Strange matter equation of state in the quark mass-density-dependent model

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We study the properties and stability of strange matter at $T = 0$ in the quark mass-density-dependent model for noninteracting quarks. We found a wide "stability window" for the values of the parameters (C, M_{s0}) and the resulting equation of state at low densities is stiffer than that of the MIT bag model. At high densities it tends to the ultrarelativistic behavior expected because of the asymptotic freedom of quarks. The density at zero pressure is near the one predicted by the bag model and *not* shifted away as stated before; nevertheless, at these densities the velocity of sound is $\approx 50\%$ larger in this model than in the bag model. We have integrated the equations of stellar structure for strange stars with the present equation of state. We found that the mass-radius relation is very much the same as in the bag model, although it extends to more massive objects, due to the stiffening of the equation of state at low densities.

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I. INTRODUCTION

Since the proposal by Witten [1] that strange matter (SM) may be the actual ground state of baryon matter at densities of the order of nuclear saturation density ($n_0 = 0.16$ fm^{-3}), several investigations have been carried out in order to study its plausibility and the implications for the SM in astrophysics, cosmology, and heavy ion collisions. For a general reference on this topic, see [2].

Until now, most of the thermodynamical treatments of the SM have been carried out in the framework of the MIT bag model. On the other hand, an alternative description of confinement in which the mass of the quarks depends upon the baryon number density was initially introduced by Fowler, Raha, and Weiner [3]. This approach was employed to study the properties of quark matter [3] and was later applied by Chakrabarty and coworkers [4–6] to the case of SM.

In both cases the properties of the quark and SM were found to be very different from those predicted by the MIT bag model. However, these features are consequences of an incorrect thermodynamical treatment of the problem. In deriving the energy density and the pressure in the present model, an extra term appears (which is not usual in the case of the free Fermi relativistic gas) due to the dependence of the quark masses on the baryon density. These extra terms produce significant changes in the energy per baryon, make the pressure take negative values in the low density regime with the well known implications, shift the stability window of SM, and change the properties of strange stars. In almost all cases we find that the properties of the SM in the quark mass-density-dependent model are nearly the same as those obtained in the MIT bag model.

It is the aim of this paper to construct the SM equation of state (EOS) in the quark mass-density-dependent model at zero temperature, and to investigate the conditions for its stability and some of their astrophysical consequences.

This paper is organized in the following manner. In Sec. II we describe the formalism applied in calculating the EOS of the SM in this approach. In Sec. III we present the integration of the structure of stars made up of this matter and in Sec. IV we address our main conclusions.

II. THE EQUATION OF STATE

We assume the SM to be a free Fermi gas mixture of quarks u,d,s, antiquarks $\bar{u}$,$\bar{d}$,$\bar{s}$, electrons, and positrons, where the mass of the quarks and antiquarks is parametrized with the baryon number density n_B as follows [4]:

$$m_u = m_d = \frac{C}{3n_B}, \quad m_s = m_{s0} + \frac{C}{3n_B}, \tag{1}$$

where m_{s0} is the strange current mass and C is a constant to be constrained by stability arguments.

The thermodynamical potential density is

$$\Omega = \sum_i \Omega_i = -\sum_i \frac{g_i T}{(2\pi)^3} \int d^3p \ln(1 + e^{-\beta(\epsilon_i - \mu_i)}) \tag{2}$$

where $i = u, \bar{u}, d, \bar{d}, d, \bar{d}, s, \bar{s}, e, \bar{e}$, g_i is the degeneracy fac-

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tor ($g_i = 2 \times 3 = 6$ for quarks and antiquarks, $g_i = 2$ for electrons and positrons), $\epsilon_i = (p^2 + m_i^2)^{1/2}$ is the single particle energy, and μ_i the chemical potential. For antiparticles $\bar{\mu}_i = -\mu_i$. At zero temperature we have (see, e.g., [7])

$$\Omega_i = -\frac{g_i}{48\pi^2} m_i^4 F(x_i), \quad (3)$$

where

$$F(x) = x(x^2+1)^{1/2}(2x^2-3) + 3 \arg\sinh(x) \quad (4)$$

and

$$x = \left[\left(\frac{\mu}{m}\right)^2 - 1\right]^{1/2}. \quad (5)$$

The number density of each particle and the total pressure are obtained from Eq. (2) by means of

$$n_i = -\left.\frac{\partial \Omega_i}{\partial \mu_i}\right|_{T,n_B}, \quad (6)$$

$$P = -\left.\frac{\partial(\Omega/n_B)}{\partial(1/n_B)}\right|_{T,\mu_i} = n_B \left.\frac{\partial \Omega}{\partial n_B}\right|_{T,\mu_i} - \Omega = \sum_i P_i. \quad (7)$$

