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Emergence of an effective two-dimensional quantum description from the study of critical phenomena in black holes

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We study the occurrence of critical phenomena in four-dimensional, rotating, and charged black holes, derive the critical exponents, and show that they satisfy the scaling laws. Correlation function critical exponents and renormalization group considerations assign an effective (spatial) dimension, $d = 2$, to the system. The two-dimensional Gaussian approximation of the order parameter is shown to reproduce all the black hole's critical exponents. Higher order corrections (which are always relevant) are discussed. Identifying the two-dimensional surface with the event horizon and noting that a generalization of scaling leads to conformal invariance and then to string theory, we arrive at 't Hooft's string interpretation of black holes. From this, an effective model for dealing with a coarse-grained black hole quantization is proposed. We also give simple arguments that lead to a (first) quantization of the black hole mass in units of the Planck mass, i.e., $M \simeq \frac{1}{\sqrt{2}} M_{\rm Pl}\sqrt{l}$ with l a positive integer, and then, from this result, to the proportionality between quantum entropy and area.

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I. INTRODUCTION

The scaling of critical phenomena [1–4] applies to a great variety of thermodynamical systems, those ranging from the internal structure of elementary particles to ferroelectricity and turbulent fluid flow, passing through superconductivity and superfluidity. The scaling is found to hold (within experimental error) in almost every case. The renormalization group approach [2,5] uses the scaling hypothesis and provides a sound mathematical foundation to the concept of universality. On the other hand black hole dynamics is governed by analogues of the ordinary four laws of thermodynamics [6–8]. These two facts lead us to conjecture that black holes also obey the scaling laws or fourth law of thermodynamics [9,10].

Let us suppose that a rotating charged black hole is held in equilibrium at some temperature T, with a surrounding heat bath. If we consider a small, reversible transfer of energy between the hole and its environment, this absorption will be isotropic, and will occur in such a way that the angular momentum J and charge Q remain unchanged, on the average. The full thermal capacity (not per unit mass) corresponding to this energy transfer can be computed by eliminating M (the total mass of the black hole) between the equations for the temperature and the area of the black hole, and differentiating the entropy S keeping J and Q constant [11,12]:

$$C_{J,Q} = T\frac{\partial S}{\partial T}\bigg|_{J,Q} = \frac{MTS^3}{\pi J^2 + \frac{\pi}{4}Q^4 - T^2S^3} . \qquad (1)$$

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This heat capacity goes from negative values for a Schwarzschild black hole, $C_{\rm Sch} = -M/T$, to positive values for a nearly extreme Kerr-Newman black hole, $C_{\rm EKN} \sim \sqrt{M^4 - J^2 - M^2Q^2} \to 0^+$. Thus, $C_{J,Q}$ has changed sign at some value of J and Q in between. In fact, the heat capacity passes from negative to positive values through an infinite discontinuity. This feature has led Davies [11] to classify the phenomenon at the critical values of J and Q as a second-order phase transition. The critical values J_c and Q_c at which the transition occurs are obtained by making the denominator on the right-hand side of Eq. (1) vanish. We can then define the parametrization

$$j = \frac{J_c^2}{M^4} \quad \text{and} \quad q = \frac{Q_c^2}{M^2} .$$

Eliminating S and T in Eq. (1) by use of the expressions for the temperature and entropy of a black hole [9], the infinite discontinuity in $C_{J,Q}$ takes place at [11]

$$j_{JQ}^2 + 6j_{JQ} + 4q_{JQ} = 3 . \qquad (2)$$

For an uncharged, i.e., Kerr, hole, $q_{JQ} = 0$. Thus, $j_{JQ} = 2\sqrt{3} - 3$. While for a nonrotating, i.e., Reissner-Nordström, hole, $j_{JQ} = 0$. Thus, $q_{JQ} = 3/4$.

It can also be shown [13] that the four isothermal compressibility-related derivatives K^{-1} are divergent as their corresponding heat capacities. For example,

$$K_{T,Q}^{-1} = J\frac{\partial \Omega}{\partial J}\bigg|_{T,Q}$$

$$\sim \frac{\pi(2\Phi Q - M)(1 - 4\pi TM)}{S^2[1 - 12\pi TM + 4\pi^2T^2(6M^2 + Q^2)]} \qquad (3)$$

diverges as $C_{J,Q}$. Also $K_{T,J}^{-1} = C_{J,Q}(\partial\Phi/\partial Q\big|_{S,J})/C_{J,\Phi}$

diverges as $C_{J,Q}$ on the singular segment given by Eq. (2).

By use of the expressions for the temperature and entropy of black holes [9], the heat capacity C_{JQ} can be expressed as [13]

$$C_{JQ} = \frac{4\pi TSM}{1 - 8\pi TM - 4\pi ST^2} \sim \frac{1}{T - T_c} , \qquad (4)$$

where the critical temperature is given by $T_c^{JQ} = \{2\pi M[3 + \sqrt{3 - q_{JQ}}]\}^{-1}$, and q_{JQ} is given by the critical curve Eq. (2).

In analogy to fluid and magnetic systems near critical conditions [1], we can define the following critical exponents for black holes.

For the specific heat at constant J and Q,

$$C_{JQ} \sim \begin{cases} (T - T_c)^{-\alpha} & \text{for } J = J_c \text{ and } Q = Q_c, \\ (J - J_c)^{\varphi} \text{ or } (Q - Q_c)^{\varphi} & \text{for } T = T_c. \end{cases} \qquad (5)$$

For the equation of state

$$\Omega \text{ or } \Phi \sim \begin{cases} (T - T_c)^{\beta} & \text{for } J = J_c \text{ and } Q = Q_c, \\ (J - J_c)^{1/\delta} \text{ or } (Q - Q_c)^{1/\delta} & \text{for } T = T_c \end{cases} \qquad (6)$$

where Ω is the angular velocity and Φ is the electric potential of the event horizon.

