PHYSICAL REVIEW D VOLUME 51, NUMBER 4 15 FEBRUARY 1995

Finite field-dependent BRS transformations

Satish D. Joglekar* and Bhabani Prasad Mandal†
Department of Physics, Indian Institute of Technology, Kanpur, Kanpur 208016, India
(Received 22 June 1994)

We consider infinitesimal field-dependent BRS transformations. We show that they can be integrated to yield finite BRS field-dependent transformations and have the same BRS form. We discuss a number of applications of the latter. We show that for certain special field-dependent BRS parameters (evaluated in a closed form), these can be used to connect the Faddeev-Popov effective action in a linear gauge with a gauge parameter λ to (i) the most general BRS–anti-BRS symmetric action in linear gauges, (ii) the Faddeev-Popov effective action in quadratic gauges, and (iii) the Faddeev-Popov effective action with another distinct gauge parameter λ'. In each case, the extra terms in the latter action are shown to arise from the Jacobian for the *nonlocal* field-dependent BRS transformations. Some applications of these ideas are suggested.

PACS number(s): 11.15.−q, 11.15.Bt

I. INTRODUCTION

On account of gauge invariance, gauge theories are described by a large number of equivalent effective actions. They are equivalent in the sense that they are supposed to give identical results for the physically observable quantities. There are the usual covariant Lorent gauges [1], the axial gauges [2], radial gauges [3], nonlinear covariant gauges [4], and Becchi-Rouet-Stora–(BRS–)anti-BRS invariant formulations [5] to name a few. In each of these, there can be one or more free continuous parameters.

As the above are equivalent formulations, these should be, and are in principle, connected to each other by field transformations. To take a simple example, the effective action in Lorentz gauges with two gauge parameters λ and $\lambda + \Delta\lambda$ differing infinitesimally are related to each other by an infinitesimal transformation

$$\delta A^\alpha_\mu = D^{\alpha\beta}_\mu M^{-1}_{\beta\gamma} \partial A^\gamma \frac{\Delta\lambda}{\lambda} \ . \tag{1.1}$$

Such infinitesimal transformations have been generally used to prove the independence of physical observables from the gauge parameter in a given set of gauges. To our knowledge, constructing finite transformations explicitly [finite analog of Eq. (1.1)] that relate fields in a, say, Lorentz gauge with parameters λ and λ' (differing by finite amounts) are not constructed, nor are they easy to construct. One could also seek finite transformations connecting fields in Lorentz gauges to fields in BRS–anti-BRS invariant action, for example. Such transformations are not known.

While for the purpose of proving the gauge independence *in a given set of gauges*, infinitesimal transformations of the type (1.1) may prove sufficient, finite transformations could find applications to a number of uses. While the purpose of this work is not to study the practical applications in detail, we should mention a few to motivate our approach. It would, for example, be nice to construct a field transformation that relates axial gauges to Lorentz gauges. This could find applications in removal of discrepancy of the anomalous dimension calculation [6] in the two sets of gauges. It would also make it possible to make axial gauge calculations more rigorous. Such transformations could help in connecting results in, say, Landau and Feynman gauges. They could also be utilized in formal treatments. Thus it would certainly be nice to have finite field transformations connecting various different formulations of gauge theories.

In this work, we shall tackle the problem of finding such finite transformations connecting the usual Faddeev-Popov (FP) effective action in Lorentz gauges with the parameter λ to (i) the BRS–anti-BRS invariant effective action, (ii) the FP action with quadratic gauges, and (iii) the FP action in Lorentz gauges with the parameter λ'. We do not, however, find it profitable to generalize the field-dependent *gauge* transformation such as in Eq. (1.1). We shall find it much more profitable to find a BRS-type field transformation transforming all gauge, ghost, and antighost fields simultaneously. This is because of the property that a finite gauge transformation

$$A' = UAU^\dagger - \partial_\mu U U^\dagger, \quad U = \exp(iT^\alpha\theta^\alpha) \tag{1.2}$$

does not preserve the simple linear form of an infinitesimal transformation

$$\delta A^\alpha_\mu = D^{\alpha\beta}_\mu \theta^\beta \ . \tag{1.3}$$

However the "finite" and infinitesimal BRS transformations have the same form. The infinitesimal transformation

$$\delta\phi_i = \delta^{(i)}_{\rm BRS}(\phi)\delta\Lambda \tag{1.4}$$

(written symbolically) goes into a "finite" BRS transformation

*Electronic address: sdj@iitk.ernet.in
†Electronic address: bpm@iitk.ernet.in

$$\phi_i' - \phi_i = \delta^{(i)}_{\rm BRS}(\phi)\Lambda \; . \tag{1.5}$$

(This happens in effect because of $\Lambda^2 = 0$.) The BRS transformations we shall be using are not, however, as simple as Eqs. (1.4) and (1.5). We shall consider $\delta\Lambda$ not as a field-independent quantity but as a field-dependent quantity, preserving $\delta\Lambda^2 = 0$, however. This is much the same way as Eq. (1.1) uses a field-dependent θ^α defined in Eq. (1.3). We shall allow

$$\delta\Lambda = \Theta'[A, c, \bar{c}]d\kappa \; , \tag{1.6}$$

where $\Theta'^2 = 0$ and is a field-dependent but x-independent quantity, and κ is a parameter. We show that such transformations are indeed a symmetry of the action. We shall call (1.4) with $\delta\Lambda$ of Eq. (1.6) the "infinitesimal field-dependent BRS transformations." We show that these can be integrated in κ and that they preserve the BRS form of Eq. (1.5) with Λ given by

$$\Lambda = \Theta(\phi) \; , \tag{1.7}$$

where $\Theta[\phi]$ is derivable from $\Theta'[\phi]$. The transformations

$$\phi_i' = \phi_i + \delta^{(i)}_{\rm BRS}(\phi)\Theta(\phi) \tag{1.8}$$

though the symmetry of the FP effective action, are, however, nonlocal finite transformations generating nontrivial Jacobians. It is shown that, in fact, these Jacobians are responsible for the differences in the effective actions of various formulations. In fact, for each of three connections dealt with, we make an ansatz for Θ' and explicitly show how the Jacobians explain the difference between the effective actions in various pairs (say, FP action in Lorentz gauges and BRS–anti-BRS invariant action). These Jacobians are obtained by integrating out, in a nontrivial procedure, the infinitesimal Jacobians as done in Sec. IV.

We now explain the plan of the paper. In Sec. II, we shall review the results on the BRS–anti-BRS invariant actions and on quadratic gauges. In Sec. III we shall introduce the infinitesimal field-dependent BRS transformations and show how these can be integrated out to yield finite field-dependent BRS transformations. In Sec. IV, we show how the Jacobians for such translations can be evaluated. In Sec. V, we do the evaluation of the Jacobians for three cases of Θ' and show that they indeed explain the differences of effective actions in the three cases mentioned earlier. In Sec. VI, we summarize our results and give directions for possible applications. We do not discuss the applications in detail but hope to do it elsewhere.

II. PRELIMINARY REVIEW

A. BRS–anti-BRS symmetry

In this section, we shall review the known results on BRS and anti-BRS symmetries of the effective action in gauge theories in linear gauges and BRS symmetry in quadratic gauges.

