The wave function of a black hole and the dynamical origin of entropy

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Recently it was proposed to explain the dynamical origin of the entropy of a black hole by identifying its dynamical degrees of freedom with the physical modes propagating in the black hole interior. The present paper contains the further development of this approach. The no-boundary proposal (analogous to the cosmological no-boundary wave function) is put forward for the pure quantum state of a black hole. This is a functional on the configuration space of physical fields (including the gravitational one) inhabiting the three-dimensional space of the Einstein-Rosen bridge topology. For linearized field perturbations on the Schwarzschild-Kruskal background this no-boundary wave function coincides with the Hartle-Hawking vacuum state. The invariant definition of interior and exterior modes is proposed and the duality existing between them is discussed. The density matrix ρ_H describing the internal state of a black hole is obtained by averaging over the exterior modes. The "dynamical" entropy, determined by $-\mathrm{Tr}\rho_H \ln \rho_H$, is calculated. It is shown that the one-loop contribution to the "dynamical" entropy calculated for a given black hole background is divergent. The notion of an *instant horizon* is proposed, which separates the interior from the exterior of the black hole. It is argued that quantum fluctuations of the instant horizon inherent in the proposed formalism may give the necessary cutoff and provide a black hole with finite dynamical entropy. The relation between the "dynamical" entropy and the standard Bekenstein-Hawking ("thermodynamical") entropy is briefly discussed.

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I. INTRODUCTION

According to the thermodynamical analogy in black hole physics, the entropy of a black hole is proportional to the surface area of a black hole [1,2]. The Hawking discovery [3,4] of black hole thermal radiation confirmed the status of thermal properties of a black hole and allowed one to fix the coefficient of proportionality in the definition of the entropy. The Bekenstein-Hawking entropy of a black hole is $S^H = A^H/(4l_P^2)$, where A^H is the area of a black hole surface and $l_P = m_P^{-1} = G^{1/2}$ is the Planck length. Four laws of black hole physics [5] show that a black hole can be considered as a thermodynamical system and its entropy plays essentially the same role as the entropy in the "usual" physics, e.g., it shows to what extent the energy contained in a black hole can be used to produce work. More exactly, for thermal equilibrium of a black hole with the surrounding radiation the "thermodynamical" (Bekenstein-Hawking) entropy defines the response of the free energy of a black hole to the change of the temperature.

The black hole entropy is shown in the general case to be connected with the Noether charge associated with the Killing horizon [6]. The generalized second law (i.e., the statement that the sum $\tilde{S} = S^H + S^m$ of a black hole entropy and the entropy S^m of the external matter cannot decrease) implies that, in the case when a black hole is part of a thermodynamical system, the thermodynamical laws will be self-consistent only if the black hole entropy is considered on an equal footing with the entropy of the "usual" matter [1,2,7] (see also Refs. [8–11] and references therein). The gedanken experiment proposed by York [12] in which a black hole is located inside a heated cavity gives a good example, showing that such parameters of a black hole as a heat capacity and entropy have a well-defined physical meaning.

The formal derivation of the thermal properties of a black hole is usually performed in the framework of the Euclidean approach initiated by Gibbons and Hawking [13,14]. It implies the existence of the thermodynamical ensemble of black holes characterized by the canonical partition function at finite temperature $T = 1/\beta$

$$Z(\beta) = \mathrm{Tr}\, e^{-\beta\hat{H}}, \tag{1.1}$$

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where $\hat{H}$ is the Hamiltonian of the full gravitational system. The known functional representation of finite temperature field theory in terms of the Euclidean quantum theory, directly extrapolated to quantum gravity, allows one to rewrite (1.1) as the Euclidean path integral over four-geometries and matter fields. The evaluation of this integral by the steepest descent method determines $Z(\beta)$ and, in particular, gives $T = 1/(8\pi M)$. A refined version of this approach which emphasizes the role of boundary conditions was developed in Refs. [12,15–17]. In the framework of this approach the construction of the microcanonical partition function within the Lorentzian context was analyzed and a revised issue of stability for the gravitational ensemble at a given temperature and given boundary quasilocal characteristics was given.

Although the Euclidean approach allows us to obtain the same value for the black hole entropy as required by the thermodynamical analogy, namely, the Bekenstein-Hawking entropy, it does not elucidate a number of questions. Mainly this is a question of the origin of the thermodynamical partition (1.1), which is assumed to be given. In other words, it does not specify the physical degrees of freedom inaccessible for observation for an external observer, their tracing out in the pure quantum state of the whole gravitational system leading to the loss of information, emergence of entropy, and the density matrix corresponding to (1.1). In particular, the conventional Euclidean approach to gravitational thermodynamics simply does not leave room for a black hole interior. The Hawking instanton, which gives the leading contribution in this approach, is described by the real Euclidean section of the complex Schwarzschild geometry with the radial coordinate r taking values only greater than the gravitational radius.

Despite some promising attempts [2,8,18,19,26], the dynamical (statistical mechanical) origin of a black hole entropy has not been well understood. According to the "standard" interpretation, the entropy of a black hole is considered as a logarithm of the number of distinct ways that the hole might have been made [8,19]. This interpretation is not satisfactory. Soon after the black hole formation neither an external nor internal observer can see or affect these states and hence it does not make sense to interpret them as usual dynamical degrees of freedom which specify the state of the system at the chosen moment of time [27]. This conclusion was supported by the gedanken experiment proposed in Ref. [23], in which a traversible wormhole was used to get information about the black hole interior.

The paper [19] by Thorne and Zurek also contains a refined version of their statistical mechanical definition of the entropy, according to which the entropy of a black hole is defined as "the logarithm of the amount of information that one loses when one 'stretches the horizon' in the black-hole 'membrane formalism,' to cover up its thin thermal atmosphere." According to this viewpoint, the black hole entropy is only skin deep. Such a definition requires an additional procedure of the renormalization of the entropy. But what is more important, the states which contribute to the entropy do not exist at a given moment of time, and in order to define the entropy we need, according to this definition, to count all the possibilities, during the lifetime of a black hole (or, equivalently, the states along the stretched horizon).

In order to solve the problem of the dynamical origin of the black hole entropy, York [26] proposed to identify the dynamical degrees of freedom of a black hole with its quasinormal modes. But the entropy of the quasinormal modes excited at a given moment of time is much smaller than $S^H = A^H/(4l_P^2)$. In order to obtain the required large value for the entropy, York proposed to sum over all different possibilities to excite quasinormal modes in the process of a black hole evaporation.

't Hooft [20] proposed a "brick wall model" in which the entropy of a black hole is identified with the entropy of a thermal gas located outside a black hole and supported in equilibrium by a heated wall located at small distance outside the horizon. The value of the gap parameter in this model is chosen by equating the entropy of the gas outside the wall to the entropy of a black hole. The relation of the "brick wall" model to the results obtained from the first principles remains unclear. Among other approaches we mention an attempt to relate the dynamical degrees of freedom of a black hole with oscillations of quantum membrane representing the horizon [28], and an interesting relation of the black-hole entropy with the probability of quantum production of pairs of black holes by an external field [29].

Recently [30] a new approach to the problem of black hole entropy was proposed. According to this approach the dynamical degrees of freedom of a black hole are identified with those modes of physical fields that are located *inside* the horizon and cannot be seen by a distant observer. It was shown that the main contribution to the entropy is given by thermally excited "invisible" modes propagating inside a black hole in the close vicinity of the horizon. The so-defined one-loop entropy of a black hole is formally divergent and requires a cutoff [32]. This divergence is caused by a sharp boundary of the invisible region and it arises already in the similar flat spacetime calculations [35,36]. The natural cutoff may arise because of the quantum fluctuations of the horizon. A calculation based on a simple estimate of the horizon fluctuations [30] yields a value of the entropy which is close to the usually adopted value $A^H/(4l_P^2)$.

The calculations made in this (as well as in other dynamical approaches) assume the following steps: (1) definition of the initial state of a black hole and specifying the modes which are considered as degrees of freedom of a black hole; (2) calculation of the density matrix ρ describing a black hole by averaging over external degrees of freedom; (3) calculation of the statistical mechanical (or "dynamical") entropy $-\mathrm{Tr}\rho\ln\rho$. There are three important problems which naturally arise in connection with these calculations.

(1) How can one generalize the calculation of the entropy in order to include the quantum fluctuations of the horizon in a self-consistent way? (2) How can one combine the developed approach with the calculations of the black hole entropy based on the Euclidean space framework? (3) What is the relation of the obtained dynamical entropy to the standard "thermodynamical" entropy by

Bekenstein-Hawking?

In this paper we present an approach which might be regarded as an attempt to fill the gaps in the statistical-mechanical foundation of black hole thermodynamics. It consists of (i) the proposal for the pure quantum state of the black hole, (ii) the invariant criterion for the separation of the dynamical degrees of freedom into observable ones and those inaccessible for an exterior observer, and (iii) the averaging over the latter variables, which leads to the density matrix of a black hole and the dynamical origin of its entropy. We also briefly discuss the relation of the calculated dynamical entropy to the "thermodynamical" entropy and make some comments on the recently proposed idea [34] that the problem of entropy is related to the renormalization of the gravitational constant.

II. DYNAMICAL DEGREES OF FREEDOM OF A BLACK HOLE

The object we are interested in is a black hole which arises as a result of the gravitational collapse. For simplicity we assume that a black hole is nonrotating and spherically symmetric. A Penrose diagram for such a black hole is given in Fig. 1. Denote by Σ_0 a spacelike Cauchy surface which intersects the event horizon H^+ and denote by ∂B the intersection $\partial B = H^+ \cap \Sigma_0$. The state of our system (a black hole and fields in its vicinity) can be characterized by giving the values of gravitational and other fields on the chosen surface Σ_0. It is evident that the states of the gravitational and other fields located inside ∂B have no influence on the future evolution of the black hole exterior.

We assume that a black hole is stationary at late time

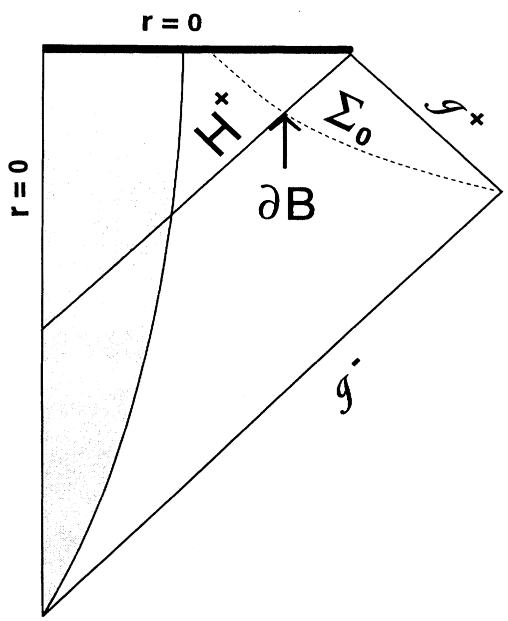

FIG. 1. This is a Penrose diagram of the forming black hole. A spacelike Cauchy surface Σ_0 goes from spatial infinity to the singularity at $r = 0$ and intersects the event horizon H^+ of the black hole at the two-dimensional surface ∂B.

and denote by ξ_t the Killing vector which is timelike at infinity in the corresponding region. For states of particles and fields which fall at late time into the black hole from the exterior region, the energy E defined by means of the Killing vector ξ_t is always positive. (For a particle $E \equiv -\xi^\mu p_\mu$, where p^μ is its momentum.) In addition to these modes there exist also states with negative total energy $E < 0$. Such states are located inside the black hole at Σ_0 and, in accordance with the definition proposed in Ref. [30], they will be considered as the internal degrees of freedom of a black hole [37].

It was shown in [30] that the main contribution to the black hole entropy is given by the states located inside the black hole in a close vicinity of the horizon. For this reason only the part of the surface Σ_0 close to the horizon is really important for the entropy calculation. But from a more general point of view the complete description of internal degrees of freedom of a black hole is complicated in such an approach because, for example, a surface Σ_0 may cross the singularity. That is why we develop another approach which greatly simplifies the consideration.

We begin with the remark that a lone black hole at late time (i.e., long after a black hole formation) is almost stationary; i.e., its state can be described as a static geometry and small perturbations (fields excitation) propagating on this background. One can formally put into correspondence with such a "real" black hole a new "unphysical" spacetime, which is obtained from the original geometry in late time region by its analytical continuation. Such an analytical continuation of a static black hole solution defines maximally extended solution which is known as an eternal black hole. The parameters of the eternal black hole (in our case mass, in more general cases also angular momentum and charge) are the same as for the initial "physical" black hole. We shall refer to such an eternal black hole which corresponds in the above sense to the "physical" black hole as to its "eternal version."

The Penrose diagram for an eternal black hole is shown in Fig. 2. If Σ_0 is chosen at late time, one can also trace back in time all the field excitations present in the vicinity of Σ_0 so that the problem of specifying the states of a black hole can be reformulated as an analogous prob-

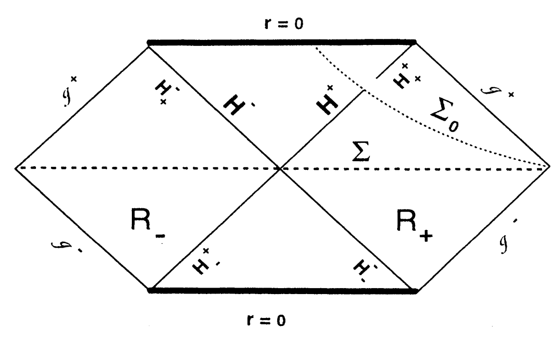

FIG. 2. This is a Penrose diagram of the eternal black hole. In Kruskal coordinates the global Cauchy surface Σ is defined by the equation $U + V = 0$. It has a wormhole topology $R \times S^2$. Both (future H^+ and past H^-) horizons consist of two parts $H^+_\pm$ and $H^-_\pm$, the boundaries of $R_\pm$.

lem for its eternal version. Technically the latter is much simpler, so that we use this approach. Such an approach is often used for simplifying the calculations of characteristics of Hawking radiation [4]. We use it for the construction of the wave function of a black hole.

The Kruskal metric for an eternal black hole reads

$$ds^2 = -\frac{32M^3}{r} \exp\left[-(r/2M - 1)\right] dU dV + r^2 d\Omega^2, \quad (2.1)$$

$$UV = (1 - r/2M) \exp(r/2M - 1) \; . \quad (2.2)$$

Denote by $\mathbf{\Sigma}$ a global Cauchy surface defined by the equation $U + V = 0$. It has a wormhole topology $\mathbf{R} \times \mathbf{S}^2$. This is a well-known Einstein-Rosen bridge connecting two asymptotically flat three-dimensional spaces [Fig. 3(a)]. The discrete isometry $U \to -U, V \to -V$ transforms the surface $\mathbf{\Sigma}$ into itself, so that one asymptotically flat region (say $\mathbf{\Sigma}_+$) is mapped onto another (say $\mathbf{\Sigma}_-$). Localized states with $E < 0$ being traced back in time in the Kruskal geometry cross $\mathbf{\Sigma}_-$, while the states with $E > 0$ cross $\mathbf{\Sigma}_+$.

A remarkable property of the Kruskal-Schwarzschild metric (2.2) is that it can be considered as a real Lorentzian-signature section of the complex manifold parametrized by the real radial r, $2M \leq r < \infty$, and complex time z coordinates:

$$z = \tau + it, \quad (2.3)$$

$$U = -\left(\frac{r}{2M} - 1\right)^{1/2} \exp\left\{\frac{1}{2}\left(\frac{r}{2M} - 1\right) + i\,\frac{z - 2\pi M}{4M}\right\}, \quad (2.4)$$

$$V = \left(\frac{r}{2M} - 1\right)^{1/2} \exp\left\{\frac{1}{2}\left(\frac{r}{2M} - 1\right) - i\,\frac{z - 2\pi M}{4M}\right\}. \quad (2.5)$$

Sectors R_+ and R_- with asymptotically flat infinities of the Kruskal spacetime are generated by the segments in the complex plane of z,

$$R_\pm : z = \pm 2\pi M + it, \; -\infty < t < \infty, \quad (2.6)$$

and analytically joined by the real Euclidean section E

$$E : z = \tau, \; -2\pi M \leq \tau \leq 2\pi M. \quad (2.7)$$

Here t is a usual Killing time coordinate in the Schwarzschild metric, while τ is its Euclidean analogue. The coordinate τ plays the role of the angular coordinate and for the Gibbons-Hawking black hole instanton it is periodic with the period $\beta_0 = 8\pi M$:

$$ds_E^2 = \left(1 - \frac{2M}{r}\right) d\tau^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2. \quad (2.8)$$

The Euclidean section (2.7) represents a half-period part of this instanton, the boundary $\mathbf{\Sigma}_+ \cup \mathbf{\Sigma}_-$ of which at $\tau_\pm = \pm 2\pi M$ is the Einstein-Rosen bridge of the above type [Fig. 3(b)]. At this boundary the Euclidean section analytically matches with the Lorentzian sectors R_+ and R_- on the Penrose diagram of the Kruskal metric.

