PHYSICAL REVIEW D VOLUME 51, NUMBER 4 15 FEBRUARY 1995

OPE's and the dilaton β function for the two-dimensional $N = 1$ supersymmetric nonlinear σ model

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(Received 19 October 1994)

Using the superspace formalism, we compute, for the two-dimensional $N = 1$ supersymmetric nonlinear σ model, the order $(\alpha')^2$ $(R_{mnpq})^2$ (three-loop) correction to the central charge via the operator product expansion of the supercurrent with itself. The contribution vanishes, in agreement with previous results obtained from the usual σ model β-function appraoch.

PACS number(s): 11.25.Hf, 11.10.Kh, 11.30.Pb

I. INTRODUCTION

Two-dimensional superconformal field theory (SCFT) has proven to be a useful means for investigating the relationship between supersymmetric nonlinear σ models and superstrings in background fields. Conformal invariance is necessary for consistent string propagation in a curved background, and this effectively forces the β functions of the corresponding σ model to be zero. The equations $\beta = 0$ so obtained can then be identified as the equations of motion of the background fields—the metric $G_{ij}(X)$, the antisymmetric tensor $B_{ij}(X)$, and the dilaton $\Phi(X)$. The results of string computations have been found to be in agreement with those from σ-model calculations [1–3].

From SCFT we get expressions for the operator product expansion (OPE) of two operators. It is possible to use these OPE's as an alternative means of obtaining the σ-model β functions, instead of using the standard renormalization-group procedures. Specifically, the expectation value $\langle J(z)J(z')\rangle$, where the operator J is the supercurrent, can be computed perturbatively for the σ model. Once computed, $\langle J(z)J(z')\rangle$ can be compared with the result of $J(z)J(z')$ from the operator product expansion. Calculations of this type have been done for the bosonic σ model [4], and for the $N = 1$ supersymmetric case (to one-loop order for β^G_{ij} and β^B_{ij}, and to two-loop order for β^Φ) [5]. In these calculations, additional terms appear in the perturbation expansion of $\langle J(z)J(z')\rangle$ that do not exist in the OPE for $J(z)J(z')$. These extra terms can be observed to be essentially the β functions of the σ model and when set to zero for consistency with the OPE's, yield the background equations of motion for the superstring fields.

In this paper we apply this alternate method of computing the β functions to the $N = 1$ supersymmetric σ-model (for the case $B_{ij} = 0$), by examing $\langle J(z)J(z')\rangle$ at three loops for extra contributions to the central charge of the form $(R_{ijkl})^2$, where R_{ijkl} is the background Riemann tensor. Terms of this form appear at two loops for the bosonic σ model when the metric β function is computed using the usual approach, but they do not appear in the supersymmetric case [6–10]. This makes the OPE calculation of particular interest. We would like to use the OPE method to obtain β^G_{ij} directly, and thus determine whether or not there are any new two-loop corrections. However, corrections to the central charge are given by β^Φ, and from the usual σ-model β-function results, $\beta^{G(L)}_{ij} \sim \delta\beta^{\Phi(L+1)}/\delta G_{ij}$, where L is the number of loops. Because the OPE generates quite a large number of two-loop diagrams that contribute to β^G_{ij}, we compute instead contributions to β^Φ at three loops (from a considerably smaller set of diagrams), of the form $(R_{mnpq})^2$. These terms are the ones which could lead to new contributions to β^G_{ij} of the form $R_i{}^{klm}R_{jklm}$. From previous calculations [6–10], the contribution from the extra $(R_{ijkl})^2$ terms is expected to be zero and the result of our calculation indicates that this is indeed the case.

The paper is organized as follows In Sec. II, we give the action of the $N = 1$ supersymmetric nonlinear σ model, the OPE for the supercurrent with itself, and the background field expansions for both of them. Section III discusses the calculation of the three-loop correction to the central charge. Our notation and conventions are those of [11] and are listed in the Appendix, along with a discussion of the techniques used and a sample diagram computation. Details of the calculation can be found in [12].