We consider the most general effective action in linear gauges given by Baulieu and Theirry-Mieg [5] that has BRS–anti-BRS invariance, when expressed entirely in terms of necessary fields $A, c, \bar{c}$ (and no auxiliary fields):

$$S_{\rm eff}[A, c, \bar{c}] = \int d^4x \left[-\tfrac{1}{4}F^\alpha_{\mu\nu}F^{\alpha\mu\nu} - \sum_\alpha \frac{(\partial \cdot A^\alpha)^2}{2\lambda} + \mathcal{L}_G \right] \tag{2.1}$$

with

$$\mathcal{L}_G = (1 - \tfrac{1}{2}\alpha)\partial^\mu \bar{c} D_\mu c + \frac{\alpha}{2} D^\mu \bar{c}\partial_\mu c + \tfrac{1}{2}\alpha(1-\tfrac{1}{2}\alpha)\frac{\lambda}{2} g^2 [f^{\alpha\beta\gamma}\bar{c}^\beta c^\gamma]^2 \tag{2.2a}$$

$$= \partial^\mu \bar{c} D_\mu c + \frac{\alpha}{2} g f^{\alpha\beta\gamma}\partial \cdot A^\alpha \bar{c}^\beta c^\gamma - \tfrac{1}{8}\alpha(1-\tfrac{1}{2}\alpha)\lambda g^2 f^{\alpha\beta\gamma}\bar{c}^\beta\bar{c}^\gamma f^{\alpha\eta\xi}c^\eta c^\xi \; . \tag{2.2b}$$

Here we are assuming a Yang-Mills theory with a simple gauge group and introduce the notation

Lie algebra:

$$[T^\alpha, T^\beta] = i f^{\alpha\beta\gamma}T^\gamma \; ,$$

Covariant derivative:

$$(D_\mu c)^\alpha \equiv D^{\alpha\beta}_\mu c^\beta = (\partial_\mu \delta^{\alpha\beta} + g f^{\alpha\beta\gamma}A^\gamma_\mu)c^\beta \; .$$

$f^{\alpha\beta\gamma}$ are totally antisymmetric.

This action has the global symmetries under the following transformations.

BRS:

$$\begin{aligned} \delta A^\alpha_\mu &= (D_\mu c)^\alpha \delta\Lambda \; , \\ \delta c^\alpha &= -\tfrac{1}{2} g f^{\alpha\beta\gamma}c^\beta c^\gamma \delta\Lambda \; , \\ \delta\bar{c}^\alpha &= \left(\frac{\partial \cdot A^\alpha}{\lambda} - \tfrac{1}{2}\alpha f^{\alpha\beta\gamma}\bar{c}^\beta c^\gamma \right)\delta\Lambda \; . \end{aligned} \tag{2.3a}$$

Anti-BRS:

$$\begin{aligned} \delta A^\alpha_\mu &= (D_\mu \bar{c})^\alpha \delta\Lambda \; , \\ \delta \bar{c}^\alpha &= -\tfrac{1}{2} g f^{\alpha\beta\gamma}\bar{c}^\beta \bar{c}^\gamma \delta\Lambda \; , \\ \delta c^\alpha &= \left(\frac{\partial \cdot A^\alpha}{\lambda} - (1-\tfrac{1}{2}\alpha) f^{\alpha\beta\gamma}\bar{c}^\beta c^\gamma \right)\delta\Lambda \; . \end{aligned} \tag{2.3b}$$

In anti-BRS transformations the roles of c and $\bar{c}$ are interchanged in addition to changes in some coefficients. Note that the $\alpha = 0$ case yields the usual Faddeev-Popov action and $\alpha = 1$ yields an action symmetric in c and $\bar{c}$.

B. Quadratic gauges

Next, we shall consider quadratic gauges. These are given by the gauge function $f^\alpha[A]$ that is quadratic in fields. For example,

$$f^\alpha[A] = \partial \cdot A^\alpha + d^{\alpha\beta\gamma}A^\beta_\mu A^{\gamma\mu} \tag{2.4}$$

Here $d^{\alpha\beta\gamma}$ is symmetric in β and γ. The effective action according to Faddeev-Popov is given by

$$S_{\rm eff}[A,c,\bar{c}] = \int d^4x\left[-\tfrac{1}{4}F^\alpha_{\mu\nu}F^{\alpha\mu\nu} - \frac{1}{2\lambda}\sum_\alpha f^\alpha[A]^2 + \mathcal{L}_G\right] \quad (2.5)$$

with

$$\mathcal{L}_G[A,c,\bar{c}] = -\bar{c}^\alpha(\partial_\mu\delta^{\alpha\beta} + 2d^{\alpha\beta\gamma}A^\gamma_\mu)(D_\mu c)^\beta \ . \quad (2.6)$$

This action has only a BRS invariance given by

$$\delta A^\alpha_\mu = (D_\mu c)^\alpha \delta\Lambda \ ,$$
$$\delta c^\alpha = -\tfrac{1}{2}g f^{\alpha\beta\gamma}c^\beta c^\gamma \delta\Lambda \ , \quad (2.7)$$
$$\delta\bar{c}^\alpha = \frac{f^\alpha[A]}{\lambda}\delta\Lambda \ .$$

Unlike linear gauges, this Faddeev-Popov action has no anti-BRS invariance nor can it be generalized to another local functional of $A, c, \bar{c}$ so as to have a double BRS invariance.

C. Gauge transformations that change λ

Next we recall how a transformation of gauge parameter is performed on a path integral involving the Faddeev-Popov action [$\alpha = 0$ in (2.1) or (2.5)]. To change the gauge parameter λ by an infinitesimal amount $\delta\lambda$, one performs an infinitesimal gauge transformation on the gauge field in which the gauge parameter is field dependent:

$$A^\alpha_\mu \to A'^\alpha_\mu = A^\alpha_\mu + D^{\alpha\beta}_\mu \theta^\beta(x) \quad (2.8)$$

with

$$\theta^\beta(x) = -\tfrac{1}{2}M^{-1}_{\beta\gamma}[A]f^\alpha[A]\frac{\delta\lambda}{\lambda} \ , \quad (2.9)$$

where

$$M_{\alpha\beta} = \frac{\delta f^\alpha[A]}{\delta A^\gamma_\mu}D^{\alpha\beta}_\mu \ . \quad (2.10)$$

It is easily seen that this leads to

$$-\sum_\alpha \frac{f^\alpha[A]^2}{2\lambda} = -\sum_\alpha \frac{f^\alpha[A']^2}{2(\lambda+\delta\lambda)} + O(\delta\lambda^2) \ . \quad (2.11)$$

It is also known that [1]

$$\mathcal{D}A\int \mathcal{D}c\mathcal{D}\bar{c}\exp\left(-i\int \bar{c}Mcd^4x\right) = \mathcal{D}A\,\det M = \mathcal{D}A'\det M[A'] = \mathcal{D}A'\int \mathcal{D}c\mathcal{D}\bar{c}\exp\left(-i\int \bar{c}M[A']cd^4x\right) \ . \quad (2.12)$$

All discussion of infinitesimal change in the gauge parameter is then based on the infinitesimal gauge transformations of Eq. (2.9). It leads to the parameter change $\lambda \to \lambda + \delta\lambda$ via

$$\mathcal{D}A\int \mathcal{D}c\mathcal{D}\bar{c}\exp\left[\int\left(-\frac{f^\alpha[A]^2}{2\lambda} + \bar{c}Mc\right)d^4x\right]$$
$$= \mathcal{D}A'\int \mathcal{D}c\mathcal{D}\bar{c}\exp\left[\int\left(-\frac{f^\alpha[A']^2}{2(\lambda+\delta\lambda)} + \bar{c}M[A']c\right)d^4x\right] \ . \quad (2.13)$$

When one wants to change the gauge parameter by a finite amount $\lambda \to \lambda + \Delta$ one must perform a finite gauge transformation:

$$A \to A' = U[U^{-1}\partial_\mu U + A]U^{-1} \quad (2.14)$$

with $U = e^{iT^\alpha\theta^\alpha[A]}$, whose infinitesimal form must yield Eq. (2.9). Such a transformation is not easy to construct explicitly by integration of Eq. (2.8).