As we mentioned above, the propagation of perturbations in a real black hole can be reduced to the analogous problem in its eternal version. For the latter one must specify the initial data on $\mathbf{\Sigma}$. It is evident that the data on $\mathbf{\Sigma}_-$ do not influence the black hole exterior. In a spacetime of an eternal version of a black hole, such perturbations are propagating to the future entirely inside the horizon. It is evident that the corresponding perturbations in a physical black hole also always remain under the horizon. That is why these data should be identified with internal degrees of freedom of a black hole. Such

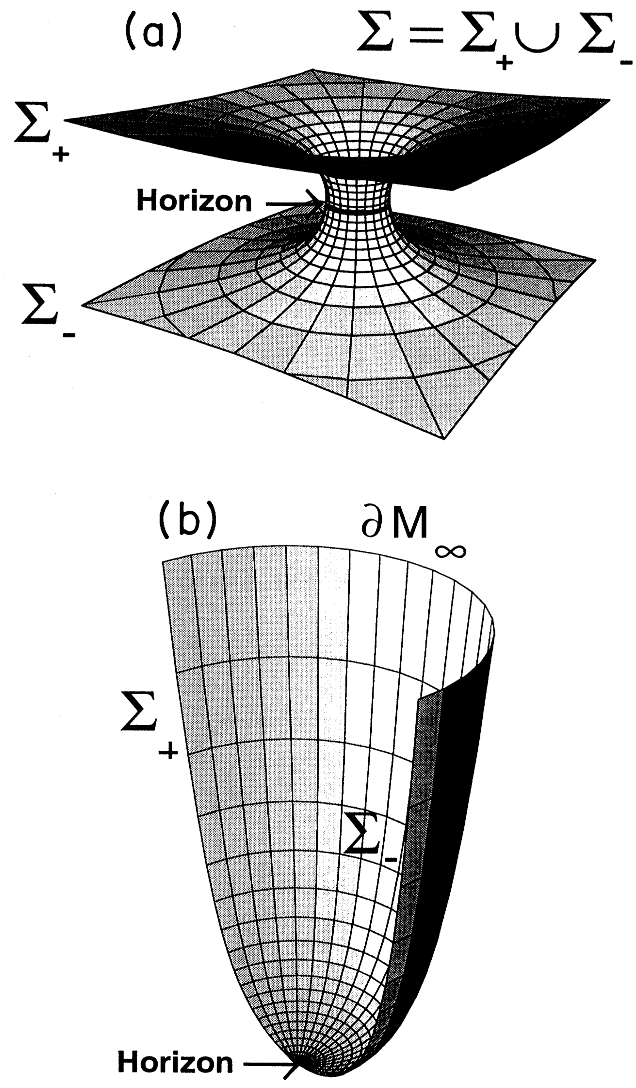

FIG. 3. (a) This figure depicts the embedding diagram of the global Cauchy surface $\mathbf{\Sigma}$. This is a well-known Einstein-Rosen bridge connecting two asymptotically flat three-dimensional spaces. The bifurcation two-sphere of the future and past horizons separates the interior $\mathbf{\Sigma}_-$ of a black hole from external space $\mathbf{\Sigma}_+$. (b) Here the half of the Gibbons-Hawking gravitational instanton is depicted. The global Cauchy surface $\mathbf{\Sigma} = \mathbf{\Sigma}_+ \cup \mathbf{\Sigma}_-$ is one boundary of this Euclidean manifold and spatial infinity $\partial \mathbf{M}_\infty$ is another. The arguments of the wave function $\mathbf{\Psi}(\varphi_+, \varphi_-)$ are the boundary values of quantum fields on the two asymptotically flat parts of the Einstein-Rosen bridge $\mathbf{\Sigma}_\pm$, respectively.

an approach can be generalized for the case when the perturbations are not small.

Among all perturbations describing the propagation of physical fields special role is played by gravitational perturbations. The corresponding initial data on the Einstein-Rosen bridge can be considered either as tensor field, or, equivalently, as small deformations of the initially spherical geometry of the Einstein-Rosen bridge. In this sense one can relate different configurations of the gravitational fields in the system with deformations of the Einstein-Rosen bridge, obeying the necessary constraints, existing in the theory. For physical excitations with finite energy the condition of asymptotic flatness at both infinities of the Einstein-Rosen bridge should be preserved. To summarize, in classical physics the space of initial physical configurations of a system including a black hole can be related to the space of "deformations" of the Einstein-Rosen bridge of the eternal black hole and possible configurations of other fields on it (in addition to the gravitational one), which obey the constraints and preserve asymptotic flatness. We use now this configuration space to define a wave function of a black hole.

Our main idea can be described as follows. Fix a three-dimensional manifold with a wormhole topology $\boldsymbol{R} \times \boldsymbol{S}^2$ and consider any three-dimensional metrics on it which possess two asymptotically flat regions. Consider also the configuration of matter fields on this manifold. The space of three-geometries and matter fields will be considered as a configuration space for our problem. We introduce a wave function of a black hole as a functional on this configuration space. It should be stressed that the metric and fields at the "internal" part $\boldsymbol{\Sigma}_-$ of space are to be considered as defining the internal state of a black hole and hence they will be identified with its dynamical degrees of freedom.

Our proposal for the quantum state of a black hole is a "no-boundary" wave function of three-geometry and matter fields on such a surface $\boldsymbol{\Sigma} = \boldsymbol{R} \times \boldsymbol{S}^2$ given by the Euclidean path integral of Hartle and Hawking over four-geometries and spacetime matter fields bounded by Σ and four-dimensional asymptotically flat and empty infinity.

Obviously, the above picture is only an illustration of the general method we shall propose here. In the full theory of quantum gravity incorporating the coupling of matter with the gravitational field (what is usually called a self-consistent back reaction of quantized matter on semiclassical background), many features of the Schwarzschild solution do not persist. Generically there are no symmetries and the very notion of the bifurcation surface of the Killing horizon separating physical variables into observable and unobservable ones does not exist and should be dynamically determined on the ground of some invariant criterion. In this paper we propose such a criterion. It is based on the notion of *instant horizon* of a black hole defined for instantaneous realization of the Einstein-Rosen bridge. The instant horizon is subject to quantum fluctuations (the horizon *zitterbewegung*) and is characterized by its quantum dispersion. The latter quantity is very important in gravitational thermodynamics [26], for it, apparently, provides a self-consistent high energy cutoff for the one-loop entropy [30].

We have to stress that the proposed no-boundary wave function describes only one special state of a black hole which in some sense is the simplest one. In many aspects the no-boundary wave function is similar to the ground state of the system. The full quantum theory of black holes must allow many different states. We are not constructing here the full quantum theory. That is why many fundamental questions remain unanswered. For example, how the local observables calculated with the help of our wave function are connected with the local observables in a spacetime of a real "physical" black hole. For the particular calculations of the entropy of a black hole, we explain later why the calculations based on the no-boundary wave function give the same answer as the calculations done directly with physical black hole [30].

It should also be emphasized that the quantum state of the black hole we advocate here is merely a *proposal*, and we must verify its validity by comparing its consequences with the known properties of the conventional gravitational thermodynamics. For this purpose we first show that, semiclassically, this state generates the black-hole Hartle-Hawking vacuum [38] for the particle excitations of all spins (including graviton) and produces by the procedure of the above type the thermal density matrix with the temperature $T = 1/8\pi M$.

III. NO-BOUNDARY WAVE FUNCTION OF A BLACK HOLE

The no-boundary wave function was first proposed by Hartle and Hawking [39,40] in the context of quantum cosmology as a path integral

$$\boldsymbol{\Psi}({}^3g(\boldsymbol{x}), \varphi(\boldsymbol{x})) = \int \mathcal{D}\,{}^4g\; \mathcal{D}\boldsymbol{\phi}\; e^{-I[{}^4g,\phi]} \tag{3.1}$$

of the exponentiated gravitational action $\boldsymbol{I}[\,{}^4g, \boldsymbol{\phi}]$ over Euclidean four-geometries and matter-field configurations on a compact spacetime $\boldsymbol{M}$ with a boundary $\partial\boldsymbol{M}$. The integration variables are subject to the conditions $({}^3g(\boldsymbol{x}), \varphi(\boldsymbol{x}))$, $\boldsymbol{x} \in \partial\boldsymbol{M}$, the collection of three-geometry and boundary matter fields on $\partial\boldsymbol{M}$, which are just the argument of the wave function (3.1).

This construction was also applied in the asymptotically flat case [41] when $\boldsymbol{M}$ represents a noncompact four-dimensional half-space whose boundary consists of two components, $\partial\boldsymbol{M} = \boldsymbol{R}^3 \bigcup \partial\boldsymbol{M}_\infty$: infinite three-dimensional hyperplane $\boldsymbol{R}^3$ carrying the field argument of the wave function and the asymptotically flat and empty infinity $\partial\boldsymbol{M}_\infty$. The latter is a singular component of the spacetime boundary and its boundary conditions are, in a certain sense, trivial and do not enter the argument of the wave function.

We propose the quantum state of a black hole which is a modification of this asymptotically flat, no-boundary wave function of Hartle [41]. It is given by Eq. (3.1), where the total boundary

$$\partial\boldsymbol{M} = \boldsymbol{\Sigma} \cup \partial\boldsymbol{M}_\infty \tag{3.2}$$

has instead of the hyperplane above the hypersurface with the topology of the Einstein-Rosen bridge

$$\boldsymbol{\Sigma} = \boldsymbol{R} \times \boldsymbol{S}^2 \tag{3.3}$$

connecting two asymptotically flat three-dimensional spaces [see Fig. 3(a)].

Quantum gravity and its path integral formulation should, as is widely recognized, include the topology change transitions. In the language of quantum states of gravitating systems, this means that these states should be determined not only on the space of three-geometries, but also on the space of different topologies of the three-dimensional spatial hypersurface. So the definition of the quantum state of a black hole might also include the non-simply-connected hypersurfaces with the topologies more complicated than that of the Einstein-Rosen bridge, sharing in common only the asymptotically flat behavior at infinities. One of the examples is shown in Fig. 4, depicting the three-dimensional handle modifying the Einstein-Rosen wormhole. Such topological modifications can play an important role in the problem of gravitational entropy, loss of information, etc., but at present they are too far from technical and conceptual implementation, and we shall restrict ourselves to the Einstein-Rosen case of the above type.

The construction (3.1)–(3.3) forms a topological part of the definition for the no-boundary wave function. Apart from that, the expression (3.1) signifies nothing unless we specify the meaning of the integration measure $\mathcal{D}^4 g\, \mathcal{D}\phi$. We also need to determine the physical inner product with respect to which one can calculate the expectation values and matrix elements for a given wave function. In the context of the Lorentzian spacetime these problems have a solution which is based on the quantization of true physical variables [42–44] and can be constructively realized at least within the semiclassical loop expansion [45]. This quantization leads to the

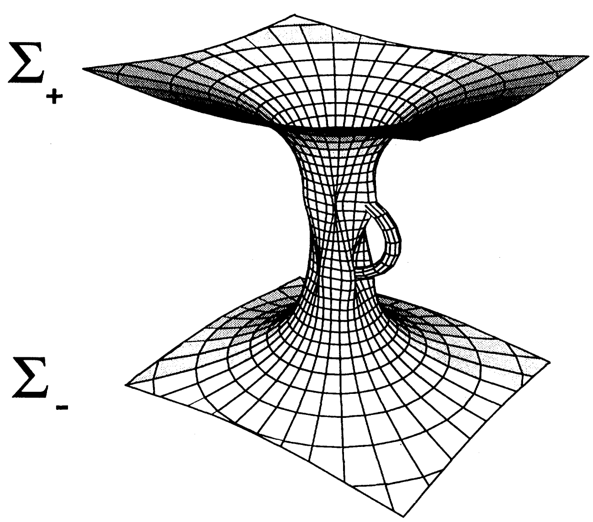

FIG. 4. Possible modification of the three-dimensional topology in the definition of the no-boundary wave function of a black hole accounting for topological transitions in quantum gravity.

standard Faddeev-Popov integration measure [46] in the functional integral (3.1) and to its analogue in the physical inner product for the wave function $\Psi({}^3g(\boldsymbol{x}), \varphi(\boldsymbol{x}))$ in the representation of local spatial three-metric tensor and matter fields. The measure in this physical inner product is nontrivial. It is roughly the Faddeev-Popov measure in the configuration space of fields taken on a single spatial surface of the spacetime. The measure incorporates the gauge-fixing procedure and effectively restricts the integration to the subset of true configuration-space coordinates among the dynamically redundant set $({}^3g(\boldsymbol{x}), \varphi(\boldsymbol{x}))$ [44,45]:

$${}^3g(\boldsymbol{x}),\, \varphi(\boldsymbol{x}) \to \varphi = (\, g^T(\boldsymbol{x}),\, \varphi(\boldsymbol{x})\,). \tag{3.4}$$

The geometrical content of the local gravitational variables can be very different, depending on the choice of gauge, and it generally represents certain two dynamically independent degrees of freedom $g^T(\boldsymbol{x})$ per spatial point. They originate from solving the gravitational constraints and imposed gauge conditions for the original gravitational phase-space variables ${}^3g(\boldsymbol{x})$, ${}^3p(\boldsymbol{x})$ in terms of $g^T(\boldsymbol{x})$ and physical conjugated momenta $p_T(\boldsymbol{x})$ [47].

The wave function can be constructed directly in the representation of physical variables (3.4), $\Psi(\varphi)$. In this representation the physical inner product has a trivial form

$$\langle \Psi_1 \,|\, \Psi_2 \rangle = \int D\varphi\, \Psi_1^*(\varphi)\, \Psi_2(\varphi), \tag{3.5}$$

that provides the unitary dynamics of $\Psi(\varphi) = \Psi(\varphi, t)$ with the physical Hamiltonian whose functional form arises from the Arnowitt-Deser-Misner (ADM) reduction (3.4) [48]. For this reason, we shall formulate our proposal (3.1)–(3.3) for the black hole wave function in the representation of physical variables [49–51]. In this representation the wave function of a black hole is given by the path integral of the form (3.1), but with the physical configuration coordinates (3.4) fixed at $\partial \boldsymbol{M}$ instead of the three-metric components of the dynamically redundant set $({}^3g(\boldsymbol{x}), \varphi(\boldsymbol{x}))$

$$\Psi(\varphi) = \int_{(\phi|_{\Sigma} = \varphi)} \mathcal{D}\phi\; e^{-I[\phi]}. \tag{3.6}$$

Here the integration goes over those spacetime histories of physical ADM fields $\phi = \phi(x)$ that generate the Euclidean four-geometries asymptotically flat at the infinity $\partial \boldsymbol{M}_\infty$ of spacetime and match φ on its "dynamically active" boundary (3.3). $\boldsymbol{I}[\phi]$ is the Lagrangian gravitational action in terms of these fields. The integration measure $\mathcal{D}\phi$ involves the local functional measure [51], the structure of which is not very important for our purposes.

As mentioned above, the nature of physical degrees of freedom depends on the choice of gauge in the ADM reduction procedure. To effectively operate with the physical wave function, we have to fix this gauge and perform the reduction (3.4). Here we use a York gauge [53] which consists of the condition

$$\mathrm{tr}\, {}^3p(\boldsymbol{x}) \equiv g^{ab}(\boldsymbol{x})\, p_{ab}(\boldsymbol{x}) = 0, \tag{3.7}$$

selecting a spacetime foliation by minimal surfaces [of vanishing mean extrinsic curvature $\mathrm{tr}\, K(\boldsymbol{x}) = 0$], and three other conditions fixing the coordinates on these surfaces. A distinguished nature of this gauge consists in the fact that, in contrast with a majority of other gauges, it does not suffer from the problem of Gribov copies invalidating the physical reduction when the latter is considered globally in phase space of the theory [52]. This property of the York gauge follows from a strong theorem of [54] on the uniqueness of a solution of the Lichnerowicz equation for the conformal factor in the conformal decomposition of a three-metric [53] provided positive-energy condition holds for matter fields.

