Nambu–Jona-Lasinio model with the homogeneous background gluon field

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Starting with the Euclidean generating functional of QCD we construct a generalization of the Nambu–Jona-Lasinio model, taking into account the homogeneous background gluon field, which ensures an analytical quark confinement. Colorless modes are determined by the confined gluons and are described by the nonlocal quark currents with appropriate radial and angular quantum numbers. The effective Lagrangian for the local meson fields corresponds to ultraviolet finite theory. The spectrum of the radial and orbital excitations is asymptotically equidistant; i.e., it has a qualitatively correct Regge behavior. It is found that in the heavy quark limit the mass of quarkonium tends to be equal to the sum of the masses of the constituent quarks.

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I. INTRODUCTION

It is now widely accepted that at low energies the complicated structure of the QCD vacuum plays an important role. Vacuum self-dual gluon fields such as instanton solutions [1–4], stochastic fields [5], or the fields with a constant strength [6–8] are widely used to explain various features of low-energy hadron physics. On the other hand, the Numbu–Jona-Lasinio (NJL) model [9–11] gives a mechanism of bosonization. Nonlocal extensions of the NJL model provide qualitatively new possibilities to investigate the effects associated with the quark structure of hadrons [12].

In this paper we construct a kind of generalization of the NJL model and show that both nonlocal quark interaction and color singlet currents arise naturally in QCD with the homogeneous (anti-)self-dual background gluon field.

As Leutwyler has shown, the (anti-)self-dual homogeneous gluon field provides an analytical quark confinement [6]. It means that the quark propagator in the momentum representation is an entire analytical function. There are no poles corresponding to free quarks. The situation with gluons is more complicated. There are modes corresponding to free massless gluons, in addition to the confined degrees of freedom of the gluon field. However, one can determine the contribution of the confined gluon modes to the propagator and investigate the sector of QCD dynamics caused only by the confined gluons. This sector of QCD manifests interesting properties which are discussed below. In contrast with pure chromomagnetic or chromoelectric configurations, self- and anti-self-dual fields are stable in a sense that the effective potential for these fields is a real function. Unfortunately, numerous attempts to estimate the field strength, minimizing the effective potential, have not given definite results. A general reason for this is quite clear: Phase transitions in quantum field systems accompanied by the appearance of nonzero vacuum fields occur out of the perturbation region and their successful investigation is damped by a lack of nonperturbative methods. At the present time, the task of proving the existence of the vacuum field and to estimate its strength, starting with first principles, seems to be quite complicated.

We will follow another, in some sense phenomenological, point of view. Namely, we suppose that a self- or anti-self-dual homogeneous gluon field realizes the QCD vacuum at low energies and find out the points in hadron physics where this vacuum field can play an important role. Our consideration is based on the bosonization procedure of the standard NJL model [9, 10]. At the same time, taking into account the background field in both the quark and gluon propagators requires an essential modification of this procedure.

Starting with the Euclidean generating functional of QCD with a background gluon field [2, 13] we construct the color-singlet bilocal quark currents. Confined gluon fields ensure a natural expansion of the bilocal quark currents over the nonlocal ones with appropriate radial and orbital quantum numbers. An idea of such an expansion was discussed in general form in [12]. The realization of this idea implies the existence of a set of orthonormalized functions. The particular form of these functions reflects the specific physical peculiarities of the system. We show that the homogeneous (anti-)self-dual vacuum field determines a quite definite set—generalized Laguerre polynomials. As a result of the expansion, an interaction of quarks is realized by the current-current terms in the effective Lagrangian; the currents are nonlocal and carry radial and orbital quantum numbers. In contrast with the nonrenormalizable local NJL model, our generalization leads to effective four-fermion theory, which is superrenormalizable due to the nonlocality of the currents.

By means of the standard NJL bosonization we get a representation of a generating functional in terms of local meson fields, interacting with nonlocal quark currents. These meson fields have a complete set of quantum numbers including radial n and orbital ℓ ones. Effective meson theory is ultraviolet finite due to nonlocal meson interactions. It should be noted that we use a representation for generating functional of QCD which implies an averaging over some parameters of the background field (for details see [2, 13]). In the case of an homogeneous field we have to average over self- and anti-self-dual con-

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figurations and over directions of the field. Because of this averaging the color, space rotation, and parity symmetries are restored at the hadron level.

Parametrization of the generalized model is quite natural. The quark masses, the four-fermion coupling constant and the background field strength are the free parameters of the model.

We calculated the asymptotic behavior of the spectrum of excited meson states in the cases $\ell \gg n$ and $n \gg \ell$. It turns out to be equidistant both over the radial n and orbital ℓ quantum numbers; i.e., it has a qualitatively right Regge character. Moreover, effective meson-quark coupling constants decrease very fast for large n, ℓ. This suggests that the higher-order corrections should not change the above-stated asymptotics, obtained at the lowest order. Regge behavior is a consequence of the nonlocality of the quark propagator and meson-quark vertices that is conditioned by the presence of the background field.

Another asymptotic regime that is considered in the paper is the limit of heavy quarks. It is found that the mass of quarkonium in this limit tends to be equal to the sum of the masses of constituent quarks: $M \to 2m_q$.

The paper is organized as follows. In Secs. II and III we introduce all preliminary definitions and discuss quark and gluon dynamics in the homogeneous background field. Collective modes induced by this background are considered in Sec. IV. Asymptotic behavior of the spectrum of excited meson states is evaluated in Sec. V. Heavy quarkonium is considered in Sec. VI.

II. EUCLIDEAN GENERATING FUNCTIONAL

The generating functional for QCD with a background gluon field B_μ^a in the Euclidean metrics is [14]

$$Z = N \int DqD\bar{q}DA\ \Delta[A,B] \exp\Big\{ \int d^4x \Big[L_{\rm QCD}(x) + L_{\rm gf}(x) \Big] \Big\}, \tag{1}$$

where

$$\begin{aligned}
L_{\rm QCD} &= -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_f^{N_f} \bar{q}_f \left[i\gamma_\mu \hat{\nabla}_\mu - m_f + g A^a_\mu \gamma_\mu t^a \right] q_f \ , \\
G^a_{\mu\nu} &= \partial_\mu \Big(A^a_\nu + B^a_\nu \Big) - \partial_\nu \Big(A^a_\mu + B^a_\mu \Big) + g f^{abc} \Big(A^b_\mu + B^b_\mu \Big) \Big(A^c_\nu + B^c_\nu \Big) \ , \\
L_{\rm gf} &= \frac{1}{2\xi} \Big(\breve{\nabla}^{ab}_\mu A^b_\mu \Big)^2 \ .
\end{aligned}$$

$\Delta[A,B]$ is the Faddeev-Popov determinant. The gauge fixing term $L_{\rm gf}$ corresponds to the background gauge condition [14]. Dirac matrices are taken to be anti-Hermitian. The following notation is used:

$$\hat{\nabla}_\mu = \partial_\mu - ig\hat{B}_\mu \ , \quad \hat{B}_\mu = B^a_\mu t^a \ , \quad \breve{\nabla}_\mu = \partial_\mu - ig\breve{B}_\mu \ , \quad \breve{B}_\mu = B^a_\mu C^a \ ,$$

where the matrices t^a and C^a are the generators of the color group $SU_c(3)$ in the fundamental and adjoint representations, so that

$$[t^a, t^b] = if^{abc} t^c \ , \quad [C^a, C^b] = if^{abc} C^c \ , \quad (C^a)_{bc} = -if^{abc} \ .$$

The fields B^a_μ obey the classical Yang-Mills equations. The functional (1) is invariant under the transformations

$$\begin{aligned}
& q(x) \to e^{-i\hat{\omega}(x)} q(x) \ , \quad \bar{q}(x) \to \bar{q}(x) e^{i\hat{\omega}(x)} \ , \\
& \hat{A}_\mu + \hat{B}_\mu \to e^{-i\hat{\omega}(x)} \Big[\hat{A}_\mu + \hat{B}_\mu \Big] e^{i\hat{\omega}(x)} + \frac{i}{g} e^{-i\hat{\omega}(x)} \partial_\mu e^{i\hat{\omega}(x)} \ .
\end{aligned} \tag{2}$$

We will consider the last transformation as a background gauge one [14]:

$$\begin{aligned}
& \hat{B}_\mu \to e^{-i\hat{\omega}(x)} \hat{B}_\mu e^{i\hat{\omega}(x)} + \frac{i}{g} e^{-i\hat{\omega}(x)} \partial_\mu e^{i\hat{\omega}(x)} \ , \\
& \hat{A}_\mu \to e^{-i\hat{\omega}(x)} \hat{A}_\mu e^{i\hat{\omega}(x)} \ .
\end{aligned} \tag{3}$$

To make representation (1) correct a gauge-invariant regularization should be introduced and the fields, masses, and coupling constant should be considered as the bare ones.

Not only is the total action manifestly invariant under background gauge transformations (2),(3), but its part quadratic over the quantum fields $q, \bar{q}$ and A is invariant also. Meanwhile, the vacuum configuration with the background field $B^a_\mu(x)$ is not invariant under both color and rotational groups. From the viewpoint of the operator formalism of quantum field theory it means that the initial and the transformed vacuum states belong to unitary nonequivalent Hilbert spaces (see, e.g., [15, 16]). Such a situation corresponds to spontaneous breaking of the color and rotational symmetries. A detailed discussion of these problems for the case of the constant background field can be found in Ref. [8].

