# Implications of a heavy top quark for the two Higgs doublet model

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Constraints on the two Higgs doublet model are presented, assuming a top quark mass of  $174\pm17$ GeV. We concentrate primarily on the "type II" model, where up-type quarks receive their mass from one Higgs doublet, and down-type quarks receive their mass from the second doublet. High energy constraints derived from the W mass, the full width of the Z, and the  $b\bar{b}$  partial width of the Z are constraints derived from the *w* mass, the fun width of the *z*, and the *oo* partial width of the *z* are combined with low energy constraints from  $\Gamma(b \to s\gamma)$ ,  $\Gamma(b \to c\tau\bar{\nu}_\tau)$  and  $B^0$ - $\bar{B}^0$  mixing to determine the experimentally favored configurations of the model. This combination of observables rules out small charged Higgs boson masses and small values of  $tan \beta$ , and provides some information about the neutral Higgs boson masses and the mixing angle  $\alpha$ . In particular, constraints derived from the  $\rho$  parameter rule out configurations where the charged Higgs boson is much heavier or much lighter than the neutral Higgs bosons. The agreement of the model with experiment is roughly as good as that found for the minimal standard model. We discuss a scenario where  $\Gamma(Z \to b\bar{b})$  is enhanced relative to the standard model result, which unfortunately is on the verge of being ruled out by the combination of  $\Gamma(b \to s\gamma)$  and  $\rho$  parameter constraints. Implications for various extensions of the standard model are briefly discussed.

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# I. INTRODUCTION

Precise measurements of electroweak observables are beginning to place significant constraints on extensions of the standard model. Measurements of the  $W$  mass [1], the partial widths and forward-backward asymmetries of the Z [2], and the branching ratio for  $B \to X_s \gamma$  [3] all provide significant information concerning unknown parameters of the standard model as well as constraining various types of new physics. Furthermore, recent experimental evidence for a top quark mass in the range  $174 \pm 17$  GeV [4] eliminates much of the uncertainty in these constraints. To date, the minimal standard model has succeeded in its description of virtually all phenomena of electroweak origin [5,6]. At the same time, however, there are some sectors of the theory about which we know very little. In particular, predictions of the standard model depend very weakly on the mass of the Higgs boson, and as a consequence we can say very little about the Higgs sector of the theory. In part for this reason, the extension of the standard model to include a second Higgs doublet is at present far from being ruled out experimentally. Furthermore, the inclusion of a second Higgs doublet is a common feature of many extensions of the standard model, such as supersymmetry or certain versions of the  $SO(10)$  and  $SU(5)$  grand-unified theories. It is therefore of interest to determine whether such a simple extension of the standard-model Higgs sector is compatible with experiment, and what further extensions of the model are indicated if it is not. In addition, we would like to determine whether the discovery of a charged or pseudoscalar Higgs boson is necessarily an indication of supersymmetry, or whether such a discovery could simply be an indication of an extended Higgs sector.

At tree level the two Higgs doublet model is identi-

cal, in most respects, to the standard model. It reproduces the important relation  $M_W^2 = M_Z^2 \cos^2 \theta_W$ , and furthermore the Yukawa couplings of the quarks can be chosen so as to eliminate Bavor-changing neutral Higgs interactions [7]. The two Higgs doublet model is also in agreement with experiment at the level of radiative corrections. The two Higgs doublet model differs from the standard model, however, in that radiative corrections often depend rather sensitively on the details of the Higgs sector. For this reason, experimental data are beginning to significantly constrain the parameters of the Higgs sector of the two-doublet model. Low-energy data such as  $\Gamma(b \to s\gamma)$ ,  $\Gamma(b \to c\tau\bar{\nu}_{\tau})$ , and the  $B^0$ - $\bar{B}^0$  mixingamplitude rule out small values of the charged Higgs boson mass, and significantly constrain  $tan \beta$ , the ratio of the vacuum expectation values. High-energy data such as the  $W$  mass and the full width of the  $Z$  constrain the Higgs boson masses and mixing angles; in particular, configurations which give a positive contribution to the  $\rho$  parameter are disfavored by these data. Additional stringent constraints are provided by

$$
R_b \equiv \Gamma(Z \to b\bar{b})/\Gamma(Z \to \mathrm{hadrons}) \ .
$$

Our aim here is to combine these constraints and to discuss the implications of a moderately heavy top quark for the model.

In Sec. II, we present a general overview of the two Higgs doublet model, with particular emphasis on the structure of the Yukawa couplings. In Sec. III, we discuss oblique corrections in the model, and discuss their implications concerning the top quark mass. In Sec IV, we introduce constraints on the model from high-energy data such as  $R_b$  and the full Z width. As a by-product, we demonstrate that the combination of  $\rho$  parameter constraints with constraints from  $R_b$  makes the model incompatible with a top mass larger than about 200 GeV. In Sec. V, we introduce constraints from low-energy data such as  $\Gamma(b \to s\gamma)$ ,  $\Gamma(b \to c\tau\bar{\nu}_\tau)$ , and  $B^0$ - $\bar{B}^0$  mixing, and illustrate how these rule out small charged Higgs boson masses and small values of  $tan\beta$ . In Sec. VI we combine all of the various constraints to determine the configurations of the model that are compatible with experiment for a top mass of 174 GeV. In Sec. VII we conclude.

### II. THE TWO HIGGS DOUBLET MODEL

In this section we briefly review some generalities of the two Higgs doublet model; for a more complete treatment, we refer the reader to Refs.  $[8,9]$ . The two Higgs doublet model is essentially the  $SU(2) \times U(1)$  standard model, modified by the extension of the Higgs sector to include two scalar isodoublets  $\Phi_1, \Phi_2$ , which we write in component form as

$$
\Phi_i = \begin{pmatrix} \phi_i^+ \\ (\phi_i^0 + i\chi_i^0)/\sqrt{2} \end{pmatrix} . \tag{1}
$$

The Yukawa couplings of the scalars and fermions are constrained by the requirement that there be no flavorchanging neutral Higgs interactions. It was shown in Ref. [7] that a necessary and sufficient condition for the elimination of Havor-changing neutral interactions in the Higgs sector is that each quark of a given charge must receive its mass from at most one Higgs field. One finds then that there are two possible arrangements for the Yukawa couplings: one may either couple all of the fermions to a single Higgs doublet (giving what is known as the "type-I" model), or one may couple  $\Phi_1$  to the right-handed down-type quarks, and  $\Phi_2$  to the righthanded up-type quarks (giving what is known as the "type-II" model). Here we will consider the latter possibility. Imposing the discrete symmetry  $\Phi_2 \rightarrow -\Phi_2$ ,  $(u, c, t)_R \rightarrow -(u, c, t)_R$  is enough to ensure that the Yukawa couplings have the correct form. The Yukawa Lagrangian has the form

$$
L_Y = \sum_{i=1}^{3} \bar{l}_{i,L} \Phi_1 e_{i,R} + \sum_{i,j=1}^{3} D_{ij} \bar{q}_{i,L} \Phi_1 d_{j,R}
$$
  