As known [53,56], the physical degrees of freedom in the York gauge can be represented by the two variables characterizing the conformally invariant part $\tilde{g}_{ab}(\boldsymbol{x})$ of the three-metric (in some gauge fixing of the three-dimensional spatial diffeomorphisms) and the conjugated transverse traceless momenta $\tilde{p}^{ab}(\boldsymbol{x})$, while the conformal mode $\Phi(\boldsymbol{x})$ of the full three-metric,

$$g_{ab}(\boldsymbol{x}) = \Phi^4(\boldsymbol{x})\, \tilde{g}_{ab}(\boldsymbol{x})\,, \qquad (3.8)$$

follows from the solution of the Lichnerowicz equation, which is just the Hamiltonian gravitational constraint rewritten in the conformal decomposition of the above type,

$$\left(\tilde{\Delta} - \frac{1}{8}\, {}^3\tilde{R}\right) \Phi + \frac{1}{8}\, C\Phi^{-7} + 2\pi\, \tilde{T}^*_* \, \Phi^{-3} = 0, \qquad (3.9)$$

$$C \equiv \tilde{p}^{ab}\, \tilde{p}_{ab}/\tilde{g}. \qquad (3.10)$$

Here $\tilde{T}^*_* = \Phi^{-8} T^*_*$ is a conformally rescaled energy, Hamiltonian density, of matter fields and tilde denotes the quantities calculated in the conformal metric $\tilde{g}_{ab}$ (in the geometrically invariant language, the physical content of $\tilde{g}_{ab}$ can be described by the conformally invariant transverse-traceless tensor of York [53] β^{ab}). In the linearized approximation the physical gravitational variables in the York gauge are the transverse-traceless part of the linear excitations h_{ab} and their conjugated transverse-traceless momenta [55]:

$$(g^T,\, p_T) = (h^T_{ab},\, p^{ab}_T). \qquad (3.11)$$

In a semiclassical approximation the wave function of a black hole

$$\Psi(\varphi) = P\, e^{-I[\phi(\varphi)]} \qquad (3.12)$$

is dominated by the classical action at the extremal of equations of motion $\phi(\varphi)$ subject to boundary conditions φ on $\boldsymbol{\Sigma}$. It also includes the preexponential factor P accumulating the result of integration over quantum field deviations from the extremal. The physical variables φ given by Eqs. (3.4) and (3.11) are treated by perturbations and the Euclidean action $\boldsymbol{I}[\,\phi(\varphi)\,]$ is to be expanded in powers of φ. To obtain the lowest order term $\boldsymbol{I}[\,\phi(0)\,]$, notice that the boundary three-geometry on $\boldsymbol{\Sigma}$ (3.8) has, by virtue of (3.9), a conformal factor satisfying the linear homogeneous conformally invariant equation in three dimensions. As shown in Appendix A, for asymptotically flat boundary conditions it gives exactly the spherically symmetric metric of the Einstein-Rosen bridge, characterized by a single constant—the mass M of the black hole. The extremal of the Euclidean vacuum Einstein equations $\phi(0)$ satisfies asymptotically flat boundary conditions at $\partial\boldsymbol{M}_\infty$. The corresponding solution is just one-half of the Schwarzschild gravitational instanton of mass M with the four-dimensional metric (2.8) for $-2\pi M \leq \tau \leq 2\pi M$ [see Fig. 3(b)]. The classical action on this half of instanton reduces to the contribution of the surface term at $\partial\boldsymbol{M}_\infty$ of the classical Einstein gravitational action

$$\boldsymbol{I}[\,\phi(0)\,] = \frac{1}{8\pi} \int_{\partial\boldsymbol{M}_\infty} K\sqrt{h} d^3x = 2\pi M^2. \qquad (3.13)$$

The expansion of $\boldsymbol{I}[\,\phi(\varphi)\,]$ in powers of φ on the background of $\phi(0)$ shows that the linear order term vanishes due to the equations of motion for the background and the vanishing of the extrinsic curvature of $\boldsymbol{\Sigma}$ (the latter property guarantees the absence of the corresponding surface terms). Therefore the leading contribution to the semiclassical wave function (3.12) takes the form

$$\Psi(\varphi, M) = P\, e^{-2\pi M^2 - I_2[\phi(\varphi)]}\,, \qquad (3.14)$$

where $\boldsymbol{I}_2[\,\phi(\varphi)\,]$ is a quadratic term of the action in the linearized physical fields (3.4) and (3.11).

Thus, our no-boundary wave function of a black hole turns out to be a functional of the local gravitational and matter degrees of freedom $\varphi(\boldsymbol{x})$, parametrized by a global variable—the gravitational mass of the Einstein-Rosen bridge M. Obviously, if we include M in the configuration space of the black hole, the dependence of the wave function on it will describe the probability distribution of black holes with different masses in this quantum state. A naive inclusion of M into the ADM phase space of the theory in the York gauge does not seem to be fully consistent. However, it was recently performed in a more general context by Kuchar [57], who, in the framework of a spherically symmetric minisuperspace model, persuasively advocated that M has a conjugated momentum P_M, so that (M, P_M) can be subject to standard canonical quantization and can incorporate as their quantum state an *arbitrary* function of the black-hole mass M. Thus, the proposed M-dependent no-boundary wave function can be regarded as a first example of such a quantum state of a black hole (or, more precisely, of the quantum Einstein-Rosen bridge) [58]. In what follows, however, we shall consider M as an external parameter not entering the argument of the wave function and, correspondingly, excluded from the phase space and the Hilbert space of the theory. Therefore, up to M-dependent normalization, the semiclassical wave function of the black hole will be dominated by its $\exp\{-\boldsymbol{I}_2[\,\phi(\varphi)\,]\}$ part, describing the dynamics of local degrees of freedom. In the next section we show that it represents their Hartle-Hawking vacuum on the background of the Schwarzschild-Kruskal geometry.

It should be emphasized, however, that even with the global mass parameter M excluded from the Hilbert space of the theory, the above construction describes the wave function of *an eternal black hole*, but not just the quantum state of matter fields on the nondynamical Schwarzschild-Kruskal background considered in Ref. [59]. In particular, the semiclassical Gaussian approximation (3.14) includes the contribution of quantum gravitational perturbations in $\boldsymbol{I}_2[\phi(\varphi)]$ describing possible distortions of a black hole. Technically, in this approximation, this wave function reduces to the case of Ref. [59] (provided the latter includes the graviton contribution), but the topological contents of our proposal as well as the analysis of this section (and Appendix A) show that its scope extends far beyond the quantum dynamics of matter fields on a fixed Schwarzschild-Kruskal background. In particular, the Lichnerowicz equation (3.8) can be used for the study of the back reaction phenomena and quantum fluctuations of the horizon (see discussion below). We illustrated here the calculations of the no-boundary wave function in the semiclassical approximation. But certainly the proposed definition (3.6) is much more general. For example, the functional-integral representation for the no-boundary wave function in principle allows one to develop perturbative calculations of higher order corrections or to study nonperturbative effects.

IV. NO-BOUNDARY WAVE FUNCTION OF LINEARIZED FIELDS

We show now that the no-boundary wave function for linearized fields on the Schwarzschild-Kruskal background coincides with Hartle-Hawking vacuum state. For this purpose we assume that the mass parameter M of a black hole is fixed and consider only the φ-dependent part of the black hole wave function (3.14), which we denote by $\Psi(\varphi)$. To simplify the formulas we work out a simple example of a scalar field $\phi\,(x)=\phi\,(\tau,\boldsymbol{x})$ with the quadratic action

$$\frac{1}{2}\int d^4x\, g^{1/2}\Big(g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\,\phi+\xi\,R\,\phi^2\Big). \tag{4.1}$$

The generalization to fields of higher spins in the quadratic approximation is obvious. The action (4.1) generates on the Euclidean section (2.7) with the metric (2.8) the linear equations of motion

$$\left\{-g^{1/2}g^{\tau\tau}\frac{d^2}{d\tau^2}-\partial_a g^{1/2}g^{ab}\partial_b\right\}\phi\,(\tau,\boldsymbol{x})=0,\qquad \boldsymbol{x}=x^a,\;\; a=1,2,3, \tag{4.2}$$

which must be solved subject to boundary conditions $\varphi=\varphi\,(\boldsymbol{x})$ on its boundary $\boldsymbol{\Sigma}$ to give the extremal $\phi(\varphi)$ of Eq. (3.14). On the Schwarzschild-Kruskal background with $R=0$ the nonminimal interaction does not contribute to the equations. In what follows we denote the boundary fields on the two asymptotically flat components of the Einstein-Rosen bridge $\boldsymbol{\Sigma}_\pm$ by $\varphi_\pm$:

$$\phi\,(x)\Big|_{\boldsymbol{\Sigma}_\pm}\equiv\phi\,(\pm\beta_0/4,\boldsymbol{x})=\varphi_\pm(\boldsymbol{x}),\;\; \beta_0\equiv 8\pi M. \tag{4.3}$$

With this notation the solution to (4.2) can be written as a decomposition,

$$\phi(\tau,\boldsymbol{x})=\sum_\lambda\Big\{\varphi_{\lambda,+}u_{\lambda,-}(\tau,\boldsymbol{x})+\varphi_{\lambda,-}u_{\lambda,+}(\tau,\boldsymbol{x})\Big\}, \tag{4.4}$$

in the basis functions of this equation

$$u_{\lambda,\pm}(\tau,\boldsymbol{x})=\frac{\sinh\left[(\beta_0/4\mp\tau)\,\omega\right]}{\sinh\left(\beta_0/2\right)}\,R_{\omega lmA}(\boldsymbol{x})\,,\qquad \lambda=(\omega,l,m,A). \tag{4.5}$$

This decomposition contains the set of spatial harmonics $R_{\omega lmA}(\boldsymbol{x})$—eigenfunctions of the eigenvalue problem

$$\partial_a\Big(g^{1/2}g^{ab}\partial_b R_{\omega lmA}(\boldsymbol{x})\Big)=-g^{\tau\tau}g^{1/2}\omega^2R_{\omega lmA}(\boldsymbol{x}) \tag{4.6}$$

originating from the separation of variables in (4.2). The eigenfunctions are enumerated by a set of continuous $\omega>0$ and discrete (l,m,A) labels, among which l and m are the usual quantum numbers of spherical harmonics and the label $A=1,2$ is responsible for two possible directions of propagation along the radial coordinate. As shown in Appendix B, these spatial harmonics can be chosen real. They are required to be regular at the horizon $r=2M$ and at spatial infinity, have a positive definite spectrum $\omega^2>0$ and satisfy the orthonormality and completeness conditions

$$\int d^3x\, g^{\tau\tau}g^{1/2}R_\lambda(\boldsymbol{x})\,R_{\lambda'}(\boldsymbol{x})=\delta_{\lambda\lambda'}, \tag{4.7}$$

$$\sum_\lambda R_\lambda(\boldsymbol{x})\,R_\lambda(\boldsymbol{x}')=\frac{\delta(\boldsymbol{x}-\boldsymbol{x}')}{g^{\tau\tau}g^{1/2}}. \tag{4.8}$$

Here, as in (4.5), we use a condensed notation λ for the full collection of quantum numbers, the summation over which implies the measure

$$\sum_\lambda(\cdots)\equiv\int_0^\infty d\omega\sum_{l,m,A}(\cdots)\,,\qquad \delta_{\lambda\lambda'}\equiv\delta(\omega-\omega')\,\delta_{ll'}\delta_{mm'}\delta_{AA'}\,. \tag{4.9}$$

In view of these relations the coefficients $\varphi_{\lambda,\pm}$ in (4.4) are just the decomposition coefficients of the fields (4.3) in the basis of spatial harmonics

$$\varphi_\pm(\boldsymbol{x})=\sum_\lambda\varphi_{\lambda,\pm}R_\lambda(\boldsymbol{x}). \tag{4.10}$$

Substituting (4.4) into (4.1), integrating by parts with respect to the Euclidean time and spatial coordinates and taking into account the equations of motion (4.2), one finds that the Euclidean action reduces to the following quadratic form in $\varphi_{\lambda,\pm}$ (cf. a similar derivation in Ref. [59]):

$$\boldsymbol{I}_2(\varphi_+,\varphi_-)=\frac{1}{2}\sum_\lambda\left\{\frac{\omega_\lambda\cosh(\beta_0\,\omega_\lambda/2)}{\sinh(\beta_0\,\omega_\lambda/2)}\,(\varphi_{\lambda,+}^2+\varphi_{\lambda,-}^2)-\frac{2\,\omega_\lambda}{\sinh(\beta_0\,\omega_\lambda/2)}\,\varphi_{\lambda,+}\varphi_{\lambda,-}\right\}. \tag{4.11}$$

We show now that the functional

$$\Psi(\varphi_+,\varphi_-) = P\exp[-\boldsymbol{I}_2(\varphi_+,\varphi_-)] \tag{4.12}$$

describes the Hartle-Hawking vacuum state [60] of the field ϕ in the spacetime of the Schwarschild-Kruskal black hole. The scheme of the proof is simple. First of all we note that this functional is Gaussian. It is easy to construct the linear combinations of $\varphi_\pm$ and partial derivatives with respect to $\varphi_\pm$ which annihilate this functional and hence play the role of the annihilation operators in this representation. In order to obtain the physical interpretation of this state it is convenient to find out its relation to the so-called Boulware vacuum states. The latter are defined in each of the regions $R_\pm$ independently and, as we show, have the form

$$\Psi_{B\pm} = P_{B\pm}\exp\left[-\frac{1}{2}\sum_\lambda \omega_\lambda\,\varphi^2_{\lambda,\pm}\right]. \tag{4.13}$$

By comparing the operators of annihilation and creation for no-boundary vacuum state with the annihilation and creation operators for the Boulware vacuum states we finally show that the no-boundary vacuum state coincides with the Hartle-Hawking vacuum state.

We begin the proof by noting that the action $\boldsymbol{I}_2(\varphi_+,\varphi_-)$ in terms of new variables $f_{\lambda,\pm}$,

$$\varphi_\pm = \frac{f_+\pm f_-}{\sqrt{2}}, \tag{4.14}$$

takes the diagonal form

$$\begin{aligned}\boldsymbol{I}_2(\varphi_+,\varphi_-) &= \bar{\boldsymbol{I}}_2(f_+,f_-)\\ &= \frac{1}{2}\sum_\lambda \omega_\lambda\left\{\tanh(\beta_0\,\omega_\lambda/4)\,f^2_{\lambda,+}\right.\\ &\quad\left.+\frac{1}{\tanh(\beta_0\,\omega_\lambda/4)}\,f^2_{\lambda,-}\right\}.\end{aligned} \tag{4.15}$$

The wave function (4.12) rewritten in the new representation (4.14) is a Gaussian state which is obviously a vacuum,

$$\Psi\,(\varphi_+,\varphi_-) = \bar{\Psi}(f_+,f_-) = P\,e^{-\bar{I}_2(f_+,f_-)} \tag{4.16}$$

$$\tilde{a}_\pm\,\bar{\Psi}\,(f_+,f_-) = 0, \tag{4.17}$$

of the following creation-annihilation operators (we omit for brevity the label λ in the definition of $\tilde{a}_\pm$ below as well as in $\omega=\omega_\lambda$):

$$\begin{aligned}\tilde{a}_+ &= \frac{1}{\sqrt{2}}\left[\left(\omega\tanh\frac{\beta_0\omega}{4}\right)^{-1/2}\frac{\partial}{\partial f_+}\right.\\ &\quad\left.+\left(\omega\tanh\frac{\beta_0\omega}{4}\right)^{1/2}f_+\right],\end{aligned} \tag{4.18}$$

$$\begin{aligned}\tilde{a}^\dagger_+ &= \frac{1}{\sqrt{2}}\left[-\left(\omega\tanh\frac{\beta_0\omega}{4}\right)^{-1/2}\frac{\partial}{\partial f_+}\right.\\ &\quad\left.+\left(\omega\tanh\frac{\beta_0\omega}{4}\right)^{1/2}f_+\right],\end{aligned}$$

$$\begin{aligned}\tilde{a}_- &= \frac{1}{\sqrt{2}}\left[\left(\frac{1}{\omega}\tanh\frac{\beta_0\omega}{4}\right)^{1/2}\frac{\partial}{\partial f_-}\right.\\ &\quad\left.+\left(\frac{1}{\omega}\tanh\frac{\beta_0\omega}{4}\right)^{-1/2}f_-\right],\end{aligned} \tag{4.19}$$

$$\begin{aligned}\tilde{a}^\dagger_- &= \frac{1}{\sqrt{2}}\left[-\left(\frac{1}{\omega}\tanh\frac{\beta_0\omega}{4}\right)^{1/2}\frac{\partial}{\partial f_-}\right.\\ &\quad\left.+\left(\frac{1}{\omega}\tanh\frac{\beta_0\omega}{4}\right)^{-1/2}f_-\right],\end{aligned}$$

subject to standard commutation relations

$$\left[\tilde{a}_{\lambda,\pm},\tilde{a}^\dagger_{\lambda'\pm}\right] = \delta_{\lambda\lambda'} \tag{4.20}$$

(all the other commutators are vanishing). For our purposes another choice of creation-annihilation operators is more useful, differing from (4.18) and (4.19) by the linear transformation not mixing the positive and negative frequencies

$$a_{\lambda,\pm} = \frac{\tilde{a}_{\lambda,+}\pm\tilde{a}_{\lambda,-}}{\sqrt{2}},\quad a_{\lambda,\pm}\bar{\Psi}\,(f_+,f_-) = 0. \tag{4.21}$$