The effective action, arising from the generating functional (1), is invariant under the rotational and gauge group. Such a conclusion is proved rigorously within the background field method, which can be applied directly to representation (1) [14, 17]. Particularly, this suggests that the vacuum energy (or the free energy of a system) does not depend on the direction of the background field in color and x space. It means that the vacuum state with a background field, if it exists, would degenerate with respect to space rotations and color gauge transformations. In order to take into account this fact one has to average all physical amplitudes over the directions of the background field. In the general case, one can say that the vacuum field depends on a set of parameters $\{\sigma_{\rm vac}\}$. If the vacuum states corresponding to different values of these parameters are degenerate, then all observable amplitudes must be averaged over $\{\sigma_{\rm vac}\}$. This idea was first realized in Ref. [2] (see also [13]), where the following representation for the generating functional (1) was evaluated:

$$Z = N\int d\sigma_{\rm vac} DqD\bar{q}\exp\Big\{\int d^4x\sum_f^{N_f}\bar{q}_f(x)\big(i\gamma_\mu\hat{\nabla}_\mu - m_f\big)q_f(x) + \sum_{n=2}^{\infty}L_n\Big\}, \tag{4}$$

where

$$L_n = \frac{g^n}{n!}\int d^4y_1\cdots\int d^4y_n j^{a_1}_{\mu_1}(y_1)\cdots j^{a_n}_{\mu_n}(y_n)G^{a_1\cdots a_n}_{\mu_1\cdots\mu_n}(y_1,\ldots,y_n\mid B),$$
$$j^a_\mu(y) = \bar{q}_f(x)\gamma_\mu t^a q_f(x).$$

The function $G^{a_1\cdots a_n}_{\mu_1\cdots\mu_n}$ is the exact (up to the quark loops) n-point gluon Green function in the external field B^a_μ. Below we shall specify the form of the measure $d\sigma_{\rm vac}$ for an (anti-)self-dual field with a constant strength.

Representation (4) is a convenient starting point for the investigation of hadronization in QCD. We will be interested in $(q\bar{q})$ collective modes and consider the form of Eq. (4) truncated up to the term L_2:

$$Z = N\int d\sigma_{\rm vac} DqD\bar{q}\exp\Big\{\int d^4x\sum_f^{N_f}\bar{q}_f(x)\big(i\gamma_\mu\hat{\nabla}_\mu - m_f\big)q_f(x) + \frac{g^2}{2}\iint d^4xd^4y\; j^a_\mu(x)G^{ab}_{\mu\nu}(x,y\mid B)j^b_\nu(y)\Big\}. \tag{5}$$

Representations (4) and (5) are manifestly invariant under the gauge transformations (2) and (3).

The exact two-point Green function is unknown and a suitable approximation should be introduced. The standard NJL model corresponds to the case

$$B^a_\mu = 0,\quad G^{ab}_{\mu\nu}(x,y\mid 0) = \delta_{\mu\nu}\delta^{ab}\delta(x-y)\ .$$

The influence of the background field was investigated within the NJL model in Ref. [11], where the homogeneous background field was taken into account only in the quark propagator, but the interaction was kept local.

We will approximate the two-point Green function in Eq. (5) by a gluon propagator in an external field without taking into account higher-order corrections. In other words, our generalization of the NJL model consists in taking into account the influence of the background field on both the gluon and quark propagators. As soon as the quark currents corresponding to such a generalization are constructed, further steps in obtaining the effective meson Lagrangian will be explicitly the same as in the NJL model.

III. QUARKS AND GLUONS IN THE SELF-DUAL GLUON FIELD WITH A CONSTANT STRENGTH

A. Background field

The properties of the above-stated background field were discussed by many authors (e.g., see [6, 7, 18, 19]). We confine ourselves to the brief formulation of main results. An homogeneous (anti-)self-dual gluon field has the form

$$B^a_\mu(x) = B^a_{\mu\nu}x_\nu = n^a B_{\mu\nu}x_\nu,\qquad n^2 = 1\ ,$$

where the vector n defines a direction in color space. The constant tensor $B_{\mu\nu}$ satisfies the conditions

$$B_{\mu\nu} = -B_{\nu\mu}\ ,\quad B_{\mu\rho}B_{\rho\nu} = -B^2\delta_{\mu\nu}\ ,$$
$$\tilde{B}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}B_{\alpha\beta} = \pm B_{\mu\nu}\ . \tag{6}$$

This field is a solution of the classical Yang-Mills equations [7]. The gauge-invariant quantity B,

$$B^2 = \frac{1}{4}B^a_{\mu\nu}B^a_{\mu\nu},$$

is the tension of the background field.

Any matrix $\hat{n}\in {\rm SU}_c(3)$ can be reduced to the general form

$$\hat{n} = t^3\cos\xi + t^8\sin\xi\ ,\quad 0\le\xi<2\pi\ , \tag{7}$$

by an appropriate global gauge transformation. As soon as the chromomagnetic $\mathbf{H}$ and chromoelectric $\mathbf{E}$ fields relate to each other like $\mathbf{H}=\pm\mathbf{E}$ for the (anti-)self-dual configuration, one has only two spherical angles (θ,φ) defining a direction of the fields in x space. Now we can write down the explicit form of the measure $d\sigma_{\rm vac}$:

$$\int d\sigma_{\rm vac}=\frac{1}{(4\pi)^2}\int_0^\pi d\theta\sin\theta\int_0^{2\pi}d\varphi\int_0^{2\pi}d\xi\sum_{\pm}\ , \qquad (8)$$

where $\sum_{\pm}$ denotes averaging over the self- and anti-self-dual configurations, which are assumed to be equiprobable.

The averaging over ξ can be easily included into the formalism described below. However, in this paper we would like to investigate asymptotic regimes such as the limits $\ell \gg 1$, $n \gg 1$ (exited meson masses) or $m_f^2 \gg B$ (heavy quarkonium). These regimes are determined just by the behavior of quark loops at large external momenta that is conditioned by a character of nonlocality of the quark and gluon propagators. Qualitatively the propagators are the same functions for any ξ in Eq. (7). An averaging over directions of n [integration over ξ in Eq. (8)] does not change qualitatively the momentum behavior of the quark loops. In order to simplify further calculations and clarify the contribution of the background field under consideration into forming the bound quark systems we will omit the integral over ξ in Eq. (8) and fix the particular vector $n^a=\delta^{a8}$, so that

$$\hat n = t^8=\mathrm{diag}\Big(\frac{1}{\sqrt3},\frac{1}{\sqrt3},-\frac{2}{\sqrt3}\Big)\ ,\quad \hat B_{\mu\rho}\hat B_{\rho\nu}=-(t^8)^2B^2\delta_{\mu\nu}\ .$$

In the adjoint representation one has

$$\breve n = C^8=\frac{\sqrt3}{2}K\ ,\quad \breve B_{\mu\rho}\breve B_{\rho\nu}=-\frac34 K^2B^2\delta_{\mu\nu}\ ,$$
$$K_{54}=-K_{45}=K_{76}=-K_{67}=i\ ,\ K^2=\mathrm{diag}(0,0,0,1,1,1,1,0)\ . \qquad (9)$$

The rest of the elements of the matrix K are equal to zero. It is convenient to define the mass scale Λ:

$$\Lambda^2=\sqrt3 B\ ,$$

where the factor $\sqrt3$ is introduced for calculation convenience, so that

$$\breve B_{\mu\rho}\breve B_{\rho\nu}=-\frac14K^2\Lambda^4\delta_{\mu\nu}\ ,$$
$$\hat B_{\mu\rho}\hat B_{\rho\nu}=-v^2\Lambda^4\delta_{\mu\nu}\ ,\quad v=\mathrm{diag}\left(\frac13,\frac13,\frac23\right)\ . \qquad (10)$$

It will be clear below that quantity Λ defines the scale of confinement.

B. Quark and gluon propagators

The Lagrangian looks like (in Feunman gauge $\xi=1$)

$$L_{\rm QCD}+L_{\rm gf}=-\frac{4}{g^2}B^2+\frac12A^a_\mu(\breve\nabla^2\delta_{\mu\nu}+4i\breve B_{\mu\nu})^{ab}A^b_\nu$$
$$+\bar q_f(i\gamma_\mu\hat\nabla_\mu-m_f)q_f+L_{\rm int}\ ,$$

where we denote $gB^a_\mu\equiv B^a_\mu$.

The quark and gluon propagators satisfy the equations

$$(i\gamma_\mu\hat\nabla_\mu-m_f)S_f(x,y\mid B)=-\delta(x-y)\ , \qquad (11)$$

$$(\breve\nabla^2\delta_{\mu\nu}+4i\breve B_{\mu\nu})G_{\nu\rho}(x,y\mid B)=-\delta_{\mu\rho}\delta(x-y)\ . \qquad (12)$$

Solutions of Eqs. (11) and (12) were considered in many papers (e.g., see [6,18,19]). The self-dual or anti-self-dual configuration, obeying condition (6), is remarkable in the sense that there are negative modes (negative eigenvalues of the differential operators) neither in Eq. (11) nor in Eq. (12). Zero modes are in Eq. (12) for the gluon propagator, but they are absent in Eq. (11) unless the quark masses are nonzero.