II. ACTION, OPE, AND THE BACKGROUND FIELD EXPANSION

The use of OPE's for obtaining the supersymmetric σ-model β functions was first presented in [5]. From superconformal field theory, the supercurrent $J_+{}^=$ must satisfy the OPE

$$\begin{aligned} J_+{}^=(z)J_+{}^=(z') \sim \frac{c/4}{S^{\neq 3}} + \frac{3}{2}\frac{\lambda^+}{S^{\neq 2}}J_+{}^=(\Sigma) \\ +\frac{1}{2}\frac{D_+J_+{}^=(\Sigma)}{S^{\neq}} \\ +\text{finite terms}\ , \end{aligned} \tag{2.1}$$

where $S^{\neq} = x^{\neq} - x'^{\neq} - i\theta^+\theta'^+$ is the supersymmetric coordinate difference, $\lambda^\pm = \frac{1}{2}(\theta - \theta')^\pm$, the midpoint is $\Sigma = \{\frac{1}{2}(x+x')^\pm, \frac{1}{2}(\theta+\theta')^\pm\}$, and c is the central charge.

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The procedure we follow now is just as in the previous bosonic [4] and supersymmetric cases [5]—calculate the expectation value $\langle J_+{}^{=}(z)J_+{}^{=}(z')\rangle$, and demand that (2.1) hold true.

We start with the action for the nonlinear σ model coupled to two-dimensional (2D) supergravity

$$S=-\frac{1}{4\pi\alpha'}\int d^2x\,d^2\theta E^{-1}\Big(G_{ij}(X)\mathcal{D}_+X^i\mathcal{D}_-X^j+\frac{\alpha'}{2}R^{(2)}\Phi(X)\Big)\ , \tag{2.2}$$

where $\mathcal{D}_\pm$ are the supergravity covariant derivatives, the superfield $R^{(2)}$ is the scalar curvature of the 2D world sheet, and E^{-1} is the superdeterminant of the inverse superzweibein. The supercurrent $J_+{}^{=}$ is defined to be

$$J_+{}^{=}=\frac{2\delta S}{\delta H_+{}^{=}}\bigg|_{H_+{}^{=}=0}\ . \tag{2.3}$$

$H_+{}^{=}$ is the supergravity gauge field, and the supercurrent is the variation of the action with respect to this field. We find

$$J_+{}^{=}=-\frac{1}{2\pi\alpha'}[G_{ij}(X)\partial_{\neq}X^iD_+X^j-\alpha'\partial_{\neq}D_+\Phi(X)]\ . \tag{2.4}$$

We use the background field method [13] as it applies to superfields to perform the perturbative expansion of $\langle J_+{}^{=}(z)J_+{}^{=}(z')\rangle$. This involves expanding the action and the supercurrent in terms of Riemann normal coordinates ξ^i on the background manifold. The relevant expansions are

$$\begin{aligned}\partial_{\neq}X^i&=\partial_{\neq}X^i_B+\nabla_{\neq}\xi^i+\tfrac{1}{3}R^i{}_{lmn}(X_B)\partial_{\neq}X^n_B\xi^l\xi^m+\tfrac{1}{12}D_jR^i{}_{lmn}(X_B)\xi^j\xi^l\xi^m\partial_{\neq}X^n_B\\&\quad+\tfrac{1}{60}D_jD_kR^i{}_{lmn}(X_B)\partial_{\neq}X^n_B\xi^j\xi^k\xi^l\xi^m-\tfrac{1}{45}R^i{}_{jkp}R^p{}_{lmn}(X_B)\partial_{\neq}X^n_B\xi^j\xi^k\xi^l\xi^m+\cdots\ ,\\ D_+X^j&=D_+X^i_B+\nabla_+\xi^i+\tfrac{1}{3}R^i{}_{lmn}(X_B)D_+X^n_B\xi^l\xi^m+\tfrac{1}{12}D_jR^i{}_{lmn}(X_B)D_+X^n_B\xi^j\xi^l\xi^m\\&\quad+\tfrac{1}{60}D_jD_kR^i{}_{lmn}(X_B)D_+X^n_B\xi^j\xi^k\xi^l\xi^m-\tfrac{1}{45}R^i{}_{jkp}R^p{}_{lmn}(X_B)D_+X^n_B\xi^j\xi^k\xi^l\xi^m\cdots\ ,\\ G_{ij}(X)&=G_{ij}(X_B)-\tfrac{1}{3}R_{ikjl}(X_B)\xi^k\xi^l-\frac{1}{3!}D_lR_{imjk}(X_B)\xi^l\xi^m\xi^k\\&\quad+\frac{1}{5!}[-6D_kD_lR_{imjn}(X_B)+\tfrac{16}{3}R_{kjl}{}^pR_{minp}(X_B)]\xi^k\xi^l\xi^m\xi^n+\cdots\ ,\\ \Phi(X)&=\Phi(X_B)+D_i\Phi(X_B)\xi^i+\tfrac{1}{2}D_iD_j\Phi(X_B)\xi^i\xi^j+\cdots\ ,\end{aligned}\tag{2.5}$$

