Interactions between U(1) cosmic strings: An analytical study

L. M. A. Bettencourt and R. J. Rivers
The Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom
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We derive analytic expressions for the interaction energy between two general U(1) cosmic strings as the function of their relative orientation and the ratio of the coupling constants in the model. The results are relevant to the statistic description of strings away from critical coupling and shed some light on the mechanisms involved in string formation and the evolution of string networks.

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I. INTRODUCTION

Phase transitions associated with the spontaneous breaking of symmetries in gauge theories are expected to have played a crucial role in the history of the early Universe. In particular, they imply the production of topologically stable defects [1]. Such defects, produced at the energy scale of grand unification, lead to a panoply of cosmological effects. The most important consequence of their existence, however, is their potential to create energy distribution anisotropies, necessary for seeding the large scale structure of the present Universe, and compatible with those observed in the cosmic microwave background [2].

Different classes of defects lead to distinct cosmological consequences. The most dramatic is that walls and monopoles are known to come to dominate the energy density of the Universe, if inflation does not occur subsequently to their formation or some other mechanism intervenes to enhance their annihilation. Strings by contrast, under certain general conditions, seem to be entirely viable on their own. The fundamental difference arises from the fact that a network of cosmic strings possesses natural mechanisms to reduce its own contribution to the total energy density. Moreover, this is done in a way such that the effective evolution of the energy fraction in strings decreases in time faster than would be expected from the expansion alone and scenarios of string domination are thereby naturally precluded.

The mechanisms that allow for the viable evolution of a string network are essentially motivated on topological grounds and are consequently expected to be model independent [3,4]. They consist of a two step process. Firstly, when two segments of a long string intersect a closed loop of string will be formed. This happens when the two segments exchange ends after collision, which in turn is a consequence of winding number (i.e., vortex topological charge) conservation on the plane. This is designated intercommuting. Finally, because closed loops of string are not globally topologically stable, they can radiate away the energy trapped in their field configuration as gravitational waves and shrink until they disappear. The final stage of collapse is probably characterized by vortex annihilation and is expected to produce extremely energetic cosmic rays.

The realization of this scenario rests crucially upon the efficiency of the intercommuting and subsequent string separation. Strings, however, are known to experience interactions which depend, in general, both on the underlying field theory and on the specific region of its parameter space. In its simplest and most widely used model strings are the classical nontrivial solutions of the Abelian Higgs model. In particular, for certain ratios of coupling constants, they are analogous to the vortex solution in type II superconductors and to vortices in superfluid ^{4}He. This latter case corresponds to the vanishing of the gauge sector in the model and the corresponding string solutions are naturally known as global strings.

Typically, when estimating how string networks evolve, numerical simulations only invoke the underlying field theory in the initial string formation. Thereafter, they are taken to obey the equations of classical Nambu-Goto strings, interaction-free but for empirical assumptions about intercommuting. For type II and global strings these assumptions are buttressed by a number of numerical studies. They reveal, in particular, that strings indeed exchange ends and separate in these cases. Studies regarding the outcome of collisions of type I strings, which feel an attractive interaction regardless of orientation, are much scarcer [5]. On the other hand, analytical studies concerning the derivation and generalization of such a range of behavior having the field theory as the starting point are scarce and tend to concentrate on specific regions of parameter space [6,7]. In this paper we attempt a more general analytic analysis.

The article is organized in two parts. The longer part is concerned with classical vortex interactions. The first sections essentially review the approximate field solutions for vortices and derive the interaction potential between two vortices or strings, for all values of coupling constant ratios and arbitrary orientations. Models for string production typically lead to initial high string densities. The latter part of the paper examines the initial stability of string networks, both classically and quantum mechanically. This part of the paper is rather more speculative. Type I and type II strings are a consequence of theories that, essentially, undergo either first-order or second-order transitions at the time of string formation. We make a preliminary attempt to relate this to the nature of string forces.

II. THE INTERACTION ENERGY BETWEEN TWO STRINGS: GENERALITIES

Prior to considering multiple string solutions we briefly review the isolated string field solutions, with particular attention to the field behavior at large distances from their axes of symmetry. Although well known, we need the details for later sections.

U(1) strings are the extrapolation to three dimensions of the well-known vortex solution of the Abelian Higgs model [8]. These are finite energy nontrivial static solutions on the plane. The Lagrangian density takes the form

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}|(\partial_\mu + ieA_\mu)\phi|^2 - \frac{\lambda}{8}(|\phi|^2 - \eta^2)^2, \tag{1}$$

where e is the gauge coupling constant to the scalar field, λ the scalar field self-coupling constant, and η the vacuum expectation value of the modulus of the scalar field in the broken phase, as usual.

The finiteness of the energy alone determines the asymptotic form of the fields at infinity. In order for the scalar field potential to vanish at infinity, ϕ is constrained to lie on a circle of radius η. Its magnitude becomes then well defined but its phase remains arbitrary. On the other hand, the modulus of the covariant derivative must also vanish asymptotically which forces the gauge field to assume the form [9]

$$\lim_{|\mathbf{x}|\to+\infty} A_\mu(x) = -\frac{1}{e}\partial_\mu \arg[\phi(x)]. \tag{2}$$

This also ensures that the energy contribution arising from the field strength tensor will be finite. Moreover, if we work in polar coordinates, we see that the problem of parametrizing the behavior of the scalar field at infinity reduces to the mapping of a circle, in coordinate space, onto another, in field space. These mappings, as is well known, fall into an infinite number of homotopy classes, each labeled by an integer $n \in Z$. Each solution, characterized by a given n, cannot in turn, be continuously deformed into another with $m \neq n$, without changing the boundary conditions for the fields at infinity, since this would imply expending an infinite amount of energy strings that are topologically stable.

Even though the above arguments supply us with with some information about vortex solutions the way in which the fields tend to their asymptotic forms is only revealed by studying the Euler-Lagrange equations. These can be solved numerically in all detail but, for certain purposes, this does not suffice. Such is the case when we want to discuss analytically the detailed energetics of a multiple string field configuration.

Exact single solutions exist only for the case of critical coupling ($b = \frac{e^2}{\lambda} = 1$) and positive winding number [10]. This is a rather special case since the interactions between vortices disappear and the equations of motion for the fields become first order; the well-known Bogomol'nyi equations. In general, however, approximate solutions in certain regimes can be derived. In order to show this we take the Euler-Lagrange equations, assuming cylindrical symmetry, in the radial temporal gauge $A_r = A_0 = 0$,

$$\nabla_r^2\varphi - \left[\left(eA_\theta - \frac{n}{r}\right)^2 + \frac{\lambda}{2}\left(\varphi^2 - \eta^2\right)\right]\varphi = 0 \tag{3}$$

$$\nabla\times\nabla\times A_\theta + e\varphi^2\left(eA_\theta - \frac{n}{r}\right) = 0, \tag{4}$$

where φ is the modulus of the scalar field and n its winding number. The field ϕ is therefore assumed to be of the form $\phi(r,\theta) = \varphi(r)e^{-in\theta}$, where θ is the angular coordinate on the plane. To probe how the fields approach their asymptotic values at infinity, one can expand φ around η as $\varphi(r) = \eta - f(r)$, where f is an auxiliary field. Then the Euler-Lagrange equations become

$$\nabla^2 f = m_S^2 f + f\left[e^2Q^2 - \tfrac{\lambda}{2}f\left(3\eta - f\right)\right] - e^2\eta Q^2 \tag{5}$$

$$\nabla\times(\nabla\times Q) = m_A^2 Q - e^2(2\eta f - f^2)Q, \tag{6}$$

where $m_S^2 = \eta^2\lambda$, $m_A = \eta e$ and $Q = A_\theta - \frac{n}{er}$.