Note that the term $n_B \frac{\partial \Omega}{\partial n_B}|_{T,\mu_i}$ arises from the baryon density dependence of the quark masses and it makes the pressure zero at a certain baryon density n_B. The energy density is obtained from the well known thermodynamic relationship

$$e = \sum_i e_i = -P + \sum_i \mu_i n_i - T \left.\frac{\partial \Omega}{\partial T}\right|_{n_B,\mu_i}. \quad (8)$$

At zero temperature we have

$$n_i = \frac{g_i}{6\pi^2} m_i^3 x_i^3, \quad (9)$$

$$P_i = \frac{g_i}{48\pi^2} m_i^4 \left[F(x_i) - \frac{C}{n_B}\frac{4}{m_i} G(x_i)\right], \quad (10)$$

$$e_i = \frac{g_i}{48\pi^2} m_i^4 \left[3H(x_i) + \frac{C}{n_B}\frac{4}{m_i} G(x_i)\right], \quad (11)$$

where

$$G(x) = x(x^2+1)^{1/2} - \arg\sinh(x), \quad (12)$$

$$H(x) = x(x^2+1)^{1/2}(1+2x^2) - \arg\sinh(x), \quad (13)$$

and we have no antiparticle contributions. Under these conditions, the baryon number density n_B is given by

$$n_B = \frac{1}{3}(n_u + n_d + n_s). \quad (14)$$

Imposing charge neutrality and chemical equilibrium (we assume that neutrinos leave the system freely) we have [8]

$$2n_u = n_d + n_s + 3n_e, \quad (15)$$

$$\mu_s = \mu_d = \mu, \quad \mu_s = \mu_u + \mu_e. \quad (16)$$

Solving Eqs. (14), (15), and (16), we can determine μ_i for a given n_B. The other quantities are obtained straightforwardly.

We now want to establish the conditions under which the SM is the true hadronic ground state. Following Farhi and Jaffe [8] we must require, at $P = 0$, $E/n_B \leq M_{^{56}\mathrm{Fe}}/56 = 930$ MeV for the SM and $E/n_B > 930$ MeV for two-flavor quark matter (where $M_{^{56}\mathrm{Fe}}$ is the mass of ^{56}Fe) in order not to contradict standard nuclear physics. The EOS will describe stable SM only for a set of values of (C, m_{s0}) satisfying these two conditions.

We first assume the current mass of the strange quark to be zero ($m_{s0} = 0$). In this limit, electrons are not present and the zero pressure condition reads

$$\begin{aligned} F(x) - 12G(x) &= x(x^2+1)^{1/2}(2x^2-15) \\ &\quad + 15 \arg\sinh(x) \\ &= 0, \end{aligned} \quad (17)$$

which is satisfied only for $x_0 = 2.347\,385$. In this approximation $n_B = n_i$ and the baryon density for which $P = 0$ is

$$\begin{aligned} n_B(P=0) &= \left(\frac{3}{\pi}\right)^{1/4} \left(\frac{C x_0}{3}\right)^{3/4} \\ &\approx 1.747 \left(\frac{C}{100 \text{ MeV fm}^{-3}}\right)^{3/4} n_0. \end{aligned} \quad (18)$$

So the stability of the SM gives us an upper bound to C:

$$C^{1/4} \leq \frac{8}{3^{7/4}\sqrt{\pi}} \frac{x_0^{15/4}}{I(x_0)} \frac{M_{^{56}\mathrm{Fe}}}{56}, \quad (19)$$

where

$$I(x) = x(x^2+1)^{1/2}(5+2x^2) - 5 \arg\sinh(x). \quad (20)$$

This condition implies $C \leq 111.6$ MeV fm^{-3}.