+ 
$$
\sum_{i,j=1}^{3} U_{ij} \bar{q}_{i,L} \Phi_2^c u_{j,R} + \text{H.c.} ,
$$
 (2)

where  $l_{i,L}, q_{i,L}$  are the left-handed lepton and quark doublets,  $U$  and  $D$  are the quark mass matrices, and  $\Phi_2^c = -i\sigma_2\Phi_2^*$ . The Higgs potential, as in the standard model, has its minimum at nonzero values of the fields  $\Phi_1, \Phi_2$ . After applying the requirement that there be no flavor-changing neutral interactions, one finds that the vacuum expectation values (VEV's) can be chosen both real and positive:

$$
\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1/\sqrt{2} \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2/\sqrt{2} \end{pmatrix} . \tag{3}
$$

After diagonalizing the Higgs mass matrix, one finds that the fields  $\phi_i^+$  mix to form a charged Nambu-Goldstone boson  $G^+$  and a charged physical scalar  $H^+$  of mass  $m_+$ .

$$
G^{+} = \cos \beta \phi_1^{+} + \sin \beta \phi_2^{+} ,
$$
  

$$
H^{+} = -\sin \beta \phi_1^{+} + \cos \beta \phi_2^{+} .
$$
 (4)

Similarly, the imaginary parts of the neutral components  $\chi_{1,2}^{0}$  mix to form a neutral Nambu-Goldstone boson  $G^{0}$ and a  $CP$ -odd physical scalar  $H_3^0$ , again with mixing angle  $\beta$ :

$$
G^{0} = \cos \beta \chi_{1}^{0} + \sin \beta \chi_{2}^{0} ,
$$
  
\n
$$
H_{3}^{0} = -\sin \beta \chi_{1}^{0} + \cos \beta \chi_{2}^{0} .
$$
\n(5)

Finally, the scalars  $\phi_{1,2}^0$  mix to form a pair of neutral  $CP$ -even scalars, now with mixing angle  $\alpha$ .

$$
H_1^0 = \cos \alpha \phi_1^0 + \sin \alpha \phi_2^0 ,
$$
  

$$
H_2^0 = -\sin \alpha \phi_1^0 + \cos \alpha \phi_2^0 .
$$
 (6)

The masses  $m_{1,2}$  of  $H_{1,2}^0$  obey  $m_1 \geq m_2$ .

The mixing angle  $\beta$  is given simply by

$$
tan\beta = \frac{v_2}{v_1} \ . \tag{7}
$$

The quantity  $tan\beta$  plays a central role in the theory because the Yukawa couplings are often proportional to either tan $\beta$  or cot $\beta$ . The Yukawa couplings of the various quarks to the various scalars have been summarized in Table I. We see in particular that couplings of up-type quarks are enhanced for small values of  $tan\beta$ , while couplings of down-type quarks are enhanced for large values

TABLE I. Quark Yukawa couplings in the "type-II" two doublet model. Overall factors of  $i-jg/2M_W, -g/2M_W, igV_{ud}/M_W$  have been omitted in the couplings of  $H_{1,2}^0$ ,  $H_3^0$ , and  $H^+$ , respectively.

Higgs boson	$\bm{u}\bar{\bm{u}}$	dd	ūd
$\overline{H_1^0}$ $H_2^0$ $H_3^0$ $H^+$	$m_{\bm u} \sin \alpha / \sin \beta$ $m_u \cos \alpha / \sin \beta$ $m_u \cot \beta \gamma_5$	$m_d {\rm cos}\alpha/\cos\beta$ $-m_d \sin \alpha / \cos \beta$ $m_d \tan \beta \gamma_5$	$m_u \cot\beta \left(\frac{1-\gamma_5}{2\sqrt{2}}\right) + m_d \tan\beta \left(\frac{1+\gamma_5}{2\sqrt{2}}\right)$

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of tan $\beta$ . Furthermore, one can decouple  $H_1^0$  from the up-type quarks by choosing  $\alpha = 0$ , and so on. The large top quark mass, in combination with the enhancement of the top Yukawa coupling for small values of  $tan\beta$  makes it possible to derive a lower bound on  $tan \beta$ . Furthermore, we note that since up-type quark masses are proportional to  $v_2$  and down-type masses to  $v_1$ , the mass hierarchy  $m_t \gg m_b$  tends to favor large tan $\beta$ . In the type-I model, where all of the quarks receive their masses from (say)  $\Phi_1$ , the Yukawa couplings of the fermions are all proportional to  $cot\beta$  or  $csc\beta$ ; consequently the Yukawa couplings of the bottom quark are negligible compared to those of top quark regardless of the value of  $tan\beta$ .

### III. OBLIQUE CORRECTIONS IN THE TWO HIGGS DOUBLET MODEL

In this section we discuss some features of oblique corrections to electroweak observables in the two Higgs doublet model. It has been known for quite some time that oblique radiative corrections from the Higgs sector can be significantly larger in the two Higgs model than in the minimal standard model  $[9-11]$ . In particular, the contribution of the Higgs sector to  $\Delta \rho$ , defined by

$$
\Delta \rho = \frac{\Sigma_{WW}(0)}{M_W^2} - \frac{\Sigma_{ZZ}(0)}{M_Z^2} \;, \tag{8}
$$

where  $\Sigma_{WW,ZZ}$  are the W and Z self-energies, can be significantly larger in the two doublet model than in the standard model. The nonstandard. contributions to the vector boson self-energies in the two doublet model have been presented in Refs.  $[9,12]$ . In the minimal standard model, the contribution of the Higgs boson to  $\Delta \rho$  is given, in the limit  $m_{\text{Higgs}} \rightarrow \infty$ , by

$$
\Delta \rho_{\text{Higgs}}(\text{MSM}) \simeq -\frac{3G_F M_W^2}{8\sqrt{2}\pi^2} \tan^2 \theta_W \left[ \ln \frac{m_{\text{Higgs}}^2}{M_W^2} - \frac{5}{6} \right] \tag{9}
$$

and grows only logarithmically with the Higgs boson mass. By contrast, the top quark contribution to  $\Delta \rho$ is given by

$$
\Delta \rho_{\rm top} \simeq \frac{3G_F}{8\sqrt{2}\pi^2} m_t^2 \;, \tag{10}
$$

which grows quadratically with  $m_t$ . It is the weak dependence of  $\Delta \rho$  on  $m_{\text{Higgs}}$  that excludes top quark masses larger than 200  $\text{GeV}$  in the minimal standard model. Since  $\Delta \rho$  grows quadratically with  $m_t$ , but falls off only logarithmically with  $m_{\text{Higgs}}$ , one must have an exponentially large Higgs boson mass to prevent  $\Delta \rho$  from becoming too large when the top is heavy; since we expect on general grounds that  $m_{\text{Higgs}} \leq 1 \text{ TeV}$ , the standard model cannot accommodate a top mass larger than about 200 GeV. In the two Higgs doublet model, the situation is different. Here the leading behavior (for Higgs boson masses much larger than the W mass) of  $\Delta \rho$  is given by [11]