Equation (6) has an obvious solution for small f, when the second term, on the right hand side, can be neglected in the face of the first[1] [8]:

$$A_\theta(r) = \frac{n}{er} - k_A K_1(m_A r), \tag{7}$$

where k_A is a constant and $K_1(m_A r)$ the modified Bessel function of order 1. Equation (7) shows us how A_θ approaches its asymptotic value. This result allows us to estimate the corresponding behavior for the scalar field. Neglecting the quadratic and cubic terms on the fields in (5) we obtain

$$f = k_S K_0(m_S r), \tag{8}$$

where k_S is another constant and $K_0(m_S r)$ the modified Bessel function of order 0. For large arguments the modified Bessel functions of order 0 and 1 have the same leading behavior: namely,

$$K_n(mr) \sim \sqrt{\frac{\pi}{2mr}}e^{-mr}\left[1 + O\left(\frac{1}{mr}\right)\right]. \tag{9}$$

The approximate solutions (7) and (8) when taken in the limit (9) indeed reproduce the exact asymptotic form of the fields at critical coupling [10]. As we have seen above, however, they are valid for all values of the coupling constants in the model.

For short distances, when $(\eta - \varphi) \simeq \eta$, Eq. (6) becomes essentially that for a free gauge field. The corresponding form for A_θ is then

$$A_\theta(r) \propto \frac{n}{er} + O(r). \tag{10}$$

[1]This is a special case of the most general solution $A_\theta(r) = \frac{n}{er} - k_A K_1(e\varphi r)$, when $\varphi \simeq$ const.

This form is actually compatible with (7), when the small argument for the Bessel function is assumed. Moreover, if we impose that the magnetic flux should vanish at the origin we will determine $k_A = n\eta$ [8]. This is certainly an acceptable procedure for strongly type II vortices. An approximate solution for the scalar field close to the origin can equally be found by using (10) in (5). We obtain

$$\varphi(r) \propto J_n\left(\frac{m_S}{\sqrt{2}}r\right), \tag{11}$$

where J_n is the Bessel function of order n, which is the usual winding number. This implies the well-known small distance behavior for φ

$$\varphi(r) \propto \frac{1}{2}\left(\frac{m_S}{\sqrt{2}}r\right)^n. \tag{12}$$

In particular for $n = 1$ the field is linear close to the origin. Because the behavior of the scalar field in these two quite different limits is not directly relatable, k_S cannot be computed with generality. In the rather special case of critical coupling ($\frac{e^2}{\lambda} = 1$), however, the dynamical equations for the fields reduce to

$$\partial_r\varphi \pm (eA_\theta - \frac{n}{r}) = 0, \tag{13}$$

$$\partial_r A_\theta \pm \frac{\sqrt{\lambda}}{2}(\varphi^2 - \eta^2) = 0, \tag{14}$$

where $\pm = \mathrm{sgn}(n)$. These are the Bogomol'nyi equations for critically coupled vortices in polar coordinates. It is easy to see that they allow us to relate f and A_θ directly and consequently also k_S and k_A. Using the identity of the two coupling constants we obtain $k_S = |n|\eta$. In principle there is no reason for this result to hold in other regions of parameter space other than the good agreement of its consequences with the numerical results, as we shall see later. Nevertheless, unless otherwise stated we will assume it henceforth.

We are now in a position to derive the interaction potential between two string segments, under the assumption that the solution for the two string field configuration can be successfully approximated by the superposition ansatz [11,12]

$$\Phi(r, r_1, r_2) = \frac{\phi(|r - r_1|)\phi(|r - r_2|)}{\eta}, \tag{15}$$

$$A_\theta(r, r_1, r_2) = A_\theta(|r - r_1|) + A_\theta(|r - r_2|). \tag{16}$$

Here ϕ and A_θ are the isolated string field configurations of Sec. II. In order to estimate the interaction energy between the two vortices one then simply substitutes (15) and (16) in the energy functional

$$E[\phi, A_\mu] = \int dV\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}|\partial_r\phi|^2 + \frac{1}{2}|eA_\theta\phi - \partial_\theta\phi|^2 + \frac{\lambda}{8}(|\phi|^2 - \eta^2)^2\right]. \tag{17}$$

We then obtain

$$\begin{aligned}E[\Phi, A] = {}& E[\phi_1, A_1] + E[\phi_2, A_2] + \int dV\frac{1}{2}\Bigg\{2\nabla_{r_1}\times A_\theta(r_1)\cdot\nabla_{r_2}\times A_\theta(r_2) + \frac{1}{\eta^2}\Bigg[[\varphi(r_1)^2 - \eta^2](\nabla_{r_2}\varphi(r_2))^2 \\ & +[\varphi(r_2)^2 - \eta^2](\nabla_{r_1}\varphi(r_1))^2 + 2\varphi(r_1)\varphi(r_2)\nabla_{r_1}\varphi(r_1)\cdot\nabla_{r_2}\varphi(r_2) + [\varphi(r_2)^2 - \eta^2]\varphi(r_1)^2\left(eA_\theta(r_1) - \frac{n_1}{r_1}\right)^2 \\ & +[\varphi(r_1)^2 - \eta^2]\varphi(r_2)^2\left(eA_\theta(r_2) - \frac{n_2}{r_2}\right)^2 + 2\varphi(r_1)^2\varphi(r_2)^2\left(eA_\theta(r_1) - \frac{n_1}{r_1}\right)\left(eA_\theta(r_2) - \frac{n_2}{r_2}\right)\Bigg] \\ & +\frac{\lambda}{4\eta^4}([\varphi(r_1)^2 - \eta^2][\varphi(r_2)^2 - \eta^2]\{[\varphi(r_1)^2 + \eta^2][\varphi(r_2)^2 + \eta^2] - 2\eta^4\})\Bigg\}.\end{aligned} \tag{18}$$

Finally, we subtract the contributions due to the isolated vortices. We obtain

$$\begin{aligned}E_{\mathrm{int}}[\varphi, A_\theta] = {}& \int dV\frac{1}{2}\Bigg\{2\nabla_{r_1}\times A_\theta(r_1)\cdot\nabla_{r_2}\times A_\theta(r_2) + \frac{1}{\eta^2}\Bigg[[\varphi(r_1)^2 - \eta^2](\nabla_{r_2}\varphi(r_2))^2 + [\varphi(r_2)^2 - \eta^2](\nabla_{r_1}\varphi(r_1))^2 \\ & +2\varphi(r_1)\varphi(r_2)\nabla_{r_1}\varphi(r_1)\cdot\nabla_{r_2}\varphi(r_2) + [\varphi(r_2)^2 - \eta^2]\varphi(r_1)^2\left(eA_\theta(r_1) - \frac{n_1}{r_1}\right)^2 \\ & +[\varphi(r_1)^2 - \eta^2]\varphi(r_2)^2\left(eA_\theta(r_2) - \frac{n_2}{r_2}\right)^2 + 2\varphi(r_1)^2\varphi(r_2)^2\left(eA_\theta(r_1) - \frac{n_1}{r_1}\right)\cdot\left(eA_\theta(r_2) - \frac{n_2}{r_2}\right)\Bigg] \\ & +\frac{\lambda}{4\eta^4}([\varphi(r_1)^2 - \eta^2][\varphi(r_2)^2 - \eta^2]\{[\varphi(r_1)^2 + \eta^2][\varphi(r_2)^2 + \eta^2] - 2\eta^4\})\Bigg\}.\end{aligned} \tag{19}$$