where the background covariant derivatives are

$$\nabla_{\neq}\xi^i=\partial_{\neq}\xi^i+\Gamma^i_{jk}(X_B)\xi^i\partial_{\neq}X^k_B$$

and

$$\nabla_+\xi^i=D_+\xi^i+\Gamma^i_{ji}(X_B)\xi^jD_+X^k_B\ ,$$

and X_B is the background field. The expansions of the action and the supercurrent (where the subscript B has been dropped) are

$$\begin{aligned}S=\frac{1}{8\pi\alpha'}\int d^2x\,d^2\theta\{&G_{ij}(X)D_\alpha X^iD^\alpha X^j+2G_{ij}D_\alpha X^i\nabla^\alpha\xi^j\\&+G_{ij}\nabla_\alpha\xi^i\nabla^\alpha\xi^j+R_{ijkl}D_\alpha X^iD^\alpha X^j\xi^k\xi^l\\&+\tfrac{4}{3}R_{ijkl}D_\alpha X^i\nabla^\alpha\xi^j\xi^k\xi^l+\tfrac{1}{3}R_{ijkl}\nabla_\alpha\xi^i\nabla^\alpha\xi^l\xi^j\xi^k+\cdots\}\ ,\end{aligned}\tag{2.6}$$

$$\begin{aligned}J_+^{=}=-\frac{1}{2\pi\alpha'}\{&G_{ij}\partial_{\neq}X^iD_+X^j+G_{ij}D_+X^i\nabla_{\neq}\xi^j+G_{ij}\partial_{\neq}X^i\nabla_+\xi^i\\&+G_{ij}\nabla_+\xi^i\nabla_{\neq}\xi^j+R_{ilmj}\xi^l\xi^mD_+X^i\partial_{\neq}X^j+\tfrac{1}{3}R_{ijkl}\nabla_+\xi^i\nabla_{\neq}\xi^l\xi^j\xi^k\\&+\tfrac{2}{3}R_{ilmn}\partial_{\neq}X^n\nabla_+\xi^i\xi^l\xi^m+\tfrac{2}{3}R_{ilmn}D_+X^n\nabla_{\neq}\xi^i\xi^l\xi^m\\&-\alpha'[D_iD_j\Phi D_+X^i\nabla_{\neq}\xi^j+D_iD_j\Phi\partial_{\neq}X^j\nabla_+\xi^j\\&+D_i\Phi\nabla_{\neq}\nabla_+\xi^i+\tfrac{1}{2}D_iD_j\Phi\nabla_{\neq}\nabla_+(\xi^i\xi^j)+\cdots]\}\ .\end{aligned}\tag{2.7}$$

Following the standard procedure, we refer ξ to the tangent frames on the manifold, $\xi^a = E^a \xi^i$, so that ξ^a is the quantum field used in the calculation and $\nabla_\alpha \xi^a = D_\alpha \xi^a + \omega^a_{ib} D_\alpha X^i \xi^b$, where ω^a_{ib} is the spin connection.

III. THE THREE-LOOP CORRECTION TO THE CENTRAL CHARGE

We calculate the order $(\alpha')^2 (R_{abcd})^2$ contributions to the central charge, and these involve three-loop graphs. *A priori* we expect the result to be zero because as mentioned previously, we know that the usual method of calculating β^G_{ij} indicates that it receives no two-loop corrections in the supersymmetric case.

It turns out that the expansions given previously in Sec. II are sufficient to generate the four different types of graphs that yield $(R_{abcd})^2$ contributions. We isolate below the relevant interaction vertices from the Lagrangian and the supercurrent that make up these graphs.