Equations (11) and (12) can be solved using the Schwinger's proper time technique. We present the results, omitting the well-known details. The quark propagator takes the form [13]

$$S_f(x,y\mid B)=e^{\frac i2(x\hat By)}H_f(x-y\mid B)e^{\frac i2(x\hat By)}\ , \qquad (13)$$

$$\tilde H_f(p\mid B)=\frac{1}{v\Lambda}\int_0^1 dt e^{-\frac{p^2}{2v\Lambda^2}t}\left(\frac{1-t}{1+t}\right)^{\frac{\alpha_f^2}{4v}}\left[\alpha_f+\frac1\Lambda p_\mu\gamma_\mu-it\frac1\Lambda(\gamma fp)\right]\left[P_\pm+P_\mp\frac{1+t^2}{1-t^2}-\frac i2(\gamma f\gamma)\frac{t}{1-t^2}\right]\ , \qquad (14)$$

where [see also Eq. (6)]

$$P_{\pm} = \frac{1}{2}(1 \pm \gamma_5) \ , \quad \alpha_f = \frac{m_f}{\Lambda} \ , \quad (xBy) = x_\mu B_{\mu\nu} y_\nu \ ,$$
$$(pf\gamma) = p_\mu f_{\mu\nu}\gamma_\nu \ , \quad f_{\mu\nu} = \frac{t^8}{v\Lambda^2} B_{\mu\nu} \ , \quad f_{\mu\rho} f_{\rho\nu} = -\delta_{\mu\nu} \ .$$

The function $\tilde{H}_f$ is the Fourier-transformed H_f; the diagonal matrix v is defined in Eq. (10). The upper (lower) sign in the matrix P corresponds to the self-dual (anti-self-dual) field.

Let us introduce the variable $\mu = p_\mu \gamma_\mu$. The function $\tilde{H}_f(\mu \mid B)$ is an entire analytical function in the complex μ plane. This means that there are no poles corresponding to the free quarks. In other words, so-called analytical quark confinement is manifest. The following asymptotical behavior takes place:

$$\tilde{H}_f(\mu \mid B) \to \begin{cases} \frac{m_f+\mu}{-\mu^2} = \frac{m_f+\gamma_\nu p_\nu}{p^2} & \text{if } \mu \to \pm\infty \ , \text{ i.e., } p^2 \to \infty \ , \\ O(\exp(\frac{\mu^2}{2v\Lambda^2})) = O(\exp(\frac{-p^2}{2v\Lambda^2})) & \text{if } \mu \to \pm i\infty, \text{ i.e., } p^2 \to -\infty. \end{cases} \tag{15}$$

It should be remembered that the momentum p is considered in these formulas in Euclidean metrics. The behavior of the function $\tilde{H}_f(p \mid B)$ is defined by the scale Λ (tension B of the gluon vacuum field). In other words, the parameter Λ defines a characteristic region of the variation of this function; i.e., it defines the scale of confinement.

The solution of Eq. (12) for the gluon propagator can be represented in the form

$$G_{\mu\nu}(x,y \mid B) = e^{\frac{i}{2}(x\check{B}y)} D_{\mu\nu}(x-y \mid B) e^{\frac{i}{2}(x\check{B}y)} \ , \tag{16}$$
$$D^{ab}_{\mu\nu}(z \mid B) = \delta_{\mu\nu}\{(1-K^2)D(z \mid 0) + K^2[D(z \mid \Lambda^2) + D_0(z \mid \Lambda^2)]\}]^{ab} + 2i\,[f_{\mu\nu}K]^{ab} D_1(z \mid \Lambda^2) \ , \tag{17}$$

where

$$D(z \mid \Lambda^2) = \frac{\Lambda^2}{4(4\pi)^2} \int_0^\infty \frac{ds}{\sinh^2(s)} \exp\Big\{-\frac{\Lambda^2 z^2}{4}\coth(s)\Big\} = \frac{1}{(2\pi)^2 z^2}\exp\Big\{-\frac{\Lambda^2 z^2}{4}\Big\}, \tag{18}$$
$$D_0(z \mid \Lambda^2) = \frac{\Lambda^2}{4(4\pi)^2} \operatorname{reg} \int_0^\infty ds \exp\Big\{-\frac{\Lambda^2 z^2}{4}\coth(s)\Big\}. \tag{19}$$

The term with the function D_1 is not important for our further consideration.

The Fourier transform of the function $D(z \mid \Lambda^2)$ is an entire analytical function in momentum space. This function describes a propagation of the confined modes of the gluon fields.

One can see that not all degrees of freedom of the gluon field are confined. The term $D(z|0)$, arising due to zero eigenvalues of the matrix K [see Eq. (9)], corresponds to free massless gluons. This is not an artifact of our choice of the vector n but it is a general feature of the background field under consideration. The divergent integral D_0 in Eq. (19) corresponds to the contribution of zero modes. An appropriate regularization of D_0 should be introduced.

To finish this section we will note that an homogeneous vacuum field modifies the quark and gluon propagators in the infrared region. At the same time, the UV behavior of the propagators has the same character as in the case of a zero background field [see Eqs. (15), (18)]. This modification has important consequences. The first one is the analytical confinement of quarks and the existence of confined modes of the gluon fields. Another remarkable consequence is concerned with colorless collective modes.

IV. COLLECTIVE MODES INDUCED BY THE HOMOGENEOUS BACKGROUND FIELD

A. Color-singlet bilocal quark currents

Let us return to representation (5) and consider the second term in the exponential. According to Eq. (16) it can be rewritten in the form

$$L_2 = \frac{g^2}{2}\iint d^4x d^4y \{\, \bar{q}_f(x)\gamma_\mu t^a [e^{\frac{i}{2}(x\check{B}y)}]^{aa'} q_f(x)\} D^{a'b'}_{\mu\nu}(x-y \mid B)\{\bar{q}_{f'}(y)\gamma_\nu [e^{\frac{i}{2}(x\check{B}y)}]^{b'b} t^b q_{f'}(y)\} \ . \tag{20}$$

Using the identity

$$t^a_{kj}[e^{-i\check{\omega}}]^{aa'} = [e^{-i\hat{\omega}} t^{a'} e^{i\hat{\omega}}]_{kj} \quad (\check{\omega} = \omega^a C^a \ , \ \hat{\omega} = \omega^a t^a \ ) \ ,$$

one can get

$$L_2 = \frac{g^2}{2}\iint d^4x d^4y \{\, \bar{Q}_f(x,y)\gamma_\mu t^a Q_f(x,y)\} D^{ab}_{\mu\nu}(x-y \mid B)\{\bar{Q}_{f'}(y,x)\gamma_\nu t^b Q_{f'}(y,x)\} \ ,$$

where $Q_f(x,y) = e^{-\frac{i}{2}(x\hat{B}y)}q_f(x)$, $\bar{Q}_f(x,y) = \bar{q}_f(x)e^{\frac{i}{2}(x\hat{B}y)}$.

Then we make the Fierz transformation of the color, flavor (we consider $N_f = 3$) and Dirac matrices. Keeping only the scalar J^{aS}, pseudoscalar J^{aP}, vector J^{aV}, and axial-vector J^{aA} colorless currents, we obtain the expression

$$L_2 = \frac{g^2}{2}\sum_{a=0}^{8}\frac{1}{4\times 18}\iint d^4x d^4y D^{cb}_{\mu\nu}(x-y\mid B)\Big\{\delta^{cb}\delta_{\mu\nu}(J^{aS}(x,y)J^{aS}(y,x) + J^{aP}(x,y)J^{aP}(y,x)) \\ -\delta^{cb}\Big(\delta_{\mu\sigma}\delta_{\nu\rho} + \delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\nu}\delta_{\rho\sigma}\Big)\Big[J^{aV}_\sigma(x,y)J^{aV}_\rho(y,x) + J^{aA}_\sigma(x,y)J^{aA}_\rho(y,x)\Big]\Big\}. \tag{21}$$

Before we deal with the currents, let us use the relations following from the structure of the gluon propagator (17),

$$\delta^{cb}\delta_{\mu\nu}D^{cb}_{\mu\nu}(z) = 16\big[D(z\mid 0) + D_0(z\mid\Lambda^2) + D(z\mid\Lambda^2)\big] \ ,$$

$$\delta^{cb}\Big[\delta_{\mu\sigma}\delta_{\nu\rho} + \delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\nu}\delta_{\rho\sigma}\Big]D^{cb}_{\mu\nu}(z) = -8\big[D(z\mid 0) + D_0(z\mid\Lambda^2) + D(z\mid\Lambda^2)\big] \ ,$$

to rewrite Eq. (21) in the form

$$L_2 = \frac{g^2}{2}\sum_{aJ}C_J\iint d^4x d^4y J^{aJ}(x,y)J^{aJ}(y,x)\big[D(x-y\mid 0) + D_0(x-y\mid\Lambda^2) + D(x-y\mid\Lambda^2)\big] \ , \tag{22}$$

where

$$J^{aJ}(x,y) = \bar{q}_f(x)M^a_{ff'}\Gamma^J e^{i(x\hat{B}y)}q_{f'}(y) \ ,$$
$$\Gamma^S = \mathbf{1} \ , \quad \Gamma^P = i\gamma_5 \ , \quad \Gamma^V = \gamma_\mu \ , \quad \Gamma^A = \gamma_5\gamma_\mu \ ,$$
$$C_S = C_P = \frac{1}{9} \ , \quad C_V = C_A = \frac{1}{18} \ , \tag{23}$$

M^a-flavor mixing matrices ($a = 0, \ldots, 8$) being equal to the matrices λ^a or their linear combinations.