This last expression can also be written in terms of the auxiliary field f. Assuming cylindrical symmetry the energy per unit length of the strings becomes

$$E_{\rm int} = \int dS \left(B_1 B_2 \mathbf{e}_{z_1}\cdot\mathbf{e}_{z_2} + m_A^2 Q_1 Q_2 \mathbf{e}_{\theta_1}\cdot\mathbf{e}_{\theta_2}\right) - \left(\nabla_r f_1 \nabla_r f_2 \mathbf{e}_{r_1}\cdot\mathbf{e}_{r_2} + m_S^2 f_1 f_2\right)$$
$$-\frac{e^2}{2}\left[f_1^2 Q_2^2 + 2f_1^2 Q_1\cdot Q_2 + f_2^2 Q_1^2 + 2f_2^2 Q_1\cdot Q_2 + 2f_1 f_2 (Q_1^2+Q_2^2) + 4f_1 f_2 Q_1\cdot Q_2\right]$$
$$+\lambda(3f_1^2 f_2 + 3f_2^2 f_1)\left[\eta - \frac{3}{8}(f_1+f_2)\right] + \frac{3}{8}\lambda(f_1^3 f_2 + f_2^3 f_1), \tag{20}$$

where $B_i = \nabla_{r_i}\times A_\theta(r_i)$, $i=1,2$. To obtain the detailed behavior for the interaction energy one then simply needs to perform the integrations in (19) or (20) assuming given functional forms for the fields. This procedure is absolutely straightforward but a few cautionary remarks are in order.

Firstly, unlike the case of critical coupling, the proof for the existence and uniqueness of multivortex solutions for general couplings does not exist. In particular, any such solution will not be static. However, it is known from experimental evidence in type II superconductors that multivortex configurations indeed do occur.

Secondly, the above procedure is clearly not exact. The superposition ansatz effectively is not an exact solution of the Euler-Lagrange equations (3) and (4). The resulting discrepancy on the value of the interaction energy can nevertheless be computed. This can be achieved by assuming residual additive terms to (15) and (16) such that the resulting fields would solve (3) and (4) exactly. Their behavior and contribution to the energy can then be estimated whenever the functional form for the individual vortex solutions is known [12]. In particular for large distances, d, away from the strings' axes, when the modified Bessel function behavior for f and Q applies, such contribution is of the order $\frac{e^{-2d}}{d}$ [12]. This is a second order subleading effect as we shall verify in the next sections. This whole procedure allows us consequently not only to state the validity of the superposition ansatz for a given functional form of the fields but also to compute the magnitude of the approximation involved.

Finally, the integrations in (19) or (20) necessary to obtain the total energy can, in general, also only be performed approximately. This will be the subject of the two ensuing sections where expressions for the interaction energy are indeed obtained, for all values of $b=\frac{e^2}{\lambda}$ and arbitrary orientation of the two string elements.

III. THE INTERACTION ENERGY BETWEEN PARALLEL STRINGS AS A FUNCTION OF b

In this section we derive the integrated expressions for the interaction energy, for all values of $b=\frac{e^2}{\lambda}$. For the sake of simplicity we will restrict ourselves to the case of two parallel strings and leave the study of arbitrary geometries to the next section. The problem then reduces to the study of two coplanar vortices. Because different values of b imply quite distinct field configurations we will analyze the cases of critical coupling ($b=1$), type II ($b<1$) and type I vortices ($b>1$) separately.

A. The case $b=1$

In view of the approximate field solutions obtained in Sec. II and the expressions for the interaction energy (19) and (20) derived in Sec. II, this is the simplest case to approach. In general there are two length scales associated with each vortex. They are simply the inverse classical masses for the gauge and scalar fields i.e., $r_A = m_A^{-1}$ and $r_S = m_S^{-1}$, respectively. When the scalar field acquires a nonzero expectation value ($\langle\varphi\rangle \to_{r\to\infty} \eta$) both fields become massive and the modified Bessel function solutions for the fields constitute a good approximation. This should happen for distances R, measured from the zero of the scalar field, such that $R > r_S$.

These approximate solutions tell us how the fields approach their asymptotic values, which in turn correspond to the vacuum of the theory. At distances R larger than r_S a vortex only perturbs the vacuum by effectively acting as a source for the fields Q and f. The interaction between two vortices, separated by a distance $d > 2r_S$ then reduces, in the first approximation, to the interaction between these two fields. This picture can indeed be obtained from (20) if we keep only the terms linear on the fields of each vortex. Then we have

$$E_{\rm int} = \eta^2 \int dS n_1 n_2 [m_A^2 K_0(m_A r_1) K_0(m_A r_2)\mathbf{e}_{z_1}\cdot\mathbf{e}_{z_2}$$
$$+m_A^2 K_1(m_A r_1)K_1(m_A r_2)\mathbf{e}_{\theta_1}\cdot\mathbf{e}_{\theta_2}]$$
$$-|n_1||n_2|[\nabla_{r_1}K_0(m_S r_1)\nabla_{r_2}K_0(m_S r_2)\mathbf{e}_{r_1}\cdot\mathbf{e}_{r_2}$$
$$+m_S^2 K_0(m_S r_1)K_0(m_S r_2)]. \tag{21}$$

Higher order terms will be much smaller due to the fact that the fields themselves are exponentially decreasing. Moreover, their contribution is of the order of the approximation involved in adopting the superposition ansatz for the two-string field configuration.

To integrate (21) we first note that $\mathbf{e}_{z_1}.\mathbf{e}_{z_2}$ is independent of the point on the plane. Then, using the fact that the modified Bessel functions of order zero satisfy

$$\left(\nabla^2 - m^2\right) K_0(mr) = -2\pi\delta(r) \tag{22}$$

in two dimensions, we can write

$$E_{\rm int} = 2\pi\eta^2 \left[n_1 n_2 K_0(m_A d) - |n_1||n_2| K_0(m_S d)\right] + \left[\int dS n_1 n_2 \left[K_0(m_A r_2)\nabla^2_{r_1} K_0(m_A r_1) + m_A^2 K_1(m_A r_1) K_1(m_A r_2) \mathbf{e}_{\theta_1}\cdot \mathbf{e}_{\theta_2}\right] - |n_1||n_2| \left[K_0(m_S r_2)\nabla^2_{r_1} K_0(m_S r_1) + \nabla_{r_1} K_0(m_S r_1)\nabla_{r_2} K_0(m_S r_2)\mathbf{e}_{r_1}\cdot \mathbf{e}_{r_2}\right]\right]. \quad (23)$$

Now by noting that

$$\nabla_r K_0(mr) = -m K_1(mr),$$
$$\nabla_{r_1} K_0(m r_2) = -m \frac{r_1 - d\cos(\theta_1)}{r_2} K_1(m r_2), \quad (24)$$

and that

$$\mathbf{e}_{\theta_1}\cdot\mathbf{e}_{\theta_2} = \mathbf{e}_{r_1}\cdot\mathbf{e}_{r_2} = \frac{r_1 - d\cos\theta_1}{r_2}, \quad (25)$$

we see that the integral in (23) vanishes and the interaction energy becomes exactly [7]

$$E_{\rm int}(d) = 2\pi\eta^2 \left[n_1 n_2 K_0(m_A d) - |n_1 n_2| K_0(m_S d)\right]. \quad (26)$$

This shows in particular that, in the limit of our approximations, two vortices at critical coupling are free. This is equally true for a pair of parallel cosmic strings, in agreement with well-known numerical results [9].

B. The case $b < 1$

Away from critical coupling the two length scales r_S and r_A become different and the fields are expected to change accordingly. In the specific case when $e^2 < \lambda$, r_A will be larger than r_S. These two lengths now define a set of three coaxial cylinders, around the axis of the string, where the fields may behave in quite a different fashion. Essentially, for distances R larger than r_A or smaller than r_S the picture of the last subsection remains qualitatively unchanged. Indeed, for $R < r_S$ the fields must vary substantially from their values at the origin to a value not too far from their asymptotic forms.