To give a particle interpretation for the obtained vacuum state we must construct the propagating physical modes corresponding to $a_{\lambda,\pm}$. For this purpose consider the $\boldsymbol{\Sigma}_\pm$ parts of $\boldsymbol{\Sigma}$ as the initial Cauchy surfaces in the right (R_+) and left (R_-) wedges of the Lorentzian Schwarzschild-Kruskal spacetime. In two causally disconnected regions lying in $R_\pm$ to the future of $\boldsymbol{\Sigma}_\pm$ one can construct two scalar field theories with the Lagrangians—the Lorentzian versions of (4.1),

$$\begin{aligned}L_\pm &= \int_{\boldsymbol{\Sigma}_\pm} d^3x\,\mathcal{L}\,(\phi,\partial\phi)\\ &= \frac{1}{2}\sum_\lambda(\dot{\varphi}^2_{\lambda,\pm}-\omega^2_\lambda\varphi^2_{\lambda,\pm}),\end{aligned} \tag{4.22}$$

which take such a form provided the corresponding spacetime fields evolving correspondingly in R_+ and R_- are decomposed in spatial harmonics with time-dependent coefficients $\varphi_{\lambda,\pm}(t)$, $\dot{\varphi}_{\lambda,\pm}\equiv d\varphi_{\lambda,\pm}(t)/dt$. At the quantum level, in the coordinate representation of $\varphi_{\lambda,\pm}$ the creation-annihilation operators $b_{\lambda,\pm}$ of these two theories associated with positive-negative frequency decomposition in the Killing time t look like

$$\sqrt{2}\,b_{\lambda,\pm} = \frac{1}{\sqrt{\omega}}\frac{\partial}{\partial\varphi_{\lambda,\pm}} + \sqrt{\omega}\,\varphi_{\lambda,\pm}, \tag{4.23}$$

$$\sqrt{2}\,b^\dagger_{\lambda,\pm} = -\frac{1}{\sqrt{\omega}}\frac{\partial}{\partial\varphi_{\lambda,\pm}} + \sqrt{\omega}\,\varphi_{\lambda,\pm}\,, \tag{4.24}$$

and correspond to the following choice of positive-frequency basis functions

$$\begin{aligned}w_{\lambda,+}(x)\,\Big|_{R_+} &= e^{-\omega_\lambda t}R_\lambda(\boldsymbol{x})\,,\\ w_{\lambda,+}(x)\,\Big|_{R_-} &= 0\end{aligned} \tag{4.25}$$

$$w_{\lambda,-}(x)\Big|_{R_-} = e^{i\omega_\lambda t}R_\lambda(\boldsymbol{x}),$$
$$w_{\lambda,-}(x)\Big|_{R_+} = 0 \tag{4.26}$$

(one should remember that the Killing time coordinate t is past pointing in R_- and $w_\pm$ by construction have zero initial data on $\Sigma_\mp$). The corresponding (Boulware) vacuum states $\Psi_{B\pm}$ are defined by Eq. (4.13).

This is a matter of a simple algebra, using the reparametrization (4.14), to show that the operators (4.23) are related to (4.21) by a nontrivial Bogolyubov transformation which mixes the positive and negative frequencies

$$b_\pm = \left(2\sinh\frac{\beta_0\omega}{2}\right)^{-1/2}\left[e^{\beta_0\omega/4}a_\pm + e^{-\beta_0\omega/4}a^\dagger_\mp\right] \tag{4.27}$$

and generates, in terms of $w_\pm$, the basis functions $v_{\lambda,\pm}(x)$ associated with the creation annihilation operators $a_{\lambda,\pm}$ of our vacuum quantum state (4.21)

$$v_\pm = \left(2\sinh\frac{\beta_0\omega}{2}\right)^{-1/2}\left[e^{\beta_0\omega/4}w_\pm + e^{-\beta_0\omega/4}w^*_\mp\right]. \tag{4.28}$$

This is a well-known transformation relating the Boulware vacua, $(b_{\lambda,\pm},\ w_{\lambda,\pm}(x))$, in the right (R_+) and left (R_-) wedges of the Kruskal diagram to the Hartle-Hawking vacuum, $(a_{\lambda,\pm},\ v_{\lambda,\pm}(x))$, of quantum fields on the maximally extended black hole spacetime [60,62]. The latter is defined by the condition that its basis functions $v_{\lambda,\pm}(x)$ contain only positive frequencies with respect to affine parameter on both horizons of the black hole metric. This property follows from Eqs. (4.25), (4.26), (4.28) and the asymptotic behaviors of $w_{\lambda,\pm}(x)$ at the horizon (see Appendix B):

$$w_{\omega lmA,+}(x)\Big|_{R_+} = (16\pi\omega M^2)^{-1}\begin{cases}\overline{A^+_{\omega lA}}(-U)^{4M\omega i}\hat{Y}_{lm}(\vartheta,\phi), & x\to H^-_+,\\ A^+_{\omega lA}(V)^{-4M\omega i}\hat{Y}_{lm}(\vartheta,\phi), & x\to H^+_+,\end{cases} \tag{4.29}$$

$$w_{\omega lmA,-}(x)\Big|_{R_-} = (16\pi\omega M^2)^{-1}\begin{cases}A^+_{\omega lA}(U)^{-4M\omega i}\hat{Y}_{lm}(\vartheta,\phi), & x\to H^-_-,\\ \overline{A^+_{\omega lA}}(-V)^{4M\omega i}\hat{Y}_{lm}(\vartheta,\phi), & x\to H^+_-,\end{cases} \tag{4.30}$$

where $A^+_{\omega lA}$ and $\overline{A^+_{\omega lA}}$ are complex coefficients defined in the Appendix B. We use the notations $H^-_\pm$ and $H^+_\pm$ for parts of past H^- and future H^+ horizons located in the $R_\pm$ wedges of the Kruskal diagram (superscripts $\pm$ correspond to $t\to\pm\infty$), so that we have $H^+ = H^+_- \cup H^+_+$ and $H^- = H^-_- \cup H^-_+$ (see Fig. 2). Substituting the asymptotics into (4.28) one finds

$$v_{\omega lmA,+}(x)\Big|_{H^-} = (16\pi\omega M^2)^{-1}\overline{A^+_{\omega lA}}\left(2\sinh\frac{\beta_0\omega}{2}\right)^{-1/2} \times\left[\theta(-U)\,e^{2\pi M\omega}(-U)^{4M\omega i} + \theta(U)\,e^{-2\pi M\omega}U^{4M\omega i}\right]\hat{Y}_{lm}(\vartheta,\phi). \tag{4.31}$$

It means that the basis functions v are of positive frequency with respect to the affine parameter U on the horizon H^+ and hence are analytic in the lower half of the complex U plane. The analogous property holds for another horizon H^- of the Kruskal diagram. Those are exactly the properties which single out the positive frequency basic functions used for the definition of the Hartle-Hawking vacuum state [60]. As was mentioned above, similar considerations apply to fields of higher spins (massless as well as massive). Thus, the proposed no-boundary wave function of a black hole represents the Hartle-Hawking vacuum state of linearized field excitations of all physical fields.

At the end of this section we would like to discuss the remarkable duality relations between interior and exterior of a black hole. We remarked already in Section II that the Kruskal metric (2.1) possesses a discrete symmetry $U\to -U$, $V\to -V$, which on the Einstein-Rosen bridge is reduced to the isometry between its external Σ_+ and internal Σ_- parts. As the result of this isometry, one can use in the decomposition (4.10) the same spatial harmonics for exterior Σ_+ and interior Σ_-. Moreover, the Euclidean action is symmetric with respect to the change $\varphi_+\to\varphi_-$, and the no-boundary wave function (4.14) of a black hole is symmetric with respect to the transposition of the interior and exterior parts of the Einstein-Rosen bridge. We call this property *duality*. The duality of the no-boundary wave function of a black hole is evident for perturbations, because we have the explicit expression for it. But this property is of more general nature. The functional integral representation (3.5) for the no-boundary wave function of a black hole does not distinguish between the exterior and interior of a black hole; that is why it possesses the duality property even if we consider the contribution of fluctuations which are not small. We should stress that the duality property is the consequence of the symmetry between interior and exteror of an eternal black hole. For a "real" black hole formed as a result of the gravitational collapse, this exact symmetry is broken. Nevertheless, since there exists a close relation between physics of a real black hole and its eternal version, the duality of the above type plays an important role and allows one, for example, to explain why the approach based on identifying the dynamical degrees of freedom of a black hole with its external modes gives formally the same answer for the dynamical entropy of a black hole as our approach. We shall return to this point at the end of the next section.

V. ONE-LOOP CONTRIBUTION TO ENTROPY OF A BLACK HOLE

Now we return to the problem of a black hole entropy. According to the above procedure of separating the physical variables into observable and unobservable ones, the proposed wave function generates the density matrix of a black hole interior as a functional trace Tr_+ over the values of the field $\varphi_+(\boldsymbol{x})$ outside the horizon (Fig. 5),

$$\rho(\varphi'_-,\varphi_-) = \mathrm{Tr}_+ \,|\Psi\rangle\langle\Psi| \equiv \int D\varphi_+\, \Psi^*(\varphi_+,\varphi'_-)\, \Psi(\varphi_+,\varphi_-). \tag{5.1}$$

We define the "dynamical" (or "statistical mechanical") entropy of a black hole by the standard relation $\boldsymbol{S} = -\mathrm{Tr}\,[\,\hat\rho\ln\hat\rho]$. In the functional coordinate representation we use, the expression for the dynamical entropy of the black hole reads

$$\boldsymbol{S} = -\mathrm{Tr}\,[\,\hat\rho\ln\hat\rho\,] = -\int D\varphi\langle\varphi(\boldsymbol{x})\mid\hat\rho\ln\hat\rho\mid\varphi(\boldsymbol{x})\rangle. \tag{5.2}$$

Up to normalization, the wave function defined by the path integral over physical degrees of freedom (3.6) in the τ foliation of the Euclidean spacetime, $-\beta_0/4<\tau<\beta_0/4$, actually represents the heat kernel or the matrix element between the configurations φ_- and φ_+ of the Euclidean "evolution" operator $\exp(-\beta_0\hat H/2)$,

$$\Psi\,(\varphi_+,\varphi_-) = \exp\,(\boldsymbol{\Gamma}/2)\,\langle\varphi_+\mid\exp\,(-\beta_0\hat H/2)\mid\varphi_-\rangle, \tag{5.3}$$

where $\hat H$ is a physical Hamiltonian of the system and $\beta_0 = 8\pi M$. In the full nonperturbative treatment of the problem this Hamiltonian is a complicated functional of physical degrees of freedom, numerically coinciding with the ADM surface integral, while in the linearized approximation (relevant to the one-loop order of semiclassical expansion) it is just an additive sum of quadratic Hamiltonians of fields of all spins on the Schwarzschild-Kruskal background. In particular, for a scalar field we consider here it is the following expression generated by the Lagrangian (4.1):

$$H(\varphi,p) = \frac{1}{2}\int d^3x\left[\frac{p^2}{g^{\tau\tau}g^{1/2}} + g^{1/2}\left(g^{ab}\partial_a\varphi\partial_b\varphi + \xi R\varphi^2\right)\right], \tag{5.4}$$

where p is the momentum conjugated to φ. Using the composition law for Euclidean evolution kernels and the completeness condition, $\int D\varphi_+\,|\varphi_+\rangle\langle\varphi_+| = \hat 1$, we obtain from (5.3) the following representation for the density matrix:

$$\rho(\varphi'_-,\varphi_-) = \exp\,(\boldsymbol{\Gamma})\,\langle\varphi'_-\mid\exp\,(-\beta_0\hat H)\mid\varphi_-\rangle. \tag{5.5}$$

Here $\boldsymbol{\Gamma}$ is defined from the normalization conditions

$$\mathrm{Tr}_-\hat\rho = \int D\varphi_-\,\rho(\varphi_-,\varphi_-) = 1 \tag{5.6}$$

and, thus, is generated by the Euclidean path integral over the fields on the full Schwarzschild-Kruskal instanton obtained from the manifold on Fig. 5 by gluing together the two shores $\boldsymbol{\Sigma}_-{}'$ and $\boldsymbol{\Sigma}_-$ of its cut. It should be emphasized here that the above procedure of gluing together the two Euclidean spacetimes with boundaries naturally arises as a graphical representation of calculating the expectation value with respect to the no-boundary quantum state (or the generalization thereof). This procedure is opposite to that of Ref. [59] in which

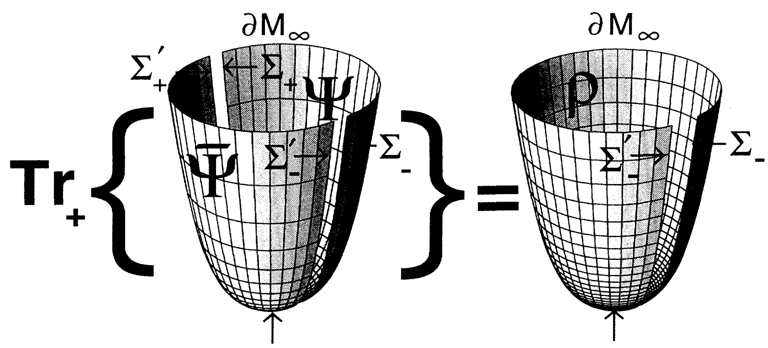

FIG. 5. The wave function $\Psi(\varphi_+,\varphi_-)$ is defined as a path integral over the physical degrees of freedom in the τ foliation of the Euclidean spacetime on a half-instanton. The density matrix $\rho(\varphi'_-,\varphi_-) = \mathrm{Tr}_+\,|\Psi\rangle\langle\Psi|$ is proportional to the analogous functional integral, but on the whole instanton, since the proposed wave function can be interpreted as an amplitude of an Euclidean evolution from the initial state φ_- to the final state φ_+ during the time interval $\beta/2$. Then $\overline{\Psi}(\varphi_+,\varphi'_-)$ implies the evolution from φ_+ to φ'_- and the density matrix is the kernel of the evolution from φ_- to φ'_- during the Euclidean time interval $0<\tau<\beta$. The arguments of the density matrix $\varphi'_-(\boldsymbol{x}')$ and $\varphi_-(\boldsymbol{x})$ are the values of the fields on different sides $\boldsymbol{\Sigma}_-{}'$ and $\boldsymbol{\Sigma}_-$ of the cut in the instanton.

the division of the Euclidean manifold into two disconnected parts was used for introducing the wave function of matter fields on a gravitational background and establishing the formalism of thermofields. Thus, in contrast with Ref. [59], in our approach we start with the definition of the black hole wave function and arrive at the notion of the full gravitational instanton (without dynamically active boundaries) as an ingredient of calculating various expectation values and dynamical entropy, in particular.

In addition to the density matrix $\hat{\rho}$ it is convenient also to define a more general object $\hat{\rho}_\beta$ which depends on the arbitrary parameter β independent of the black hole mass

$$\rho_\beta\,(\varphi'_-,\varphi_-) = \langle \varphi'_- \mid \hat{\rho}_\beta \mid \varphi_-\rangle = \exp\left(\mathbf{\Gamma}_\beta\right) \langle \varphi'_- \mid \exp\left(-\beta\hat{H}\right) \mid \varphi_-\rangle, \quad (5.7)$$

so that the density matrix of a black hole $\hat{\rho}$ (5.1) coincides with $\hat{\rho}_{\beta_0}$. In the one-loop approximation we have

$$\langle \varphi'_- \mid \exp\left(-\beta\hat{H}\right) \mid \varphi_-\rangle = \left[\det \frac{1}{2\pi}\frac{\partial^2 \boldsymbol{I}_\beta(\varphi'_-,\varphi_-)}{\partial\varphi'_-\partial\varphi_-}\right]^{1/2} \exp\left[-\boldsymbol{I}_\beta(\varphi'_-,\varphi_-)\right], \quad (5.8)$$

where the Euclidean Hamilton-Jacobi function $\boldsymbol{I}_\beta(\varphi'_-,\varphi_-)$ is given by the expression (4.11) with the only replacement $\beta_0 \to 2\beta$. In the coordinate representation one has the following expression for the kernel of the Van Vleck-Morette functional matrix:

$$\frac{\partial^2 \boldsymbol{I}_\beta(\varphi'_-,\varphi_-)}{\partial\varphi'_-(\boldsymbol{x})\partial\varphi_-(\boldsymbol{y})} = g^{\tau\tau}g^{1/2}\frac{\hat{\omega}}{\sinh\beta\hat{\omega}}\delta(\boldsymbol{x}-\boldsymbol{y}) \quad (5.9)$$

with the operator of frequency $\hat{\omega}$ defined on a spatial three-dimensional hypersurface as

$$\hat{\omega} = \left[-\frac{1}{g^{\tau\tau}g^{1/2}}\partial_a g^{1/2}g^{ab}\partial_b + \frac{1}{g^{\tau\tau}}\xi R\right]^{1/2}, \quad a,b = 1,2,3, \quad (5.10)$$

$$g \equiv \det g_{\mu\nu} = g_{\tau\tau}\det g_{ab}, \quad \mu,\nu = 0,1,2,3. \quad (5.11)$$

The normalization factor of Eq. (5.7) is, therefore, given by the following functional determinant on the space of functions of three spatial coordinates:

$$\mathbf{\Gamma}_\beta = -\ln\left[\int D\varphi_- \langle \varphi_- \mid \exp\left(-\beta\hat{H}\right) \mid \varphi_-\rangle\right] \quad (5.12)$$

$$= -\frac{1}{2}\ln\det\left[\frac{1}{2(\cosh\beta\hat{\omega}-1)}\delta(\boldsymbol{x}-\boldsymbol{y})\right]. \quad (5.13)$$

It is worth emphasizing that all the quantities and operators entering the WKB approximation of the wave function and density matrix depend on a three-geometry of space and values of fields on it. The whole information about four-dimensional manifold is contained in the interval β of Euclidean time between the points with the same spatial coordinates $\boldsymbol{x}$ of spacelike slices $\mathbf{\Sigma}_+$ and $\mathbf{\Sigma}_-$. We stress once again that according to our definition we need to consider β as an arbitrary parameter and only at the end to put it equal to $\beta_0 \equiv 8\pi M$.