The currents (23) are the scalars under color group transformations (2) and (3). This becomes obvious if one notices that

$$\exp\{i(x\hat{B}y)\} = \exp\Big\{-i\int_x^y dz_\mu\hat{B}_\mu(z)\Big\} \ ,$$

where the path z_μ is parametrized, for example, as

$$z_\mu(t) = (1-t)x_\mu + ty_\mu \ , \quad 0\le t\le 1 \ .$$

Path ordering is not necessary. One has, under the background transformation (3),

$$\exp\{i(x\hat{B}y)\} \to e^{-i\hat{\omega}(x)}\exp\{i(x\hat{B}y)\}e^{i\hat{\omega}(y)} \ .$$

This ensures the invariance of currents (23) under color transformations (2), (3).

B. Nonlocal quark currents with radial and orbital quantum numbers

Let us change variables in the integrals in Eq. (21):

$$x \to x + \tfrac{1}{2}y \ , \quad y \to x - \tfrac{1}{2}y.$$

After this substitution the currents (23) can be represented in the form

$$J^{aJ}(x,y) \to J^{aJ}\left(x + \frac{1}{2}y, x - \frac{1}{2}y\right) = \bar{q}(x)M^a\Gamma^J e^{\frac{1}{2}y\overleftrightarrow{\nabla}(x)}q(x) \ ,$$
$$J^{aJ}(y,x) \to J^{aJ}\left(x - \frac{1}{2}y, x + \frac{1}{2}y\right) = \bar{q}(x)M^a\Gamma^J e^{-\frac{1}{2}y\overleftrightarrow{\nabla}(x)}q(x) \ ,$$

where

$$\overleftrightarrow{\nabla}_\mu(x) = \overleftarrow{\nabla}_\mu(x) - \overrightarrow{\nabla}_\mu(x) \ , \quad \overrightarrow{\nabla}_\mu(x) = \overrightarrow{\partial}_\mu - i\hat{B}_\mu(x) \ ,$$
$$\overleftarrow{\nabla}_\mu(x) = \overleftarrow{\partial}_\mu + i\hat{B}_\mu(x) \ .$$

The operators $\overrightarrow{\nabla}_\mu$ and $\overleftarrow{\nabla}_\mu$ are the right and left covariant derivatives. The representation (22) takes the form

$$L_2 = g^2\sum_{aJ}C_J\iint d^4x d^4y\big[D(y\mid 0) + D_0(y\mid\Lambda^2) + D(y\mid\Lambda^2)\big] \\ \times\Big\{\bar{q}(x)M^a\Gamma^J e^{\frac{1}{2}y\overleftrightarrow{\nabla}(x)}q(x)\Big\}\Big\{\bar{q}(x)M^a\Gamma^J e^{-\frac{1}{2}y\overleftrightarrow{\nabla}(x)}q(x)\Big\}. \tag{24}$$

The next step consists in integrating out the variable y in Eq. (24). It can be done by decomposition of the bilocal currents over some appropriate orthogonal set of polynomials. From a quantum mechanical viewpoint, this orthogonal set provides a description of the radial and orbital excitations. The decomposition of the bilocal currents has the form

$$J^{aJ}(x,y) = \sum_{n\ell}(y^2)^{\ell/2} f^{n\ell}_{\mu_1\cdots\mu_\ell}(y) J^{aJ\ell n}_{\mu_1\cdots\mu_\ell}(x) ,$$

$$f^{\ell n}_{\mu_1\cdots\mu_\ell}(y) = L_{n\ell}(y^2) T^{(\ell)}_{\mu_1\cdots\mu_\ell}(n_y) \quad (n_y = y/\sqrt{y^2}) . \tag{25}$$

The irreducible tensors of the four-dimensional rotational group $T^{(\ell)}_{\mu_1\cdots\mu_\ell}$ are orthogonal,

$$\int_\Omega \frac{d\omega}{2\pi^2} T^{(\ell)}_{\mu_1\cdots\mu_\ell}(n_y) T^{(k)}_{\nu_1\cdots\nu_k}(n_y) = \frac{1}{2^\ell(\ell+1)} \delta^{\ell k} \delta_{\mu_1\nu_1} \cdots \delta_{\mu_\ell\nu_\ell} , \tag{26}$$

and they are subjected to the conditions

$$T^{(\ell)}_{\mu_1\cdots\mu\cdots\nu\cdots\mu_\ell}(n_y) = T^{(\ell)}_{\mu_1\cdots\nu\cdots\mu\cdots\mu_\ell}(n_y) , \quad T^{(\ell)}_{\mu\cdots\mu\cdots\mu_\ell}(n_y) = 0,$$
$$T^{(\ell)}_{\mu_1\cdots\mu_\ell}(n_y) T^{(\ell)}_{\mu_1\cdots\mu_\ell}(n_{y'}) = \frac{1}{2^\ell} C^{(1)}_\ell (n_y n_{y'}) . \tag{27}$$

The measure $d\omega$ in Eq. (26) relates to integration over angles of the unit vector n_y and $C^{(1)}_\ell$ in Eq. (27) are Gegenbauer's (ultraspherical) polynomials. The polynomials $L_{n\ell}(u)$ obey the condition

$$\int_0^\infty du \rho_\ell(u) L_{n\ell}(u) L_{n'\ell}(u) = \delta_{nn'} .$$

The weight function $\rho_\ell(u)$ has to be determined by the terms in Eq. (24), arising from the gluon propagator. The function $D(y|\Lambda^2)$ leads to the weight function

$$\rho_\ell(u) = u^\ell e^{-u} ,$$

corresponding to the generalized Laguerre's polynomials $L_{n\ell}(u)$. Other terms do not provide any possibility for decomposition such as (25). In other words, such a situation suggests that only confined gluons [contributing to $D(y|\Lambda^2)$] provide the generation of bound states. At the same time, it is the function $D(y|\Lambda^2)$ that accumulates the non-Abelian nature of gluons—their self-interaction. This is the point where QCD differs from quantum electrodynamics essentially. As a result we have a mechanism of the generation of bound states different from the potential picture.

This is the reason why we will consider the part of Eq. (24) concerning the confined gluon modes; i.e., we will deal with the function $D(y|\Lambda^2)$, given in Eq. (18), and omit the terms with $D(z|0)$ and D_0. In other words, only the function $D(y|\Lambda^2)$ is responsible for $(q\bar{q})$ bound state formation.

The details of calculation of the currents $J^{aJ\ell n}_{\mu_1\cdots\mu_\ell}(x)$ in Eq. (25) can be found in Appendix A. As a result we obtain the following representation for L_2:

$$L_2 = \sum_{aJ\ell n} \frac{1}{2\Lambda^2} G^2_{J\ell n} \int d^4x \left[J^{aJ\ell n}_{\mu_1\cdots\mu_\ell}(x)\right]^2 , \tag{28}$$

where

$$G^2_{J\ell n} = C_J \, g^2 \, \frac{(\ell+1)}{2^\ell n! (\ell+n)!} , \tag{29}$$

$$J^{aJ\ell n}_{\mu_1\cdots\mu_\ell}(x) = \bar{q}(x) V^{aJ\ell n}_{\mu_1\cdots\mu_\ell}(x) q(x) , \tag{30}$$

$$V^{aJ\ell n}_{\mu_1\cdots\mu_\ell}(x) \equiv V^{aJ\ell n}_{\mu_1\cdots\mu_\ell}\left(\frac{\overleftrightarrow{\nabla}(x)}{\Lambda}\right) = M^a \Gamma^J \left\{\!\!\left\{ F_{n\ell}\left(\frac{\overleftrightarrow{\nabla}^2(x)}{\Lambda^2}\right) T^{(\ell)}_{\mu_1\cdots\mu_\ell}\left(\frac{1}{i}\frac{\overleftrightarrow{\nabla}(x)}{\Lambda}\right)\right\}\!\!\right\} , \tag{31}$$

$$F_{n\ell}(s) = \left(\frac{s}{4}\right)^n \int_0^1 dt t^{\ell+n} \exp\left\{\frac{1}{4}st\right\} . \tag{32}$$

The double brackets in Eq. (31) mean that the covariant derivatives commute inside these brackets. The functions $F_{n\ell}(s)$ are entire analytical functions in the complex s plane; they obey the confinement condition. The representation (28)–(31) is one of the main results of this paper.

The vertex functions (31) have the following asymptotic behavior for large Euclidean momentum:

$$\lim_{p^2\to\infty} F_{n\ell}[p^2] T^{(\ell)}_{\mu_1\cdots\mu_\ell}[p] \sim \frac{1}{(p^2)^{1+\ell/2}} . \tag{33}$$

Therefore, there is a single divergent diagram given in Fig. 1. This divergency can be removed by the counter-term $-2J(x)\mathrm{Tr}VS$ inserted into the interaction Lagrangian (28). After this renormalization we arrive at a truncated approximate QCD functional:

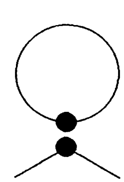

FIG. 1. The divergent bubble diagram.

$$Z = \int d\sigma_{\rm vac} Dq D\bar{q} \exp\left\{ \iint d^4x d^4y \bar{q}(x) S^{-1}(x,y|B) q(y) + \sum_{aJ\ell n} \frac{1}{2\Lambda^2} G^2_{J\ell n} \int d^4x \left[J^{aJ\ell n}_{\mu_1\cdots\mu_\ell}(x) - {\rm Tr} V^{aJ\ell n}_{\mu_1\cdots\mu_\ell} S \right]^2 \right\} . \tag{34}$$

The measure $d\sigma_{\rm vac}$ is given by Eq. (8).