$$
\Delta \rho_{\text{Higgs}} = \frac{3G_F}{8\sqrt{2}\pi^2} [\sin^2(\alpha - \beta) F(m_+, m_3, m_1) + \cos^2(\alpha - \beta) F(m_+, m_3, m_2)] , \quad (11)
$$

where

$$
F(m_+, m_3, m_{1,2}) = m_+^2 - \frac{m_+^2 m_3^2}{m_+^2 - m_3^2} \ln\left(\frac{m_+^2}{m_3^2}\right)
$$

$$
-\frac{m_+^2 m_{1,2}^2}{m_+^2 - m_{1,2}^2} \ln\left(\frac{m_+^2}{m_{1,2}^2}\right)
$$

$$
+\frac{m_3^2 m_{1,2}^2}{m_3^2 - m_{1,2}^2} \ln\left(\frac{m_3^2}{m_{1,2}^2}\right).
$$
 (12)

The Higgs contribution to  $\Delta \rho$  can be either positive or negative depending on the ordering of the Higgs boson masses, and in general grows quadratically with the largest Higgs boson mass. If the Higgs boson masses are ordered as

$$
m_{1,2} < m_+ < m_3
$$
 or  $m_3 < m_+ < m_{1,2}$ , (13)

then  $\Delta \rho_{\text{Higgs}}$  will be negative and grow quadratically with the largest Higgs boson mass; if the charged Higgs boson is heavier or lighter than all of the neutral Higgs bosons, then  $\Delta\rho_{\rm Higgs}$  will be positive. The negative contribution to  $\Delta \rho$  is largest if  $m_+ \simeq 0.562 m_{\rm heavy}$ , where  $m_{\mathrm{heavy}}$  is the largest Higgs boson mass. For appropriate values of  $\alpha - \beta$ , one then has

$$
\Delta \rho_{\text{Higgs}} \simeq -\frac{3G_F}{8\sqrt{2}\pi^2} \times 0.216 m_{\text{heavy}}^2 \ . \tag{14}
$$

Since the top quark contribution to  $\Delta \rho$  is quite large, the Higgs contribution to  $\Delta \rho$  must be either small (if it is positive) or negative.

## IV. CONSTRAINTS FROM THE FULL  $Z$  WIDTH AND  $R_b$

In this section we introduce constraints derived from the full Z width and the  $b\bar{b}$  partial width. The measurements of these quantities at the CERN  $e^+e^-$  collider LEP [2] are

(10) 
$$
\Gamma(Z \to \text{all}) = 2.4974 \pm 0.0038 \text{ GeV}
$$
 (15)

 $and$ 

$$
R_b \equiv \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})} = 0.2202 \pm 0.0020 \ . \tag{16}
$$

The quantity  $R_b$  is particularly convenient to work with, since oblique and QCD corrections to  $\Gamma(Z \to bb)$  and  $\Gamma(Z \to \text{hadrons})$  cancel to a large extent in the ratio  $R_b$ . On the other hand, the full  $Z$  width is rather sensitive to oblique corrections through the  $\rho$  parameter. We begin with a general discussion of the radiative corrections to  $\Gamma(Z \to f\bar{f}).$ 

The Z width in the two Higgs doublet model has been studied previously in Ref. [13], and, in the context of the supersymmetric standard model, in Ref. [14]. We have independently carried out the calculation, and our results agree with those found previously. The vertex connecting the  $Z$  to a pair of fermions may be written as

$$
\Gamma_{\mu} = ie\gamma_{\mu}(v - a\gamma_5) , \qquad (17)
$$

where at the tree level the vector and axial vector couplings  $v$  and  $a$  are

$$
v = \frac{I_3 - 2Qs^2}{2sc} \text{ and } a = \frac{I_3}{2sc} \ . \tag{18}
$$

We use the abbreviations  $s \equiv \sin \theta_W$ ,  $c \equiv \cos \theta_W$ . Radiative corrections modify these couplings through selfenergy and vertex corrections. The self-energy corrections arise from the  $Z$  self-energy and from the  $Z$ -photon mixing. The Z self-energy insertion introduces an overall factor of the  $Z$  wave-function renormalization constant. The Z-photon mixing modifies the vector coupling of the  $Z$  to a fermion pair: we make the replacement

$$
v \rightarrow \frac{I_3 - 2Qs^2}{2sc} + Q \frac{\Sigma_{AZ}(M_Z^2)}{M_Z^2}
$$
 (19)

to include this effect. Vertex diagrams introduce further corrections to the vector and axial couplings of the  $Z$ , which we denote by  $\delta v, \delta a$ . Neglecting fermion masses, we find that the partial  $Z$  width into a particular final state may then be written

$$
\Gamma(Z \to f\bar{f}) = Z_Z \frac{n_c \alpha M_Z}{3} \left( v^2 + a^2 + 2v \operatorname{Re}\delta v + 2a \operatorname{Re}\delta a + 2vQ \operatorname{Re} \frac{\Sigma_A Z(M_Z^2)}{M_Z^2} \right), \tag{20}
$$

where  $n_c$  is the number of colors and  $Z_Z$  is the Z wavefunction renormalization constant, which cancels in the ratio  $R_b$ . We have employed the renormalization scheme of Hollik [15] throughout our calculations.

In addition to these purely electroweak corrections, partial widths into quarks are modified by QCD corrections. For quarks other than the  $b$ , the QCD corrections enhance the partial width by about 4%: we have

$$
\Gamma(Z \to q\bar{q})
$$
  
=  $\Gamma^0(Z \to q\bar{q}) \left[ 1 + \frac{\alpha_s(M_Z^2)}{\pi} + 1.41 \left( \frac{\alpha_s(M_Z^2)}{\pi} \right)^2 \right],$  (21)

where  $\Gamma^0$  is the uncorrected width. In the case of b quarks, the finite  $b$  mass and the large mass of the top quark modify the QCD corrections. The relevant correctgions have been calculated in Refs.  $[16-18]$ . In Ref.  $[16]$ , it was pointed out that the vector and axial parts of the  $Z$  width into  $b$  quarks are affected differently by QCD. In particular, the vector-current contribution to the  $Z$  width into  $b$  quarks is corrected by the same QCD factor appearing in Eq. (21), while the axial part  $\Gamma_A$  is corrected by a factor

$$
\Gamma_A = \Gamma_A^0 \left[ 1 + \frac{\alpha_s(M_Z^2)}{\pi} + [1.41 - I(z)/3] \left( \frac{\alpha_s(M_Z^2)}{\pi} \right)^2 \right], \qquad (22)
$$

where  $z = M_Z^2/4m_t^2$ , and  $-I(z)/3$  is negative and increases in magnitude for increasing top mass. For a top mass of 100 GeV,  $-I(z)/3 \sim -3$ ; for 250 GeV,  $-I(z)/3 \sim -5$ . The impact of these QCD corrections is to slightly reduce  $R_b$ . A second set of QCD corrections has been discussed in Refs.  $[17,18]$ , where corrections to  $\Gamma(Z \to b\bar{b})$  due to the finite b mass have been calculated. These corrections enhance  $\Gamma(Z \to b\bar{b})$  by about 2 MeV, giving a small increase in  $R_b$ . The combined effect of all of these QCD corrections is to enhance  $R_b$  by about  $0.4\%$ .