The density matrix $\hat{\rho}_\beta$ satisfies the equation

$$\frac{\partial\hat{\rho}_\beta}{\partial\beta} = \left(\frac{\partial\mathbf{\Gamma}_\beta}{\partial\beta} - \hat{H}\right)\hat{\rho}_\beta. \quad (5.14)$$

Using this relation one can easily show that the entropy of the system in question can be obtained from the effective action $\mathbf{\Gamma}_\beta$:

$$\boldsymbol{S} = \boldsymbol{S}_\beta\Big|_{\beta=8\pi M}, \quad (5.15)$$

$$\boldsymbol{S}_\beta \equiv -\mathrm{Tr}\left[\hat{\rho}_\beta\ln\hat{\rho}_\beta\right] = -\mathrm{Tr}\left[\left(\mathbf{\Gamma}_\beta - \beta\hat{H}\right)\hat{\rho}_\beta\right] = \beta\frac{\partial\mathbf{\Gamma}_\beta}{\partial\beta} - \mathbf{\Gamma}_\beta. \quad (5.16)$$

Note that it would be incorrect to differentiate $\mathbf{\Gamma}$ directly over M in order to obtain the entropy $\boldsymbol{S}$, since the total effective action is an integral over the whole space and depends also on its geometry. The Hawking temperature $T_{\mathrm{BH}} = 1/8\pi M$ depends both on the space-geometry and on $g_{\tau\tau}$ of the four-dimensional metric, and hence operations of differentiation over M and integration over volume do not commute in general case. In order to avoid these complications we introduced the generalized density matrix $\hat{\rho}_\beta$.

In order to calculate Tr ln operation entering the expression for the effective action, it is convenient once again to use the expansion the functions $\varphi(\boldsymbol{x})$ in terms of eigenfunctions $R_\lambda(\boldsymbol{x})$ of the operator $\hat{\omega}$,

$$\varphi(\boldsymbol{x}) = \sum_\lambda \varphi_\lambda R_\lambda(\boldsymbol{x}), \qquad \hat{\omega}^2 R_\lambda(\boldsymbol{x}) = \omega_\lambda^2 R_\lambda(\boldsymbol{x}). \quad (5.17)$$

Here $\sum_\lambda$ denotes the sum over all quantum numbers λ. Substitution of the expansion of δ function in terms of eigenfunctions of the operator $\hat{\omega}$ gives

$$\boldsymbol{S} = \int d\boldsymbol{x}\left(\beta\frac{\partial}{\partial\beta} - 1\right)\ln\left(2\sinh\frac{\beta\hat{\omega}}{2}\right)\delta(\boldsymbol{x}-\boldsymbol{y})\Big|_{\boldsymbol{y}=\boldsymbol{x},\beta=\beta_0}$$
$$= \int d\boldsymbol{x}\left[\frac{\beta_0\hat{\omega}_y}{2}\coth\frac{\beta_0\hat{\omega}_y}{2} - \ln\left(2\sinh\frac{\beta_0\hat{\omega}_y}{2}\right)\right]\sum_\lambda g^{\tau\tau}(\boldsymbol{x})g^{\frac{1}{2}}(\boldsymbol{x})R_\lambda(\boldsymbol{x})R_\lambda(\boldsymbol{y})\Big|_{\boldsymbol{y}=\boldsymbol{x}}$$
$$= \int d\boldsymbol{x}g^{\tau\tau}g^{1/2}\sum_\lambda R_\lambda^2(\boldsymbol{x})\left[\frac{\beta_0\omega_\lambda}{2}\coth\frac{\beta_0\omega_\lambda}{2} - \ln\left(2\sinh\frac{\beta_0\omega_\lambda}{2}\right)\right].$$

Thus we have

$$S = \int dx \sum_{\lambda} \mu_{\lambda}(x) s(\beta_0 \omega_{\lambda}), \qquad (5.18)$$

$$s(\beta\omega) = \frac{\beta\omega}{e^{\beta\omega} - 1} - \ln(1 - e^{-\beta\omega}), \qquad (5.19)$$

where $s(\beta\omega)$ is a well-known expression for the entropy of a single oscillator with the frequency ω at temperature $T = 1/\beta$ and

$$\mu_{\lambda}(x) = g^{\tau\tau} g^{1/2} [R_{\lambda}(x)]^2 \qquad (5.20)$$

is a phase space density of quantum modes. It should be emphasized that according to our definition the integration in (5.16) is over the interior part Σ_- of the Einstein-Rosen bridge.

In order to estimate the contribution of spatial regions in the vicinity of the horizon we should find an asymptotic solution for the mode functions $R_{\lambda}(x)$ near the horizon. Eigenfunctions $R_{\lambda}(x)$ for a massless scalar field in the Schwarzschild spacetime are of the form [see Eq. (B2) of Appendix B]

$$R_{\omega lmA}(r, \vartheta, \phi) = R_{\omega lA}(r) \hat{Y}_{lm}(\vartheta, \phi). \qquad (5.21)$$

Here radial functions $R_{\omega lA}(r)$ are real and obey the equation

$$\left[\frac{d}{dr}(r^2 - 2Mr)\frac{d}{dr} - l(l+1) + \omega^2 \frac{r^3}{r - 2M}\right] R_{\omega lA}(r) = 0. \qquad (5.22)$$

Additional index $A = 1, 2$ is used to numerate two linearly independent solutions. The expression for entropy of such a system takes the form

$$S = \int_{2M}^{\infty} dr r^2 (1 - 2M/r)^{-1} \times \int_0^{\infty} d\omega \sum_A \sum_{l=0}^{\infty} (2l+1) [R_{\omega lA}(r)]^2 s(8\pi M\omega). \qquad (5.23)$$

By using Eq. (B23) we can write near the horizon the relation

$$\sum_{l=0}^{\infty} \sum_{A=1}^{2} (2l+1)|R_{\omega lA}|^2 \approx \sum_{l=0}^{\infty} (2l+1)|R^{up}_{\omega l}|^2 + \frac{1}{8\pi M^2} \sum_{l=0}^{\infty} (2l+1)|t_{\omega l}|^2 \approx \frac{4\omega^2}{\pi} \frac{M}{r - 2M} \qquad (5.24)$$

with complex basis functions $R^{up}_{\omega l}$ and absorption coefficients $t_{\omega l}$ defined in Appendix B. To obtain the latter equality we used the result of Ref. [31] for the sum $\sum_{l=0}^{\infty} (2l+1)|R^{up}_{\omega l}|^2$ and the fact that $t_{\omega l}$ are exponentially decreasing functions of l, so that the sum $\sum_{l=0}^{\infty} (2l+1)|t_{\omega l}|^2$ remains finite at the horizon. [The functions $\vec{R}_l(\omega|r)$ in the notations of Ref. [31] are related to our $R^{up}_{\omega l}$ as $\vec{R}_l(\omega|r) = \sqrt{2\pi} R^{up}_{\omega l}$.] The relation (5.24) can be used to find out the contribution of the internal modes located near the horizon to the entropy of a black hole.

We discuss now the contribution to S of large distances. Formally this contribution is divergent. The divergency is directly related to the divergency of the entropy of thermal radiation if a black hole is located inside infinite thermal bath. It is well known that the latter problem is unphysical because the system is unstable. Stability can be provided if a black hole is located inside a cavity of size $r_0 \leq 3M$ (see, e.g., Ref. [12]). For such a case the above calculations are to be modified. Instead of the modes $R_{\omega lA}$ one must use the modes $R_{\omega l0}$, obeying given boundary conditions at the boundary $r = r_0$. (See Appendix B for their definition.) The expression for the entropy of a black hole will remain the same (5.23) with the replacement of $\sum_A |R_{\omega lA}(r)|^2$ by $|R_{\omega l0}|^2$ and restriction of integration with respect to r by the upper limit r_0. In order to estimate this integral we remark that the entropy of an oscillator with frequency ω exponentially decreases for frequencies much larger than the black hole temperature $T = 1/8\pi M$. In the vicinity of the horizon $| r/2M - 1 | \ll 1$ a regular solution for radial modes $R_{\omega l0}(r)$ takes a simple form

$$R_{\omega l0}(r) \simeq Q(M, \omega, l) K_{i4M\omega}\left[\sqrt{2l(l+1)(r/M - 2)}\right]. \qquad (5.25)$$

The normalization factor $Q(M, \omega, l)$ depends on the coefficient of penetration of modes through the potential barrier. All the modes in the range of frequencies in question and with angular quantum numbers $l \geq 3$ are trapped. For such modes the penetration coefficient is exponentially small and normalization factor does not depend on l. The larger l, the closer to the horizon a return point lies and the better the approximation becomes. The evaluation of normalization factor gives

$$Q(M, \omega, l) \simeq \left[\frac{2\omega \sinh(4\pi M\omega)}{M\pi^2}\right]^{1/2}, \qquad (5.26)$$

where the relation

$$\int_0^{\infty} dy \, \frac{1}{y} \, K_{ix}(y) K_{ix'}(y) = \frac{\pi^2}{2} \frac{1}{x \sinh(\pi x)} \delta(x - x') \qquad (5.27)$$

was used. We also use the fact that the modified Bessel functions decrease very fast with an increase of their argument and, hence, with a good accuracy the integration along the radius can be extended to infinity. One can see that the main contribution to the integral of entropy near the horizon comes from large l. Replacement in Eq. (5.23) of summation over l by integration leads to the expression

$$\sum_{l=0}^{\infty} (2l+1) [R_{\omega l0}(r)]^2 \simeq 2 \int_0^{\infty} dl \, l \, [R_{\omega l0}(r)]^2 = \frac{4\omega^2}{\pi} \frac{M}{r - 2M}. \qquad (5.28)$$

This asymptotic formula coincides with (5.24). This means that in both cases (a black hole in infinite space and a black hole inside a cavity) the contribution to entropy of the region near the horizon is the same. In what follows we assume that the black hole is located inside a reflecting boundary sphere and restrict the integration over r by the value r_0.

Thus we have

$$\begin{aligned} \boldsymbol{S} &= \frac{4M}{\pi}\int_{2M}^{r_0} dr \frac{r^3}{(r-2M)^2}\int_0^\infty d\omega\omega^2 s(8\pi M\omega) \\ &\simeq \frac{512M^5}{\pi}\int_0^{z_0} dz \frac{1}{z^3}\int_0^\infty d\omega\omega^2 s(8\pi M\omega) \\ &= \frac{4M^2}{45}\int_0^{z_0} dz \frac{1}{z^3}, \end{aligned} \tag{5.29}$$

where

$$z_0 = r_0\sqrt{1-2M/r_0} + M\ln\left[(r_0/M) - 1 + (r_0/M)\sqrt{1-2M/r_0}\right] \tag{5.30}$$

is a proper distance from the horizon to the point r_0. This result shows that one-loop entropy of a black hole $\boldsymbol{S}$ diverges near the horizon (at $z=0$). The expression (5.29) gives the leading divergent term

$$\boldsymbol{S} = \frac{A^H}{360\pi l^2}, \tag{5.31}$$

where A^H is the surface area of a black hole and l is the proper-distance cutoff parameter. It reproduces the result by Frolov and Novikov [30]. This divergence is physical and its origin does not depend on particular properties of quantum fields surrounding a black hole. The analogous divergence evidently occurs for higher spins and for rotating and charged black holes. Hence quantum corrections can never be neglected in a description of thermodynamical properties of black holes.

The dynamical entropy S of a black hole is obtained by summing over the contributions of fields of different spins. In this respect all the matter fields inside a black hole contribute to its dynamical entropy. But even if we consider a hypothetical case when there is no other physical field but the gravitational one, we have a nonzero answer due to the inevitable presence of gravitational perturbations.

It is worth emphasizing that shifting a position of the horizon as a whole due to the back reaction effect of quantum fields on the black hole geometry does not remove the divergence. Spatially inhomogeneous fluctuations of the horizon are to be taken into account to provide the necessary cutoff.

To conclude this section we make some general remarks concerning the relation of our result to the results obtained in other dynamical approaches to the black hole entropy. First of all, we explain why our result based on the calculations for the eternal version of a black hole reproduces the results of [30] for a real physical black hole. The reason is the following. The main contribution to the black hole entropy in that approach originates from the modes of field perturbations propagating inside a black hole in a close vicinity of its event horizon. These modes are thermally excited and their density matrix with very high accuracy is thermal and does not depend on the particular choice of the initial state, provided this state is regular at the horizon. That is why the leading divergent term in the dynamical entropy calculated for the Unruh vacuum, or its modification, which differs by additional incoming particles with finite energy (less than the mass of a black hole) is the same as for the Hartle-Hawking state. But the Hartle-Hawking state, as we have shown, coincides with our no-boundary wave function. That is why the calculations of the dynamical entropy for the real black hole and its eternal version give the same answer.

In our calculations we begin with a pure quantum state—the no-boundary wave function of a black hole. The dynamical entropy S of a black hole arises as the result of splitting the system into two parts (in our case internal and external states) and averaging over one part of the system (in our case external states). One can define another entropy S' by averaging over another part of the system. It can be shown that these two entropies S and S' in fact coincide [22]. The easiest way to prove it is to use a biorthogonal or Schmidt canonical basis [24] in which the spectra of the reduced density matrices of the two subsystems explicitly coincide [25]. In application to our problem this means that the entropy of the internal excitations of an eternal black hole is formally equal to the entropy of its external excitations. Because of the presence of the divergence one cannot use this property directly. But because of the duality of the interior and exterior of a black hole, not only are the total values of the entropies S and S' the same, but also the explicit expressions for them. Namely the dynamical entropy as given by Eq. (5.16) is identical to one which is obtained by reversing the proposed procedure and averaging at first over internal modes and after making summation in the expression for the entropy over the external modes. In the latter case the expression will contain the spatial integration over the exterior. That is why for a symmetric choice of the cutoff, the finite values of both entropies (external and internal one) are the same. This explains why the calculations based on identifying the dynamical degrees of freedom of a black hole with its external excitations (as it was done by 't Hooft [20] and others) give formally the same result as our approach. The difference in the interpretations arises when we relate the calculations for the eternal version of a black hole to the entropy of a real physical black hole.

VI. ENTROPY AND EFFECTIVE ACTION

In the preceding section we used the proposal for a wave function of a black hole in the calculation of the contribution to the entropy of a scalar field. Only the properties of a three-dimensional space and fields on it were used in these calculations. It is instructive to compare this result with that of the four-dimensional Euclidean action approach. This also allows one to generalize the result of the preceding section to arbitrary static black holes.

