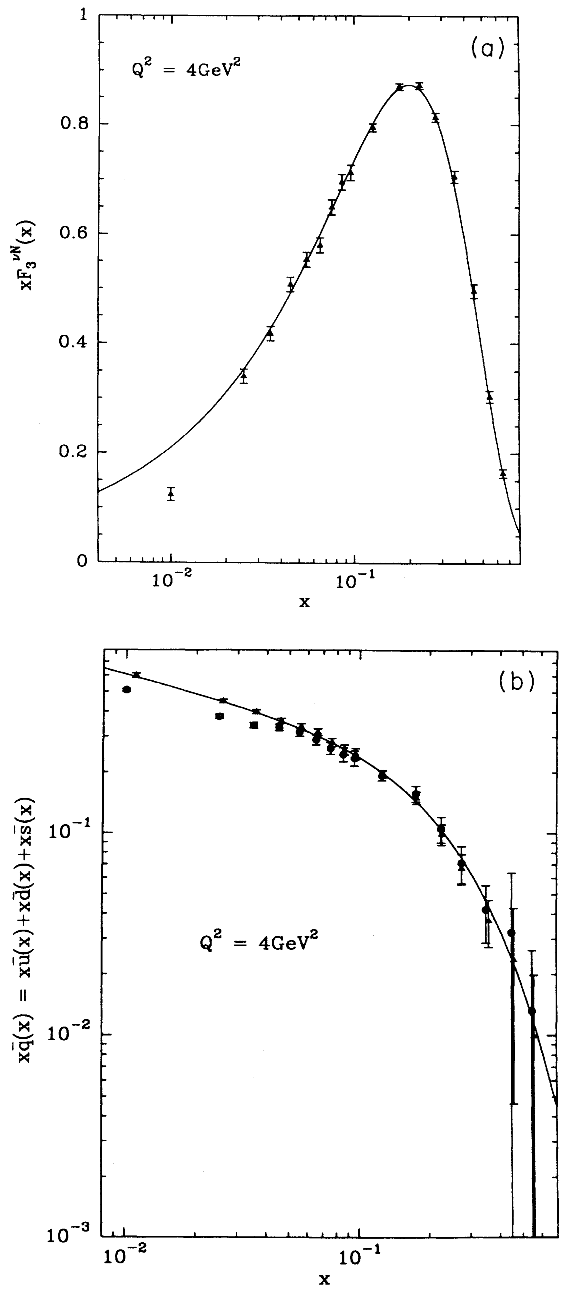

FIG. 2. (a) The structure function $xF_3^{\nu N}(x)$ vs x. Data are from [13] at Q^2=3 GeV^2 and the solid line is the result of our fit. (b) The antiquark contribution $x\bar{q}(x) = x\bar{u}(x) + x\bar{d}(x) + x\bar{s}(x)$ at $Q^2 = 3$ GeV^2 (solid circles) and $Q^2 = 5$ GeV^2 (solid triangles) vs x. Data are from [12] and solid line is the result of our fit.

we find $I_G = 0.228$ in beautiful agreement with Eq. (1). The steady rise of $x\bar{q}(x)$ at small x leads to a rise of F_2^p which is consistent with the first results from the DESY *ep* collider HERA [14]. We show in Fig. 3 the data compared to our calculations down to the small-x region. For the fraction of the total momentum carried by quarks and antiquarks we find

$$\int_0^1 xu(x)dx = 0.304, \quad \int_0^1 xd(x)dx = 0.148 \,, \qquad (18)$$
$$\int_0^1 x[\bar{u}(x) + \bar{d}(x) + s(x) + \bar{s}(x)]dx = 0.088 \,.$$

Concerning the gluon distribution, we find $a_G = 8.073$ and $xG(x)$ is fairly consistent with some preliminary indirect experimental determination from direct photon production [15], from neutrino deep-inelastic scattering [16] at $Q^2 = 5$ GeV^2, and at high Q^2 and smaller x from NMC [17].

Let us now turn to the polarized structure functions $xg_1^{p,n}(x,Q^2)$ which will allow to test our simple relations (6)–(10). Since the proton data from the EMC [5] and Spin Muon Collaboration (SMC) [6] are at $Q^2 = 10$ GeV^2 and the neutron data from[3] SLAC [18] is at $Q^2 = 2$ GeV^2, we have to consider the Q^2 evolution in order to use our parton distributions determined at $Q^2 = 4$ GeV^2. For this purpose we have used a numerical solution [20] of the Altarelli-Parisi equations [21] which lead to relatively small corrections in the Q^2 range we are dealing with. In Figs. 1(a) and 1(b) the dashed lines are the theoretical predictions at $Q^2 = 10$ GeV^2. As expected we see that for low x, $F_2^p(x) - F_2^n(x)$ increases with Q^2 whereas it decreases with Q^2 for high x, leaving the integral unchanged. The Q^2 dependence of the ratio $F_2^n(x)/F_2^p(x)$ has the right trend although probably a bit too weak compared to experimental observation which has been attributed to different higher twist effects for proton and neutron [22].