In Fig. 1, we have plotted  $R_b$  as a function of the top mass for both the minimal standard model and the two Higgs model for a few values of  $tan\beta$ . The Z width into b quarks is of particular interest because of the presence of virtual top quarks in various vertex corrections. These corrections can be used to constrain  $tan\beta$  and the

0.225 0.2225  $\widetilde{R_d}$  (MSM) 0.22 0.2175  $0.215$  $0.2125$  $R_h$  (2HD  $0.21$ 150 100  $125$ 175 200 225 250  $m_{top}$  (GeV)

FIG. 1.  $R_b$  as a function of the top mass in the minimal standard model and in the two Higgs doublet model. The solid curve is the standard model result for  $m_{\text{Higgs}} = 100$ GeV. The upper and lower dashed curves are the two Higgs doublet results for  $m_2 = m_3 = 50$  GeV,  $m_1 = 875$  GeV,  $m_+ = 422$  GeV, and  $tan\beta = 70$ , 1, respectively. For comparison, the standard-model result for  $R_d$ , the corresponding quantity for d quarks, is also shown. The error bars indicate the  $1\sigma$  experimental measurements of  $m_t$  and  $R_b$ .

charged Higgs mass, and, to a lesser extent, the mixing angle  $\alpha$  and the neutral Higgs masses. The Higgs boson exchange vertex diagrams contributing to  $\Gamma(Z \to b\bar{b})$ are shown in Fig. 2. These break down into two classes: those involving charged Higgs boson exchange, and those involving neutral Higgs boson exchange. The dependence of  $R_b$  on the top mass results from the diagrams involving charged Higgs boson exchange, since these involve the top quark Yukawa couplings of Table I. These diagrams tend to suppress the decay of the  $Z$  into  $b$  quark pairs by an amount that grows quadratically with the top mass. In the minimal standard model, the effect of a heavy top is to suppress the  $Z$  width into  $b$  quarks by an amount  $\left[19\right]$ 

$$
\Delta\Gamma(Z \to b\bar{b})_{\text{MSM}} \simeq -\frac{\alpha_{\text{QED}}^2 M_Z}{8\pi s^2} \frac{1}{4s^2 c^2} \left(1 - \frac{2s^2}{3}\right)
$$

$$
\times \left[\frac{m_t^2}{M_W^2} + \left(\frac{8}{3} + \frac{1}{6c^2}\right) \ln \frac{m_t^2}{M_W^2}\right] \tag{23}
$$

in the limit  $m_t \gg M_W$ . In the two Higgs doublet model, diagrams involving charged Higgs exchange further suppress the Z width into b quarks by an amount (for  $m_+$ ,  $m_t \gg M_W)$ 

$$
\Delta\Gamma(Z \to b\bar{b})_H \pm \simeq -\frac{\alpha_{\text{QED}}^2 M_Z}{8\pi s^2} \frac{1}{4s^2 c^2} \frac{m_t^2}{M_W^2 \tan^2\beta} \times \left(1 - \frac{2s^2}{3}\right) \times \left(-\frac{x}{(x-1)^2}\ln x + \frac{x}{x-1}\right), \quad (24)
$$



FIG. 2. Vertex diagrams involving charged and neutral Higgs boson exchange in the two doublet model. Diagrams involving neutral Higgs boson exchange that do not contain potentially large trigonometric factors (such as  $\cot^2\beta$ ,  $\sec^2\beta$ ) have been neglected.

where  $x = m_t^2/m_+^2$ . This correction falls off for large  $m_+$  and grows with increasing  $m_t$ . The corrections due to charged Higgs boson exchange are always negative, and increase with increasing top mass. This makes it dificult to accommodate a 150—200 GeV top quark mass, particularly for small values of  $tan \beta$ ; for a top quark mass of 174 GeV, these diagrams exclude small values of  $tan\beta$ .

The situation is somewhat different in the case of diagrams involving neutral Higgs boson exchange. Here we find that the contribution to  $\Gamma(Z \to b\bar{b})$  can be either positive or negative, depending on the masses of the neutral scalars. These diagrams have no top mass dependence, of course, since they involve only b quarks. Their explicit forms have been reported in Ref. [13], and so we do not reproduce them here. These diagrams are only important for large values of  $tan \beta$ , for it is in this situation that the bottom quark's Yukawa coupling becomes large. In Fig. 3, we have plotted the correction to  $\Gamma(Z \to b\bar{b})$  resulting from neutral Higgs boson exchange as a function of  $m_3$  for the special case  $\alpha = \pi/2, m_2 = 50$ GeV, and  $tan\beta = 70$ . In this case  $H_1^0$  decouples from the vertex. The correction is positive for small and roughly equal masses  $m_2 \simeq m_3$ , but becomes negative for large mass splittings. This is in fact one of the few situations where vertex corrections significantly enhance the Z partial width into b quarks. These positive corrections make it possible to accommodate a heavy top quark, particularly if the neutral Higgs bosons are light and  $tan\beta$  is large. As we will show below, however, large top quark masses are not admissible in the model.

We have also studied the impact of these vertex corrections on the partial widths for  $Z \to \tau^+\tau^-$  and  $Z \to \nu_\tau \bar{\nu}_\tau$ . Here the Higgs boson exchange vertex corrections are important only for large values of  $tan\beta$  and when two or more neutral scalars are light. Both of the partial widths are enhanced by a small amount by these corrections;



FIG. 3. Correction to  $\Gamma(Z \to b\bar{b})$  from  $H_{2,3}^0$  exchange as a function of  $m_3$  for  $m_2 = 50$  GeV,  $\tan\beta = 70$ , and  $\alpha = \pi/2$ .

however, the effect is less than 1 MeV in both cases.