Consider the Euclidean effective action $\mathbf{\Gamma}_\beta$ of a conformal scalar field $\phi(\tau, \boldsymbol{x})$ with the Hamiltonian Eq. (5.4) on a manifold periodic in Euclidean time with the period β. Up to a contribution of a local functional measure it can be represented in the form

$$\mathbf{\Gamma}_\beta = +\frac{1}{2}\mathrm{Tr}\ln F + \delta^4(0)(\cdots) \ , \tag{6.1}$$

$$F = -\Box + \frac{1}{6}R \ . \tag{6.2}$$

This effective action and the corresponding free energy $\boldsymbol{F}_\beta = \mathbf{\Gamma}_\beta/\beta$ have ultraviolet divergences. Note that, although the last term in Eq. (6.1) diverges, it is proportional to β and hence the free energy does not depend on β and its contribution into entropy vanishes. The same argument remains valid for all ultraviolet divergences in the effective action. Thus we have

$$\boldsymbol{S} = \beta^2 \frac{\partial}{\partial\beta}\boldsymbol{F}_\beta = \beta^2 \frac{\partial}{\partial\beta}\boldsymbol{F}_\beta^{\mathrm{Ren}}. \tag{6.3}$$

The scalar field effective action and thermodynamic potential at finite temperature in a static curved spacetime were calculated by Dowker and Schofield [65]. They proved that in the case of the conformal scalar field the two renormalized effective actions in two conformally related spaces with metrics $g_{\mu\nu}$ and $\bar g_{\mu\nu} = e^{-2\Omega}g_{\mu\nu}$ are related by

$$\Delta\mathbf{\Gamma}_\beta[g,\Omega] = \mathbf{\Gamma}_\beta^{\mathrm{Ren}}[\bar g] - \mathbf{\Gamma}_\beta^{\mathrm{Ren}}[g] \ , \tag{6.4}$$

$$\Delta\mathbf{\Gamma}_\beta[g,\Omega] = -\frac{1}{2880\pi^2}\int_0^\beta d\tau \times \int g^{\frac{1}{2}} d^3x \left[+3(\Box\Omega)^2 - 4\Omega^\sigma\Omega_\sigma\,\Box\,\Omega + 2(\Omega^\sigma\Omega_\sigma)^2 - 2R_{\mu\nu}\Omega^\mu\Omega^\nu + \Omega\left\{R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - R_{\alpha\beta}R^{\alpha\beta} + \Box R\right\}\right] \ ,$$

where $\Omega_\mu = \nabla_\mu\Omega$ and $\mathbf{\Gamma}_\beta^{\mathrm{Ren}}[\bar g]$ is the renormalized value of the effective action in the space with the metric $\bar g_{\mu\nu}$,

$$\mathbf{\Gamma}[\bar g] = \frac{1}{2}\mathrm{Tr}\ln\bar F, \quad \bar F = -\bar\Box + \frac{1}{6}\bar R \ . \tag{6.5}$$

The difference $\Delta\mathbf{\Gamma}_\beta[g,\Omega]$ for two conformally related theories is proportional to β and hence does not contribute to the entropy. In order to obtain the leading (divergent near the horizon) contribution to the black hole entropy, we apply these relations to the particular case of an ultrastatic metric $\bar g$ corresponding to the choice $\Omega \equiv \frac{1}{2}\ln g_{\tau\tau}$. We show that the three-dimensional part

$$d\bar l^2 = q_{ab}(\boldsymbol{x})dx^a dx^b \tag{6.6}$$

of the ultrastatic metric

$$d\bar s^2 = d\tau^2 + d\bar l^2 \tag{6.7}$$

conformal to the metric of an arbitrary (distorted) static black hole metric, can be approximated by the three-metric of constant negative curvature. We use this property to analyze the leading contribution to the entropy divergent near the horizon.

For the calculation of the effective action in the ultrastatic metric $\bar g$ we apply the heat-kernel technique. In the proper time representation, the thermal effective action of a scalar field takes the form

$$\mathbf{\Gamma}_\beta[\bar g] = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\mathrm{Tr}\bar K_\beta(s) \ , \tag{6.8}$$

where the heat kernel $\bar K_\beta$ is a periodic in τ with a period β solution of the problem

$$\begin{aligned} \frac{\partial}{\partial s}\bar K_\beta(s|\tau,\boldsymbol{x};\tau',\boldsymbol{x}') &= \bar F \bar K_\beta(s|\tau,\boldsymbol{x};\tau',\boldsymbol{x}') \ , \\ \bar K_\beta(s|\tau,\boldsymbol{x};\tau',\boldsymbol{x}') &= \bar K_\beta(s|\tau+\beta,\boldsymbol{x};\tau',\boldsymbol{x}') \ , \\ \bar K_\beta(0|\tau,\boldsymbol{x};\tau',\boldsymbol{x}') &= \delta(\tau-\tau')\delta(\boldsymbol{x}-\boldsymbol{x}') \ . \end{aligned} \tag{6.9}$$

It can be obtained by the method of images

$$\bar K_\beta(s|\tau,\boldsymbol{x};\tau',\boldsymbol{x}') = \sum_{n=-\infty}^{\infty} \bar K(s|\tau,\boldsymbol{x};\tau'+\beta n,\boldsymbol{x}') \tag{6.10}$$

from the nonperiodic heat kernel $\bar K = \bar K_\infty$ defined on a complete interval $-\infty < \tau,\tau' < \infty$. Because of the separation of variables in the operator $\bar F = -\partial^2/\partial\tau^2 - \bar\Delta + 1/6\bar R$, the heat kernel $\bar K$ takes the form

$$\bar K(s|\tau,\boldsymbol{x};\tau',\boldsymbol{x}') = (4\pi s)^{-\frac{1}{2}}\exp\left[-\frac{(\tau-\tau')^2}{4s}\right]^3 \times \bar K(s|\boldsymbol{x};\boldsymbol{x}') \ , \tag{6.11}$$

where ${}^3\bar K(s|\boldsymbol{x};\boldsymbol{x}')$ is a three-dimensional analogue of the heat kernel corresponding to the operator $-\bar\Delta + 1/6\bar R$. From Eqs. (6.10) and (6.11) we have

$$\bar K_\beta(s|\tau,\boldsymbol{x};\tau',\boldsymbol{x}') = \theta_3\left(-i\frac{(\tau-\tau')\beta}{4\pi s}, \exp\left[-\frac{\beta^2}{4s}\right]\right) \times {}^3\bar K(s|\tau,\boldsymbol{x};\tau',\boldsymbol{x}') \ , \tag{6.12}$$

where θ_3 is a Riemann theta function. The "zero-temperature" heat kernel $\bar K(s|\tau,\boldsymbol{x};\tau',\boldsymbol{x}')$ can be expanded in the nonlocal series in powers of spacetime curvatures [63,66]. To calculate the effective action we need the trace of the heat kernel with coincident points $(\tau = \tau',\ \boldsymbol{x} = \boldsymbol{x}')$. In the notation of Ref. [63] it reads

$$\mathrm{Tr}\bar K_\beta(s|\tau,\boldsymbol{x};\tau,\boldsymbol{x}) = \theta_3\left(0, \exp\left[-\frac{\beta^2}{4s}\right]\right) \times \mathrm{Tr}\bar K(s|\tau,\boldsymbol{x};\tau,\boldsymbol{x}) \ . \tag{6.13}$$

Here

$$\theta_3(0,\exp[-b]) = \sum_{n=-\infty}^{\infty} \exp[-bn^2] \,, \tag{6.14}$$

$$\mathrm{Tr}\bar{K}(s|\tau,\boldsymbol{x};\tau,\boldsymbol{x}) = \frac{1}{(4\pi s)^2}\int_0^\beta d\tau \int d^3x\sqrt{\bar{g}}\Bigg\{1 + s\bar{P} + s^2\Bigg[\bar{R}_{\mu\nu} f_1(-s\,\bar{\Box})\bar{R}^{\mu\nu} + \bar{R} f_2(-s\,\bar{\Box})\bar{R} + \bar{P} f_3(-s\,\bar{\Box})\bar{R} + \bar{P} f_4(-s\,\bar{\Box})\bar{P}\Bigg]\Bigg\} + O(\text{curvatures}^3), \tag{6.15}$$

$f_i(-s\,\bar{\Box})$ are nonlocal form factors [63] and $\bar{P} = 0$ for conformal scalar field. The first two terms in this expression are local. Nonlocalities appear only in quadratic and higher orders in curvature terms.

Consider a static black hole. In the general case its Euclidean metric can be written in the form [67]

$$ds^2 = X d\tau^2 + \frac{1}{4\kappa^2 X} dX^2 + r_{AB} dx^A dx^B, \tag{6.16}$$

where $A,B = 2,3$, $\kappa = \kappa(X,x^A)$, $r_{AB} = r_{AB}(X,x^A)$, and $X = 0$ is the equation of the horizon. The vacuum Einstein equations imply that near the horizon one has [68]

$$\kappa = \kappa_0 - (1/2)k_0(x^A)X + O(X^2), \tag{6.17}$$

$$r_{AB} = r_{0AB}[1 + (1/2\kappa_0)k_0(x^A)X] + O(X^2), \tag{6.18}$$

and κ_0 =const is the surface gravity of a black hole. The corresponding conformal ultrastatic metric reads

$$d\bar{s}^2 = d\tau^2 + d\bar{l}^2, \tag{6.19}$$

$$d\bar{l}^2 = \frac{\kappa_0^2}{\kappa^2(\boldsymbol{x})} dz^2 + \exp(-2\kappa_0 z) r_{AB} dx^A dx^B, \tag{6.20}$$

where $z = (2\kappa_0)^{-1}\ln X$. In these coordinates the horizon corresponds to $z = -\infty$. Near the horizon $\kappa \to \kappa_0$ =const , and two-dimensional surfaces z=const (which have infinitely growing radius) can be locally approximated by planes. In any finite region of space the metric (6.19) with high accuracy can be approximated by the four-dimensional metric with three-dimensional section H^3 of constant negative curvature $R_H = -6\kappa_0^2$:

$$d\bar{s}_H^2 = d\tau^2 + dl_0^2 \,, \tag{6.21}$$

$$dl_0^2 = dz^2 + \exp(-2\kappa_0 z)\left[(dx^2)^2 + (dx^3)^2\right] . \tag{6.22}$$

The difference between the metric $\bar{g}$ and $\bar{g}_H$ is characterized by

$$h^\mu_\nu = \bar{g}_H^{\mu\sigma}[\bar{g}_{\nu\sigma} - g_{H\nu\sigma}]. \tag{6.23}$$

The difference between the invariants of the metric (6.20) and (6.21) can be always expanded in the series of invariants constructed from the tensor h^μ_ν and its covariant derivatives of arbitrary order with respect to the homogeneous metric dl_0^2. One can show that $\nabla\cdots\nabla h^\mu_\nu = O\,(X^2)$, $X\to 0$, whence it follows that the integrand of (6.15),

$$\bar{K}(s|\tau,\boldsymbol{x};\tau',\boldsymbol{x}) = \bar{K}_H(s|\tau,\boldsymbol{x};\tau',\boldsymbol{x}) + O\,(X^2),\ \ X\to 0, \tag{6.24}$$

because the nonlocal form factors can be expanded in powers of derivatives and thus reduce the right-hand side of (6.15) to the series of local invariants of the above type.

By using (6.24) and the known exact expression

$$\bar{K}_H(s|\tau,\boldsymbol{x};\tau',\boldsymbol{y}) = \frac{1}{(4\pi s)^2}\frac{\kappa_0\sigma(\boldsymbol{x},\boldsymbol{y})}{\sinh\left[\,\kappa_0\sigma(\boldsymbol{x},\boldsymbol{y})\,\right]} \times \exp\left[\frac{\tau^2+\sigma^2(\boldsymbol{x},\boldsymbol{y})}{4s}\right]\sqrt{g_H} \tag{6.25}$$

for the heat kernel on the space of constant negative curvature H^3 [with $\sigma(\boldsymbol{x},\boldsymbol{y})$—the world function on the space section τ=const], we have

$$\mathrm{Tr}\bar{K}_\beta(s|\tau,\boldsymbol{x};\tau,\boldsymbol{x}) = \frac{1}{(4\pi s)^2}\theta_3\left(0,\exp\left[-\frac{\beta^2}{4s}\right]\right)\int_0^\beta d\tau\int d^3x\sqrt{\bar{g}}[1+O(X^2)] \ . \tag{6.26}$$

Composition of this expression, Eqs. (6.4) and (6.8) gives for the free energy of a conformal scalar field the expression

$$\boldsymbol{F}_\beta^{\rm Ren} - \boldsymbol{F}_\infty^{\rm Ren} = -\frac{\pi^2}{90}\int d\boldsymbol{x} g^{\frac{1}{2}}\left[(g_{\tau\tau})^{-2}\frac{1}{\beta^4}\right] + \cdots \ , \tag{6.27}$$

where we used the integral relation

$$\int_0^\infty dx x^{a-1}\left[\theta_3(0,e^{-x})-1\right] = 2\Gamma(a)\zeta(2a)\Big|_{a=2} = \frac{\pi^4}{45} \tag{6.28}$$

and restored the physical metric $g_{\mu\nu}$. The ellipsis designates terms which are less divergent or finite at the horizon. The entropy of an arbitrary static black hole reads

$$\begin{aligned} S &= \beta^2 \frac{\partial}{\partial \beta} F_\beta^{\rm Ren} = \beta^2 \frac{\partial}{\partial \beta} \left[F_\beta^{\rm Ren} - F_\infty^{\rm Ren} \right] \\ &= \frac{2\pi^2}{45} \frac{1}{\beta^3} \int d\boldsymbol{x} (g_{\tau\tau})^{-2} g^{\frac{1}{2}} + \cdots \ . \end{aligned} \tag{6.29}$$

For the particular case of Schwarzschild black hole this formula reproduces the result Eq. (5.29).

It is worth emphasizing that, as a result of the restoration of the physical metric,

$$\begin{aligned} \bar{g}_{\mu\nu} &= e^{-2\Omega} g_{\mu\nu} = \frac{1}{g_{\tau\tau}} g_{\mu\nu} \ , \\ \sqrt{\bar{g}} &= e^{-4\Omega} \sqrt{g} = \frac{1}{(g_{\tau\tau})^2} \sqrt{g} \ , \\ \bar{R}^\nu_\mu &= e^{2\Omega} \left[R^\nu_\mu + 2\Omega^{;\nu}_\mu + \Box\Omega\delta^\nu_\mu + 2\Omega_\mu\Omega^\nu - 2\Omega^\sigma\Omega_\sigma\delta^\nu_\mu \right] \ , \end{aligned} \tag{6.30}$$

in the general expansion Eq. (6.13) we get a nonlocal expansion of the effective action in terms of curvature, "acceleration" Ω_μ and their derivatives. One can use this effective action in order to get $\langle T_{\mu\nu} \rangle^{\rm Ren}$. The action can be written in a completely invariant form if we substitute $g_{\tau\tau} = g_{\mu\nu}\xi^\mu\xi^\nu$ and consider ξ^μ as external field, which is fixed during the variations over $g_{\mu\nu}$ and is taken to coincide with the Killing vector field after the variations were performed [70]. An additional (external) vector field ξ in the effective action for thermal state is required because such a state is possible only in a stationary spacetime, i.e., the spacetime with additional geometric structure.

VII. FROM WAVE FUNCTION OF A BLACK HOLE TO ITS DENSITY MATRIX: INSTANT HORIZON

In all our one-loop statistical mechanical considerations above we heavily relied upon the Killing properties of the underlying Schwarzschild-Kruskal spacetime. The crucial moment was a separation of the physical degrees of freedom into interior and exterior ones—the procedure closely related to the existence of the black hole horizon and its bifurcation unseparable from the Killing symmetry. The question arises: is it possible to generalize the above transition from the wave function to a density matrix for a general case of a black hole setting of the problem?

This problem is very complicated if one works in the physical spacetime of a black hole which arises as a result of the gravitational collapse. The main reason is that the event horizon is a nonlocal notion and it depends on the boundary conditions at future infinity. From the viewpoint of quantization, on the other hand, it is desirable to have a definition local in time that is determined entirely in terms of objects specified at a given spacelike hypersurface, like other phase-space observables in any local quantum field theory or quantum mechanics. We discuss now an interesting possibility which arises in the framework of the approach based on the consideration of an eternal version of a black hole. This setting, according to Sec. III, implies only the statement of the Einstein-Rosen (or wormhole) topology of the spatial section (3.3) and its asymptotic flatness at both ends of the wormhole. For an eternal version of a black hole, a natural criterion for separating the dynamical modes into external and internal ones consists in finding the generalization of the bifurcation two-sphere of the Killing horizon, which is applicable to the case of a deformation of the geometry of the Einstein-Rosen bridge. For this purpose we propose here the notion of *instant horizon* formulated entirely on a spatial section of spacetime and coinciding with the bifurcation sphere in case of a spherically symmetric eternal black hole.

Because of the locality in time, this definition can be efficiently used in calculating the quantum averages with a given wave function of a black hole and in the transition to its quantum statistics and gravitational thermodynamics.

Take a two-dimensional submanifold S of spherical topology from the first nontrivial homotopic class $\pi_2(R^1 \times S^2) \equiv \mathbf{Z}$ of the three-dimensional space (3.3) and define $A[S]$ as the surface area of S. Because S is noncontractable, the functional $A[S]$ has a nonvanishing minimum at some S_0. In the case where there exist several minima, we chose S_0 to be the surface of absolute minimum of area. This quasispherical surface S_0 which we call an *instant horizon* can serve as a needed time-local notion of the horizon. It is obvious that this notion does not involve Killing symmetries and defines a horizon at a given instant of "time," that is not in four-dimensional spacetime, but entirely on the current spatial section. Remarkably, this definition of the instant horizon again brings us back into the scope of York gauge formalism and Lichnerowich equation. Indeed, the minimality of $A[S]$ at S_0 implies the minimality of this surface, which is just the York gauge in the two-dimensional (one dimension less) context. This observation might lead to even deeper parallels with the York-gauge framework, while calculating the quantum and quantum-statistical averages on the basis of this definition of the instant horizon.