It should be remembered that the approximation consists in the use of the gluon propagator in the form (16) instead of the exact one. The truncation means that Eq. (34) corresponds to the sector of QCD determined by the two circumstances. First, we consider only a term L_2 in Eq. (4). Second, we take into account only confined degrees of freedom of the gluon fields associated with the function $D(y|\Lambda^2)$ in the gluon propagator.

Representation (34),(29)–(31) manifests the following main properties. The quarks are confined due to the background field. The nonlocal quark currents (30) are invariant under transformations (2),(3); i.e., they are color singlets. These currents have a suitable set of quantum numbers including the radial n and orbital ℓ ones. The effective theory determined by Eqs. (34) and (29)–(31) is UV finite due to the nonlocality of the vertex (31). This suggests that all counterterms contained in the initial representation (1) should be related to the part of the QCD dynamics which was omitted within the above-mentioned truncation. To simplify our notation we will not introduce a special notation for the renormalized quantities. Below the quark fields, their masses m_f and the gauge coupling constant g [see Eq. (29)] are considered as renormalized ones. These renormalized quark masses m_f, constant g, and the scale Λ are the parameters of the effective theory, defined in Eqs. (32)–(34).

The tensors $T^{(\ell)}_{\mu_1\cdots\mu_\ell}$ contained in the currents $J^{aJ\ell n}_{\mu_1\cdots\mu_\ell}$ realize the irreducible representations of the group $O(4)$ of rotations in four-dimensional Euclidean space. This point leads to the question, is there any possibility to interpret the number ℓ as the Minkowski space $O(3)$ angular momentum? An answer can be formulated in terms of collective mesonlike variables.

C. Local mesonlike fields

Now we show how the Lagrangian describing bosonic fields arises. Starting with representation (34), one can introduce the local mesonlike fields by means of the standard bosonization procedure. The exponential with the term L_2 is represented in the form of the functional integral over Bose fields like

$$\begin{aligned} Z = N \int d\sigma_{\rm vac} Dq D\bar{q} \prod_{a\kappa} D\varphi^{a\kappa}_{\mu} \exp\Big\{ - \iint d^4x d^4y \bar{q}_f(x) S_f^{-1}(x,y \mid B) q_f(y) \\ + \int d^4x \Big[-\frac{\Lambda^2}{2} \varphi^{a\kappa}_\mu(x) \varphi^{a\kappa}_\mu(x) + G_\kappa \varphi^{a\kappa}_\mu(x) \left[J^{a\kappa}_\mu(x) - {\rm Tr} V^{a\kappa}_\mu S \right] \Big] \Big\} , \end{aligned} \tag{35}$$

where we introduced the condensed index

$$\kappa \equiv (J\ell n) , \quad \mu \equiv (\mu_1 \cdots \mu_\ell) .$$

After integration over the quark fields Eq. (35) takes the form

$$\begin{aligned} Z = N \int \prod_{a\kappa} D\varphi^{a\kappa}_\mu \exp\Big\{ - \int d\sigma_{\rm vac} \int d^4x \Big[\frac{\Lambda^2}{2} \varphi^{a\kappa}_\mu(x) \varphi^{a\kappa}_\mu(x) - G_\kappa \varphi^{a\kappa}_\mu(x) {\rm Tr} V^{a\kappa}_\mu S \Big] \\ + \int d\sigma_{\rm vac} {\rm Tr} \ln \Big\{ 1 - G_\kappa \varphi^{a\kappa}_\mu(x) \left[V^{a\kappa}_\mu(x) S(x,y \mid B) \right] \Big\} \Big\} . \end{aligned} \tag{36}$$

Below we restrict ourselves to the one-loop approximation. The integral over $d\sigma_{\rm vac}$ in Eq. (36) has been transferred to the exponential that is accurate at the one-loop level and suitable for our nearest purposes.

The fields $\varphi^{a\kappa}_\mu$ in Eq. (36) look like the auxiliary ones. We have to find conditions providing an interpretation of $\varphi^{a\kappa}_\mu$ as the physical fields. These conditions can be obtained by considering the part of the action quadratic in fields $\varphi^{a\kappa}_\mu$. Expanding the logarithm in Eq. (36) we get the quadratic part in the form

$$I_2 = -\frac{1}{2} \iint d^4x d^4y \varphi^{a\kappa}_\mu(x) \Big[\Lambda^2 \delta_{\mu,\mu'} \delta(x-y) + G^2_\kappa \Pi^{a\kappa,a\kappa}_{\mu,\mu'}(x-y) \Big] \varphi^{a\kappa}_{\mu'}(y), \tag{37}$$

where

$$\delta_{\mu,\mu'} \equiv \delta_{\mu_1\mu'_1}\cdots\delta_{\mu_\ell\mu'_\ell} ,$$
$$\Pi^{a\kappa,a'\kappa'}_{\mu,\mu'}(x-y) = \int d\sigma_{\rm vac}{\rm Tr}\Big\{V^{a\kappa}_{\mu}(x)S(x,y\mid B)V^{a'\kappa'}_{\mu'}(y)S(y,x\mid B)\Big\} . \tag{38}$$

Because of the asymptotics (33), the polarization operator (38) is UV finite and can be computed. In the momentum representation it has the structure

$$\tilde\Pi^{a\kappa,a\kappa}_{\mu,\mu'}(p) = \delta_{\mu_1\mu'_1}\cdots\delta_{\mu_\ell\mu'_\ell}\tilde\Pi_{a\kappa}(p) + \tilde P^{a\kappa}_{\mu,\mu'}(p) , \tag{39}$$

where the nondiagonal part $\tilde P^{a\kappa}_{\mu,\mu'}$ can be represented as

$$\tilde P^{a\kappa}_{\mu,\mu'}(p) = p_{\mu_1}p_{\mu'_1}\delta_{\mu_2\mu'_2}\cdots\delta_{\mu_\ell\mu'_\ell}\tilde P^{(1)}_{a\kappa}(p^2) + \cdots + p_{\mu_1}p_{\mu'_1}\cdots p_{\mu_\ell}p_{\mu'_\ell}\tilde P^{(\ell)}_{a\kappa}(p^2) . \tag{40}$$

Let us write the Euler-Lagrange equations for the fields $\tilde\varphi^{a\kappa}_{\mu}(p)$, which minimize the quadratic (free) part of the action (37) (in the momentum representation):

$$\Big[\Lambda^2\delta_{\mu,\mu'} + G^2_\kappa\tilde\Pi^{a\kappa,a\kappa}_{\mu,\mu'}(p)\Big]\,\tilde\varphi^{a\kappa}_{\mu'}(p) = 0 . \tag{41}$$

The Euclidean momentum p in Eq. (41) is arbitrary. Since we have neglected the interaction between the fields, Eqs. (41) should be considered as the equations of motion for the free fields. This interpretation is self-consistent if there exist the values $p^2 = -m^2_{a\kappa}$ in the physical Minkowski region for which Eqs. (41) are reduced to the standard Klein-Gordon form. Therefore, we have to demand that the following relation must be realized for $p^2 \to -m^2_{a\kappa}$:

$$\Big[\Lambda^2\delta_{\mu,\mu'} + G^2_\kappa\tilde\Pi^{a\kappa,a\kappa}_{\mu,\mu'}(p)\Big]\,\tilde\varphi^{a\kappa}_{\mu'}(p) = 0 \quad\longrightarrow\quad \left[p^2+m^2_{a\kappa}\right]\tilde\varphi^{a\kappa}_{\mu}(p) = 0 , \tag{42}$$

where $m_{a\kappa}$ are some unknown masses of the fields $\varphi^{a\kappa}$. Taking into account Eqs. (39) and (40) one can represent Eqs. (41) in the limit $p^2 \to -m^2_{a\kappa}$ as

$$\Big\{\Big[\Lambda^2 + G^2_\kappa\tilde\Pi_{a\kappa}(-m^2_{a\kappa})\Big]\,\delta_{\mu,\mu'} + G^2_\kappa\left[\tilde P^{(1)}_{a\kappa}(-m^2_{a\kappa})p_{\mu_1}p_{\mu'_1}\delta_{\mu_2\mu'_2}\cdots\delta_{\mu_\ell\mu'_\ell} + \cdots + \tilde P^{(\ell)}_{a\kappa}(-m^2_{a\kappa})p_{\mu_1}\cdots p_{\mu'_\ell}\right]$$
$$+G^2_\kappa\tilde\Pi'_{a\kappa}(-m^2_{a\kappa})\left(p^2+m^2_{a\kappa}\right)\delta_{\mu,\mu'} + G^2_\kappa\tilde\Pi^R_{a\kappa}(p^2)\Big\}\tilde\varphi^{a\kappa}_{\mu'}(p) = 0 , \tag{43}$$

where

$$\tilde\Pi^R_{a\kappa}(p^2) \equiv \tilde\Pi_{a\kappa}(p^2) - \tilde\Pi_{a\kappa}(-m^2_{a\kappa}) - \tilde\Pi'_{a\kappa}(-m^2_{a\kappa})(p^2+m^2_{a\kappa}) = O((p^2+m^2_{a\kappa})^2),$$
$$\tilde\Pi'_{a\kappa}(-m^2_{a\kappa}) = \frac{d}{dp^2}\tilde\Pi_{a\kappa}(p^2)\mid_{p^2=-m^2_{a\kappa}} . \tag{44}$$

One can see that the requirement (42) leads to the conditions

$$\Lambda^2 + G^2_\kappa\tilde\Pi_{a\kappa}(-m^2_{a\kappa}) = 0 , \tag{45}$$
$$p_{\mu_j}\tilde\varphi^{a\kappa}_{\mu_1\cdots\mu_j\cdots\mu_\ell}(p)|_{p^2=-m^2_{a\kappa}} = 0 . \tag{46}$$

Equations (45) define the masses $m_{a\kappa}$ of the fields. The on-shell conditions (46) exclude nonphysical degrees of freedom of the fields and treat the quantum number ℓ as the Minkowski space O(3) angular momentum.