In light of the present discrepancy between the standard-model prediction of  $R_b$  and the measured value, one interesting feature of the two doublet model is the enhancement of  $R_b$  relative to the standard-model result when  $tan\beta$  is large and two or more of the neutral scalars are light. Although we will show below that such configurations of the model are on the verge of being ruled out by  $\Gamma(b \to s\gamma)$ , we feel that direct experimental tests of this scenario are worth brieQy discussing. Since the enhancement of  $R_b$  requires small masses for the neutral scalars, it may be rather easy to test at LEP II. The cross section for the process  $e^+e^- \rightarrow Z^*H_2^0$  is suppressed relative to the standard-model result for  $e^+e^- \rightarrow Z^*H^0$  by a factor of  $\sin^2(\alpha - \beta)$ . If  $\sin^2(\alpha - \beta)$  is not too small, it should be possible to detect a light scalar in the mass range  $m_2 \sim 50{\text -}100 \text{ GeV}$  at LEP II using data from a standard-model Higgs boson search. Barring extremely small values of  $\sin^2(\alpha - \beta)$ , this should provide a fairly conclusive experimental test of this scenario. In the event that  $\sin^2(\alpha - \beta)$  does turn out to be small, the process that sm  $(\alpha - \beta)$  does turn out to be small, the process<br> $e^+e^- \rightarrow Z^* \rightarrow H_2^0H_3^0$ , for which the cross section is proportional to  $\cos^2(\alpha - \beta)$ , may provide a complementary experimental probe of this scenario. The current lower bound on  $m_2$  [20] ranges from about 58.4 GeV for  $\sin^2(\alpha - \beta) = 1$  to about 30 GeV for  $\sin^2(\alpha - \beta) = 0.05$ . Finally, we note that this scenario is relevant only to the type-II model. In a type-I model, the Yukawa couplings of both top and bottom quarks are proportional to  $cot\beta$ . Consequently the positive vertex corrections from neutral Higgs boson exchange are far smaller than the negative corrections from charged Higgs boson exchange, and so one does not expect significant enhancements of  $\Gamma(Z \to b\bar{b})$  at large values of tan $\beta$ .

The various corrections to the  $Zb\bar{b}$  vertex also modify the  $b\bar{b}$  forward-backward asymmetry of the Z. At tree level,  $A_{FB}(b\bar{b})$  is given by

$$
A_{\rm FB}(b\bar{b}) = \frac{3}{4} \frac{2v_e a_e}{v_e^2 + a_e^2} \frac{2v_b a_b}{v_b^2 + a_b^2} , \qquad (25)
$$

where  $v_f$ ,  $a_f$  are the vector and axial vector couplings of the electron and b quark. Vertex corrections tend to modify the left-handed coupling of the  $b$  for small values of  $tan\beta$ , and the right-handed coupling of the b for large values of tan $\beta$ . We have calculated the  $b\bar{b}$  forward-backward asymmetry of the Z including oblique and vertex corrections; additional contributions from box diagrams are known to be numerically small and so are neglected here. The forward-backward asymmetry shows much the same behavior as  $R_b$  as a function of the Higgs boson masses and mixing angles:  $A_{FB}(b\bar{b})$  is suppressed at small values of  $tan\beta$  as a result of charged Higgs effects, while in the case of large  $tan\beta$  there are several possibilities. If all of the neutral Higgs bosons are light (with masses less than 100 GeV) and  $tan\beta$  is large, then  $A_{FB}$  is enhanced slightly; if  $H_2^0,H_3^0$  are light, but  $H_1^0$  is much heavier then  $A_{\text{FB}}$  is enhanced for  $|\alpha| \simeq \pi/2,$  but suppressed for  $\alpha \simeq 0$ . In addition to its sensitivity to vertex corrections, AFB is also sensitively dependent on oblique corrections through the value of  $\sin^2 \theta_W$ . The enhancement of  $A_{\text{FB}}$ at large values of  $tan\beta$  limits the extent to which  $R_b$  can be enhanced by neutral Higgs efFects.

To illustrate the range of top masses that can be accommodated by the two doublet model, we can attempt to construct a scenario which allows a very large top mass by arranging for negative contributions to  $\Delta \rho$  and positive contributions to  $R_b$  from the Higgs sector. In order to have positive contributions to  $R_b$ ,  $m_3$  must be small. The requirement of a small mass for  $H_3^0$  together with the fact that  $m_2 \leq m_1$  implies that we must have the mass hierarchy  $m_2, m_3 < m_+ < m_1$ . We will then have a negative contribution to  $\Delta \rho$  for  $|\alpha - \beta| = \pi/2$ . Here, however, we run into a conflict with the constraint from  $R_b$ : in order to have simultaneously tan $\beta$  large and  $|\alpha - \beta| = \pi/2$ , we must have  $\beta \simeq \pi/2$ , and  $\alpha \simeq 0$ . Referring to Table I, we see that for this value of  $\alpha$  the Yukawa coupling of  $H_2^0$  is small; hence it is not possible to simultaneously obtain a negative contribution to the  $\rho$  parameter and still have positive vertex corrections to  $R_b$ . Although either constraint separately can permit a very large top mass (on the order of 250 GeV or larger), when the two are combined the top mass is constrained to be less than about 200 GeV, consistent with both standard-model expectations and recent evidence for top quark production at the Collider Detector at Fermilab (CDF) [4]. The existence of this upper bound has been noted previously by the authors of Ref. [21]. The mass limit of 200 GeV has been established by a systematic search of the entire parameter space of the two doublet model.

The full Z width provides constraints on the charged Higgs boson mass. The  $Z$  width is quite sensitive to oblique corrections through the  $\rho$  parameter: if the  $Z$ width is expressed in terms of  $G_F$ , the effect of oblique corrections is to shift the value of  $\sin^2\theta_W$  and to renormalize the width by a factor  $\rho$ . As a result, positive contributions to  $\Delta \rho$  tend to enhance the Z width. This constrains the mass splittings in Higgs sector of the two doublet model; in particular, at the  $1\sigma$  level and with a 174 GeV top quark, we find that the charged Higgs boson mass is constrained by

$$
m_{\text{light}} - 130 \text{ GeV} \le m_+ \le m_{\text{heavy}} + 130 \text{ GeV} , \quad (26)
$$

where  $m_{\rm heavy, light}$  are the masses of the heaviest and lightest neutral Higgs bosons, respectively. These mass limits have again been derived by a systematic search of the entire parameter space of the two doublet model, allowing each of the Higgs boson masses to vary between 50 and 1000 GeV, and each of the mixing angles to vary over their full range.