The quantum state of a black hole is characterized by an amplitude of different realizations of dynamical variables on the spatial section. Thus for any particular realization one acquires its own instant horizon S_0 (Fig. 6). Since the position and shape of the instant horizon change from one realization to another, such a dependence of S_0 on the realization can be interpreted as quantum fluctuations of the instant horizon. The effect of quantum fluctuation (*zitterbewegung*) of the horizon is important for the problem of entropy discussed in the paper and, apparently, can serve as a cutoff for the (otherwise) divergent entropy of the black hole calculated above in the approximation of a frozen (classical) horizon. The entropy divergence has a universal law near the black hole horizon for all fields (massless and massive, with and without spin). It arises because in our one-loop approximation the background geometry (and hence the position of the horizon) is fixed. Quantum fluctuations result in its spreading. Because of spreading we no longer can once and for all split the states of quantum fields into the "visible" and "invisible" ones. The splitting of states into internal and external states of a black hole begins depending on the realization. Averaging over different realizations (which effectively takes into account the zit-

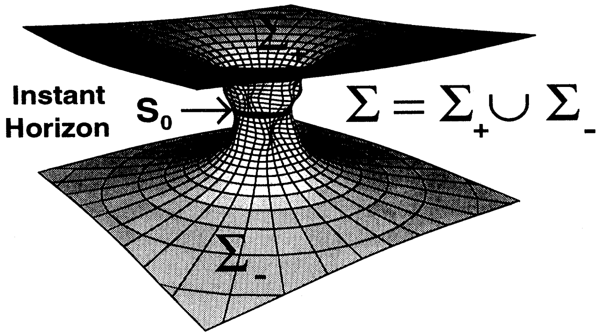

FIG. 6. Here a virtual realization of deformed Einstein-Rosen bridge is schematically depicted, corresponding to a particular realization of dynamical quantum fields on a black hole background. The *instant horizon* S_0 is a two-dimensional surface of minimal area with a topology of sphere S^2, which cannot be contracted to a point. It separates the interior of a black hole $\mathbf{\Sigma}_-$ from the external space $\mathbf{\Sigma}_+$.

terbewegung of the horizon) may produce the required cutoff for the entropy.

It should be stressed that the notion of the instant horizon is naturally defined only for the eternal black hole. One can expect that this notion may be especially useful for the discussion of those characteristics, which are identical for a real black hole and its eternal version (such as entropy). From a more general point of view, the relation of the fluctuations of the *instant* horizon to quantum properties of the *event* horizon requires a special consideration.

VIII. CONCLUSIONS

In conclusion, we summarize the main results and list the open problems. We argue that dynamical degrees of freedom of a black hole must be connected with its internal states, i.e., the states of matter and gravitational field located inside a black hole. The proposed no-boundary construction of a black hole wave function is shown to describe both internal degrees of freedom of a black hole and matter surrounding it. For small perturbations the no-boundary wave function represents the Hartle-Hawking vacuum state. The density matrix of the black hole is defined by averaging over the states of the external matter and gravitational fields. The resulting density matrix is used to define the dynamical (or statistical mechanical) entropy of a black hole. The so-defined entropy is divergent. The divergence occurs due to the contribution of the states located in a close vicinity of the horizon. It is argued that the effect of the horizon zitterbewegung may provide the necessary cut-off. It should be stressed that the dynamical entropy of a black hole in such an approach is mainly defined by the one-loop contribution. It means that all the fields (including the gravitational one) must contribute additively. The natural question is how this observation can be in agreement with the fact that in the framework of the thermodynamical analogy the entropy of a black hole is universal and does not depend on the number of fields N. One of the possible explanations proposed in [30] is that the fluctuations of the horizon which provide the cut-off for the entropy is also dependent on the number of fields. Even in the case when the resulting one-loop entropy does not grow with N, there remains the problem of calculating its exact value. In other words, what is the mechanism which gives for the entropy of a black hole the standard universal value $A/4$? We should emphasize that in the framework of our approach the dynamical degrees of freedom of a black hole contribute to the entropy only on the one-loop level, while there is no tree-level contributions. A remarkable fact is that, in the standard Euclidean approach, the "correct" answer for the entropy ($A/4l_P^2$) is obtained by calculating the tree-level contribution into the Euclidean gravitational action. On the other hand there is no direct connection of this contribution with some internal dynamical degrees of freedom. This problem is not specific for our approach. It is more general and common for different dynamical approaches to the black hole entropy.

The relationship between "dynamical" and "topological" contributions to the entropy, as well as the origin of the universality of the expression for the entropy of a black hole, is an important question. We are not going to provide here the complete answer but we just indicate a possible solution. Recently one of the authors [71] drew attention to the fact that the dynamical entropy [defined as $-\mathrm{Tr}(\rho\ln\rho)$] differs from the Bekenstein-Hawking (or thermodynamical) entropy. The difference arises because the internal states of a black hole depend on its mass, the parameter which in thermal equilibrium is directly connected with the external temperature. For this reason, when one considers the variation of the free energy with temperature (which defines the thermodynamical entropy), besides the standard term proportional to the dynamical entropy there arise additional terms, originating from the change of the internal parameters of the system induced by the change of the temperature. It is shown in [71] that this effect is sufficient to explain the required universality of the Bekenstein-Hawking entropy and clarify its relation with the dynamical entropy, considered in this paper.

Recently, another proposal [34] has been given for a mechanism maintaining the exact relation between the black hole entropy and its horizon area. This mechanism holds on the nonperturbative level in the limit of heavy black holes. Briefly it looks as follows. Suppose we have the gravitational effective action $\mathbf{\Gamma}[g]$, possibly generated by the fundamental theory of (super)strings and, therefore, finite. It may have a very general structure about which only one assumption is made: it is supposed to be analytic in the curvature and free from the effective cosmological term (thus admitting the existence of the asymptotically flat solutions of effective Einstein equations)

$$\mathbf{\Gamma}[g] = \sum_{n=1}^{\infty} \int dx_1 ... dx_n \, \mathbf{\Gamma}_n(x_1, ..., x_n) R(x_1) ... R(x_n). \tag{8.1}$$

Here $R(x)$ is a collective notation for the curvature and Ricci tensors and $\mathbf{\Gamma}_n(x_1,\ldots,x_n)$ is a set of (generally nonlocal) form factors accumulating all the information about the quantum and statistical effects in the theory. Since these form factors represent the coordinate kernels of some nonlocal operators constructed of derivatives, the only covariant expression available for $\mathbf{\Gamma}_1(x)$ is just the local density

$$\mathbf{\Gamma}_1(x) = -\frac{1}{16\pi^2 l_{\text{eff}}^2}\, g^{1/2}(x) \tag{8.2}$$

with a purely numerical coefficient which can be identified with the effective (renormalized) gravitational constant or Planck length l_{eff} (all the covariant derivatives in $\mathbf{\Gamma}_1$ contract to form a total derivative which disappears when integrated over asymptotically flat spacetime).

According to Eq. (5.16) the calculation of entropy involves the effective action $\mathbf{\Gamma}_\beta = \mathbf{\Gamma}[\, g^\beta\,]$ calculated on the conical spacetime with metric g^β having a conical singularity with $\beta \neq 8\pi M$. On such a manifold the curvature has a form

$$R_\beta(x) = (\beta - 8\pi M)\, f(x) + R_{\text{reg}}(x), \tag{8.3}$$

where $R_{\text{reg}}(x)$ is a regular part of the curvature bounded by $1/M^2$ and, therefore, negligible for heavy black holes $M \to \infty$. The singular part caused by conical structure for $\beta \neq 8\pi M$ involves the generalized function $f(x)$ which, when regulated, can be even nonsingular one, but having the compact support in the vicinity of the tip of the cone (black hole horizon) and satisfying the relation

$$\int dx\, g^{1/2}(x)\, f(x) = -8\pi M\ . \tag{8.4}$$

Substituting the structure (8.3) into (8.1) and using (5.16), we immediately find that the entropy is entirely generated by the effective Einstein term of the action, because the expansion in powers of the curvature becomes the expansion in powers of the angle deficit $(\beta - 8\pi M)$ of the conical manifold:

$$\begin{aligned} \boldsymbol{S} &= \left(\beta\frac{\partial}{\partial\beta} - 1\right)\mathbf{\Gamma}_\beta \\ &= \beta\int dx\,\mathbf{\Gamma}_1(x)\,R(x) \\ &= \frac{A}{4\,l_{\text{eff}}^2}\ . \end{aligned} \tag{8.5}$$

The above arguments could have been even generalized to the case of the finite-mass black hole by noting that in asymptotically flat spacetime the actual expansion of the effective action can be performed in powers of the Ricci curvature $R_{\mu\nu}$ only [63,64], for which $R_{\mu\nu\,\text{reg}}(x) \equiv 0$ in Eq. (8.3). However, there is a serious objection to this mechanism which apparently invalidates this proposal. If it were correct, then the perturbative calculations of entropy would maintain the universal relation between the entropy and one-quarter of the horizon area, the quantum corrections to the classical entropy being compensated for by the simultaneous renormalization of this length. But this is definitely not the case for the dominant divergent contribution (6.29) obtained in the one-loop approximation. Indeed, as it follows from Eq. (6.27), this contribution involves the invariant of the Killing vector field $\xi^\mu\xi_\mu = g_{\tau\tau}$. This invariant can be regarded as a restriction of some nonlocal functional of metric to the manifold with Killing symmetries. Killing field ξ^μ as a functional of the metric does not have a unique continuation off the symmetric (Killing) points in the configuration space of metric, but it is undoubtedly nonlocal and most likely has a structure of the solution of the Killing equation

$$\Box\xi^\mu + R^\mu_\nu\,\xi^\nu = 0 \tag{8.6}$$

as a functional of the metric and boundary conditions: $\xi^\mu = \xi^\mu[\,g, \text{boundary data}]$. This data is nontrivial; it nontrivially depends on β, and induces extraneous structures in the functional argument of the effective action, which do not reduce to the local metric or curvature. By iteratively solving the equation (8.6) we can obtain ξ^μ as a nonlocal expansion in curvatures, but there will be a zero-order term ξ_0^μ independent of the metric and pointing out the direction in spacetime in which it is periodically compactified with a circumference given by β at spatial infinity. This vector field generates new "noncovariant" curvature structures in the effective action, such as $g_{\mu\nu}\xi_0^\mu\xi_0^\nu$, $R_{\mu\nu}\xi_0^\mu\xi_0^\nu$, etc., that do not reduce to the renormalization of the cosmological or gravitational constant even in the lowest orders of curvature expansion. Therefore, the dependence of $\mathbf{\Gamma}_\beta$ on β will be induced not only by the metric argument of $\mathbf{\Gamma}[\,g\,]$: $\mathbf{\Gamma}_\beta = \mathbf{\Gamma}[\,g^\beta, \xi_0^\mu(\beta), \beta\,]$ and the above mechanism will break down, since the first-order term in $(\beta - 8\pi M)$ will no longer be generated by the Einstein term of the effective action. On the contrary, if the mechanism proposed in [71] does work, it is simply not necessary to relate the difference between the Bekenstein-Hawking ("thermodynamical") and the dynamical entropies of a black hole with the renormalization of the gravitational constant.

We would like to conclude the paper by reminding readers that its main purpose was to develop an approach which gives an adequate description of the internal degrees of freedom of a black hole. This approach makes a black hole very close to the usual body with real dynamical degrees of freedom. Certainly, there are special properties of a black hole which single it out as a thermodynamical system. The detailed study of these properties, especially in the general context of black hole thermodynamics, requires further investigation.

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APPENDIX A: LICHNEROWICZ EQUATION AND THE GEOMETRY OF THE EINSTEIN-ROSEN BRIDGE

Here we show that the three-geometry of a spatial section on which we define the no-boundary wave function of true physical variables (3.11), $(g^T, p_T) = (h^T_{ab}, p^{ab}_T$, matter variables) coincides with the geometry of the Einstein-Rosen bridge in the lowest order of the perturbation theory in (g^T, p_T). This approximation corresponds to a ground state of physical excitations (of both matter and gravitational fields) on the spatial section with the topology (3.3).

Consider three-geometry ${}^3g_{ab}$ and define

$$\tilde{\beta}^{ab} = \epsilon^{aef}\nabla_e \left[\sqrt{{}^3g}\,({}^3R^b_f - \frac{1}{4}\delta^b_f\ {}^3R)\right], \tag{A1}$$

where ∇_e is a spatial covariant derivative. York [53] showed that $\tilde{\beta}^{ab}$ gives a pure spin-two representation of intrinsic geometry. Conditions $\tilde{\beta}^{ab} = 0$ together with $p_{ab} = 0$ specify the state where no dynamical gravitational perturbations are present. For this case in the absence of matter the Lichnerowicz equation (3.9) reduces to the equation

$$^3R = 0. \tag{A2}$$

Condition $\tilde{\beta}^{ab} = 0$ implies that the three-metric is conformally flat,

$$dl^2 = \Phi^4 d\tilde{l}_0^2 = \Phi^4(dx^2 + dy^2 + dz^2). \tag{A3}$$

The Lichnerowicz equation (A2) in this case is equivalent to the equation

$$\triangle\Phi = 0 \tag{A4}$$

for the conformal factor Φ. Here $\triangle$ is the Laplace operator in the flat three-dimensional flat metric $d\tilde{l}_0^2$. A solution which is regular everywhere is constant and the corresponding geometry is a flat three-dimensional space R^3. Nontrivial solutions have singularities. A solution with one simple pole generates a three-dimensional space $S^2 \times R^1$ with the Einstein-Rosen bridge geometry. We choose coordinates so that the pole is located at the origin of coordinates

$$\Phi = 1 + \frac{M}{2\rho}, \tag{A5}$$

where $\rho^2 = x^2 + y^2 + z^2$. For this conformal factor the metric dl^2 can be written as

$$dl^2 = \frac{dr^2}{1 - 2M/r} + r^2 d\Omega^2, \tag{A6}$$

where $r \equiv \rho\left(1 + \frac{M}{2\rho}\right)^2$. A point $\rho = \infty$ corresponds to a spatial infinity of Σ_+, while a point $\rho = 0$ labels the spatial infinity of Σ_-, the constant M being the mass $(M_+ = M_- = M)$. The important property of the obtained solution describing the state without excitation is that the corresponding three-metric is spherically symmetric.

The metric (A3) with (A5) can be identically rewritten in the form in which both spatial infinities are represented in the completely symmetric way. To do this we call that the flat metric is conformally related with a metric on a three-sphere S^3, so that we have

$$dl^2 = \tilde{\Phi}_0^4(d\chi^2 + \sin^2\chi d\Omega^2) = \frac{dr^2}{1 - 2M/r} + r^2 d\Omega^2, \tag{A7}$$

where $\tilde{\Phi}_0$ is a solution of the conformal invariant equation on the three-sphere,

$$\left(\tilde{\Delta} - \frac{1}{8}{}^3\tilde{R}\right)\tilde{\Phi}_0 = 0, \tag{A8}$$

which is of the form $\tilde{\Phi}_0 = \phi_0/\sin\chi$, with $\phi_0 = M^{1/2}[\sin(\chi/2) + \cos(\chi/2)]$.

In the presence of gravitational perturbations and matter, the Lichnerowicz equation (3.9) reads

$$\left(\tilde{\Delta} - \frac{1}{8}{}^3\tilde{R}\right)\tilde{\Phi} = J, \tag{A9}$$

where the source J in terms of conformally transformed variables looks like

$$J = -\frac{1}{8}({}^3\tilde{g})^{-1}\ {}^3\tilde{p}^{ab}\,{}^3\tilde{p}_{ab}\Phi^{-7} - 2\pi\,\tilde{T}^*_*\,\Phi^{-3}. \tag{A10}$$

Denote by $G(\boldsymbol{x}, \boldsymbol{x}')$ the Green function defined as a solution of the equation

$$\left(\tilde{\Delta} - \frac{1}{8}{}^3\tilde{R}\right)G(\boldsymbol{x}, \boldsymbol{x}') = -\delta^3(\boldsymbol{x}, \boldsymbol{x}'). \tag{A11}$$

The solution of the equation (A9) can be presented in the form

$$\tilde{\Phi} = \tilde{\Phi}_0 + \int G(\boldsymbol{x}, \boldsymbol{x}')J(\boldsymbol{x}')\tilde{g}^{\frac{1}{2}}d\boldsymbol{x}'. \tag{A12}$$

The first term $\tilde{\Phi}_0$ is invariant with respect to the reflection $\chi \to \pi - \chi$. In the general case, J does not obey this property and the solution $\tilde{\Phi}$ is not invariant under reflection and asymptotic values of $M_\pm$ of masses at infinities of $\Sigma_\pm$ are different. In order to illustrate this general property we consider here a simple case when J is spherically symmetric.