For the nonstrange case we have, because of charge neutrality, $2n_u = n_d$; then $x_d = 2^{1/3} x_u$ and the condition for zero pressure is

$$F(x_u) + F(x_d) - 12[G(x_u) + G(x_d)] = 0, \quad (21)$$

which also has only one solution: $x_{u0} = 2.005\,834$. Then, the condition of nonstability for nonstrange matter reads

$$C^{1/4} > \frac{8}{3^{3/4}\sqrt{\pi}} \frac{{x_{u0}}^{15/4}}{I(x_{u0}) + I(x_{d0})} \frac{M_{^{56}\mathrm{Fe}}}{56}, \quad (22)$$

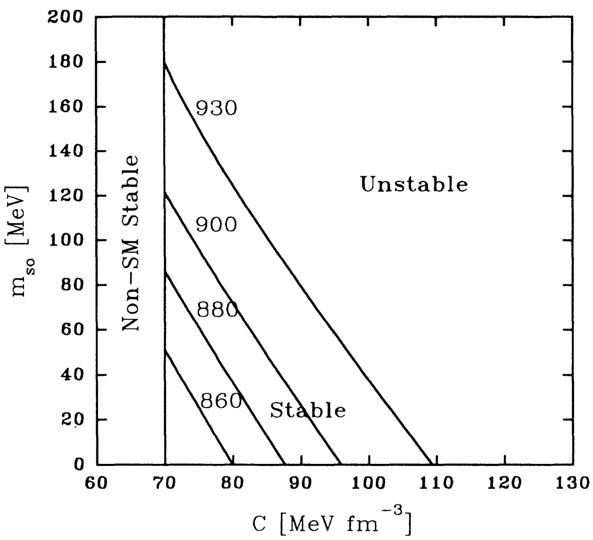

FIG. 1. The stability window of the SM in the quark mass-density-dependent model at zero pressure. The stability region is where the energy per particle is lower than 930 MeV and two-flavor quark matter is unstable. We denoted the curves of constant energy per particle for the case of $E/n_B \leq 930$ MeV.

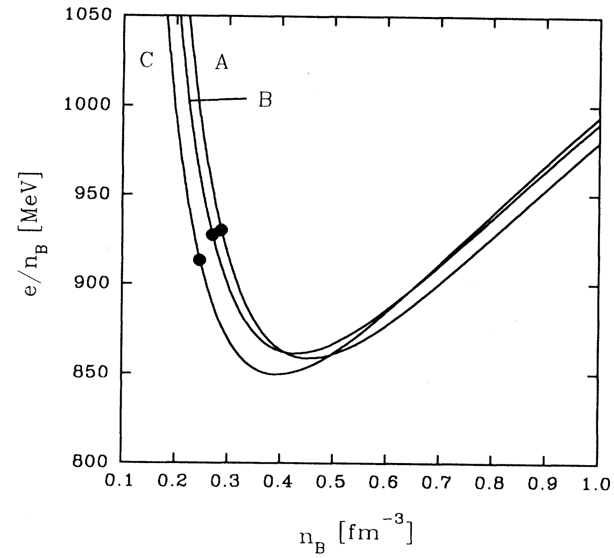

FIG. 3. The energy per baryon vs baryon number density in the quark mass-density-dependent model for the cases A, B, and C. The zero pressure density does not correspond to the minimum of the energy per baryon (as in the usual case) but occurs at the solid dots. It is a consequence of the assumed dependence of the quark masses upon the baryon density.

so $C > 69.05$ MeV fm^{-3} (this condition is an *exact* one because we have assumed that u and d quarks have zero current masses). Then C must be in the range $69.05 < C < 111.6$ MeV fm^{-3}.

In the case of $m_{s0} > 0$ we have a "stability window" (in the spirit of Farhi-Jaffe work [8]) given in Fig. 1. In this model, the stability window is trianglelike and noticeably wide. Note that with increasing m_{s0} the range of C for which we have stable SM becomes narrower because it is less favorable to convert d into s quarks and the SM tends to be nonstrange, and so unstable.