III. FINITE FIELD-DEPENDENT BRS TRANSFORMATION

In this section, we shall discuss finite field-dependent BRS transformations that can be obtained by integration of infinitesimal (field-dependent) BRS transformations.

We note that the invariance of the gauge invariant Lagrange density under infinitesimal gauge transformations

$$A'^\alpha_\mu = A^\alpha_\mu + D^{\alpha\beta}_\mu\theta^\beta(x) \quad (3.1)$$

does not depend on whether the local parameters $\theta^\beta(x)$ are field dependent or not as long as they are infinitesimal. [In fact, we mention a field-dependent gauge transformation in Eqs. (2.8) and (2.9)]. In a similar manner the BRS invariance of the Faddeev-Popov action,

$$S_{\rm eff} = \int\left[-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{(\partial\cdot A^\alpha)^2}{2\lambda} - \bar{c}Mc\right]d^4x \ , \quad (3.2)$$

under the global BRS transformations of Eq. (2.3a), depends only on the global and anticommuting nature of $\delta\Lambda$ and not on its infinitesimal nature or on whether $\delta\Lambda$ is field independent or not. Thus we are at liberty to choose $\delta\Lambda$ finite and field dependent as long as it is x independent and anticommuting. [For example, $\delta\Lambda = \int d^4y f^{\alpha\beta\gamma}\zeta^\alpha(y)\zeta^\beta(y)c^\gamma(y)$.] Thus a finite BRS transformation has the same form as an infinitesimal one. The BRS transformation which we will be interested in will have the general structure

$$A'^\alpha_\mu(x) = A^\alpha_\mu(x) + D^{\alpha\beta}_\mu c^\beta(x)\Theta[\phi] \ ,$$
$$c'^\alpha(x) = c^\alpha(x) - \tfrac{1}{2}g f^{\alpha\beta\gamma}c^\beta(x)c^\gamma(x)\Theta[\phi] \ , \quad (3.3)$$
$$\bar{c}'^\alpha(x) = \bar{c}^\alpha(x) + \frac{\partial\cdot A^\alpha}{\lambda}\Theta[\phi] \ ,$$

where $\Theta[\phi]$ is an x-independent anticommuting functional of fields $A, c, \bar{c}$ (not necessarily arbitrary, however).

If we are interested in using such a finite field-dependent Θ in the BRS transformation, however, the integration measure is nontrivially affected by it; i.e., the Jacobian may be nontrivial. [Note that if Θ were field independent the Jacobian would be as if for an infinitesimal transformation (as $\Theta^2 = 0$) in which case antisymmetry of structure constants and/or vanishing of $\delta^{(n)}(0)$ in dimensional regularization makes the Jacobian trivial.] As we shall see, the Jacobian for such a finite BRS transformation is indeed nontrivial generally, and one needs a procedure for obtaining it. Rather than calculating the superdeterminant involved, we shall find it easier to follow the following method.

First, consider the fields as a function of the parameter $\kappa : 0 \leq \kappa \leq 1$: For a field $\phi(x,\kappa)$, $\phi(x,0) = \phi(x)\phi(x,\kappa = 1) = \phi'$. We define infinitesimal BRS transformations

$$\frac{d}{d\kappa} A_\mu^\alpha(x,\kappa) = (D_\mu^{\alpha\beta} c^\beta)(x,\kappa)\Theta'[\phi(x,\kappa)] ,$$
$$\frac{d}{d\kappa} c^\alpha(x,\kappa) = -\tfrac{1}{2} g f^{\alpha\beta\gamma} c^\beta(x,\kappa) c^\gamma(x,\kappa)\Theta'[\phi(x,\kappa)] ,$$
$$\frac{d}{d\kappa} \bar{c}^\alpha(x,\kappa) = \frac{\partial \cdot A^\alpha(x,\kappa)}{\lambda}\Theta'[\phi(x,\kappa)] , \tag{3.4}$$

with boundary conditions (BC's) $A_\mu^\alpha(x,0) = A_\mu^\alpha(x)$, $c^\alpha(x,0) = c^\alpha(x)$, and $\bar{c}^\alpha(x,0) = \bar{c}^\alpha(x)$. We shall now solve these equations and, by integrating from $\kappa = 0$ to 1, show that fields $A_\mu^\alpha(x,\kappa=1)$, $c^\alpha(x,\kappa=1)$, $\bar{c}^\alpha(x,\kappa=1)$ are precisely related to $A_\mu^\alpha(x), c^\alpha(x), \bar{c}^\alpha(x)$ by finite BRS equations (3.3) (with Θ related to Θ'); thus showing that Eqs. (3.3) for a certain choice of $\Theta[\phi]$ can be obtained by a succession of infinitesimal BRS transformations of Eq. (3.4). We shall show that the transformation (3.3) can be broken up as a succession of infinitesimal field-dependent BRS transformations. The Jacobian for an infinitesimal transformation is easy to calculate and will be used to obtain, by integration, the net Jacobian. We shall first of all derive a result that gives us $\Theta'[\phi(\kappa)]$ in terms of $\Theta'[\phi(0)]$, a result very useful at different stages of the calculation. We shall also obtain, using this result an expression for $\Theta[\phi(\kappa),\kappa]$, that, as we will see later, enters the integration of Eqs. (3.4).

For later convenience, we write Eqs. (3.4) compactly as

$$\frac{d}{d\kappa}\phi(\kappa) = \delta_{\rm BRS}[\phi(\kappa)]\Theta'[\phi(\kappa)] . \tag{3.5}$$

Now let us calculate $(d/d\kappa)\Theta'[\phi(\kappa)]$ and integrate out the result

$$\frac{d}{d\kappa}\Theta'[\phi(\kappa)] = \left[\frac{\delta\Theta'}{\delta\phi_i}\delta_{\rm BRS}\phi_i\right]\Theta'[\phi(\kappa)]$$
$$\equiv f[\phi(\kappa)]\Theta'[\phi(\kappa)] . \tag{3.6}$$

This has the obvious solution

$$\Theta'[\phi(\kappa)] = \Theta'[\phi(0)]\exp\left[\int_0^\kappa f[\phi(\kappa')]d\kappa'\right] . \tag{3.7}$$