For $R > r_A$, on the other hand, the scalar field should be very close to its asymptotic value and consequently will be a slowly varying function of r. Then both fields will have acquired their classical masses. For this case the computations of Sec. III(A) are certainly valid and it follows that two vortices will repel and a pair vortex-antivortex will attract, with magnitude given by (26).

The behavior in the intermediate region, however, turns out to be the most interesting. There, $R > r_S$ and the fields are expected to acquire mass. This implies, namely, that the modified Bessel function form for Q holds. However, because $m_A R < 1$, the Bessel function can be approximated by its small argument behavior

$$Q = -n\eta K_1(m_A R) \simeq -\frac{n}{eR}. \quad (27)$$

When this form is used in the Euler-Lagrange equation for the scalar field, assuming $\varphi = \eta - f$, we obtain

$$\phi \simeq 1 - \frac{n^2}{(m_S r)^2} + O\left(\frac{1}{(m_S r)^4}\right), \quad (28)$$

which is the solution for the global string, in the same regime ($R > r_S$) [6].

It should now be clear that, as we take $e^2 \to 0$, this intermediate region grows to fill all space (from $R = r_S$ to infinity), and the global string behavior is fully recovered. This is not very surprising, of course, since in this limit the gauge field decouples from the scalar field and the Euler-Lagrange equation for the latter is that for the theory exhibiting the global symmetry alone. What is nevertheless quite interesting is that for an ensemble of type II local strings to interact as in the global case they simply need to lie at relative distances smaller than r_A.[2]

Let us now see in detail how this picture arises from the previous expressions for the interaction energy. The part of the energy dependent on f can be integrated to give a contribution which falls with at least $\frac{1}{m_S^4 d^3}$. Because $m_S d > 1$ this contribution will be small. On the other hand, since the modified Bessel function of order 1 is still a solution for the gauge field the computations of Sec. III(A) referring to it will still hold. We then obtain, keeping the terms on the gauge field only,

$$E(d) \simeq 2\pi\eta^2 n_1 n_2 K_0(m_A d) \simeq -2\pi\eta^2 n_1 n_2 \ln(m_A d), \quad (29)$$

since $m_A R$ is small for $R < r_A$. However, our assumptions about the nature of the fields leading to (29) were only valid for $R > r_S$, whereas in (26) we implicitly took it to be applicable in all space between the origins of each vortex. We must, therefore subtract the energy corresponding to $R < r_S$. We then get

$$E(d) \simeq -2\pi\eta^2 n_1 n_2 \left[\ln(m_A d) - \ln(m_A r_S)\right] = -2\pi\eta^2 n_1 n_2 \ln(m_S d). \quad (30)$$

This expression is just what we would expect for the global string, cf. [6], and ensures the consistency of the arguments above. As is well known, this results in a strong, infinite range,[3] repulsive (attractive) force for a vortex-vortex (vortex-antivortex) pair.

[2] We should keep in mind that this result is a direct consequence of the value taken for k_A, in Sec. II.

[3] In the case of nonvanishing e, m_A^{-1} provides the natural cutoff at large r.

C. The case $b > 1$

In the preceding analysis we were able to compute estimates for the interaction energy between vortices under a few quite general assumptions. In particular we learned that the proximity of the scalar field to its asymptotic value (traduced in the criterion $R > r_S$) introduced a fundamental qualitative change in the behavior of the fields, allowing us to use the approximate solutions (7) and (8) with some confidence. The case of type I vortices is naturally characterized by the fact that $r_A < r_S$. The results for $b = 1$ for the interaction between two vortices should in particular still be valid as long as their scalar cores do not overlap, i.e., for $(d > 2r_S)$. Then, unlike what happened when $b < 1$, and because now $m_A > m_S$ the scalar field contribution will dominate the energy. Moreover, since the corresponding term in the interaction energy (26) has no dependence on the sign of the winding numbers this will always result on an attractive force regardless of the actual nature of the two vortices involved.

As may easily be anticipated we will run into a problem when we try to estimate what happens in the intermediate region $(r_A < d < r_S)$. This can be explicitly shown if we search for the behavior of the fields of a single vortex in this region of space.

In fact, the way we derived the Bessel function behavior of Q for large distances was to assume that φ would be close to η, so that the mass term in (6) would clearly be the dominant one. This always holds provided that $R \gtrsim r_S$, and is thus certainly a good approximation for the cases above.

If, on the other hand, R is small enough so that $(\varphi^2 - \eta^2) \simeq -\eta^2$, and Q exhibits its short distance behavior, then the solution of (3) becomes

$$\varphi(r) \propto J_n(\frac{m_S r}{\sqrt{2}}) \simeq \frac{1}{2}(\frac{m_S r}{\sqrt{2}})^n. \quad (31)$$

But this is the behavior we expect for the scalar field very close to the origin, independent, of any particular value of the coupling constants.

However, we also note that as we progress towards $R = r_S$, f approaches η and the Bessel function behavior (7), and (8) start being valid for Q and f, respectively. When this happens, Q automatically assumes an exponentially decreasing form since $m_A R$ is large but f should still behave approximately logarithmically since $m_S R$ is small. This should happen in a region where the distance to the string's axis is sufficiently close to r_S. Consequently, and unlike what happened for type II strings, this approximate behavior for the fields can only exist for a thin region of space around a string. Its consequences for the interaction energy between the two strings will then only constitute a transient regime between its form for large distances and the actual superposition of the two vortices. Nevertheless we can compute what the corresponding interaction energy should be. Proceeding in the same way as in Sec. III(B), but now only keeping the contribution from the scalar field we obtain

$$E(d) \simeq -2\pi\eta^2 |n_1 n_2| K_0(m_S d) \simeq 2\pi\eta^2 |n_1 n_2| \ln(m_S d), \quad (32)$$

where, again, we implicitly assumed that this regime was present in the whole region between the two axes. We should then remove the contribution from the inner region where this does not apply. In the absence of another length scale we can only subtract the energy arising from $R < r_A$. This, however, will result in an overestimate (in absolute value) for the final result. We then obtain

$$E(R) \simeq 2\pi\eta^2 |n_1 n_2| \ln(m_A d). \quad (33)$$

This always gives rise to an attractive force, as expected.

At distances smaller than r_S, the scalar cores of the two strings will superimpose. This results in a field configuration of winding number $n_1 + n_2$ as the distance between the zeros of the scalar fields vanishes. Such a transformation of the two string field configuration is accompanied by the change in behavior between the modified and unmodified Bessel functions solution for the scalar field of Sec. II. This change is dictated by the nonlinearities in the corresponding Euler-Lagrange equations which implies the breakdown of the superposition ansatz. Close to $d = 0$, however, it is known numerically [9] that the interaction energy should become approximately constant, signaling the fact that a type I string of higher winding number is a *stable* solution of the Euler-Lagrange equations relative to its lower winding number isolated constituents. (Alternatively, type II strings of higher winding number are unstable with respect to decay into strings of lower winding number.)

IV. INTERACTION BETWEEN TWO COSMIC STRINGS: GENERAL GEOMETRY

In the previous section we analyzed how the value of the ratio of the two coupling constants in the model b changes the field configurations and derived the interaction energy between two vortices, i.e., between a pair of parallel strings, per unit length.

For a general network of cosmic strings we know [13] that strings wiggle and bend, possibly on several scales, and, in general, no two strings are parallel.[4] We shall expand upon this in the next section.