The Lagrangian term is

$$\mathcal{L}_{\text{int}} = \tfrac{1}{3} R_{dfge} D_+ \xi^d D_- \xi^e \xi^f \xi^g \ . \tag{3.1}$$

The supercurrent term is

$$J = G_{cd} \partial_{\neq} \xi^c D_+ \xi^d + \tfrac{1}{3} R_{abcd} \xi^c \xi^d \partial_{\neq} \xi^b D_+ \xi^a \ . \tag{3.2}$$

The diagrams are evaluated entirely in coordinate space using propagators $G(z,z')$. We recall that a conserved quantity such as the supercurrent should not receive any renormalization counterterms. In the one-loop case [4,5], all the divergences that did not cancel out between diagrams could be isolated into a tadpole integral, $G(0)$. This $G(0)$ divergence canceled out of both sides of (2.1) for the supercurrent because, although it appeared in $\langle JJ \rangle$ on the left-hand side of the OPE, it also appeared in $\langle J \rangle$ on the right. Because we expect that ultimately the supercurrent should be finite, we evaluate the graphs in such a way that we keep only the finite pieces, while noting that all the divergent terms we discard are of the form $G(0)$, or $\delta(0)G(0)$. However, we do not explicitly check this, and likewise we do not explicitly show that contributions from the connection terms cancel among themselves. The latter was verified in the bosonic and one-loop supersymmetric cases, and is expected to be generally true. We used standard superspace techniques [11] in the actual calculation of the diagrams, paying particular attention when doing the D algebra to the fact that one is *not* computing an effective action here, but an OPE. Because of the structure of the OPE, the points z and z' are not integrated over, and this means that one cannot push D's past those points on the diagrams.

The superspace propagator in coordinate space is

$$\begin{aligned} \Delta(z,z') &= \overline{\xi^i(z)\xi^j}(z') \\ &= -\tfrac{1}{2}\alpha' \delta^{ij} \ln(S^{\neq} S^{=}) \\ &= -\tfrac{1}{2}\alpha' \delta^{ij} G(z,z') \end{aligned} \tag{3.3}$$

and in doing the D algebra we eliminate propagators from the diagrams (thus bringing two superspace points u and v together) by applying the equation

$$D^u_- D^u_+ \Delta(u,v) = -2\pi\alpha' \delta^{(4)}(u-v) \ , \tag{3.4}$$

where $\Delta(u,v)$ is understood to be suitably defined by a cutoff μ whenever this equation is used.

The four types of graphs that contribute to the $(R_{abcd})^2$ term in the central charge are shown in Fig. 1. All the graphs yield contributions of the general form

$$\frac{1}{S^{\neq 3}} R_{i(jk)l} R^{ijkl} [G(z,z') + G(z,z')^2] \ , \tag{3.5}$$

where $G(z,z') = \ln(S^{\neq} S^{=}) = \ln S^{\neq} + \ln S^{=}$, formally.

The perturbation expansion of $\langle J_+{}^{=}(z) J_+{}^{=}(z') \rangle$ generates large numbers of specific contributions for the four types of graphs shown in Fig. 1. Initially there are over 300 different permutations of the derivatives for the four graphs, but many of these can be shown to be zero by D algebra, or because $\lambda^+ \lambda^+ = 0$, or are discarded because they give contributions involving $G(0)$. Tables of the relevant diagrams and their associated contributions can be found in [12]. A sample calculation of one of the graphs of type (d) is given in the Appendix.

Combining all the separate contributions for the various types of graphs gives diagrams of type (a),

$$\frac{1}{9} \frac{(\alpha')^2}{256\pi^2} \frac{i}{S^{\neq 3}} R_{a(bc)d} R^{abcd} [4G(z,z') + 4G(z,z')^2] \ , \tag{3.6}$$

diagrams of type (b),

$$\frac{1}{9} \frac{(\alpha')^2}{256\pi^2} \frac{i}{S^{\neq 3}} R_{a(bc)d} R^{abcd} [-8G(z,z') - 8G(z,z')^2] \ , \tag{3.7}$$