We write $\tilde{\Phi}$ in the form $\tilde{\Phi} = \phi/\sin\chi$. The function ϕ obeys the equation

$$\frac{d^2\phi}{d\chi^2} + \frac{1}{4}\phi = j \equiv J\sin\chi, \tag{A13}$$

and has a general solution in terms of the Green function $G(\chi, \chi')$:

$$\phi(\chi) = \phi_0(\chi) + \int_0^\pi G(\chi, \chi')j(\chi')d\chi', \tag{A14}$$

$$G(\chi, \chi') = -2\Big\{\theta(\chi - \chi')\sin(\chi/2)\cos(\chi'/2) + \theta(\chi' - \chi)\sin(\chi'/2)\cos(\chi/2)\Big\}. \tag{A15}$$

The asymptotic masses at two spatial infinities are

$$M_+ = \phi \frac{d\phi}{d\chi}\Big|_{\chi=0}, \quad M_- = -\phi \frac{d\phi}{d\chi}\Big|_{\chi=\pi}. \tag{A16}$$

From this expression it follows that

$$\begin{aligned} &\phi(0) = M^{1/2}, \quad \phi(\pi) = M^{1/2}, \\ &\phi'(0) = M^{1/2}/2 - \alpha, \quad \phi'(\pi) = -M^{1/2}/2 + \beta, \\ &\alpha \equiv \int_0^\pi \cos(\chi'/2) j(\chi') d\chi', \\ &\beta \equiv \int_0^\pi \sin(\chi'/2) j(\chi') d\chi', \end{aligned} \tag{A17}$$

whence

$$M_+ - M_- = 2M^{1/2}(\beta - \alpha). \tag{A18}$$

This relation shows that in the general case the asymmetric distribution of matter on the Einstein-Rosen bridge results in different masses M_+ and M_- at two asymptotic infinities. For a known distribution and fixed M_+, the value of M_- can be obtained by solving the Lichnerowicz equation.

APPENDIX B: *R*-MODES

In this appendix we construct the basis of positive frequency solutions

$$w_\lambda = \frac{1}{\sqrt{2\omega}} \exp(-i\omega t) R_{\omega lmA}(r, \vartheta, \phi) \tag{B1}$$

for the scalar field in the exterior region R_+ of the eternal black hole, for which spatial functions $R_{\omega lmA}$ are real (*R modes*).

By using the separation of variables for the equation $\Box\phi = 0$ we write

$$R_{\omega lm}(r, \vartheta, \phi) = R_{\omega l}(r) \hat{Y}_{lm}(\vartheta, \phi), \tag{B2}$$

where

$$\hat{Y}_{lm}(\vartheta, \phi) = P_l^m(\vartheta) \begin{cases} \frac{1}{\sqrt{2\pi}}, & m = 0, \\ \frac{1}{\sqrt{\pi}} \cos m\phi, & 0 < m \le l, \\ \frac{1}{\sqrt{\pi}} \sin m\phi, & -l \le m < 0. \end{cases} \tag{B3}$$

We choose the spherical harmonics $\hat{Y}_{lm}$ to be real so that the R basis will be constructed if solutions $R_{\omega l}(r)$ of the radial equation (5.22) are chosen to be real. Denote $\hat{R}_{\omega l}(r) = \sqrt{2\pi} r R_{\omega l}(r)$, then the radial equation reads

$$\frac{d^2 \hat{R}_{\omega l}}{dr^{*2}} + (\omega^2 - V_l) \hat{R}_{\omega l} = 0 \,, \tag{B4}$$

where $r^* = r - 2M + 2M \ln[(r - 2M)/2M]$, and

$$V_l = \left(1 - \frac{2M}{r}\right) \left(\frac{l\,(l+1)}{r^2} + \frac{2M}{r^3}\right). \tag{B5}$$

For any two solutions of (B4) the Wronskian $W[f_1, f_2] \equiv f_1(df_2/dr^*) - f_2(df_1/dr^*) =$const.

Functions $\hat{R}_{\omega l}$ have the asymptotics $\exp(\pm i\omega r)$ at $r \to \infty$ and $\exp(\pm i\omega r^*)$ at $r^* \to -\infty$. We begin by defining so-called up modes which are specified (for $\omega > 0$) by the asymptotics

$$\hat{R}^{\rm up}_{\omega l}(r) = \begin{cases} e^{i\omega r^*} + r_{\omega l} e^{-i\omega r^*}, & r^* \to -\infty, \\ t_{\omega l} e^{i\omega r}, & r \to \infty. \end{cases} \tag{B6}$$

By comparing the Wronskians at $r^* = \pm\infty$ for $\hat{R}^{\rm up}_{\omega l}$ and its complex conjugated, one gets the standard relations between reflection and absorption coefficients

$$|r_{\omega l}|^2 + |t_{\omega l}|^2 = 1. \tag{B7}$$

The coefficients of the radial equation are real. That is why $\hat{R}^{\rm down}_{\omega l}(r) \equiv \bar{\hat{R}}^{\rm up}_{\omega l}(r)$ is again a solution. One has

$$\bar{\hat{R}}^{\rm up}_{\omega l}(r) = \hat{R}^{\rm up}_{-\omega l}(r), \tag{B8}$$

so that $\bar{r}_{\omega l} = r_{-\omega l}$ and $\bar{t}_{\omega l} = t_{-\omega l}$. The real and imaginary parts of $\hat{R}^{\rm up}_{\omega l}(r)$ (for $\omega > 0$) can be used as real basic solutions. The problem is that the corresponding solutions w_λ do not possess the proper normalization conditions. In particular, the functions $w^{\rm up}$ and $w^{\rm down}$ are not orthogonal. Namely, one has

$$(w^{\rm up}_{\omega lm}, w^{\rm up}_{\omega' l' m'}) = \delta(\omega - \omega') \delta_{ll'} \delta_{mm'}, \tag{B9}$$

$$(w^{\rm down}_{\omega lm}, w^{\rm down}_{\omega' l' m'}) = \delta(\omega - \omega') \delta_{ll'} \delta_{mm'}, \tag{B10}$$

$$(w^{\rm down}_{\omega lm}, w^{\rm up}_{\omega' l' m'}) = r_{\omega l} \delta(\omega - \omega') \delta_{ll'} \delta_{mm'}. \tag{B11}$$

Here

$$(f_1, f_2) = -i \int (\bar{f}_1 f_{2,\mu} - \bar{f}_2 f_{1,\mu}) d\sigma^\mu \tag{B12}$$

is a scalar product in the space of solutions. The proper normalization conditions (including the orthogonality conditions) can be satisfied by the following linear transformation of the basic functions:

$$\hat{R}^{\rm up'}_{\omega l} = a_{\omega l} \hat{R}^{\rm up}_{\omega l} + b_{\omega l} \hat{R}^{\rm down}_{\omega l}, \tag{B13}$$

$$\hat{R}^{\rm down'}_{\omega l} = \bar{b}_{\omega l} \hat{R}^{\rm up}_{\omega l} + a_{\omega l} \hat{R}^{\rm down}_{\omega l}, \tag{B14}$$

where

$$a_{\omega l} = \frac{\sqrt{1 + |t_{\omega l}|}}{\sqrt{2} |t_{\omega l}|}, \qquad b_{\omega l} = -\frac{r_{\omega l}}{\sqrt{2} |t_{\omega l}| \sqrt{1 + |t_{\omega l}|}}. \tag{B15}$$

The following solutions $\hat{R}_{\omega lA}$ $(A = 1, 2)$ are real and for $\omega > 0$ form a proper normalized basis

$$\hat{R}_{\omega l1} = \frac{1}{\sqrt{2}} (\hat{R}^{\rm up'}_{\omega l} + \hat{R}^{\rm down'}_{\omega l}), \tag{B16}$$

$$\hat{R}_{\omega l2} = \frac{1}{i\sqrt{2}} (\hat{R}^{\rm up'}_{\omega l} - \hat{R}^{\rm down'}_{\omega l}). \tag{B17}$$

These solutions have the asymptotics

$$\hat{R}_{\omega lA}(r) = \begin{cases} A^{+}_{\omega lA} e^{-i\omega r^*} + \overline{A^{+}_{\omega lA}} e^{i\omega r^*}, & r^* \to -\infty, \\ B^{+}_{\omega lA} e^{i\omega r} + \overline{B^{+}_{\omega lA}} e^{-i\omega r}, & r \to \infty. \end{cases} \tag{B18}$$

Here

$$A^{+}_{\omega l1} = \frac{1}{2}\left(\sqrt{1+|t_{\omega l}|} + \frac{r_{\omega l}}{\sqrt{1+|t_{\omega l}|}}\right), \tag{B19}$$

$$B^{+}_{\omega l1} = \frac{1}{2}\frac{t_{\omega l}}{|t_{\omega l}|}\left(\sqrt{1+|t_{\omega l}|} - \frac{\bar{r}_{\omega l}}{\sqrt{1+|t_{\omega l}|}}\right), \tag{B20}$$

$$A^{+}_{\omega l2} = \frac{i}{2}\left(\sqrt{1+|t_{\omega l}|} - \frac{r_{\omega l}}{\sqrt{1+|t_{\omega l}|}}\right), \tag{B21}$$

$$B^{+}_{\omega l2} = \frac{1}{2i}\frac{t_{\omega l}}{|t_{\omega l}|}\left(\sqrt{1+|t_{\omega l}|} + \frac{\bar{r}_{\omega l}}{\sqrt{1+|t_{\omega l}|}}\right). \tag{B22}$$

By using the asymptotics (B6) and (B18), one can show that near the horizon

$$\sum_{A=1}^{2} |R_{\omega lA}|^2 - |R^{up}_{\omega l}|^2 \approx \frac{1}{8\pi M^2}|t_{\omega l}|^2. \tag{B23}$$

To summarize, we constructed the basis $\{w_\lambda\}$ $(\omega > 0, A = 1, 2)$

$$w_\lambda = \frac{1}{\sqrt{2\omega}} \exp(-i\omega t) R_{\omega lmA}(r, \vartheta, \phi), \tag{B24}$$

where $R_{\omega lmA}(r,\vartheta,\phi) \equiv (\sqrt{2\pi}r)^{-1}\hat{R}_{\omega lA}(r)\hat{Y}_{lm}(\vartheta,\phi)$ are real functions, obeying the normalization conditions

$$\int d^3x\, g^{\tau\tau} g^{1/2} R_{\omega lmA}(\boldsymbol{x})\, R_{\omega' l' m' A'}(\boldsymbol{x}) = \delta(\omega - \omega')\, \delta_{ll'}\delta_{mm'}\delta_{AA'}. \tag{B25}$$

In addition to the above constructed basis we also introduce modes which are propagating inside a spherical cavity surrounding a black hole. We assume that the boundary conditions at the surface of the cavity located at $r = r_0$ are of the form

$$\left[\alpha \frac{d\hat{R}_{\omega l}}{dr} + \beta \hat{R}_{\omega l}\right]_{r=r_0} = 0, \tag{B26}$$

with real coefficients α and β. We denote the corresponding real solutions as $\hat{R}_{\omega l0}$ and radial functions $R_{\omega l0} = (\sqrt{2\pi}r)^{-1}\hat{R}_{\omega l0}$. The real solutions inside the cavity have the following asymptotics near the horizon

$$\hat{R}_{\omega l0}(r) \approx \bar{r}_{\omega l0} e^{i\omega r^*} + r_{\omega l0} e^{-i\omega r^*}, \tag{B27}$$

where $|r_{\omega l0}| = 1$.

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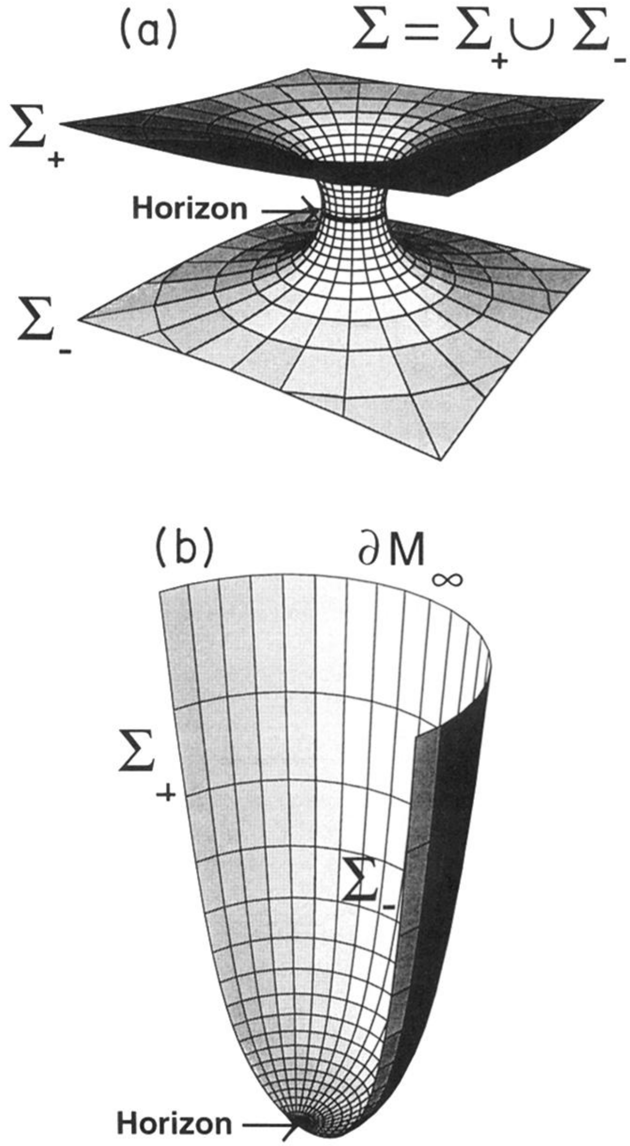

FIG. 3. (a) This figure depicts the embedding diagram of the global Cauchy surface $\mathbf{\Sigma}$. This is a well-known Einstein-Rosen bridge connecting two asymptotically flat three-dimensional spaces. The bifurcation two-sphere of the future and past horizons separates the interior $\mathbf{\Sigma}_-$ of a black hole from external space $\mathbf{\Sigma}_+$. (b) Here the half of the Gibbons-Hawking gravitational instanton is depicted. The global Cauchy surface $\mathbf{\Sigma} = \mathbf{\Sigma}_+ \cup \mathbf{\Sigma}_-$ is one boundary of this Euclidean manifold and spatial infinity $\partial \boldsymbol{M}_\infty$ is another. The arguments of the wave function $\mathbf{\Psi}(\varphi_+, \varphi_-)$ are the boundary values of quantum fields on the two asymptotically flat parts of the Einstein-Rosen bridge $\mathbf{\Sigma}_\pm$, respectively.

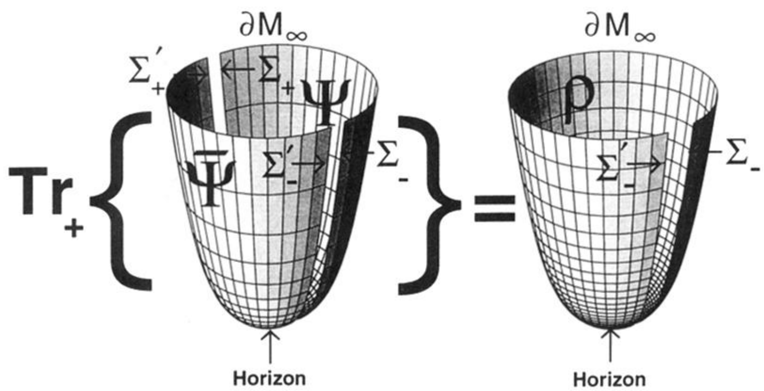

FIG. 5. The wave function $\Psi(\varphi_+, \varphi_-)$ is defined as a path integral over the physical degrees of freedom in the τ foliation of the Euclidean spacetime on a half-instanton. The density matrix $\rho(\varphi'_-, \varphi_-) = \mathrm{Tr}_+ |\Psi\rangle\langle\Psi|$ is proportional to the analogous functional integral, but on the whole instanton, since the proposed wave function can be interpreted as an amplitude of an Euclidean evolution from the initial state φ_- to the final state φ_+ during the time interval $\beta/2$. Then $\overline{\Psi}(\varphi_+, \varphi'_-)$ implies the evolution from φ_+ to φ'_- and the density matrix is the kernel of the evolution from φ_- to φ'_- during the Euclidean time interval $0 < \tau < \beta$. The arguments of the density matrix $\varphi'_-(\boldsymbol{x}')$ and $\varphi_-(\boldsymbol{x})$ are the values of the fields on different sides $\boldsymbol{\Sigma}_-{}'$ and $\boldsymbol{\Sigma}_-$ of the cut in the instanton.