The assumption of parallel strings, however, conveniently allowed us some comfortable simplifications. Because each string is locally (i.e., for any element of infinitesimal length along its axis) cylindrically symmetric we could associate the natural frame for the second string with that for the first simply by translating it by a distance d, within the same plane. This distance was then automatically defined for all pairs of string elements and

[4]Unlike cosmic strings, flux lines in superconductors and vortices in superfluids can be parallel. For those cases the results of Sec. III apply.

the interaction energy for a piece of string due to the presence of the second is simply that for one element times the length. For a general configuration of the two strings both the distance and orientation have to be specified for each pair of elements. They will, therefore, weigh differently in the evaluation of the interaction energy between two pieces of string of finite length.

To proceed further we have to assume that the local cylindrical symmetry of the fields composing each string is still, on some small enough length scale, preserved. Such a scale must naturally be defined relative to the length scales that characterize structure on the string and typically must be of the order of the smallest of these. Under this working assumption, to find the interaction energy for one string element we need only look for an element of a second string intersecting the plane it defines. The natural frame for the element of the second string will appear in the general case to have its origin at a distance d but also to have been rotated so that it no longer lies in the plane of the former. However, because of its own local cylindrical symmetry, rotations around its axis of symmetry leave it unchanged. This rotation relative to the plane of the first vortex is then generated by two angles only. This can be seen in Fig. 1, where we chose the x axis along the direction connecting the origins of the two vortices. The angles α and γ parametrize rotations around the x and y axes, respectively.

In order to be able to generalize the results of Sec. III we must be able to understand how the change in orientation of the two relative elements affects the interaction energy. This will result essentially in a change in the inner products between the unit vectors for the directions associated with the natural frame of each string.

This change can be computed by rotating one of the frames around the x and y axes of Fig. 1. We now see, in particular, that $\mathbf{e}_{z_1} \cdot \mathbf{e}_{z_2}$ will simply have the form

$$\mathbf{e}_{z_1} \cdot \mathbf{e}_{z_2} = \cos(\alpha)\cos(\gamma). \tag{34}$$

This allows us to integrate the two terms in (23) to obtain

$$\begin{aligned} E_{\rm int}(d,\alpha,\gamma) = {} & 2\pi\eta^2\left[n_1 n_2 K_0(m_A d)\cos(\alpha)\cos(\gamma) - |n_1||n_2| K_0(m_S d)\right] \\ & -\Bigg(\int dS n_1 n_2 \left[\nabla_{r_1} K_0(m_A r_2)\nabla_{r_1} K_0(m_A r_1)\cos(\alpha)\cos(\gamma) - K_1(m_A r_1)K_1(m_A r_2)\mathbf{e}_{\theta_1}.\mathbf{e}_{\theta_2}\right] \\ & -|n_1||n_2|\left[\nabla_{r_1} K_0(m_S r_2)\nabla_{r_1} K_0(m_S r_1) - \nabla_{r_1} K_0(m_S r_1)\nabla_{r_2} K_0(m_S r_2)\mathbf{e}_{r_1}\cdot\mathbf{e}_{r_2}\right]\Bigg). \end{aligned} \tag{35}$$

The two residual terms inside the integral behave somewhat differently. In the case of that arising from the scalar field the change in variable of differentiation on the first exactly generates the inner product $\mathbf{e}_{r_1} \cdot \mathbf{e}_{r_2}$. Because both these terms have opposite signs their sum vanishes.

Such is not the case for the corresponding term in the gauge sector. There we have

$$\begin{aligned} \frac{dr_2}{dr_1}\mathbf{e}_{z_1}\cdot\mathbf{e}_{z_2} = {} & \frac{1}{r_1 r_2}\cos(\alpha)\cos(\gamma)[x_1(x_1-d)\cos^2(\gamma) + y_1^2\cos^2(\alpha) + x_1 z_1\cos(\alpha)\cos(\gamma)\sin(\gamma) \\ & -x_1 y_1\sin(\alpha)\sin(\gamma)\cos(\gamma) + y_1 z_1\sin(\alpha)\cos^2(\alpha)], \end{aligned} \tag{36}$$

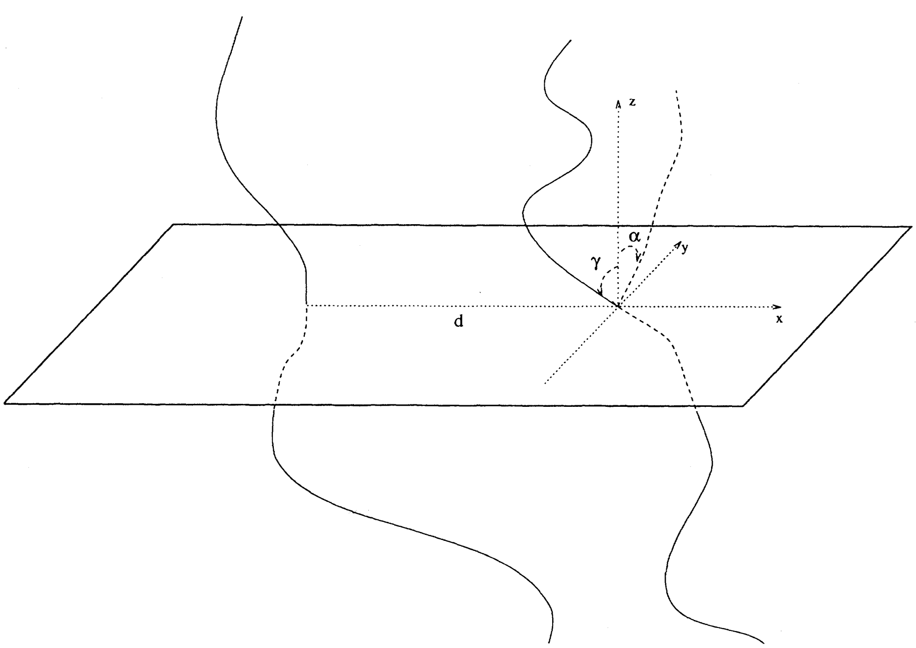

FIG. 1. A schematic example of a two-string configuration and of the relevant quantities necessary to compute the corresponding interaction energy.

which must be compared to

$$\mathbf{e}_{\theta_1}\cdot\mathbf{e}_{\theta_2}=\frac{1}{r_1r_2}\{\cos(\alpha)\cos(\gamma)[x_1(x_1-d)+y_1^2]+x_1z_1\cos^2(\alpha)\sin(\gamma)$$
$$-x_1y_1\sin(\alpha)\cos(\alpha)\sin(\gamma)+y_1z_1\sin(\alpha)\cos(\gamma)\}. \tag{37}$$

In both the above expressions we took a rotation by γ after a rotation by α.

These two terms do not cancel in general. Rewriting (35) we finally obtain

$$E_{\rm int}(d,\alpha,\gamma)=2\pi\eta^2\left[n_1n_2K_0(m_Ad)\cos(\alpha)\cos(\gamma)-|n_1||n_2|K_0(m_Sd)\right]-\left[\int dSn_1n_2\nabla_{r_2}K_0(m_Ar_2)\nabla_{r_1}K_0(m_Ar_1)\right.$$
$$\left.\times\left(\frac{1}{r_1r_2}\left[x_1(x_1-d)\sin^2(\gamma)+y_1^2\sin^2(\alpha)+\text{higher order in }\alpha,\gamma\right]\right)\right]\cos(\alpha)\cos(\gamma). \tag{38}$$

We see that the interaction energy is essentially given by the two first terms. In the limit of vanishing α and γ the residual contribution in the integral effectively goes to zero with the sine squared of the angles, in agreement with the results of Sec. III.