The resulting EOS for the SM is shown in Fig. 2 where we have included the data for cases A, B, and C (See

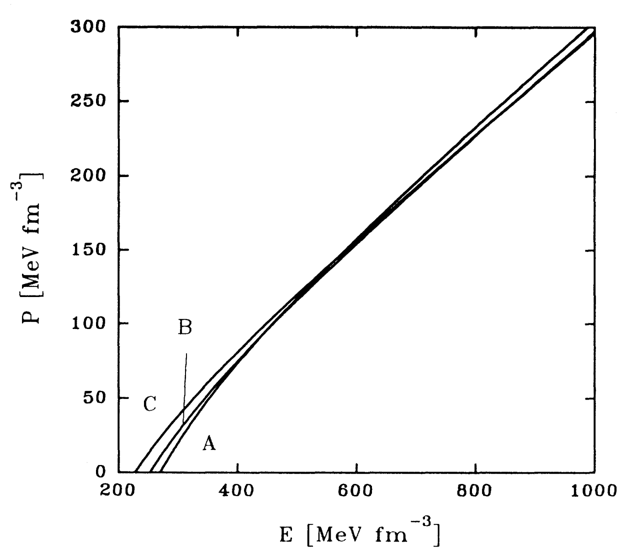

FIG. 2. The SM EOS in the quark mass-density-dependent model. It asymptotically tends to the ultrarelativistic case but this is not true at low pressures as in the MIT bag model. For the meaning of A, B, and C, see text.

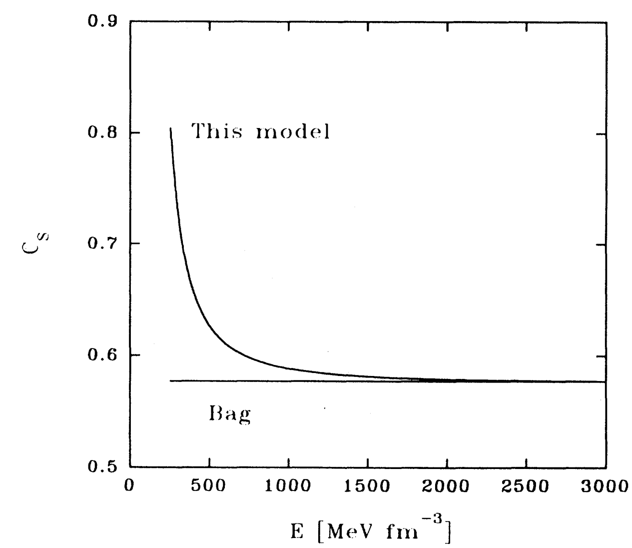

FIG. 4. The velocity of sound vs energy density in the quark mass-density-dependent model. It asymptotically tends to the ultrarrelativistic case $c_s = 1/\sqrt{3}$ but this is not true at low energy densities. The constant value of c_s corresponding to the MIT bag model is also shown. The employed parameters correspond to the case B.

TABLE I. Selected parameters sets.

Case	C (MeV fm^{-3})	m_{s0} (MeV)
A	100	40
B	90	80
C	80	100

Table I). This EOS show a similar dependence of the pressure vs energy density to that expected, but at low densities it is significatively stiffer than predicted by the MIT bag model.

The energy per baryon vs baryon number density is given in Fig. 3 for cases A, B, and C. It is interesting to note that the zero pressure density is not that corresponding to the minimum energy per baryon (as in the usual case) but occurs at the solid dots for each case. This is another consequence of the assumed dependence of the quark masses upon the baryon density.

In Fig. 4 we plot the velocity of sound C_s of the SM in this model. It is not constant as in the MIT bag model in which $C_s = 1/\sqrt{3}$ but asymptotically tends to this value as should be expected. This behavior can be interpreted as due to the fact that the effect of quark masses is negligible only at high enough densities. The C_s found here agrees nicely with the curve C of Fig. 1 of Ref. [9].

It should be noted that the range of C for stable SM is outside that given in Refs. [4–6]. This is due to the omission of the derivatives with respect to n_B, as stated above. In the case of the pressure, this derivative is a negative contribution, which is just the one that makes the quarks confined. Allowing for these contributions, we find that the zero pressure density is *near* the one predicted by the MIT bag model and not far away ($\approx 8n_0$) as stated before.

III. STRANGE STARS

It has been speculated since the paper of Witten [1] that the currently named neutron stars are in fact strange stars (i.e., stars made up not of neutrons but of the SM) [10]. From the astrophysical point of view, the structure of strange stars in the framework of the new SM EOS we present here is of obvious importance.

In order to study this problem we have numerically solved the Tolman-Oppenheimer-Volkoff (TOV) equations of the structure of general relativistic compact stars [11]. It is worth noting that in the MIT bag model the SM EOS has a *linear* dependence of the pressure vs energy density. In this case, the strange star structure can be scaled for changes in the value of the bag constant B. This is *not* true in the case of the EOS of this paper, simply because the dependence of the pressure vs energy density is not linear. This can be noted in Fig. 4 as quoted above.