Thus $\Theta'[\phi(\kappa)]$ contains $\Theta'[\phi(0)]$ as factor which is assumed to be nilpotent: $\Theta'^2 = 0$. Now in Eqs. (3.6) we carry out a Taylor expansion of $f[\phi(\kappa)]$ in κ:

$$f[\phi(\kappa)] = f[\phi(0)] + \kappa\left[\frac{\delta f}{\delta\phi_i}\frac{d\phi_i}{d\kappa}\right]_{\kappa=0} + \left[\frac{\delta f}{\delta\phi_i}\frac{d^2\phi_i}{d\kappa^2}\right]_{\kappa=0}\frac{\kappa^2}{2}$$
$$+\left[\frac{\delta^2 f}{\delta\phi_i\delta\phi_j}\frac{d\phi_i}{d\kappa}\frac{d\phi_j}{d\kappa}\right]_{\kappa=0}\frac{\kappa^2}{2} + \cdots ; \tag{3.8}$$

each term on the right hand side except the first is easily seen to contain $\Theta'[\phi(0)]$. $\Theta'[\phi(\kappa)]$ on the right hand side of Eq. (3.6) also contains $\Theta'[\phi(0)]$. Hence, in the right hand side of Eq. (3.6), we may drop all terms in the Taylor expansion of $f[\phi(\kappa)]$ and write

$$\frac{d}{d\kappa}\Theta'[\phi(\kappa)] = f[\phi(0)]\Theta'[\phi(\kappa)] \tag{3.9}$$

with the consequent simplification in Eq. (3.6)

$$\Theta'[\phi(\kappa)] = \Theta'[\phi(0)]\exp\{\kappa f[\phi(0)]\} . \tag{3.10}$$

In the future we shall need the quantity

$$\Theta[\phi(\kappa),\kappa] = \int_0^\kappa d\kappa'\Theta'[\phi(\kappa')] \tag{3.11}$$
$$= \Theta'[\phi(0)]\frac{\exp\{\kappa f[\phi(0)]\} - 1}{f[\phi(0)]} \tag{3.12}$$

formally. (Actually the right hand side is an infinite series.)

In particular, we shall see that $\Theta[\phi(0)]$ of Eq. (3.1) will turn out to be

$$\Theta[\phi(0)] = \Theta[\phi(\kappa),\kappa]|_{\kappa=1} = \Theta'[\phi(0)]\frac{\exp\{f[\phi(0)]\} - 1}{f[\phi(0)]} \tag{3.13}$$
$$= \Theta'[\phi(0)]\left[1 + \tfrac{1}{2}f[\phi(0)] + \frac{1}{3!}f[\phi(0)]^2 + \cdots\right] , \tag{3.14}$$

a known quantity. In particular, it follows that

$$\Theta[\phi(\kappa),\kappa]\Theta'[\phi(\kappa)] \propto \Theta'[\phi(0)]^2 = 0 . \tag{3.15}$$

Now, we can proceed to integrate Eq. (3.4). As was just shown below Eq. (3.7), for any local functional $g[\phi(\kappa)]$,

$$g[\phi(\kappa)]\Theta'[\phi(\kappa)] = g[\phi(0)]\Theta'[\phi(\kappa)] ; \tag{3.16}$$

hence we could write Eqs. (3.5) as

$$\frac{d\phi(\kappa)}{d\kappa} = \delta_{\rm BRS}[\phi(0)]\Theta'[\phi(\kappa)] \ . \tag{3.17}$$

This immediately integrates to

$$\phi(\kappa) = \phi(0) + \delta_{\rm BRS}[\phi(0)]\int_0^\kappa \Theta'[\phi(\kappa')]d\kappa' \ , \tag{3.18}$$

i.e.,

$$\phi(\kappa) = \phi(0) + \delta_{\rm BRS}[\phi(0)]\Theta[\phi(\kappa),\kappa] \ , \tag{3.19}$$

where Θ is defined in Eq. (3.11) and is given in terms of $\phi(0)$ by Eq. (3.12). Again, as $\Theta[\phi(\kappa),\kappa]$ is proportional to $\Theta'[\phi(0)]$ [see Eq. (3.12)] we have

$$\delta_{\rm BRS}[\phi(\kappa)]\Theta[\phi(\kappa),\kappa] = \delta_{\rm BRS}[\phi(0)]\Theta[\phi(\kappa),\kappa] \tag{3.20}$$

and we can invert (3.18) to yield

$$\phi(0) = \phi(\kappa) - \delta_{\rm BRS}[\phi(\kappa)]\Theta[\phi(\kappa),\kappa] \ . \tag{3.21}$$

Using the definition of $\Theta[\phi]$ of Eq. (3.13), Eq. (3.19) at $\kappa = 1$ reads

$$\phi' = \phi + \delta_{\rm BRS}[\phi]\Theta[\phi] \ , \tag{3.22}$$

where $\phi' = \phi(\kappa = 1)$ and $\phi = \phi(\kappa = 0)$. These are precisely the finite BRS transformations of Eq. (3.3) for $\Theta[\phi]$ obtainable from a local $\Theta'[\phi]$ via definitions (3.13) and (3.11). This thus proves the result announced earlier.

IV. METHOD FOR EVALUATION OF JACOBIANS

In this section, we shall present a procedure for evaluation for Jacobians for finite BRS transformations of the type of Eq. (3.3) for certain specific $\Theta[\phi]$'s, utilizing the fact that they can be written as a succession of infinitesimal transformations of the type (3.4). We define

$$\begin{aligned}\mathcal{D}A\mathcal{D}c\mathcal{D}\bar{c} &= J(\kappa)\mathcal{D}A(\kappa)\mathcal{D}c(\kappa)\mathcal{D}\bar{c}(\kappa)\\ &= J(\kappa+d\kappa)\mathcal{D}A(\kappa+d\kappa)\\ &\quad\times\mathcal{D}c(\kappa+d\kappa)\mathcal{D}\bar{c}(\kappa+d\kappa) \ .\end{aligned} \tag{4.1}$$

Now the transformation from $A(\kappa)$ to $A(\kappa+d\kappa)$ is an infinitesimal one and one has, for its Jacobian,

$$\frac{J(\kappa)}{J(\kappa+d\kappa)} = \sum_\phi \pm \frac{\delta\phi(x,\kappa+d\kappa)}{\delta\phi(x,\kappa)} \ , \tag{4.2}$$

where $\sum_\phi$ sums over all fields in the measure $A^\alpha_\mu, c^\alpha, \bar{c}^\alpha$ and the $\pm$ sign refers to whether ϕ is a bosonic or a fermionic field. We evaluate the right hand side as

$$\int d^4x \sum_\alpha \left[\sum_\mu \frac{\delta A^\alpha_\mu(x,\kappa+\delta\kappa)}{\delta A^\alpha_\mu(x,\kappa)} - \frac{\delta c^\alpha(x,\kappa+\delta\kappa)}{\delta c^\alpha(x,\kappa)} - \frac{\delta\bar{c}^\alpha(x,\kappa+\delta\kappa)}{\delta\bar{c}^\alpha}\right] \ . \tag{4.3}$$