At this stage we would like to examine the consequence of our simple relations (6)–(10). If the d (valence and sea) quarks were unpolarized, Eqs. (6) and (7) allow us to relate the contribution of u quarks to $xg_1^p(x)$, to the contributions of u and d to $F_2^p(x) - F_2^n(x)$, i.e.,

$$xg_1^p(x)|_u = \tfrac{2}{3}(F_2^p(x) - F_2^n(x))|_{u+d} \,. \qquad (19)$$

First, by comparing Fig. 1(a) and Fig. 4[4] one sees very clearly, the similarity of the two sets of data points.[5]

[3]There is also some SMC data [19] on the polarized structure function xg_1^d on deuterium.

[4]The vertical scales have been chosen in such a way that one absorbs the factor $\frac{2}{3}$ by superimposing the two figures.

[5]This was first noticed in [8] but, with more accurate data, it becomes now very convincing.

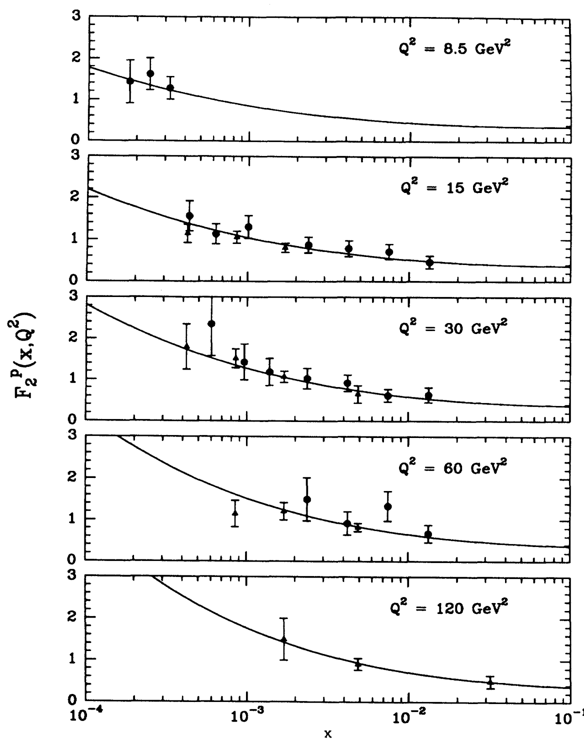

FIG. 3. The structure function $F_2^p(x)$ vs x for different Q^2 values. Data are from [14] and the solid lines are the results of our calculation.

Second, on Fig. 4 the dashed line represents simply the dashed line of Fig. 1(a) multiplied by a factor $\frac{2}{3}$, whereas the dotted line corresponds to the case of a flavor symmetric sea, i.e., $\bar{u}(x) = \bar{d}(x)$, or, in other words, to a zero polarization of the u sea quarks. This shows why we strongly suspect that the defect in the Gottfried sum rule and the defect in the proton Ellis-Jaffe sum rule [23] are closely related. We still think it has nothing to do with the polarization of the strange quarks, that we took to be zero [see Eq. (10)], which is supported by reasonable phenomenological arguments [24]. Moreover, in this approach, the strange quarks do not even participate, because they cancel in the difference $F_2^p(x) - F_2^n(x)$. Finally, if one takes into account the polarization of the d valence quarks by using Eq. (8), we get the solid line in Fig. 4 which improves the agreement with the data. In fact we found, for $Q^2 = 10$ GeV^2,

$$\int_{0.003}^{0.7} g_1^p(x)dx = 0.134 \qquad (20)$$

in beautiful agreement with Eq. (4) of [6].