Changing the field variables like

$$\varphi^{a\kappa}_{\mu} \to \Phi^{a\kappa}_{\mu} = G_\kappa\sqrt{\tilde\Pi'_{a\kappa}(-m^2_{a\kappa})}\varphi^{a\kappa}_{\mu}$$

and taking into account condition (45) we get the following representation for the generating functional (36):

$$Z = N\int\prod_{a\kappa}D\Phi^{a\kappa}_{\mu}\exp\Big\{\frac{1}{2}\iint d^4xd^4y\Phi^{a\kappa}_{\mu}(x)\Big[(\Box - m^2_{a\kappa})\delta(x-y) - h^2_{a\kappa}\Pi^R_{a\kappa}(x-y)\Big]\Phi^{a\kappa}_{\mu}(y) + I_{\rm int}[\Phi]\Big\}, \tag{47}$$

$$I_{\rm int} = -\frac{1}{2}\iint d^4xd^4yh_{a\kappa}h_{a'\kappa'}\Phi^{a\kappa}_{\mu}(x)\Big[\Pi^{a\kappa,a'\kappa'}_{\mu,\mu'}(x-y) - \delta^{a\kappa,a'\kappa'}_{\mu\mu'}\Pi(x-y)\Big]\Phi^{a'\kappa'}_{\mu'}(y)$$
$$-\sum_{m=3}^{\infty}\frac{1}{m}\int d^4x_1\cdots\int d^4x_m\prod_{k=1}^{m}h_{a_k\kappa_k}\Phi^{a_k\kappa_k}_{\mu_k}(x_k)\Gamma^{a_1\kappa_1,\ldots,a_m\kappa_m}_{\mu_1,\ldots,\mu_m}(x_1,\ldots,x_m) , \tag{48}$$

$$\Gamma^{a_1\kappa_1,\ldots,a_m\kappa_m}_{\mu_1,\ldots,\mu_m}(x_1,\ldots,x_m) = \int d\sigma_{\rm vac}{\rm Tr}\Big\{V^{a_1\kappa_1}_{\mu_1}(x_1)S(x_1,x_2\mid B)\cdots V^{a_m\kappa_m}_{\mu_m}(x_m)S(x_m,x_1\mid B)\Big\} . \tag{49}$$

One can see that terms linear in fields $\Phi_\mu^{a\kappa}$ have not appeared in the action (48) due to counterterms in Eq. (34) related to the diagram in Fig. 1. The constants

$$h_{a\kappa} = 1/\sqrt{\tilde{\Pi}'_{a\kappa}(-m^2_{a\kappa})} \tag{50}$$

play the role of the effective coupling constants of the meson-quark interaction. The relation (50) between meson-quark coupling constants and polarization operator of the meson fields agrees with the compositeness condition in quantum field theory [20]. There is no explicit dependence on the coupling constant G_κ in the representation (47)–(49). This constant enters into the formalism only through Eq. (45) for the spectrum.

According to representation (47)–(49) the meson-meson interaction is described by the vertex functions Γ given in Eq. (49). They are UV finite and can be computed. The averaging over $d\sigma_{\rm vac}$ has to restore the parity and space rotation symmetries in the vertices Γ, i.e., at the hadron level.

Free parameters, defining effective meson theory (45)–(49), have a clear physical meaning. They are the quark masses m_f, the confinement scale Λ (strength of the background field), and the gauge coupling constant g. The last one appears only in Eq. (45) defining the meson masses.

The generating functional (47) satisfies all requirements of nonlocal quantum field theory [21]. Particularly, this functional leads to the unitary S matrix.

It should be noted that obtaining effective meson theory (47) from the QCD functional integral (4) proceeds through a series of drastic approximations: elimination of the terms L_n in Eq. (4) for $n > 2$, taking into account the confined part of the gluon propagator (16) instead of the exact one (unknown), one-loop approximation in Eq. (45) for the spectrum, and some others. We are compelled "to force a way through jungle." At the same time, the approximations reflect the usual problems accompanying attempts to find a link between QCD and low-energy effective quark models. We hope that further investigations will clarify the status of these approximations.

V. ASYMPTOTIC SPECTRUM

A. Approximation of the polarization operator

Here we consider the solutions of Eq. (45),

$$\Lambda^2 + G^2_{S\ell n}\tilde{\Pi}_{aS\ell n}(-m^2_{aS\ell n}) = 0 \ , \tag{51}$$

subject to the condition

$$m^2_{aS\ell n} \gg \Lambda^2 \ , \quad \text{if} \ \ n \gg \ell \ (\text{or} \ \ell \gg n) \ . \tag{52}$$

For the sake of simplicity we take $J = S$. The function $\tilde{\Pi}_{aS\ell n}$ is defined as

$$\text{diag}\tilde{\Pi}^{aS\ell n}_{\mu_1\cdots\mu_\ell,\nu_1\cdots\nu_\ell}(k) = \delta_{\mu_1\nu_1}\cdots\delta_{\mu_\ell\nu_\ell}\tilde{\Pi}_{aS\ell n}(k^2) \ . \tag{53}$$

Equation (53) is the diagonal part (in the momentum space) of the tensor:

$$\Pi^{aS\ell n}_{\mu_1\cdots\mu_\ell,\nu_1\cdots\nu_\ell}(x-y) = \int d\sigma_{\rm vac}\text{Tr}\Big\{V^{aS\ell n}_{\mu_1\cdots\mu_\ell}(x)S(x,y\mid B)V^{aS\ell n}_{\nu_1\cdots\nu_\ell}(y)S(y,x\mid B)\Big\} \ . \tag{54}$$

Therefore we have to determine this diagonal part.

It is convenient to do some approximations. The asymptotics like (52) is defined by the behavior of the quark propagator S_f and vertex $V^{aS\ell n}_{\mu_1\cdots\mu_\ell}$ in the region of large Minkowski momenta. Hence we may omit the phase factors [such as $\exp\{i(x\hat{B}y)\}$] and use the approximation [see Eqs. (13),(31)]

$$S(x,y\mid B) \sim H(x-y),$$

$$\tilde{H}(p) = \frac{1}{v\Lambda^2}\int_0^1 ds\,[\gamma_\mu p_\mu - is(\gamma f p)]\exp\left\{-\frac{p^2}{2v\Lambda^2}s\right\} \ ,$$

$$V^{aS\ell n}_{\mu_1\cdots\mu_\ell}\Big(\frac{\overleftrightarrow{\nabla}(x)}{\Lambda}\Big) \sim V^{aS\ell n}_{\mu_1\cdots\mu_\ell}\Big(\frac{\overleftrightarrow{\partial}(x)}{\Lambda}\Big)$$

$$= M^a F_{n\ell}\Big(\frac{\overleftrightarrow{\partial}^2(x)}{\Lambda^2}\Big)T^{(\ell)}_{\mu_1\cdots\mu_\ell}\Big(\frac{1}{i}\frac{\overleftrightarrow{\partial}(x)}{\Lambda}\Big) \ , \tag{55}$$

where $v = \text{diag}\,(1/3, 1/3, 2/3)$. It is convenient to use the representations

$$T^{(\ell)}_{\mu_1\cdots\mu_\ell}\Big(\frac{1}{i}\frac{\overleftrightarrow{\partial}}{\Lambda}\Big) = T^{(\ell)}_{\mu_1\cdots\mu_\ell}\Big(\frac{1}{i}\frac{\partial}{\partial\xi}\Big)\exp\Big\{\frac{\overleftrightarrow{\partial}}{\Lambda}\xi\Big\}_{\xi=0} \ ,$$

$$F_{n\ell}\Big(\frac{\overleftrightarrow{\partial}^2}{\Lambda^2}\Big) = \int_0^1 dt t^{\ell+n}\Big[\frac{d}{dt}\Big]^n\int d\sigma_p\,\exp\Big\{\sqrt{t}\frac{\overleftrightarrow{\partial}}{\Lambda}p\Big\} \ ,$$

$$d\sigma_p = \frac{d^4p}{\pi^2}\,\exp\Big\{-p^2\Big\} \ . \tag{56}$$

After some calculations (see Appendix B) we get the following asymptotic expression for $\tilde{\Pi}_{aS\ell n}$:

$$\tilde{\Pi}^{aS\ell n}(-m^2_{aS\ell n}) \sim -\Lambda^2\frac{(2n+\ell)!2^\ell}{5^{2n+\ell}}\exp\left\{\frac{3}{4}\mu^2_{aS\ell n}\right\} \ , \tag{57}$$

where $\mu_{aS\ell n} = m_{aS\ell n}/\Lambda$. Only the factorial and exponential over n and ℓ factors are written in Eq. (57), since only they determine the asymptotic behavior of the spectrum. Let us show this.

B. Spectrum

Taking into account Eqs. (57), (29), and (51), we obtain the equation

$$\frac{(2n+\ell)!}{n!(n+\ell)!5^{2n+\ell}}\exp\left\{\frac{3}{4}\mu^2_{aS\ell n}\right\}\sim 1\ . \tag{58}$$

Let us consider two limits.