Dropping those terms which do not contribute on account of the antisymmetry of the structure constant [these terms also do not contribute in dimensional regularization on account of $\delta^4(0)$], we have the expression (4.3) as (reverting back to the summation convention)

$$1 + d\kappa \int \left[D^{\alpha\beta}_\mu c^\beta(x,\kappa)\frac{\delta\Theta'[\phi(x,\kappa)]}{\delta A^\alpha_\mu(x,\kappa)} - \tfrac{1}{2}g f^{\alpha\beta\gamma}c^\beta(x,\kappa)c^\gamma(x,\kappa)\frac{\delta\Theta'[\phi]}{\delta c^\alpha(x,\kappa)} - \frac{\partial\cdot A^\alpha(x,\kappa)}{\lambda}\frac{\delta\Theta'[\phi]}{\delta\bar{c}^\alpha(x,\kappa)}\right]$$

$$= \frac{J(\kappa)}{J(\kappa+d\kappa)} \tag{4.4}$$

$$= 1 - \frac{1}{J(\kappa)}\frac{dJ(\kappa)}{d\kappa}d\kappa \ . \tag{4.5}$$

Now consider

$$\begin{aligned}W &= \int \mathcal{D}A\mathcal{D}c\mathcal{D}\bar{c}\, e^{iS_{\rm eff}[A,c,\bar{c}]}\\ &\equiv \int \mathcal{D}\phi(x,0)\, e^{iS_{\rm eff}[\phi(x,0)]} \ ,\end{aligned} \tag{4.6}$$

where $\phi(x,0)$ generically denotes fields $A, c, \bar{c}$ at $\kappa = 0$. This equals

$$\int \mathcal{D}\phi(x,\kappa)J(\kappa)e^{iS_{\rm eff}[\phi(x,\kappa)]} \ , \tag{4.7}$$

where invariance of $S_{\rm eff}$ under $\phi(x,0) \to \phi(x,\kappa)$ of Eq. (3.21), which is a BRS transformation, has been used. This expression is further equal to

$$\begin{aligned}&\int \mathcal{D}\phi(x,\kappa+d\kappa)J(\kappa+d\kappa)e^{iS_{\rm eff}[\phi(x,\kappa+d\kappa)]}\\ &\quad= \int \mathcal{D}\phi(x,\kappa+d\kappa)J(\kappa)\left[1+\frac{1}{J}\frac{dJ}{d\kappa}d\kappa\right]\\ &\quad\quad\times e^{iS_{\rm eff}[\phi(x,\kappa+d\kappa)]} \ .\end{aligned} \tag{4.8}$$

We would now like to show that $J(\kappa)$ in Eq. (4.3) can be replaced by $\exp(iS_1[\phi(\kappa);\kappa])$ for a certain functional S_1 (to be determined in each individual case). To this end, consider expression (4.8) with $J(\kappa+d\kappa)$ replaced by $e^{iS_1[\phi(\kappa+d\kappa);\kappa+d\kappa]}$ and call this quantity $W'(\kappa+d\kappa)$:

$$W'(\kappa+d\kappa) \equiv \int \mathcal{D}\phi(\kappa+d\kappa) e^{iS_1[\phi(\kappa+d\kappa);\kappa+d\kappa]} e^{iS_{\rm eff}[\phi(\kappa+d\kappa)]} \tag{4.9}$$

$$= \int \mathcal{D}\phi(\kappa) \frac{J(\kappa)}{J(\kappa+d\kappa)} e^{iS_1[\phi(\kappa),\kappa]} \left[1 + \frac{dS_1}{d\kappa} d\kappa\right] e^{iS_{\rm eff}[\phi(\kappa)]} , \tag{4.10}$$

where $dS_1/d\kappa$ is a total derivative of S_1 with respect to κ in which dependence on $\phi(\kappa)$ is also differentiated. $S_{\rm eff}[\phi(\kappa)] = S_{\rm eff}[\phi(\kappa+d\kappa)]$ has also been used. Hence

$$W'(\kappa+d\kappa) = \int \mathcal{D}\phi(\kappa) \left[1 - \frac{1}{J}\frac{dJ}{d\kappa} d\kappa\right] \left[1 + i\frac{dS_1}{d\kappa} d\kappa\right] \times e^{iS_1 + iS_{\rm eff}} \tag{4.11}$$

$$= W'(\kappa) \tag{4.12}$$

if and only if (iff)

$$\int \mathcal{D}\phi(\kappa) \left[\frac{1}{J}\frac{dJ}{d\kappa} - i\frac{dS_1}{d\kappa}\right] e^{iS_1 + iS_{\rm eff}} = 0 . \tag{4.13}$$

Further, if Eq. (4.13) is satisfied, $dW'/d\kappa = 0$ and hence W' is independent of κ. Hence $W' = W'|_{\kappa=0}$; so that

$$\int \mathcal{D}(\kappa) e^{iS_1[\phi(\kappa),\kappa] + iS_{\rm eff}[\phi(\kappa)]} = \int \mathcal{D}\phi(0) e^{iS_{\rm eff}[\phi(0)]} , \tag{4.14}$$

provided S_1 vanishes identically at $\kappa = 0$. But the right hand side of Eq. (4.14) is just W of Eq. (4.6). Hence $W' = W$; in the expression (4.7) for W one can replace $J(\kappa)$ by $e^{iS_1[\phi(\kappa),\kappa]}$. Setting $\kappa = 1$ in (4.7) we could write, in terms of $\phi' = \phi(\kappa = 1)$.

$$W = \int \mathcal{D}\phi' e^{iS_1[\phi'] + iS_{\rm eff}[\phi']} . \tag{4.15}$$

Thus the Jacobian yeilds a new piece to the action. Our procedure, then, in evaluating Jacobians is (i) calculate the infinitesimal Jacobian change $(1/J)(dJ/d\kappa)d\kappa$ of Eq. (4.5) for the infinitesimal BRS transformations of Eq. (3.8), (ii) make an ansatz for S_1, (iii) prove the equation (4.13) for this ansatz, and (iv) replace $J(\kappa)$ by e^{iS_1} in the expression for W of Eq. (4.7). Setting $\kappa = 1$, this would then yield the new effective action $S'_{\rm eff} = S_1 + S_{\rm eff}$.

To proceed further, it is necessary to assume a particular form for $\Theta[\phi]$.

Case I:

$$\Theta'[\phi(y,\kappa)] = i\beta \int d^4y f^{\alpha\beta\gamma} \bar{c}^\alpha(y,\kappa) \bar{c}^\beta(y,\kappa) c^\gamma(y,\kappa) ; \tag{4.16}$$

clearly, from Eq. (4.5),

$$\frac{1}{J(\kappa)}\frac{dJ(\kappa)}{d\kappa} = -i\beta d\kappa \left\{ -\tfrac{1}{2} g \int d^4y\, f^{\alpha\beta\gamma} \bar{c}^\alpha(y,\kappa) \bar{c}^\beta(y,\kappa) f^{\gamma\eta\xi} c^\eta(y,\kappa) c^\xi(y,\kappa) + 2\int d^4y\, f^{\alpha\beta\gamma} \frac{\partial\cdot A^\alpha(y,\kappa)}{\lambda} \bar{c}^\beta(y,\kappa) c^\gamma(y,\kappa) \right\} . \tag{4.17}$$

Case II:

$$\Theta'[\phi(y,\kappa)] = i\beta \int d^4y\, d^{\alpha\beta\gamma} A^\beta_\mu(y,\kappa) A^{\gamma\mu}(y,\kappa) \bar{c}^\alpha(y,\kappa) , \tag{4.18}$$

$$\frac{1}{J(\kappa)}\frac{dJ(\kappa)}{d\kappa} = i\beta \left[2\int d^4y\, d^{\alpha\beta\gamma} D^{\beta\eta}_\mu c^\eta(y,\kappa) \times A^{\gamma\mu}(y,\kappa) \bar{c}^\alpha(y,\kappa) + \int d^4y\, d^{\alpha\beta\gamma} A^\beta_\mu A^{\gamma\mu}(y,\kappa) \frac{\partial\cdot A^\alpha_\mu(y,\kappa)}{\lambda} \right] . \tag{4.19}$$

Case III:

$$\Theta'[\phi(y,\kappa)] = i\gamma \int d^4y\, \bar{c}(y,\kappa) \partial\cdot A^\alpha(y,\kappa) \tag{4.20}$$

$$\frac{1}{J(\kappa)}\frac{dJ(\kappa)}{d\kappa} = i\gamma \left[\int d^4y\, \bar{c}^\alpha(y,\kappa)(Mc)^\alpha(y,\kappa) + \int d^4y \frac{1}{\lambda} [\partial\cdot A^\alpha(y,\kappa)]^2 \right] . \tag{4.21}$$

The net Jacobian, or properly $\langle J(\kappa)\rangle$, the expectation values of the Jacobians, will be evaluated in the next section by the procedure outlined earlier.