# V. CONSTRAINTS FROM  $\Gamma(b \to s\gamma),$ <br> $B^0$ - $\bar{B}^0$  MIXING, AND  $\Gamma(b \to c\tau\bar{\nu}_\tau)$

The CLEO Collaboration [3] has recently reported a value for the inclusive branching ratio for radiative B decays:

$$
\mathcal{B}(\bar{B}\to X_s\gamma)=(2.32\pm 0.51\pm 0.29\pm 0.32)\times 10^{-4}~,
$$

where the first error is statistical, and the latter two arise from systematic errors in the yield and efficiency, respectively. The 95% confidence level limits on the branching

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ratio are [3)

$$
1 \times 10^{-4} < \mathcal{B}(\bar{B} \to X_s \gamma) < 4 \times 10^{-4} \ . \tag{27}
$$

This result can be used to rule out small charged Higgs boson masses and small values of  $tan\beta$ . The relevant electroweak diagrams are shown in Fig. 4, and have been calculated in Ref. [22]. Ignoring QCD corrections, the effective Lagrangian giving rise to the  $b \to s\gamma$  transition Is

$$
L_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e}{8\pi^2} m_b \bar{s}_{L,\alpha} \sigma_{\mu\nu} b_L^{\alpha} F^{\mu\nu}
$$

$$
\times \left[ F \left( \frac{m_t^2}{M_W^2} \right) + G \left( \frac{m_t^2}{m_+^2} \right) \right], \qquad (28)
$$

where  $\alpha$  is a color index, and F, G are given by [22]

$$
F(x) = \frac{3x^3 - 2x^2}{4(x-1)^4} \ln x + \frac{-8x^3 - 5x^2 + 7x}{24(x-1)^3} ,
$$
\n(29)

$$
G(y) = \frac{3y^2 - 2y}{6(y-1)^3} \ln y + \frac{-5y^2 + 3y}{12(y-1)^2} + \cot^2 \beta \left[ \frac{3y^3 - 2y^2}{12(y-1)^4} \ln y + \frac{-8y^3 - 5y^2 + 7y}{72(y-1)^3} \right].
$$

These results are only valid when one ignores QCD effects. In order to incorporate QCD, it is necessary to resum the large logarithms  $\sim \ln(M_W^2/m_b^2)$  using the renormalization group. The leading logarithmic QCD corrections result in an overall enhancement of the  $b \to s \gamma$  transition rate by a factor of 3 to 5 over that which would be obtained using this naïve effective Lagrangian. These corrections have been discussed extensively in the literature [23—27]. Here we will use the QCD calculations of Ref. [25]. There are minor differences between the various calculations of the anomalous dimension matrix; however these result in only small (on the order of 1% or less) variations in the calculated value of  $\mathcal{B}(b \to s\gamma)$ . To extract a value of  $\mathcal{B}(\bar{B} \to X_s \gamma)$ , it is convenient to use the approximation [23]



FIG. 4. Diagrams in the two Higgs doublet model contributing to the magnetic-moment-type operator that mediates the  $b\to s\gamma$  transition.

$$
\frac{\Gamma(\bar{B}\to X_s\gamma)}{\Gamma(\bar{B}\to X_c e\bar{\nu}_e)}\simeq \frac{\Gamma(b\to s\gamma)}{\Gamma(b\to c e\bar{\nu}_e)}, \qquad (30)
$$

which reduces uncertainties due to a factor  $m_b^5$  which appears in both numerator and denominator. Including the leading logarithmic QCD corrections, we then have

$$
\frac{\Gamma(\bar{B}\to X_s\gamma)}{\Gamma(\bar{B}\to Xe\bar{\nu}_e)}\simeq \frac{6\alpha}{\pi f(m_c/m_b)}\frac{|V_{ts}^*V_{tb}|^2}{|V_{bc}|^2}|A(m_b)|^2\ ,\qquad (31)
$$

where  $A(m_b)$  is the coefficient of the operator  $(e/8\pi^2)m_b\bar{s}_L\sigma_{\mu\nu}b_LF^{\mu\nu}$  evaluated at the scale  $m_b$  and  $f(m_c/m_b) \simeq 0.316$  is the phase-space factor for the semileptonic decay. The coefficient  $A(m_b)$  is given in terms of  $\eta \equiv \alpha_s(m_b)/\alpha_s(M_W)$  by

$$
A(m_b) = \eta^{-16/23} A(M_W) + \frac{8}{3} (\eta^{-14/23} - \eta^{16/23}) B(M_W) + \sum_{i=1}^{8} \eta^{\alpha_i} x_i,
$$
 (32)

where  $A(M_W) = F(x) + G(y)$ ,  $B(M_W)$  is the coefficient of the "chromomagnetic moment operator"  $(g/8\pi^2) m_b \bar{s}_L^{\alpha} \sigma_{\mu\nu} T^a_{\alpha\beta} b_L^{\beta} G^{\mu\nu}_{\alpha}$  evaluated at the scale  $M_W$ , and the sum results from mixing with the four-quark operator  $(\bar{c}_{L\alpha}\gamma^{\mu}b_{L\alpha})(\bar{s}_{L\beta}\gamma_{\mu}c_{L\beta})$ . The numbers  $\alpha_i$  and  $x_i$ are related in a simple way to the eigenvalues and eigenvectors of the anomalous dimension matrix. We have sed  $|V_{ts}^*V_{tb}|^2/|V_{bc}|^2 = 0.95$ .

Recent analyses of the uncertainties in the leading logarithmic calculation have been given in Refs. [27,28], where it was pointed out that the theoretical uncertainty is dominated by the unknown next-to-leading logarithmic corrections. Other significant uncertainties result from the experimental error in the measurements of  $\alpha_s(M_Z) = 0.12 \pm 0.01$  and the ratio  $m_c/m_b$ , which occurs in the phase-space factor for the semileptonic decay. Combining the various theoretical uncertainties given in Ref. [27] in quadrature, one finds that the overall error for the standard-model result is on the order of 30%, the error estimate of Ref. [28] is also roughly of this magnitude. In the case of the two doublet model, the situation is complicated by the greater sensitivity of the result to the next-to-leading-order (NLO) corrections. This has been pointed out in Ref. [28]. In light of this, the constraints from  $b \rightarrow s\gamma$  presented here should be considered as qualitatively correct, but nonetheless subject to considerable theoretical uncertainty. To factor in these uncertainties, in deriving limits on  $tan \beta$  and the charged Higgs boson, we have employed a 30% estimate of the theoretical error. We require only that

 $0.7 \times$  (theoretical estimate)

## $\leq$  experimental upper bound . (33)

This has the efFect of weakening somewhat the lower bound on the charged Higgs boson mass.