$n\gg\ell$. Equation (58) takes the form

$$\exp\left\{\frac{3}{4}\mu^2_{aS\ell n}\right\}\sim\frac{(n!)^2}{(2n)!}5^{2n}\approx\left(\frac{5}{2}\right)^{2n}$$

or

$$\mu^2_{aS\ell n}=\frac{8}{3}\ln\left(\frac{5}{2}\right)n\ +O(\ln n)\ . \tag{59}$$

$\ell\gg n$. In this case we obtain, from Eq. (58),

$$\exp\left\{\frac{3}{4}\mu^2_{aS\ell n}\right\}\sim 5^\ell\ ,$$

$$\mu^2_{aS\ell n}=\frac{4}{3}\ln 5\ell\ +O(\ln\ell)\ . \tag{60}$$

One can see that Eqs. (59) and (60) manifest the equidistant character of the meson spectrum both for large n and ℓ. Comparing expression (60) with experimental data [22] for Regge trajectories one obtains the estimation $\Lambda\approx 700$ MeV.

For a conclusion let us estimate the asymptotic behavior of the meson-quark coupling constants defined by Eq. (50). Using this formula and Eq. (57) we see that for $n\gg 1$ or $\ell\gg 1$ (hence $\mu_{aS\ell n}\gg 1$) the meson-quark coupling constant decreases like

$$h^2_{aS\ell n}\sim\frac{5^{2n+\ell}}{(2n+\ell)!2^\ell}\exp\left\{-\frac{3}{4}\mu^2_{aS\ell n}\right\}\sim\frac{1}{2^\ell n!(n+\ell)!}\ .$$

This decrease of the coupling constant suggests that higher-order corrections should hardly change the asymptotic relations (59), (60).

VI. MASS OF HEAVY QUARKONIUM

Another asymptotic regime that can be easily investigated is the limit of heavy quarks. In this case one has to solve Eq. (45):

$$\Lambda^2+G^2_{J00}\tilde{\Pi}_{aJ00}(-M^2)=0\ , \tag{61}$$

supposing that both the quarkonium mass M and the quark mass m_q are much greater than the confinement scale Λ:

$$M\gg\Lambda\ ,\quad m_q\gg\Lambda\ .$$

We can use the same approximation (54) for the vertex functions, but it is necessary to take into account the factor $[(1-s)/(1+s)]^{m_q^2/4v\Lambda^2}$ [see (13)] in the approximation of the quark propagator, which is enough to keep leading terms throughout the calculation of the polarization operator $\tilde{\Pi}_{aJ00}(-M^2)$.

Evaluating the integral over the loop momentum and calculating the trace of the Dirac matrices we arrive at

$$\begin{aligned}\tilde{\Pi}_{aJ00}(-M^2)\approx&-\iint_0^1 dt_1dt_2\iint_0^1 ds_1ds_2\left[A(s_1,s_2,t_1,t_2)\frac{M^2}{\Lambda^2}+B(s_1,s_2,t_1,t_2)\frac{m_q^2}{\Lambda^2}\right]\\&\times\exp\left\{\frac{2s_1s_2+v(t_1+t_2)(s_1+s_2)}{2v(t_1+t_2)+s_1+s_2}\frac{M^2}{4v\Lambda^2}-\frac{m_q^2}{4v\Lambda^2}\ln\left(\frac{1+s_1}{1-s_1}\frac{1+s_2}{1-s_2}\right)\right\}\ .\end{aligned} \tag{62}$$

The particular form of the functions $A>0$ and $B>0$ is unimportant for the leading behavior of the integral at $M\gg\Lambda$ and $m_q\gg\Lambda$. These functions contribute to the next-to-leading terms only. The asymptotic behavior can be found by the Laplace method. The exponential in Eq. (62) is maximal for

$$s_1=s_2=s_{\max}=\sqrt{1-\frac{4m_q^2}{M^2}}\ ,$$

so that

$$\begin{aligned}\tilde{\Pi}_{aJ00}(-M^2)\approx&-\exp\left\{\frac{M^2}{4v\Lambda^2}s_{\max}-\frac{m_q^2}{2v\Lambda^2}\ln\left(\frac{1+s_{\max}}{1-s_{\max}}\right)\right\}\\&\times\iint_0^1 dt_1dt_2\iint_0^1 ds_1ds_2\left[A(s_1,s_2,t_1,t_2)\frac{M^2}{\Lambda^2}+B(s_1,s_2,t_1,t_2)\frac{m_q^2}{\Lambda^2}\right]\\&\times\exp\left\{-\frac{1}{2}F_{ij}(s_i-s_{\max})(s_j-s_{\max})\right\}\ ,\end{aligned} \tag{63}$$

where F_{ij} is a positive definite matrix. One can notice that $s_{\max} \to 0$ in the limit $M^2 \to 4m_q^2$. Therefore, in the heavy quark limit Eq. (61) with $\tilde{\Pi}_{aJ00}(-M^2)$ given by Eq. (63) has the asymptotic solution

$$M^2 = 4m_q^2 \left[1 + O\left(\frac{\Lambda^2}{m_q^2} \ln \frac{m_q^2}{\Lambda^2}\right)\right]^{\rho} \quad \text{for} \quad m_q \gg \Lambda \; . \tag{64}$$

The number ρ in Eq. (64) depends on the particular form of the functions A and B and defines the next-to-leading term, while the leading behavior $4m_q^2$ is determined by the exponential before the integral in Eq. (63).

Thus, we conclude that nonlocality of the quark and gluon propagators arising from the background field under consideration leads to a relation between the mass M of quarkonium and constituent quark mass m_q:

$$M \to 2m_q \quad \text{for} \quad m_q \gg \Lambda \; ,$$

which agrees with accepted notions about heavy quarkonia.

VII. SUMMARY

In conclusion we will summarize the main features of the model considered in the paper.

Quark confinement: usual local UV behavior of the quark propagator in the Euclidean region.

Superrenormalizable effective quark theory [Eqs. (29)–(34)] with the nonlocal colorless quark currents having a complete set of quantum numbers (including the orbital and radial ones).

UV-finite effective meson theory given by the generating functional (47).

The natural parametrization by the quark "masses" m_f, tension of the background field B (Λ), and gauge coupling constant g.

Asymptotically equidistant spectrum of the radial and orbital excitations.

In the heavy quark limit the mass of quarkonium tends to be equal to sum of the masses of constituent quarks [see Eq. (64)].

Further investigation should be undertaken to try to describe the basic points of the low-energy meson phenomenology. Our preliminary calculations of the pseudoscalar and vector meson masses as well as the masses of the excited states of π, ρ, and K^* mesons for real values of ℓ and n show satisfactory agreement with experimental data (will be published elsewhere).

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APPENDIX A

Here we calculate the currents $J^{aJ\ell n}_{\mu_1\cdots\mu_\ell}(x)$ in Eq. (25).

To be accurate with the noncommutative covariant derivatives $\overleftrightarrow{\nabla}$ in the currents, it is convenient to use the completeness condition

$$\delta^4(y-y') = \frac{1}{\pi^2\sqrt{y^2y'^2}} \sum_{n,\ell=0}^{\infty} 2^\ell(\ell+1)\sqrt{\rho_\ell(y^2)\rho_\ell(y'^2)}L_{n\ell}(y^2)L_{n\ell}(y'^2)T^{(\ell)}_{\mu_1\cdots\mu_\ell}(n_y)T^{(\ell)}_{\mu_1\cdots\mu_\ell}(n_{y'}) \; , \tag{A1}$$

where the tensors $T^{(\ell)}_{\mu_1\cdots\mu_\ell}(n_y)$ are subject to conditions (27). The polynomials $L_{n\ell}(u)$ obey the condition

$$\int_0^\infty du \rho_\ell(u) L_{n\ell}(u) L_{n'\ell}(u) = \delta_{nn'} \; , \quad \rho_\ell(u) = u^\ell \rho(u) = u^\ell e^{-u} \; .$$

After the rescaling $y \equiv y/2\Lambda$ and insertion of the δ function in Eq. (24) one gets

$$L_2 = g^2 \sum_{aJ} \frac{C_J}{\pi^2\Lambda^2} \int d^4x \iint d^4y d^4y' \frac{\sqrt{\rho(y^2)\rho(y'^2)}}{\sqrt{y^2y'^2}} \delta^4(y-y') \left\{\bar{q}(x)M^a\Gamma^J e^{iy\overleftrightarrow{P}(x)}q(x)\right\}\left\{\bar{q}(x)M^a\Gamma^J e^{-iy'\overleftrightarrow{P}(x)}q(x)\right\} \; , \tag{A2}$$

where $\overleftrightarrow{P}_\mu = \overleftrightarrow{\nabla}_\mu / i\Lambda$. Using the completeness condition (A1) we represent Eq. (A2) as

$$L_2 = g^2 \sum_{aJ\ell n} (-1)^\ell 2^\ell (\ell+1) \frac{C_J}{\Lambda^2} \int d^4x \left[J^{aJ\ell n}_{\mu_1\cdots\mu_\ell}(x)\right]^2 \; , \tag{A3}$$

where

$$J^{aJ\ell n}_{\mu_1\cdots\mu_\ell}(x) = \bar{q}(x)M^a\Gamma^J V^{\ell n}_{\mu_1\cdots\mu_\ell}(x)q(x) \; , \tag{A4}$$