V. EVALUATION OF JACOBIANS

In this section we shall evaluate the Jacobian explicitly for three particular choices of Θ'. These choices are not, however, arbitrary: they correspond to three particularly important cases. These are BRS transformations that take the Faddeev-Popov effective action in linear gauges to (i) the most general BRS–anti-BRS invariant effective action, or to (ii) quadratic gauge effective action, or to (iii) the Faddeev-Popov effective action with changed gauge parameters. We shall follow the procedure outlined in the previous section.

Before we proceed, we shall need an identity which occurs in the examples. Hence we shall derive it first.

Consider the quantity

$$A=\int \mathcal{D}\phi h^{\alpha}[\phi]M_0 c^{\alpha}\Theta'[\phi]e^{iS_{\rm eff}+iS_1}\ . \tag{5.1}$$

Here $h^{\alpha}[\phi]$ is some functional of $\phi\equiv A,c,\bar{c}$. The Faddeev-Popov ghost term in linear gauges is $-\int d^4x\,\bar{c}M_0 c$. $\Theta'[\phi]$ is the appropriate Θ' function in a given example; e^{iS_1} is the Jacobian equivalent being tried as an ansatz. We note

$$(M_0 c)^{\alpha}=i\frac{\delta}{\delta\bar{c}^{\alpha}}e^{iS_{\rm eff}}\ . \tag{5.2}$$

Hence

$$A=\int \mathcal{D}\phi h^{\alpha}[\phi]\left[i\frac{\delta}{\delta\bar{c}^{\alpha}}e^{iS_{\rm eff}}\right]\Theta' e^{iS_1}\ . \tag{5.3}$$

We now integrate by parts in $\bar{c}^{\alpha}$. The result is

$$A=-i\int \mathcal{D}\phi\left[\frac{\delta h^{\alpha}}{\delta\bar{c}^{\alpha}}\Theta'+h^{\alpha}\frac{\delta\Theta'}{\delta c^{\alpha}}+i\Theta' h^{\alpha}\frac{\delta S_1}{\delta\bar{c}^{\alpha}}\right]e^{iS_{\rm eff}+iS_1}\ . \tag{5.4}$$

Case I. Consider

$$\Theta'[\phi]=i\beta\int d^4x\, f^{\alpha\beta\gamma}\bar{c}^{\alpha}(y,\kappa)\bar{c}^{\beta}(y,\kappa)c^{\gamma}(y,\kappa)\ . \tag{5.5}$$

We shall introduce the shorthand notation $\int f\bar{c}\bar{c}c$ to mean the integral in the above quantity in Θ'. We shall also write $(fcc)^{\alpha}=f^{\alpha\beta\gamma}c^{\beta}c^{\gamma}$ and $\int f\bar{c}\bar{c}fcc=\int d^4y(f\bar{c}\bar{c})^{\alpha}(fcc)^{\alpha}$. We recall the logarithmic derivation of the Jacobian of Eq. (4.17), viz.,

$$\frac{1}{J}\frac{dJ}{d\kappa}d\kappa=-i\beta\, d\kappa\left[-\tfrac{1}{2}g\int f\bar{c}\bar{c}fcc+\frac{2}{\lambda}\int f\partial\cdot A\bar{c}c\right]\ . \tag{5.6}$$

This suggests the terms linear in κ in S_1. But S_1 could have higher powers of κ. We make an ansatz for S_1 and show that it works with the choice of parameters in it. Let

$$S_1[\phi(x,\kappa);\kappa]=\left(\frac{\beta\kappa}{2}-\xi\beta\kappa^2\right)g\int f\bar{c}\bar{c}fcc-\frac{2\beta\kappa}{\lambda}\int f\partial\cdot A\bar{c}c \tag{5.7}$$

when ξ is yet to be determined. Now

$$\begin{aligned}-i\int \mathcal{D}\phi e^{iS_1+iS_{\rm eff}}\frac{dS_1}{d\kappa}=\int \mathcal{D}\phi \exp[iS_1+S_{\rm eff}]\Bigg[&\frac{-i\beta}{2}g\int f\bar{c}\bar{c}fcc+\frac{2i\beta}{\lambda}\int f\partial\cdot A\bar{c}c\\ &+2i\xi\beta\kappa g\int f\bar{c}\bar{c}fcc+i\beta(\kappa-2\xi\kappa^2)g\int f\frac{\partial\cdot A}{\lambda}\bar{c}fcc\Theta'\\ &+\frac{2i\beta\kappa}{2}(-\tfrac{1}{2}g)\int f\partial\cdot A\bar{c}(fcc)^{\gamma}\Theta'-\frac{2i\beta\kappa}{\lambda}\int f(M_0c)\bar{c}c\Theta'\Bigg]\ .\end{aligned} \tag{5.8}$$

Here (i) the BRS invariance of fcc and (ii) the antisymmetry of structure constants have been used to drop two of the conditions. In the last term in Eq. (5.7) we use Eq. (5.4) by setting $h^{\alpha}=f^{\alpha\beta\gamma}\bar{c}^{\beta}c^{\gamma}$ (and use the antisymmetry of the structure constant to set $\delta h^{\alpha}/\delta\bar{c}^{\alpha}=0$) to obtain

$$\begin{aligned}\frac{2i\beta\kappa}{\lambda}\int \mathcal{D}\phi\exp[iS_1+iS_{\rm eff}]\int f(M_0c)\bar{c}c\Theta'=\frac{2i\beta\kappa}{\lambda}\int \mathcal{D}\phi\exp[iS_1+iS_{\rm eff}]\Bigg[&\int (f\bar{c}c)^{\alpha}(f\bar{c}c)^{\alpha}2\beta\\ &+\Theta'\{(\beta\kappa-2\xi\beta\kappa^2)\int f(f\bar{c}c)\bar{c}(fcc)+\frac{2\beta\kappa}{\lambda}\int f(\partial\cdot A)(f\bar{c}c)c\}\Bigg]\ .\end{aligned} \tag{5.9}$$

Now we use

$$\begin{aligned}f(f\bar{c}\bar{c})\bar{c}(fcc)&\equiv f^{\alpha\beta\gamma}f^{\alpha\eta\xi}\bar{c}^{\eta}c^{\xi}\bar{c}^{\beta}f^{\gamma\tau\sigma}c^{\tau}c^{\sigma}\\ &\sim f^{\alpha\eta\beta}\bar{c}^{\beta}c^{\eta}f^{\alpha\xi\gamma}c^{\xi}f^{\gamma\tau\sigma}c^{\tau}c^{\sigma}=0\end{aligned}$$

on account of the repeated use of the Jacobi identity. We also note $(f\bar{c}c)(f\bar{c}c)=-\frac{1}{2}(f\bar{c}\bar{c})(fcc)$. From Eqs. (5.6), (5.8), and (5.9), we easily see that

$$\int \mathcal{D}\phi\left(\frac{1}{J}\frac{dJ}{d\kappa}-i\frac{dS_1}{d\kappa}\right)\exp[iS_1+S_{\rm eff}]=0 \tag{5.10}$$

iff

$$\frac{\beta}{\lambda}=-\xi g\ . \tag{5.11}$$

Substituting for β in (5.7) we obtain

$$S'_{\rm eff} = S_0 + S_{g\cdot f} + \int d^4x \left[-\bar{c}M_0 c + 2g\xi\kappa \int f\partial \cdot A\bar{c}c - \frac{g^2\xi\kappa\lambda}{2}(1-2\xi\kappa)\int f\bar{c}\bar{c}fcc \right] . \qquad (5.12)$$