diagrams of type (c),

$$\frac{1}{9} \frac{(\alpha')^2}{256\pi^2} \frac{i}{S^{\neq 3}} R_{a(bc)d} R^{abcd} [-8G(z,z') + 8G(z,z')^2] \ , \tag{3.8}$$

and diagrams of type (d),

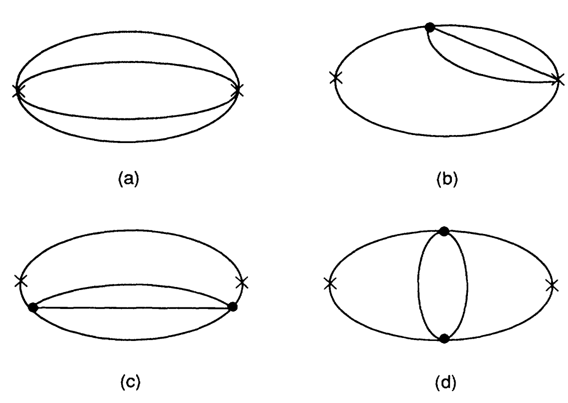

FIG. 1. Diagrams contributing to the central charge. Crosses denote supercurrent vertices while dots denote interaction vertices.

$$\frac{1}{9}\frac{(\alpha')^2}{256\pi^2}\frac{i}{S^{\neq 3}}R_{a(bc)d}R^{abcd}[12G(z,z')-4G(z,z')^2]\ . \quad (3.9)$$

By summing the above four terms, we obtain the complete result, which is zero as expected. Therefore we find that there are no additional corrections to β_{ij}^G of the form $R_i{}^{klm}R_{jklm}$ because there are no $(R_{ijkl})^2$ corrections to β^Φ.

IV. CONCLUSION

We have used results from superconformal field theory involving operator product expansions to obtain information about the β functions of the $N = 1$ supersymmetric nonlinear σ model. In particular, by computing the operator product expansion of the supercurrent with itself, we identified terms of the form $(\alpha')^2(R_{ijkl})^2$ that might contribute to the central charge, and hence give a new contribution to the dilaton β function at three loops. However, when all the terms are summed, the complete result is found to be zero, in agreement with results obtained previously using standard methods.

ACKNOWLEDGMENTS

We are indebted to Marc Grisaru for numerous discussions, and for a factor of two. We also thank the Physics Departments of the University of Toronto and Queen's University for their hospitality. This work was supported in part by the NSERC of Canada.

APPENDIX A

(1,1) superspace is four-dimensional space that is parametrized by two commuting coordinates x^a ($a = 0,1$), and two anticommuting coordinates θ^α ($\alpha = 1,2$). We use a one-component spinor notation $(+,-)$ in which $+(-)$ corresponds to the $+\frac{1}{2}$ $(-\frac{1}{2})$ helicity representation. The spinor coordinates are denoted by θ^+ and θ^-, and the light-cone components of x^a are

$$x^{\neq} = \frac{1}{\sqrt{2}}(x^0+x^1) \ \text{ and } x^{=} = -\frac{1}{\sqrt{2}}(x^0-x^1)\ .$$

Our conventions are those of [11], but in $(+,-)$ notation; the superspace derivatives become

$$D_A = (\partial_{\neq}, \partial_{=}, D_+, D_-)\ ,$$

where

$$D_+ = \frac{\partial}{\partial\theta^+} + i\theta^+\partial_{\neq} \ \text{ and } D_- = \frac{\partial}{\partial\theta^-} + i\theta^-\partial_{=}\ .$$

We use the expressions listed below, which can be obtained from those in [11] by carefully replacing a spinor index α with $+$ or $-$, and a vector index a (or a pair of spinor indices $\alpha\beta$) by $\neq$ or $=$:

$$D_\alpha = \varepsilon_{\alpha\beta}D^\beta \to D_+ = \varepsilon_{-+}D^- = -D^-$$

and

$$D_- = \varepsilon_{-+}D^+ = D^+\ ,$$

$$D^2 = \tfrac{1}{2}D^\alpha D_\alpha = \tfrac{1}{2}(D^+D_+ + D^-D_-) = D_-D_+ = D^-D^+$$