In order to cover the values of (C, m_{s0}) that gives a stable SM, we have chosen the cases A, B, and C (see Table I). The integration of the TOV equations gives strange stars whose mass-radius relation is shown in Fig. 5.

The curves for the cases A, B, and C are remarkably

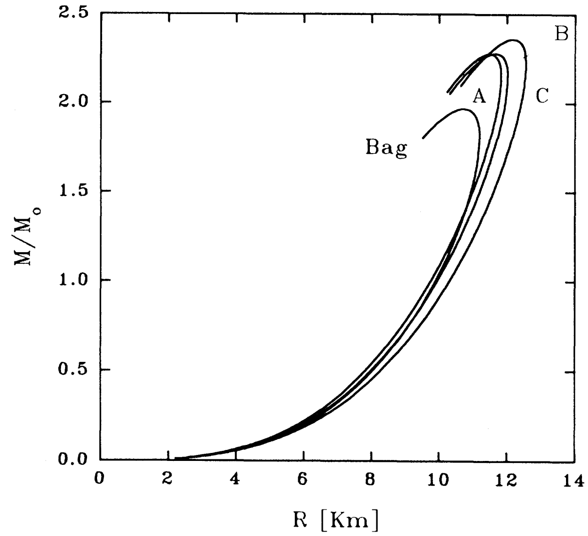

FIG. 5. The structure of strange stars in the quark mass-density-dependent model. For the meaning of A, B, and C, see text. Taking the parameters of the EOS inside the stability window the sequences of strange stars are very similar to each other. For the sake of comparison we also show the results corresponding to the MIT bag model with $B = 60$ MeV fm^{-3}.

similar. For the sake of comparison we have also included the data corresponding to the case of MIT bag model strange stars with $B = 60$ MeV fm^{-3}. These are very similar in shape to the relationships that correspond to the EOS of this paper, though the nonbagged strange stars are more massive than the bagged ones (at least for the value of B employed in the figure). This can be interpreted as due to the greater stiffness of this EOS compared to that of the MIT bag model at low densities.

The EOS we present in this work allows for the existence of objects with masses up to $\approx 2.4 M_\odot$ with radii of the order of 10 km. Then this EOS comfortably explains the mass of PSR 1913+16 of $\approx 1.44 M_\odot$ (see, e.g., Ref. [11]) and there is no obvious reason to consider the structure of these objects as more physically plausible than the MIT bag model structures, and vice versa.

Finally, we note that the structure of strange stars in the framework of the quark mass-density-dependent model EOS has been calculated by Chakrabarty [5]. However, we note that because of the omission in that paper the structure of these stars is strongly dependent upon the values of the parameter C (he names it B), a behavior we do not find if the values of (C, m_{s0}) are inside the stability window.

IV. DISCUSSION AND CONCLUSIONS

We have calculated the equation of state of strange matter in the framework of the quark mass-density-dependent model. We started with the thermodynamic potential density and carefully derived the thermodynamic quantities of interest at zero temperature.

We found a wide "stability window" such as that of

Farhi and Jaffe [8] for the MIT bag model and the resulting equation of state is stiffer at low densities (the velocity of sound is $\approx 50\%$ larger in this model than in the bag model). At higher densities it tends to the ultrarelativistic behavior expected because of the asymptotic freedom of quarks.

Noticeably, the density at zero pressure is near the one predicted by the bag model and *not* shifted away as stated before (see Fig. 3). It is an important result that, contrary to what has been stated before, starting with very different assumptions compared to the MIT bag model we found a very similar behavior. The differences between this result and the former ones in this model of confinement should be attributed to the omission of terms in the expressions of pressure and energy density.

We have integrated the relativistic equations of stellar structure for stars made up of strange matter with this equation of state. We found that the mass-radius relation is very much the same as in the bag model, although it extends to more massive objects. This is due to the greater stiffness of this equation of state. The objects have a structure that does not contradict observational data for any parameter set inside the stability window. If compact stars are indeed strange stars, it seems not an easy matter to distinguish this equation of state from the bag model one on astrophysical grounds. In this sense, it would be interesting to perform studies of vibrational properties of compact stars such as that presented in Ref. [12].

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