In view of the fact that only one combination of ξ and κ, viz. $\xi\kappa$, is appearing in Eq. (5.12), we parametrize it as

$$2\xi\kappa = \frac{\alpha}{2} . \qquad (5.13)$$

This gives

$$S'_{\rm eff} = S_0 + S_{\rm eff} + \int d^4x \left[\bar{c}M_0 c + \frac{\alpha}{2} g \int f\partial \cdot A\bar{c}c - \frac{\lambda}{8} g^2 \left(1-\frac{\alpha}{2}\right)\alpha \int f\bar{c}\bar{c}fcc \right] . \qquad (5.14)$$

This is precisely the action of Eq. (2.2b) of Thierry-Mieg and Baulieu [5] having both BRS anti-BRS invariances.

Case II.

$$\Theta' = i\beta \int d^{\alpha\beta\gamma}\bar{c}^\alpha(y,\kappa)A^\beta_\mu(y,\kappa)A^{\gamma\mu}(y,\kappa)d^4y . \qquad (5.15)$$

The discussion proceeds very much the same way as in case I; hence we only give minimal analogous steps. We note

$$\frac{1}{J}\frac{dJ}{d\kappa} = 2i\beta \int d\bar{c}(D_\mu c)A^\mu + \frac{i\beta}{\lambda}\int d(\partial\cdot A)A_\mu A^\mu , \qquad (5.16)$$

using the obvious notation $d\phi_1\phi_2\phi_3 = d^{\alpha\beta\gamma}\phi_1^\alpha\phi_1^\beta\phi_3^\gamma$. [Also we shall use $(d\phi_1\phi_2)^\alpha = d^{\alpha\beta\gamma}\phi_1^\beta\phi_2^\gamma$.] We make an ansatz

$$S_1 = 2\beta\kappa \int d\bar{c}(D_\mu c)A^\mu + \frac{\beta\kappa}{\lambda}\int d(\partial\cdot A)A_\mu A^\mu + \xi\kappa^2 \int [(dA_\mu A^\mu)^\alpha]^2 . \qquad (5.17)$$

We note that after using Eq. (5.4) with $h^\alpha = d^{\alpha\beta\gamma}A^\beta_\mu A^{\gamma\mu}$ once we can write, after straightforward cancellations,

$$\int \mathcal{D}\phi(-i)\frac{dS_1}{d\kappa}\exp\{iS_1 + iS_{\rm eff}\} = \int \mathcal{D}\phi \exp[iS_1 + iS_{\rm eff}]\Bigg\{ -2i\beta\int d\bar{c}(Dc)A - \frac{i\beta}{\lambda}\int d(\partial\cdot A)A_\mu A^\mu + 2i\xi\kappa\int (dAA)^2 + \frac{2i\beta^2\kappa^2}{\lambda}\int\int (dAA)^\alpha(dDcA)^\alpha\Theta' + \frac{i\kappa\beta^2}{\lambda}\int (dAA)^2 + 4i\xi\kappa^2\int (dAA)^\alpha(dDcA)^\alpha\Theta' \Bigg\} . \qquad (5.18)$$

Thus

$$\int \mathcal{D}\phi \exp[iS_1 + S_{\rm eff}]\left[\frac{1}{J}\frac{dJ}{d\kappa} - i\frac{dS_1}{d\kappa}\right] = 0 , \qquad (5.19)$$

$$\text{iff } 2\xi = -\frac{\beta^2}{\lambda} . \qquad (5.20)$$

We now set $\kappa = 1$. We can choose $\beta = -1$ also as it is arbitrary;

$$\Theta' = -i\int d^{\alpha\beta\gamma}\bar{c}^\alpha A^\beta_\mu A^{\gamma\mu}d^4y \qquad (5.21)$$

and

$$S_1 = -\frac{1}{\lambda}\int d(\partial\cdot A)AA - 2\int d\bar{c}(D_\mu c)A^\mu - \frac{1}{2\lambda}\int (dA_\mu A^\mu)^2 . \qquad (5.22)$$

$S'_{\rm eff} = S_{\rm eff} + S_1$ is then precisely the effective action of quadratic gauges given in Eq. (2.5).

Case III.

$$\Theta' = i\gamma\int \bar{c}^\alpha(y,\kappa)\partial\cdot A^\alpha(y\cdot\kappa)d^4y \qquad (5.23)$$

with γ to be yet determined. Here

$$\frac{1}{J}\frac{dJ}{d\kappa} = i\gamma\int \bar{c}M_0 c + i\gamma\int \frac{(\partial\cdot A)^2}{\lambda} . \qquad (5.24)$$

We expect this Θ' to take us from one gauge parameter λ to another gauge parameter λ'. So we try an ansatz

$$S_1 = \xi(\kappa)\int \frac{(\partial\cdot A)^2}{\lambda} , \qquad (5.25)$$

i.e.,

$$-i\frac{dS_1}{d\kappa} = -i\xi'(\kappa)\int \frac{(\partial\cdot A)^2}{\lambda} - 2i\xi(\kappa)\int \frac{\partial\cdot A}{\lambda}M_0 c\Theta' . \qquad (5.26)$$

Hence we have

$$\int \mathcal{D}\phi(\kappa)\exp[iS_1 + S_{\rm eff}]\left[\frac{1}{J}\frac{dJ}{d\kappa} - i\frac{dS}{d\kappa}\right] = \int \mathcal{D}\phi \exp[iS_1 + iS_{\rm eff}]\left\{ i\gamma \int \bar{c}^\alpha (M_0 c)^\alpha + i[\gamma - \xi'(\kappa)]\int \frac{(\partial\cdot A)^2}{\lambda} - 2i\xi(\kappa)\int \frac{\partial\cdot A}{\lambda} M_0 c\Theta' \right\} . \qquad (5.27)$$

We apply Eq. (5.3) with $h^\alpha = \bar{c}^\alpha, \Theta' \to 1$ to the first term on the right hand side to find it $\propto \delta^{(n)}(0)$ which we drop in dimensional regularization. We apply Eq. (5.3) to the last term (with $h^\alpha = \partial\cdot A^\alpha/\lambda$ and $\Theta' = i\gamma\int \bar{c}\partial\cdot A$) to learn that we may effect the substitution

$$\int \frac{\partial\cdot A}{\lambda} M_0 c\Theta' \to \gamma \frac{(\partial\cdot A)^2}{\lambda} .$$

Thus the right hand side of Eq. (5.27) is then seen to vanish iff

$$-i[-\gamma + \xi'(\kappa) - 2i\xi(\kappa)\gamma] = 0 ,$$

i.e., $\xi(\kappa)$ must be a solution of

$$\xi'(\kappa) + 2\gamma\xi(\kappa) - \gamma = 0$$

with the BC $\xi(0) = 0$. The unique solution is

$$\ln|1 - 2\xi(k)| = -2\gamma\kappa .$$

Thus at $\kappa = 1$ the extra term in the net effective action from the Jacobian is

$$S_1(\kappa = 1) = \tfrac{1}{2}(-e^{-2\gamma} + 1)\int \frac{(\partial\cdot A)^2}{\lambda} d^4x . \qquad (5.28)$$

The net gauge-fixing term is then

$$-\frac{(\partial\cdot A)^2}{2\lambda} e^{-2\gamma} \equiv -\frac{(\partial\cdot A)^2}{2\lambda'} . \qquad (5.29)$$

This leads to $\gamma = -\frac{1}{2}\ln(\lambda'/\lambda)$. Thus

$$\Theta' = -\tfrac{1}{2}\ln\left(\frac{\lambda'}{\lambda}\right)\int \bar{c}^\alpha \partial\cdot A^\alpha \qquad (5.30)$$

leads one from a Faddeev-Popov effective action with the gauge parameter λ to the same effective action with the gauge parameter λ'.