Additional low-energy constraints on the two doublet model can be obtained using  $B^0$ - $\bar{B}^0$  mixing. Here the relevant quantity is  $x_b \equiv \Delta m/\Gamma$ , where  $\Delta m$  is the mass

difference between the heavy and light admixtures of  $B^0$ and  $\bar{B}^0$ , and  $\Gamma$  is their average width. By observing time-dependent  $B^0$ - $\bar{B}^0$  oscillations, ALEPH and DEL-PHI have determined the mass splitting to be  $\Delta m =$  $3.41 \times 10^{-4}$  eV, which yields  $x_b = 0.797 \pm 0.128$  [29]. Measurements of the time integrated mixing probability by ARGUS and CLEO give  $x_b = 0.67 \pm 0.08$  [29]. The weighted average of these values is  $x_b = 0.706 \pm 0.068$ . In the two doublet model,  $x_b$  is given by

$$
x_b = \frac{\Delta m}{\Gamma} = \frac{G_F^2}{6\pi^2} |V_{td}^*|^2 |V_{tb}|^2 f_B^2 m_B B_B \eta_B \tau_B M_W^2
$$
  
×(I<sub>WW</sub> + I<sub>WH</sub> + I<sub>HH</sub>), (34)

where  $\tau_{B_d^0} = 1.53 \pm 0.09$  ps [29] is the *B* lifetime,  $f_B$ is the  $B$ <sup>\*</sup>meson decay constant,  $m_B$  is the  $B$  meson mass,  $B_B$  is the bag factor, and  $\eta_B$  is a QCD correction factor, whose value lies in the range from 0.55 to 0.85 [30-32]. The Cabibbo-Kobayashi-Maskawa (CKM) matrix element  $|V_{td}|$  lies in the range from 0.005 to 0.014 [33], while  $V_{tb} = 1$ . Finally, the dependence on  $m_+$ ,  $m_t$ , and  $tan\beta$  is contained in  $I_{WW}$ ,  $I_{WH}$ , and  $I_{HH}$ . We have  $\lceil 8 \rceil$ 

$$
I_{WW} = \frac{x}{4} \left[ 1 + \frac{3 - 9x}{(x - 1)^2} + \frac{6x^2 \ln x}{(x - 1)^3} \right],
$$
  
\n
$$
I_{WH} = xy \cot^2 \beta \left[ \frac{(4z - 1) \ln y}{2(1 - y)^2 (1 - z)} - \frac{3 \ln x}{2(1 - x)^2 (1 - z)} + \frac{x - 4}{(1 - x)(1 - y)} \right],
$$

and

$$
I_{HH} = \frac{xy \cot^4 \beta}{4} \left[ \frac{1+y}{(1-y)^2} + \frac{2y \ln y}{(1-y)^3} \right],
$$
 (35)

where  $x = m_t^2/M_W^2$ ,  $y = m_t^2/m_+^2$ , and  $z = M_W^2/m_+^2$ .

A final constraint on the charged Higgs boson mass and  $tan\beta$  is provided by  $\Gamma(b \to c\tau\bar{\nu}_{\tau})$ . Since in the two doublet model this process is mediated by both  $W$  and charged Higgs boson exchange, one cannot have too small a mass for the charged Higgs boson or too large a value of  $tan\beta$  without exceeding the experimental limits. This constraint has been analyzed recently in Ref. [34], where it was shown that the allowed values of  $m_+$  and  $\tan\!\beta$ must satisfy

$$
\tan\!\beta \le 0.51 m_+ [\text{GeV}] \tag{36}
$$

at the  $1\sigma$  level. In contrast with the constraints given above, this measurement provides an upper bound on  $tan \beta$ , rather than a lower bound.

In Fig. 5, we have plotted the allowed reigon in the  $m_+$ -tan $\beta$  plane after the constraints due to  $\mathcal{B}(b \to s\gamma)$ ,



FIG. 5. Allowed region in the  $m_{+}$ -tan $\beta$  plane for the combination of observables indicated. The shaded region is allowed.

 $\Gamma(b \to c\tau\bar{\nu}_{\tau}), B^0$ - $\bar{B}^0$  mixing, and  $R_b$  have been applied, with a top mass of 174 GeV. The constraints from  $R_b$ indicated are at the 2.5 $\sigma$  level; to agree with experiment to within  $2\sigma$  or less,  $\tan\!\beta$  must be large and the scalars  $H_{2,3}^0$  must be light. In this situation, the charged Higgs boson mass cannot be much larger than about 150 GeV, for reasons we will discuss in Sec. VI. We have used the 95% confidence limits for  $b \rightarrow s\gamma$ . The constraint from  $\mathcal{B}(b \to s\gamma)$  rules out small charged Higgs boson masses, while the inclusion of  $R_b$  strengthens the lower bound on  $tan\beta$ . The constraint from  $x_b$  is similar to that from  $R_b$  but somewhat weaker. Similar conclusions have been reached in Refs. [27,28,35]. From these considerations, we can constrain  $m_+$  and  $\tan\beta$  by

$$
m_+ \geq 200 \text{ GeV} \tag{37}
$$

and

$$
\tan\!\beta \ge 0.7\tag{38}
$$

at the 2.5 $\sigma$  level. At the  $2\sigma$  level, the bound on  $tan\beta$ is roughly 13 for  $m_t = 157$  GeV, increasing to 38 for  $m_t = 174 \text{ GeV}.$ 

Since experimental measurements of  $\mathcal{B}(b \to s\gamma)$  are likely to improve, we present also the charged Higgs boson mass limits that result for smaller values of  $\mathcal{B}(b \rightarrow$  $s\gamma$ ). This serves to illustrate the high sensitivity of these mass bounds to theoretical uncertainties. For comparison, we give both the mass bound derived using the leading-order QCD corrections we have employed here, as well as the mass bounds reported in Ref. [28] where the next-to-leading-order corrections were partially included:

$$
m_{+} \geq \begin{cases} 200 \text{ GeV (LO)}, 150 \text{ GeV (NLO)}, \ \mathcal{B}(b \to s\gamma) < 4 \times 10^{-4}, \\ 380 \text{ GeV (LO)}, 200 \text{ GeV (NLO)}, \ \mathcal{B}(b \to s\gamma) < 3 \times 10^{-4}, \\ 650 \text{ GeV (LO)}, 300 \text{ GeV (NLO)}, \ \mathcal{B}(b \to s\gamma) < 2 \times 10^{-4} \end{cases}, \tag{39}
$$

We see that the lower bound on the charged Higgs boson mass may move downward by a factor of 2 once the full next-to-leading QCD corrections are known. We see also that the partial inclusion of next-to-leading-order corrections relaxes the charged Higgs boson mass bound from the 200 GeV cited above to about 150 GeV.

### VI. GLOBAL CONSTRAINTS ON THE TWO DOUBLET MODEL

In this section we discuss the experimentally preferred configurations of the two doublet model by combining the constraints of the previous sections. Throughout we assume a top quark mass of 174 GeV. We first discuss those configurations of the model which are compatible with each of the various high-energy measurements  $\{R_b,$  $\Gamma(Z \to \text{all})$ , and  $M_W$ ] at the 1.5*o* level. We then add to these the low-energy constraints.