$$V^{\ell n}_{\mu_1\cdots\mu_\ell}(x)=\int_0^\infty dy^2\int_\Omega \frac{d\omega}{2\pi^2}\rho(y^2)L_{n\ell}(y^2)T^{(\ell)}_{\mu_1\cdots\mu_\ell}(y)e^{iy\overleftrightarrow{P}(x)}\ , \tag{A5}$$

the measure $d\omega$ corresponds to integration over the angles. Representation (A3)–(A5) can be reduced to a more convenient and clear form. To make this, let us consider how the vertex operator (A5) acts. Using Eq. (A5) we can easily get the result

$$f_1(x)V^{\ell n}_{\mu_1\cdots\mu_\ell}(x)f_2(x)=\iint\frac{d^4p_1d^4p_2}{(2\pi)^8}e^{ix(p_1+p_2)}\tilde f_1(p_1)\tilde f_2(p_2)I^{\ell n}_{\mu_1\cdots\mu_\ell}(p_1,p_2)\ ,$$

$$I^{\ell n}_{\mu_1\cdots\mu_\ell}(p_1,p_2)=\int_0^\infty dy^2\int_\Omega\frac{d\omega}{2\pi^2}\rho(y^2)L_{n\ell}(y^2)T^{(\ell)}_{\mu_1\cdots\mu_\ell}(y)e^{iyp}\ ,$$

$$p_\mu=\left[2\hat B_{\mu\nu}x_\nu+(p_1-p_2)_\mu\right]/\Lambda\ ,$$

where $\tilde f_1$ and $\tilde f_2$ are the Fourier transforms of the functions f_1 and f_2. Now the quantity p is a c number and we can use the relations

$$e^{iyp}=\sum_{k=0}^\infty i^k2^k(k+1)\frac{J_{k+1}(\sqrt{y^2p^2})}{\sqrt{y^2p^2}}T^{(k)}_{\mu_1\cdots\mu_k}(n_y)T^{(k)}_{\mu_1\cdots\mu_k}(n_p)\ ,$$

$$\frac{J_{k+1}(\sqrt{y^2p^2})}{(\sqrt{y^2p^2})^{k+1}}=\sum_{m=0}^\infty R_{mk}(p^2)L_{mk}(y^2)\ ,$$

$$R_{mk}(p^2)=\frac{1}{2^k\sqrt{m!(m+k)!}}\left(\frac{p^2}{4}\right)^m\int_0^1 dt t^{k+m}\exp\left\{-\frac{p^2}{4}t\right\}\ ,$$

$$\int_\Omega\frac{d\omega}{2\pi^2}T^{(\ell)}_{\mu_1\cdots\mu_\ell}(n_y)T^{(k)}_{\nu_1\cdots\nu_k}(n_y)=\frac{1}{2^\ell(\ell+1)}\delta^{\ell k}\delta_{\mu_1\nu_1}\cdots\delta_{\mu_\ell\nu_\ell}\ , \tag{A6}$$

to get the expression

$$I^{\ell n}_{\mu_1\cdots\mu_\ell}(p_1,p_2)=i^\ell R_{n\ell}(p^2)T^{(\ell)}_{\mu_1\cdots\mu_\ell}(p)\ .$$

At last, taking into account the equality

$$\exp\{ip_1x\}\left[\frac{\overleftrightarrow{\nabla}_\mu(x)}{i\Lambda}\right]\exp\{ip_2x\}=p_\mu\exp\{ix(p_1+p_2)\}\ ,$$

we conclude that Eq. (A3) can be represented in the form of Eq. (28).

APPENDIX B

Here we calculate the asymptotics of the polarization operator, keeping only the factorial and exponential over n and ℓ terms. Using formulas (54), (55), we obtain the expression

$$\begin{aligned}\Pi^{aS\ell n}_{\mu_1\cdots\mu_\ell,\nu_1\cdots\nu_\ell}(x-y)\sim\ &\mathrm{Tr}\int_0^1dt_1\int_0^1dt_2(t_1t_2)^{\ell+n}\iint d\sigma_pd\sigma_q\left(\frac{d}{dt_1}\frac{d}{dt_2}\right)^n\\ &\times T^{(\ell)}_{\mu_1\cdots\mu_\ell}\left(\frac{\partial}{i\partial\xi}\right)T^{(\ell)}_{\nu_1\cdots\nu_\ell}\left(\frac{\partial}{i\partial\eta}\right)H\left[x-y-(\xi+\eta+\sqrt{t_1}p+\sqrt{t_2}q)/\Lambda\right]\\ &\times H\left[y-x-(\xi+\eta+\sqrt{t_1}p+\sqrt{t_2}q)/\Lambda\right]\ |_{\xi=\eta=0}\ .\end{aligned}$$

The sign Tr means the trace of the Dirac matrices and summation over all elements of the diagonal matrix v [see Eq. (10)].

Taking into account Eqs. (54), we integrate over the variables p and q, then perform the Fourier transformation, and thus we get in the momentum representation

$$\begin{aligned}\tilde\Pi^{aS\ell n}_{\mu_1\cdots\mu_\ell,\nu_1\cdots\nu_\ell}(k)\sim\ &\Lambda^2\mathrm{Tr}\int_0^1dt_1\int_0^1dt_2(t_1t_2)^{\ell+n}\int\frac{d^4q}{(2\pi)^4}\times T^\ell_{\mu_1\cdots\mu_\ell}\left(\frac{\partial}{i\partial\xi}\right)T^\ell_{\nu_1\cdots\nu_\ell}\left(\frac{\partial}{i\partial\eta}\right)\left[\frac{d}{dt_1}\frac{d}{dt_2}\right]^n\\ &\times\tilde H\left(q+\frac{k}{2}\right)\tilde H\left(q-\frac{k}{2}\right)\exp\left\{-(t_1+t_2)q^2+2iq(\xi+\eta)\right\}\ |_{\xi=\eta=0}\ .\end{aligned}$$

All variables in the integrand were made dimensionless by rescaling. Differentiation with respect to ξ, η, t_1 , and t_2

leads to the expression

$$\tilde{\Pi}^{aS\ell n}_{\mu_1\cdots\mu_\ell,\nu_1\cdots\nu_\ell}(k) \sim \Lambda^2 \mathrm{Tr}\int_0^1 dt_1 \int_0^1 dt_2 \big(t_1 t_2\big)^{\ell+n} \int \frac{d^4q}{(2\pi)^4} T^{(\ell)}_{\mu_1\cdots\mu_\ell}(2q) T^{(\ell)}_{\nu_1\cdots\nu_\ell}(2q)(q^2)^{2n} \times \tilde{H}\left(q+\frac{k}{2}\right)\tilde{H}\left(q-\frac{k}{2}\right)\exp\left\{-(t_1+t_2)q^2\right\} \;.$$

Using the expression for $\tilde{H}$ [Eqs. (55)] we obtain

$$\tilde{\Pi}^{aS\ell n}_{\mu_1\cdots\mu_\ell,\nu_1\cdots\nu_\ell}(k) \sim -\Lambda^2 \mathrm{Tr}\int_0^1 dt_1 \int_0^1 dt_2 \big(t_1 t_2\big)^{\ell+n} \int_0^1 ds_1 \int_0^1 ds_2 (1-s_1 s_2) \int \frac{d^4q}{(2\pi)^4}(q^2-k^2) T^{(\ell)}_{\mu_1\cdots\mu_\ell}(2q) T^{(\ell)}_{\nu_1\cdots\nu_\ell}(2q) \times (q^2)^{2n} \exp\left\{-\frac{s_1}{2v}\left(q+\frac{k}{2}\right)^2 - \frac{s_2}{2v}\left(q-\frac{k}{2}\right)^2 - (t_1+t_2)q^2\right\} \;.$$

One can notice that the factor before k^2 in the exponential is maximal for $s_1 = s_2 = 1$. Using this fact, one gets the leading term for $k^2 \to -\infty$ in the form

$$\tilde{\Pi}^{aS\ell n}_{\mu_1\cdots\mu_\ell,\nu_1\cdots\nu_\ell}(k) \sim -\Lambda^2\ \mathrm{Tr}\ \frac{4^\ell v^{-2}}{(2+v^{-1})^{2n+\ell}} \exp\left\{-\frac{k^2}{4v}\right\} \int \frac{d^4q}{(2\pi)^4}\left(q^2\right)^{2n} T^{(\ell)}_{\mu_1\cdots\mu_\ell}(q) T^{(\ell)}_{\nu_1\cdots\nu_\ell}(q) \exp\{-q^2\} \;.$$

Using Eq. (26), we integrate over the angles and then evaluate the integral over q^2. The result has the form

$$\tilde{\Pi}^{aS\ell n}_{\mu_1\cdots\mu_\ell,\nu_1\cdots\nu_\ell}(k) \sim -\delta_{\mu_1\nu_1\cdots\mu_\ell\nu_\ell}\Lambda^2\ \mathrm{Tr}\frac{(2n+\ell+1)!2^\ell}{v^2(2+v^{-1})^{2n+\ell}} \exp\left\{-\frac{k^2}{4v}\right\} \;.$$

Here Tr means summation over the elements of the matrix v. One can see that the leading terms arise from the values $v_1 = v_2 = 1/3$. Thus we arrive at

$$\tilde{\Pi}^{aS\ell n}(-m^2_{aS\ell n}) \sim -\Lambda^2 \frac{(2n+\ell+1)!2^\ell}{5^{2n+\ell}} \exp\left\{\frac{3}{4}\mu^2_{aS\ell n}\right\},$$

where we substituted $k^2 = -\mu^2_{aS\ell n} = -m^2_{aS\ell n}/\Lambda^2$.

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