VI. CONCLUSIONS AND POSSIBLE APPLICATIONS

We have taken the infinitesimal BRS of the form normally given in obvious notation by

$$\delta\phi(x) = (\delta_{\rm BRS}\phi)\delta\Lambda \qquad (6.1)$$

and considered its modifications in which $\delta\Lambda = \Theta' d\kappa$ is a field-dependent, x-independent, anticommuting parameter involving the differential of a parameter $\kappa (0 \le \kappa \le 1)$: viz.,

$$d\phi(x,\kappa) = [\delta_{\rm BRS}\phi(x,\kappa)]\Theta'[\phi(x,\kappa)]d\kappa . \qquad (6.2)$$

We have shown that this set of three equations can be simultaneously integrated out to yield again transformations of the BRS form itself: viz.,

$$\phi'(x) = \phi(x) + \delta_{\rm BRS}(\phi)\Theta[\phi(x)] \qquad (6.3)$$

[where $\phi'(x) = \phi(x,\kappa = 1)$; $\phi(x) = \phi(x,\kappa = 0)$]. We call (6.3) the finite, field-dependent transformation as it relates $\phi(x)$ to $\phi'(x)$, differing from it not by an infinitesimal amount but by a finite amount. Here, given Θ', Θ is known in a closed form, given formally by Eq. (3.12): viz.,

$$\Theta(\phi) = \Theta'(\phi)\frac{\exp[f(\phi) - 1]}{f(\phi)} , \qquad (6.4)$$

where f is defined in terms of BRS variation of Θ' by Eq. (3.6).

We have applied transformations of the form of Eq. (6.3) to correlate different effective actions of the gauge theory, yielding the same S matrix. We have shown that the corresponding difference in effective actions is on account of the Jacobian for corresponding finite BRS transformations of Eq. (6.3). We have, by suitable conjecture duly verified in Sec. V, obtained Θ for such transformations in each case; and we have dealt with three cases in all. These are summarized below.

(A) Faddeev-Popov→BRS–anti-BRS. Here, setting $\kappa = 1$ in (5.12) and using (5.10) in (5.5) we find

$$\Theta'[\phi] = -\frac{i\lambda\alpha}{4} g \int d^4y\, f^{\alpha\beta\gamma}\bar{c}^\alpha\bar{c}^\beta c^\gamma , \qquad (6.5)$$

$$f[\phi] = -\frac{i\alpha}{2} g \int d^4y\, f^{\alpha\beta\gamma}(\partial\cdot A)^\alpha \bar{c}^\beta c^\gamma - \frac{\alpha\lambda}{8} g^2 \int f^{\alpha\beta\gamma}\bar{c}^\alpha\bar{c}^\beta f^{\alpha\eta\xi} c^\eta c^\xi , \qquad (6.6)$$

and $\Theta[\phi]$ given by Eq. (6.4). Thus in the generating function of the Faddeev-Popov the effective action given by

$$\begin{aligned} A'_\mu &= A_\mu + D_\mu c\Theta[\phi] , \\ c^{\alpha\prime} &= c^\alpha - \tfrac{1}{2} g f^{\alpha\beta\gamma} c^\beta c^\gamma \Theta[\phi] , \\ \bar{c}^{\alpha\prime} &= \bar{c}^\alpha + \frac{\partial\cdot A^\alpha}{\lambda}\Theta[\phi] \end{aligned} \qquad (6.7)$$

converts it into a generating functional of general BRS–anti-BRS invariant action having parameters α and λ.

(B) Faddeev-Popov (linear gauge)→ Faddeev-Popov (quadratic gauge). Here

$$\Theta'[\phi] = -i \int d^{\alpha\beta\gamma} \bar{c}^{\alpha} A^{\beta}_{\mu} A^{\gamma\mu} \tag{6.8}$$

and

$$f[\phi] = -i \int \frac{d^{\alpha\beta\gamma}}{\lambda} \partial \cdot A^{\alpha} A^{\beta}_{\mu} A^{\gamma\mu} - 2 \int d^{\alpha\beta\gamma} \bar{c}^{\alpha} (D_{\mu} c)^{\beta} A^{\gamma\mu} \tag{6.9}$$

A discussion similar to that below Eq. (6.6) applies here.

(C) FP covariant linear gauge parameter $\lambda \rightarrow$FP covariant linear gauge parameter λ'. Here

$$\Theta'[\phi] = \ln\left(\frac{\lambda}{\lambda'}\right) \int \bar{c}^{\alpha} \partial \cdot A^{\alpha} d^4 y \ , \tag{6.10}$$

$$f[\phi] = \ln\left(\frac{\lambda}{\lambda'}\right) \int \frac{(\partial \cdot A^{\alpha})^2}{\lambda} + \ln\left(\frac{\lambda}{\lambda'}\right) \int \bar{c} M_0 c \, d^4 y \ . \tag{6.11}$$

A discussion analogous to that following Eq. (6.6) holds here also.

We shall now briefly indicate the possible applications of our results. Our aim here is not, however, to discuss them in detail, which we hope to do elsewhere. We shall merely indicate directions which *could* be profitable.

By these methods, a finite BRS transformation could be constructed to connect linear Lorentz gauges with axial gauges. This finite BRS could be useful in comparing results in two gauges. One may be able to make an axial gauge prescription rigorous by connecting it with Lorentz gauges which are known to be rigorous. It may also be possible to compare results in covariant linear gauges with two different gauge parameters: e.g., the Feynmann gauge and the Landau gauge.

On the formal side, renormalization in quadratic gauges could perhaps be understood easily by their connection with linear gauges given in (B) above. The same applies to renormalization of BRS–anti-BRS invariant actions given by Baulieu and Thierry-Mieg [5]. Here the discussion in (A) above could be useful.

To the best of our knowledge, this is the first time a BRS transformation and/or gauge transformation connecting various actions has been written in a closed form. It should find various applications in the future.

[1] See, e.g., E. Abers and B. W. Lee, Phys. Rep. **9C**, 1 (1973).
[2] See, e.g., W. Kummer, Nucl. Phys. **B100**, 106 (1975).
[3] See, e.g., H. Weigert and U. Heinz, Z. Phys. C **56**, 145 (1992), and references therein.
[4] See, e.g., G. 't Hooft and M. Veltman, Nucl. Phys. **B50**, 318 (1972); S. D. Joglekar, Pramana J. Phys. **32**, 195 (1983).
[5] See, e.g., L. Baulieu and J. Thierry-Mieg, Nucl. Phys. **B197**, 477 (1982), and references therein.
[6] See, e.g., R. Hamberg and W. Van Neerven, Nucl. Phys. **B379**, 143 (1992).