Expression (38) stresses the quite different nature of the two contributions to the interaction energy. The vector-like character of the gauge field introduces a dependence on the relative orientation between the two strings. In particular we see that if one of the strings is rotated by π the corresponding interaction energy term changes sign. This is equivalent to changing the nature of a vortex into an antivortex or vice versa and results from the geometric nature of the winding number. The term arising from the scalar field is, in contrast, insensitive to any particular configuration for the two-string system, as could naturally be anticipated.

It also becomes clear that critical coupling ($b=1$), for an arbitrary geometric configuration, ceases to be the special case when the interactions between strings vanish since the angular dependence on α and γ destroys the balance between the gauge and scalar terms in the energy.

All expressions derived above concern elements of string with infinitesimal length, resulting from the integration of the interaction energy functional on the plane. To obtain the interaction energy for two segments of string of finite length one then has to integrate over a series of planes perpendicular to one of the strings. In particular, every time the string bends so that any of its elements will lie at an angle larger than $\frac{\pi}{2}$ it is expected to experience self-interactions. This is always the case for closed loops of string which contain part of the energy arising from self-interactions, in addition to their tension.[5]

The expressions derived above allow us to compute analytically the interactions for a variety of configurations. Let us consider two simple examples.

For a circular loop of large enough diameter D ($D>2r_S$ as always) it is straightforward to compute its self-energy. It is simply

$$E_{\rm self}(D)=\frac{\pi D}{2}E(D), \tag{39}$$

where $E(D)$ is given by expression (26), taken in the appropriate limit, for a parallel string-antistring pair ($|n_1|=-|n_2|$).

Another interesting case is to compute the interaction energy due to a circular loop of string, in the limit when the distance to a test string segment is much larger than the loop's diameter. Then, see Fig. 2, the interactions are essentially due to a dipole of string segments. The corresponding interaction energy is proportional to the sum of the Bessel functions coming from the gauge and scalar field contributions. In particular for the contribution from the former we will have

$$E_{\rm int}\propto\left[K_0(mr)-K_0(mr_2)\right], \tag{40}$$

where

$$r_2=\sqrt{D^2+r^2+2Dr\cos(\theta)}. \tag{41}$$

Now, in the limit of small diameter compared to distance $\frac{D}{r}<<1$ and for the strong type II small loops of string, when $Dm_A\cos(\theta)<1$, we can series expand the Bessel function to obtain

$$K_0\left(mr\sqrt{1+\frac{D^2}{r^2}+2\frac{D}{r}\cos(\theta)}\right)\simeq K_0(mr)\left[1+m^2D^2\cos(\theta)\right]-mD\left[\cos(\theta)-\frac{D}{r}\sin^2(\theta)K_1(mr)\right]. \tag{42}$$

The energy can then approximately be written as[6]

[5]If the string's tension is defined to be the energy/unit length for an infinite straight string.

[6]Here we have neglected the contribution from the scalar field and took the winding number of the test string segment to be one. It is also assumed to lie in a plane parallel to that of the loop.

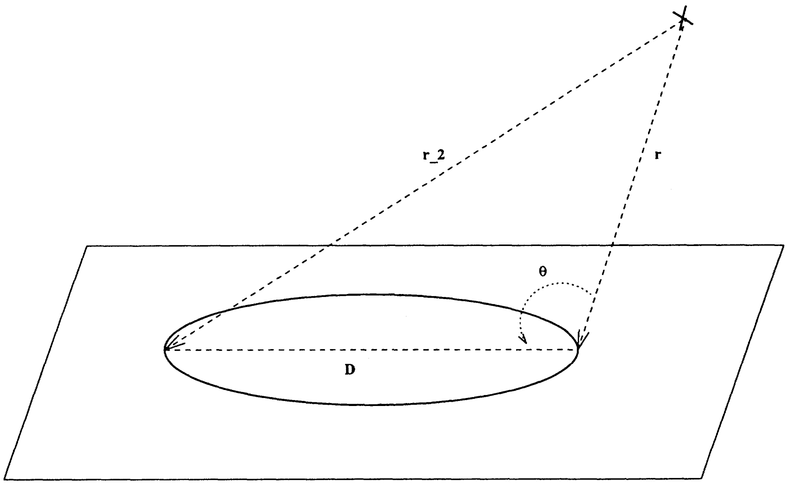

FIG. 2. The field as measured by an observer away from a circular string loop.

$$E_{\rm int} \simeq 2\pi\eta^2 n \left(mD \left[\cos(\theta) - \frac{D}{r}\sin^2(\theta) \right] K_1(mr) - m^2 D^2 \cos^2(\theta) K_0(mr) \right) . \quad (43)$$

This expression has two different limits depending on the relative value of m_A^{-1} and r. If $m_A r$ is large then we can take the asymptotic forms for the Bessel functions at large arguments (9) to obtain

$$E_{\rm int} \simeq 2\pi\eta^2 \sqrt{\frac{\pi}{2m_A r}} e^{-m_A r} \left(m_A D\cos(\theta) - \sin^2(\theta)\frac{D}{r} - m_A^2 D^2 \cos^2(\theta) \right) . \quad (44)$$

If, on the other hand, $r < m_A^{-1}$ (extreme type II case), we can use the form of the Bessel functions for small arguments to obtain

$$E_{\rm int} \simeq 2\pi\eta^2 \left[m_A D \left(\cos(\theta) - \frac{D}{r}\sin^2(\theta) \right) \frac{1}{m_A r} + m_A^2 D^2 \cos^2(\theta) \ln(m_A r) \right] . \quad (45)$$

We see in particular that in this latter case the leading order term in (45) coincides with the usual $\frac{1}{r}$ potential for a pointlike field source in three dimensions. A small loop of (quasi)global string thus behaves effectively as a monopole when seen at sufficiently large distances.

V. STRING FORMATION AND STRING FORCES

In simulations of string production (e.g., see [14]) ϕ field phases are laid down at random in a space divided into correlation volumes within which the ϕ field phase is held approximately constant. In these circumstances strings with winding number $|n| = 1$ are produced at high density. The laboratory production [15] of string defects in superfluid ^{4}He has suggested that this model is plausible [16], at least for the nonrelativistic vortices of the global U(1) Ginzberg-Landau theory.

In addition to the role string interactions have in determining the outcome of string collisions they may affect the early string evolution if, as expected, strings are produced at sufficiently high density. In the remainder of this paper we shall examine the effect of interstring forces in a newly created, and approximately static, ensemble of strings. In this section we shall investigate whether, with only attractive forces, type I strings can exist at high density. This is not a question of how two strings of lower winding number combine to form a string with higher winding number, since we expect strings, of whatever type, to appear as a tangled mess. However, we shall return to this point later. To see how type I strings could exist we adopt a classical toy model possessing some of the characteristics of real strings, in which the effect of attractive forces is more transparent.

Cosmic strings are produced at phase transitions in the grand unified theory (GUT) era but, nominally independent of this, it is known that thermodynamic ensembles of classical string naturally display transitions. Most naively, if cosmic strings are treated as noninteracting Nambu-Goto strings with modes of arbitrarily high frequency they undergo what particle physicists term a Hagedorn transition [17]. In this section we take this classical transition seriously, but we shall be more realistic by treating classical strings more as polymers. These again show transitions but, unlike the fundamental Nambu-Goto strings, permit the inclusion of string forces at low densities.

The comments that follow are a straightforward extension of earlier work of ours on global strings [18]. Since they are somewhat speculative a detailed recreation of these earlier results is inappropriate, and the reader is referred to the literature, particularly the extensive work of Kleinert [19].