To be consistent with the LEP measurement of  $R_b$  at the 1.5 $\sigma$  level, one must assume that tan $\beta$  is large and that  $H_2^0$  and  $H_3^0$  are quite light, having masses less than or of the order of 100 GeV. We have evaluated the maximum allowed masses and the minimum value of  $tan\beta$  by both a systematic search of the parameter space and by randomly sampling a large number of configurations of the model. By both methods, we have found the limits

$$
m_2, m_3 \leq 65 \text{ GeV} \tag{40}
$$

and

$$
\tan\!\beta \ge 53 \,\, , \qquad \qquad (41)
$$

which translates to a value of  $\beta$  very nearly equal to  $\pi/2$ . In addition,  $R_b$  provides a constraint on the mixing angle  $\alpha$  in the case that  $m_1$  is large; the reason is that, for  $m_1$ large and  $\alpha \simeq 0$ , the neutral Higgs contribution to  $R_b$  is negative (cf. Fig. 3). As a consequence, one has

$$
|\alpha| \ge \pi/3 \quad \text{for } m_1 \ge 100 \text{ GeV}. \tag{42}
$$

If one assumes the values of  $m_{2,3}$  and  $\tan\!\beta$  implied by  $R_b$ , then further constraints can be derived from  $M_W$  and the full  $Z$  width, through the  $\rho$  parameter. The leading

behavior of 
$$
\Delta \rho
$$
 in this case is given by  
\n
$$
\Delta \rho_{\text{Higgs}} \simeq \frac{3G_F}{8\sqrt{2}\pi^2} \left[ m_+^2 - \cos^2 \alpha \frac{m_+^2 m_1^2}{m_+^2 - m_1^2} \ln \left( \frac{m_+^2}{m_1^2} \right) \right].
$$
\n(43)

From the leading behavior of  $\Delta\rho_{\mathrm{Higgs}}$ , we see immediately that the charged Higgs boson mass cannot be too large, for this would give a large positive contribution to  $\Delta \rho$ . Furthermore, although the second term in Eq. (43) is negative, its coefficient is small due to the bound on  $\alpha$ . imposed by  $R_b$  when  $m_1$  is large. As a result, we have

$$
m_+ \leq 150 \text{ GeV} \tag{44}
$$

Due to the quadratic growth of  $\Delta \rho$  with  $m_+$ , larger values are strongly disfavored. Finally, large values of  $m_1$ 

are weakly favored by  $\rho$  parameter constraints; one has  $m_1 \geq 500$  GeV, although smaller values are not in terribly poor agreement with the data. In combination with constraints from  $R_b$ , the bound on the mass of  $m_1$  gives

$$
|\alpha| \ge \pi/3 \tag{45}
$$

This configuration of the model is also compatible with the  $b\bar{b}$  forward-backward asymmetry of the Z. In deriving these limits, we have again carried out a systematic search of the parameter space of the model, covering the full range of  $\alpha$  and  $\beta$ , and varying each of the scalar masses between 50 and 1000 GeV. If one allows larger values of  $m_1$ , then slightly larger values of  $m_+$  are admissible; for  $m_1 = 1.5$  TeV, the upper limit on the charged Higgs boson mass recedes to 170 GeV.

Finally, we can combine the constraints from highenergy data with those from  $b \to s\gamma$ ,  $b \to c\tau\bar{\nu}_{\tau}$ , and B mixing. This class of constraints also rules out small values of  $tan\beta$ , although the bounds are weaker than those derived from  $R_b$ . If we take the weaker of the two charged Higgs boson mass bounds given in Eq. (39), we find that the charged Higgs boson mass is constrained to lie in a narrow region in the vicinity of 150 GeV:

$$
m_+ \simeq 150 \text{ GeV}. \tag{46}
$$

We find that compatibility of the model with the measured value of  $R_b$ , when combined with the other relevant observables, yields a very restrictive set of constraints.

#### VII. CONCLUSIONS

We have presented constraints on the type-II two Higgs doublet model from a variety of observables. The overall agreement of the model with experiment is acceptable, and quite comparable to that found for the minimal standard model. We have shown that  $tan\beta$  and the charged Higgs boson mass are bounded by

$$
m_+ \ge 150 - 200 \text{ GeV}, \quad \tan\!\beta \ge 0.7 \tag{47}
$$

for 2.5 $\sigma$  agreement with measurements of  $R_b$ , and discussed the sensitivity of the charged Higgs boson mass bound to theoretical uncertainties. The lower bound on tan $\beta$  increases to about 13 for  $2\sigma$  agreement with  $R<sub>b</sub>$ . Oblique constraints on the model require that the charged Higgs boson mass lie in the range

$$
m_{\rm light}-130~{\rm GeV}\leq m_+\leq m_{\rm heavy}+130~{\rm GeV}~,~~(48)
$$

where  $m_{\rm heavy, light}$  are the masses of the heaviest and lightest neutral Higgs bosons. We have discussed one very tightly constrained scenario where the model (unlike the standard model) agrees with measurements of  $R_b$ at the 1 to  $1.5\sigma$  level. In this case, every parameter of the model is constrained in some way; for the mixing angles one has

$$
\tan\!\beta \ge 53, \quad |\alpha| \ge \pi/3 \tag{49}
$$

while for the Higgs boson masses one finds

$$
m_{2,3} \leq 65 \text{ GeV}, \quad m_+ \simeq 150 \text{ GeV}, \quad m_1 \geq 500 \text{ GeV} .
$$
 (50)

The preference for large values of  $m_1$  is weak, and  $\rho$  parameter constraints strongly disfavor larger values of  $m_+$ . This configuration of the model is virtually ruled out by  $b \rightarrow s\gamma$ : only a very narrow allowed region remains for the charged Higgs boson mass, and if the uncertainty on the measurement is reduced even slightly, this window is likely to disappear. Consequently, it is improbable that the two doublet model can improve on the standardmodel prediction of  $R_b$ .

The large  $tan\beta$  scenario discussed here may nonetheless be of some interest for studies of supersymmetric grand-unified theories. Indeed, if one assumes a large value of  $tan\beta$  and a charged Higgs boson mass on the order of 100 GeV, then one finds that  $H_{2,3}^0$  are light and nearly degenerate, and that  $\alpha$  is very nearly equal to  $-\pi/2$ . This configuration of the model is surprisingly similar to that found in the large  $tan\beta$  scenario. In supersymmetric SO(10) models the Yukawa couplings of the t, b, and  $\tau$  can be made to unify at the unification scale if tan $\beta \simeq 50$ –60 and  $m_t \simeq 160$ –190 GeV [36,37]. Of course, the full calculation including superpartners is necessary in order to study the viability of this scenario.

We have shown that the type-II two Higgs doublet model is roughly consistent with numerous experimental observations at the level of radiative corrections. The measured values of the  $W$  mass, the  $Z$  width, and lowenergy data such as  $\Gamma(b \to s\gamma)$ ,  $\Gamma(b \to c\tau\bar{\nu}_\tau)$ , and the  $B^0$ - $\bar{B}^0$  mixing amplitude can all be accommodated within the model. The partial width of the  $Z$  into  $b$  quarks poses a significant challenge to the model, which will become increasingly restrictive as data on this and other observables improve. We conclude, then, that the minimal two doublet model without supersymmetry cannot be ruled out on the basis of current data. Although the discovery of a light neutral Higgs boson or a charged Higgs boson would be indicative of supersymmetry, it is not an ironclad guarantee that the world is indeed supersymmetric.

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