Concerning the neutron polarized structure function $xg_1^n(x)$ we show in Fig. 5 a comparison of the SLAC data [18] at $Q^2 = 2$ GeV^2 with our theoretical calculations. The dashed line corresponds to the case where d quarks are assumed to be unpolarized and it clearly disagrees

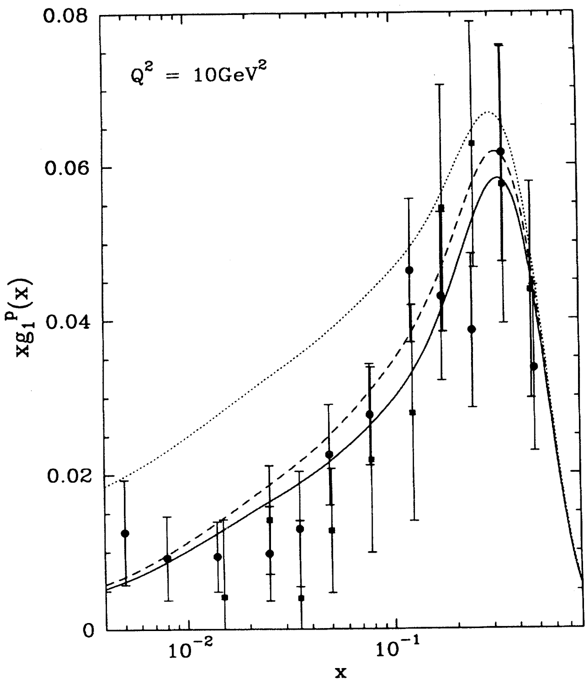

FIG. 4. $xg_1^p(x)$ at $\langle Q^2\rangle = 10$ GeV^2 vs x. Data are from [5] (solid squares) and [6] (solid circles) together with our predictions at $Q^2 = 10$ GeV^2. [The dotted line is the contribution of $\Delta u_{\rm val}(x)$ only, the dashed line is the contribution of $\Delta u(x)$ and $\Delta\bar{u}(x)$, and the solid line is the contribution of $\Delta u(x)$, $\Delta\bar{u}(x)$, and $\Delta d_{\rm val}(x)$.]

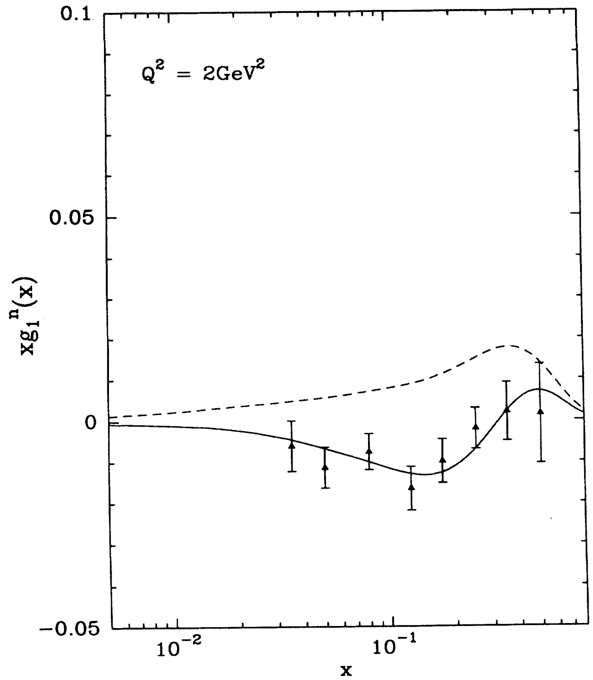

FIG. 5. $xg_1^n(x)$ at $\langle Q^2\rangle = 2$ GeV^2 vs x. Data are from [18] together with our predictions at $Q^2 = 2$ GeV^2 . [The dashed line is the contribution of $\Delta u(x)$ and $\Delta\bar{u}(x)$ only and the solid line is the contribution of $\Delta u(x)$, $\Delta\bar{u}(x)$, and $\Delta d_{\rm val}(x)$.]

with the data. However by including the d valence quark polarization according to Eq. (8), we obtain the solid line in perfect agreement with the data and we find for $Q^2 = 2$ GeV^2

$$\int_0^1 g_1^n(x)dx = -0.020\ . \tag{21}$$

To summarize we have given an accurate description of deep-inelastic-scattering data at low Q^2 in terms of Fermi-Dirac distributions parametrized with only eight free parameters for quarks and antiquarks. Although we have some understanding of their meaning, much remains to be done for a more fundamental theoretical interpretation, in terms of new information for the nucleon structure. We have proposed a set of simple relations between unpolarized and polarized quark (antiquark) distributions for which, so far, there is a striking experimental evidence. Of course our approach has to be further tested with more accurate deep-inelastic-scattering data and in particular the important issue of the validity of the Bjorken sum rule [25]. Polarized proton collisions at high energies will also provide independent tests which will be most welcome in the near future [26].

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