Strings produced in the way indicated above behave like random walks of high density [20]. It is well known that an ensemble of noninteracting random walks at temperature $T = \beta^{-1}$, step length a, can be described by a dual field theory of a free complex field $\chi(\underline{x})$ [21]. The "loop field" $\chi(\underline{x})$ has the action (Hamiltonian) [19,21]

$$S_0 = \int d\underline{x}\,\left[|\nabla\chi|^2 + M^2|\chi|^2\right], \tag{46}$$

where

$$M^2 \propto \frac{(\epsilon - Ts)}{aT}, \tag{47}$$

in which ϵ is the string energy/unit length and $s = O(a^{-1})$ is the entropy/unit length. The vanishing of M^2 at $\epsilon = Ts$ defines a Hagedorn transition at $T = T_H$, at which string is produced copiously. For $T > T_H$ the theory is unstable.[7]

This is already enough to indicate why, even if the string formation mechanism naturally generated strings with higher winding number $|n| > 1$, the thermal energy would be sufficient to unpeel them to $|n| = 1$ strings. The problem is akin to that of adhesion in macromolecules. As a first step we adopt the simplifying assumption that, once separated from a string of higher winding number, $|n| = 1$ strings experience no forces. A straightforward extension [22] of the work of Wiegel [23] shows that, provided the energy required to split an $n = 2$ string into two $n = 1$ strings is less than the cost of creating the string, the Hagedorn transition survives at the same temperature T_H. Moreover, there is a temperature range $T_0 < T < T_H$ in which the separation into $n = 1$ strings is total. We shall not consider the problem of higher winding number further.

The forces between the strings modify (46). Orientable forces are characterized by a dual vector field $\mathcal{A}$. It is not possible to mimic our cosmic strings exactly. However, for a strong type II theory of static strings at low density, a plausible dual field theory that correctly describes the long-range forces between strings is given by an action [18,19]

$$S_{II} = \int d\underline{x}\left[\frac{1}{4}\left(\nabla\wedge\mathcal{A}\right)^2 + \frac{1}{2}m_A^2\mathcal{A}^2 + |\left(\nabla + if\mathcal{A}\right)\chi|^2 + |\nabla\chi|^2 + M^2|\chi|^2\right], \tag{48}$$

where $m_A \simeq 0$, and $f^2 = O(\beta^2\epsilon)$ is the dimensionless coupling strength.[8] M^2 is as before.

We have seen that, in this case, there is no nonorientable force at short distance that would be the consequence of an explicit $g|\chi|^4$ term in (48). However, such a term is induced by the $\mathcal{A}$ field. At its simplest, a one-loop approximation to the effective density potential in which the $\mathcal{A}$ field is integrated over in (48) gives

$$V(\rho) = M^2\rho + 3T\int d\!\!\!{}^-\underline{k}\,\ln(\underline{k}^2 + m_A^2 + 2f^2\rho). \tag{49}$$

In (49) we have identified the dimensionful string density $\rho = L/V$, the length/volume, with $|\chi|^2$ [21]. Expanding the logarithm generates all powers of ρ.

Now consider $b \gg 1$. Then m_A^{-1}, which characterizes the range of the orientable force, becomes small. The string forces are largely the consequence of the nonorientable second term in (38), with larger (but still finite) range m_S^{-1}. In this regime the dual gauge field can almost be neglected, whereas the *attractive* nature of the nonorientable force can be described by the inclusion in (48) of a $g|\chi|^4$ term (or $g\rho^2$ term) with g *negative.* The stability of the system will be expected to be preserved by the existence of a $h|\chi|^6$ (or $h\rho^3$) term that, for a three-dimensional theory, is possible but usually irrelevant near the phase transition. As b decreases the magnitude of the negative ρ^2 term decreases until a value of b is reached at which it cancels the ρ^2 effects. The dual theory is at a tricritical point above which (for negative ρ^2 term) it displays a first-order transition in ρ, as temperature T (and hence M^2 and its corrections) is varied. There is no *a priori* reason why this should occur at $b = 1$ exactly, although that is the value at which there are no attractive forces. (However, in a lattice model for superconductors it has been argued that the ρ^2 term vanishes for $b \simeq 1$ [19].) Whatever the details, type I strings can coexist at nonzero density.

VI. STRING FORMATION FROM QUANTUM FLUCTUATIONS

The main reason for halting our analysis of the dual theory is not so much its incompleteness as the fact that, initially, cosmic string production arises from quantum fluctuations at the phase transition possessed by the underlying local U(1) field theory. Without reference to string, in approximate thermal equilibrium, the theory displays a second-order transition at the temperature $T_c = O(\eta)$ if $b \ll 1$. On the other hand, for $b \gg 1$ it is reliably expected to display a first-order transition. This accords with our terminology of type II and type I strings, respectively. Yet again, there is no reason to believe that the transition changes from second to first order exactly at $b = 1$.

Whatever the case, in quantum field theory there cannot be a Hagedorn transition, with its characteristic maximum temperature. Rather, there is a critical temperature T_c above which the $U(1)$ symmetry is restored and strings cannot occur. In thermal equilibrium at temperature T, this is best seen by working in Euclidean time, in which the fields are periodic with period $\beta = T^{-1}$. Provided T is larger than the mass scales the "heavy" ($n \neq 0$) modes in the Fourier series can be integrated out, to give an effective three-dimensional theory derived

[7] And at which all energy goes into the production of a single string.

[8] The coupling f should not be confused with the field $f(\underline{x})$ of Sec. II.

from (1), with the action [24,25]

$$\beta S_3 = \int d\underline{x}\left[\frac{1}{4}\left(\underline{\nabla}\wedge\underline{A}\right)+\frac{1}{2}|\left(\underline{\nabla}+ie\underline{A}\right)\phi|^2 -\frac{1}{2}m^2(T)|\phi|^2+\frac{\lambda}{8}|\phi|^4\right]$$
$$+\text{terms containing } A_0(\underline{x}) + \text{counterterms}, \quad (50)$$

where $A_0(\underline{x})$ is the temporal component of the "light" ($n=0$) mode component of the gauge-$\underline{A}$ field, and

$$m^2(T) = \frac{\lambda\eta^2}{2}\left(1-\frac{T^2}{T_c^2}\right), \quad (51)$$

is the effective scalar mass of the theory.

There are two points to note about the action (51). Firstly, when $A_0 = 0$, it is extremized by the same (static) vortex solutions as was the action based upon (1), but for the fact that the Higgs boson mass is temperature dependent. The second is that the Higgs potential in (51) shows a second-order transition, prior to integrating over the gauge field $\underline{A}$, whatever the value of b. A first-order transition can only occur for $b \gg 1$ as a consequence of gauge-field radiative corrections.

Returning to the first point, for a type II theory we could use the "classical" solutions to (51) as the basis of a dual field theory, as in the previous section. This would now be a dual theory in the more usual sense of the word [19], a rewriting of the original quantum theory in terms of its excitations. The instability of this theory is, in the first instance, characterized again by the vanishing of

$$\mathcal{U} = \epsilon - Ts, \quad (52)$$

except that ϵ and a, the step length, are now expressed in terms of $m(T)$. With $\epsilon = O(\frac{m^2(T)}{\lambda})$ and $s = O(m(T))$ the condition $\mathcal{U} = 0$ becomes $\frac{\lambda T}{m(T)}$ =const [24], or

$$\left(1-\frac{T^2}{T_c^2}\right) = O(\lambda). \quad (53)$$

This is the Ginzburg criterion for the onset of large fluctuations in the vicinity of T_c, at which (51) becomes an unreliable basis for calculation. Thus, the Hagedorn transition of the previous section becomes subsumed in the conventional phase transition for the quantum field and there is no dichotomy. The high string density in the naive Kibble mechanism [20] is commensurate with the high density at the Hagedorn transition.