$$\{D_\alpha, D_\beta\} = 2i\partial_{\alpha\beta} \to \{D_+,D_-\} = 0,\ \{D_+,D_+\} = 2i\partial_{\neq},$$

$$\{D_-,D_-\} = 2i\partial_{=}$$

$$(D^2)^2 = \Box \quad \Box = \partial_{=}\partial_{\neq} = -p^2 \ \text{ if } i\partial \to p$$

$$\theta^2 = \tfrac{1}{2}\theta^\alpha\theta_\alpha = \theta_-\theta_+ = \theta^-\theta^+\ ,$$

$$D^2D_+ = -D_+D^2 = i\partial_{\neq}D^+$$

and

$$D^2D_- = -D_-D^2 = i\partial_{=}D^-$$

In the actual computation of the diagrams, we only need terms with the derivatives $(\partial_{\neq}, D_+)$ acting on propagators, and so we can effectively drop the $\ln S^{=}$ term in the propagator, $G = \ln S^{\neq}S^{=}$. We need the identities

$$D_+^z S^{\neq} = i(\theta-\theta')^+ \equiv i\lambda^+ = D_+^{z'}S^{\neq}\ ,$$

$$\partial_{\neq}^z G(z,z') = \frac{1}{S^{\neq}} = -\partial_{\neq}^{z'}G(z,z')\ ,$$

$$D_+^z G(z,z') = \frac{i\lambda^+}{S^{\neq}} = D_+^{z'}(z,z')\ ,$$

$$(\partial_{\neq}^z)^2G(z,z') = -\frac{1}{S^{\neq 2}},\quad (\partial_{\neq}^z)^3G(z,z') = \frac{2}{S^{\neq 3}}\ ,$$

$$\partial_{\neq}^{z'}G(z,z')\overleftarrow{\partial}_{\neq}^{z'} = \frac{1}{S^{\neq 2}},\quad D_+^zG(z,z')\overleftarrow{D}_+^{z'} = \frac{i}{S^{\neq}}\ .$$

Graphs of the kind shown in Fig. 1(d) are the most complicated. They involve integration over two vertices, u and v. We calculate the diagram in Fig. 2 as an example.

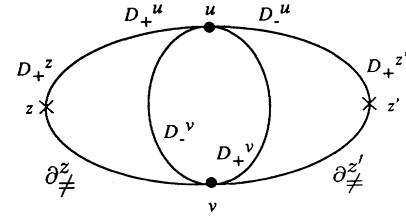

FIG. 2. Sample diagram for D algebra.

Figure 2 gives

$$2R_{i(jk)l}R^{ijkl}\int d^4u\, d^4v\, \partial^z_{\neq}G(z,v)D^z_+D^u_+G(z,u)D^v_+G(u,v)D^v_-G(u,v)\partial^{z'}_{\neq}G(v,z')D^u_-D^{z'}_+G(u,z') \ .$$

Using (3.4) to integrate over u and then integrating the D^v_- by parts gives

$$2R_{i(jk)l}R^{ijkl}D^z_+D^{z'}_+G(z,z')\int d^4v\, \partial^z_{\neq}D^v_-G(z,v)D^v_+G(z',v)\partial^{z'}_{\neq}G(v,z')G(z',v)$$

$$= 2R_{i(jk)l}R^{ijkl}\frac{i}{S^{\neq}}\int d^4v(-i)D^v_+\delta(z-v)D^{z'}_+G(z',v)\partial^{z'}_{\neq}G(v,z')G(z',v) \ .$$

Now we integrate off the D^v_+ and then do the v integration to get

$$2R_{i(jk)l}R^{ijkl}\frac{i}{S^{\neq}}\int d^4v i\delta(z-v)(-i)D^z_+D^{z'}_+G(z',v)\partial^{z'}_{\neq}G(v,z')G(z',v)$$

$$= 2R_{i(jk)l}R^{ijkl}\frac{i}{S^{\neq}}i\delta(z-v)(-i)D^z_+D^{z'}_+G(z',z)\partial^{z'}_{\neq}G(z,z')G(z',z)$$

$$= 2R_{i(jk)l}R^{ijkl}\frac{i}{S^{\neq 3}}G(z,z')(\times\text{overall factors}) \ .$$

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