For type I strings the situation is more complicated. *A priori* the Kibble mechanism does not discriminate between type I and type II strings, producing a high initial density that may not be compatible with the density after the tricritical point. A pointer comes from a naive model for dislocation melting in solids, based upon action $S_{\rm II}$ [26]. Surprisingly, the "quantum" fluctuations of the $\mathcal{A}$-field in (48) can indeed induce a first-order transition in the order parameter ρ if the coupling f^2 of the orientable forces is strong enough.[9] Calculations have been performed [22] that extend the work of [18] to include the temperature dependence of the mass $m(T)$ and the coupling strength f for strings with short-range orientable forces. They show that, should a first-order transition occur, it too will be buried in the Ginzburg temperature range. The effect of this is that the string density can be very much lower (e.g., a factor 10^{-3}) than we would have had otherwise. Such a low density is unlikely to have any problems for stability.

As a final comment, it might be argued that the use of static networks, even initially, is inappropriate. However, the difference between an ensemble of static strings and an ensemble of relativistic strings, while present, is not as much as might be thought. For example, relativistic Nambu-Goto loops are described in terms of right-moving and left-moving modes. This doubling of modes changes the power behavior of the prefactor to the Hagedorn exponential in the counting of states, in comparison to a static ensemble.[10] However, this change is exactly cancelled by integrating over the loop center-of-mass momenta, which effectively makes their centers of mass static. In the same sense, the time-independent action (51), if used as a basis for string saddle points, would give rise to zero-frequency modes that could be interpreted as center-of-mass loop dynamics.

Of course, we have been very simplistic in our neglect of quantum fluctuations about the strings. This has been considered elsewhere (last reference, [18]) and does not derail our conclusions to date. However, it does compound our inability to be quantitative with the dual theory.

VII. CONCLUSIONS

We have shown how the interaction energy for U(1) cosmic strings depends on the parameter space of the model as well as on the relative orientations of two interacting strings.

One of the crucial assumptions in all simulations of the evolution of networks of cosmic strings is that they intercommute in all cases. This property, however, should depend both on the topology of the problem which ensures the exchange of ends of the two strings and is model independent, but also on the dynamics of the system under the effect of its interactions.

Our analysis reveals that type I strings experience attractive and, in the limit where the contribution from the gauge sector is negligible, nonorientable interactions. This is corroborated by several numerical studies and implies that in the absence of a strong effect at the superposition of their scalar cores two such strings will form a

[9] There is no contradiction in type-II strings having a first order transition once the strength of the string forces can be chosen arbitrarily, and are not fixed by the underlying field theory.

[10] It is this power which determines critical indices.

bound state of higher winding number, at a sufficiently low energy collision and for small relative angles. Such higher winding number states are well known to be stable. A more detailed study concerning this problem is presented elsewhere [27], where we attempt to quantify the circumstances in which such configurations may occur.

Type I strings are especially interesting for scenarios of thermal string production after a period of inflation, since the critical temperature associated with the phase transition is lowered by taking $b \gg 1$ [29]. If the intercommuting of type I strings could be shown to be very inefficient such scenarios would have to be ruled out on the grounds of being cosmologically unacceptable. The most likely effect of the interactions, however, would probably be that of modifying the evolution parameters of a network of strings to some extent without jeopardizing the approach to a scaling regime [27].

Type II, global, and critically coupled strings in our picture would intercommute. This is absolutely consistent with several numerical studies [6,28].

Finally, the knowledge of explicit forms for the interaction energy of strings is a crucial element for the construction of a realistic statistical description of strings, as well as of vortices away from critical coupling. In the last sections we have indicated, through the methods of a dual field theory, how strings with attractive forces do not destabilize the initial string network. Further, we see why $b = 1$ ceases to be critical once arbitrary string orientations are taken into account. Of course, strings produced by fluctuations have to be frozen in by subsequent out-of-equilibrium development, and this has been omitted from our discussion. The whole mechanism of string production from a quantum theory, both with and without initial approximate thermal equilibrium, is considered elsewhere [30]. Nonetheless, the mechanisms proposed by Kibble [1] for string production from fluctuations, from which we have been quoting, seem substantially correct.

ACKNOWLEDGMENTS

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[1] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).
[2] Ya. B. Zeldovich, Mon. Not. R. Astron. Soc. **192**, 663 (1980); A. Vilenkin, Phys. Rev. Lett. **46**, 1169 (1981); **46**, 1496(E) (1981).
[3] E. P. S. Shellard, in *Cosmic Strings: The Current Status*, edited by F. S. Accetta and L. M. Kraus (World Scientific, Singapore, 1988).
[4] E. Copeland and N. Turok (unpublished).
[5] P. Laguna and R. A. Matzner, Phys. Rev. Lett. **62**, 1948 (1989).
[6] E. P. S. Shellard, Nucl. Phys **B283**, 624 (1987).
[7] P. Bhattacharjee, Ph.D. thesis, Imperial College, 1984.
[8] H. B. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).
[9] L. Jacobs and C. Rebbi, Phys. Rev. B **19**, 4486 (1979).
[10] H. de Vega and F. Shaposnik, Phys. Rev. D **14**, 1100 (1976).
[11] A. A. Abrikosov, Sov. Phys. JETP **5**, 1174 (1957).
[12] E. Müller-Hartmann, Phys. Lett. **23**, 521 (1966).
[13] D. B. Bennett and F. R. Bouchet, Phys. Rev. Lett. **60**, 257 (1988); in *The Formation and Evolution of Cosmic Strings*, edited by G. W. Gibbons, S. W. Hawking, and T. Vachaspati (Cambridge University Press, Cambridge, 1990).
[14] T. Vachaspati and A. Vilenkin, Phys. Rev. D **31**, 3052 (1985).
[15] P. C. Hendy, N. S. Lawson, R. A. M. Lee, P. V. E. McLintock, and C. D. H. Williams, Nature (London) **368**, 315 (1994).
[16] W. H. Zurek, Nature (London) **368**, 292 (1994).
[17] N. Turok, Report No. Fermilab-Conf-88/116-A (unpublished).
[18] E. Copeland, D. Haws, S. Holbraad, and R. J. Rivers, in *The Formation and Evolution of Cosmic Strings* [13]; E. Copeland, G. Cheetham, T. S. Evans, and R. J. Rivers, Phys. Rev. D **47**, 5316 (1993).
[19] H. Kleinert, *Gauge Fields in Condensed Matter,* Vol. 1 (World Scientific, Singapore, 1989).
[20] R.J. Scherrer and J. A. Frieman, Phys. Rev. D **33**, 3556 (1986).
[21] P. R. Thomas and M. Stone, Nucl. Phys. **B144**, 503 (1978); H. Kleinert, Phys. Lett. **97A**, 51 (1983).
[22] S. Holbraad, Ph.D thesis, University of London, 1990.
[23] F.W. Wiegel, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and J. Lebowitz (Academic Press, New York, 1973), Vol. 7.
[24] E. Copeland, D. Haws, and R. J. Rivers, Nucl. Phys. **B319**, 687 (1989).
[25] A. Jakovac, K. Kajantie, and A. Patkos, Phys. Rev. D **49**, 6819 (1994).
[26] S. F. Edwards and M. Warner, Philos. Mag. **40**, 257 (1979); E. Copeland, D. Haws, S. Holbraad, and R. J. Rivers, Physica A **179**, 507 (1991).
[27] L.M.A. Bettencourt and T.W.B. Kibble, Phys. Lett. B **332**, 297 (1994).
[28] R. A. Matzner, Comput. Phys. **2**, 51 (1989).
[29] H. M. Hodges and J. R. Primack, Phys. Rev. D **43**, 3155 (1991).
[30] R. J. Rivers, in Proceedings of the NATO Advanced Workshop on Electroweak Physics and the Early Universe, Sintra, Portugal, 1994 (Plenum, New York, in press).