For the isothermal susceptibility-related derivative,

$$K_{TQ}^{-1} \sim \begin{cases} (T - T_c)^{-\gamma} & \text{for } J = J_c \text{ and } Q = Q_c, \\ (J - J_c)^{1-1/\delta} \text{ or } (Q - Q_c)^{1-1/\delta} & \text{for } T = T_c \end{cases} \qquad (7)$$

and entropy

$$S \sim \begin{cases} (T - T_c)^{1-\alpha} & \text{for } J = J_c \text{ and } Q = Q_c, \\ (J - J_c)^{\psi} \text{ or } (Q - Q_c)^{\psi} & \text{for } T = T_c. \end{cases} \qquad (8)$$

We can obtain the first two critical exponents (that characterize the approach to the divergence in the heat capacity at J, Q, and T fixed, respectively [9]) directly by inspection of Eq. (4):

$$\alpha = 1 \quad , \quad \varphi = 1 . \qquad (9)$$

Analogously, from the expression for the compressibility, Eq. (3) (that diverges as $C_{J,Q}$), we obtain the critical exponents corresponding to the approach to the divergence at J, Q, and T fixed, respectively [9]:

$$\gamma = 1 \quad , \quad 1 - \delta^{-1} = 1 \Rightarrow \delta^{-1} \to 0 . \qquad (10)$$

To obtain the critical exponents corresponding to the equation of state and entropy, we choose a path either along a critical isotherm or at constant angular momentum $J = J_c$ or constant charge $Q = Q_c$. However, in this case the black hole equations of state just reproduce the critical values, and we can formally assign a zeroth power corresponding to the critical exponents associated with Ω and S, respectively [9]:

$$\beta \to 0 \quad , \quad \delta^{-1} \to 0 ,$$

$$1 - \alpha = 0 \quad , \quad \psi \to 0 . \qquad (11)$$

One can easily check that the set of critical exponents given by Eqs. (9)–(11) satisfy the scaling laws [1]:

$$\alpha + 2\beta + \gamma = 2 \quad , \quad \alpha + \beta(\delta + 1) = 2 ,$$

$$\gamma(\delta + 1) = (2 - \alpha)(\delta - 1) \quad , \quad \gamma = \beta(\delta - 1) , \qquad (12)$$

$$(2 - \alpha)(\delta\psi - 1) + 1 = (1 - \alpha)\delta \ , \quad \varphi + 2\psi - \delta^{-1} = 1 .$$

Five other heat capacities can be computed, of which $C_{\Omega,Q}$ and $C_{J,\Phi}$ also exhibit a singular behavior. The remaining $C_{\Phi,Q} = C_{J,\Omega}$, and $C_{\Omega,\Phi}$ being regular functions in the allowed set of values of the parameters [13]. Heat capacities and isothermal compressibilities at fixed (Ω, Q) and (J, Φ) give the same critical exponents as in the previous case where we held (J, Q) constant. This result can in fact be understood as a realization of the *universality hypothesis*: For a continuous phase transition the static critical exponents depend only on the following three properties: (a) the dimensionality of the system, d; (b) the internal symmetry dimensionality of the order parameters, D; (c) whether the forces are of short or long range.

The critical curves for the three cases studied [9] are all different, but the critical exponents, according to the above-mentioned hypothesis, are the same within each class as just specified. We also observe that the equality between the primed ($T \to T_c^-$) and unprimed ($T \to T_c^+$) critical exponents is verified in each one of the three transitions studied.

The lack of qualitative change in the properties of the black hole can be understood as in analogy to what happens in the case of a liquid-vapor system, where near criticality no qualitative distinction between phases can be made. Note that in this case there is no such thing as a latent heat [14] (since M remains continuous through the transition), as it happens in magnetic critical transitions. In addition, it can be seen that the critical transitions occur when we cool down the black hole with respect to the corresponding Schwarzschild temperature, $T_S = 1/(8\pi M)$, by increasing its charge or angular momentum at fixed total mass. Further, we have seen how black holes satisfy the scaling laws and universality hypothesis, both characteristics of critical phase transitions.

It is worth noting [14] that although this phase transition does not affect the internal state of the system it is physically important as it indicates the transition from a region ($C_{JQ} < 0$) where only a microcanonical ensemble is appropriate (stable equilibrium if the system is isolated from the outside world) to a region ($C_{JQ} > 0$) where a canonical ensemble can be also used (stable equilibrium with an infinite heat bath).

This paper is organized as follows. In Sec. II we analyze two further critical exponents defined for the static correlation functions. We find that all critical exponents correspond to those of a Gaussian model in two dimensions. Renormalization group arguments are given to establish $d = 2$ as the effective dimension of the system. In Sec. III we develop the idea of a two-dimensional effective representation for the black hole horizon as the fundamental object to quantize and make connection with string theory in a description similar to that of 't Hooft. We end the paper with a discussion and simple derivation, using the two-dimensional representation of black holes, of a mass quantization, and a quantum originated entropy-area relation.

II. CORRELATION FUNCTIONS, THE GAUSSIAN MODEL, AND THE RENORMALIZATION GROUP

Not only relations among critical exponents corresponding to thermodynamic functions can be obtained, but also relations concerning correlation function exponents.

The static two-point (at distance $|\mathbf{r}|$) connected correlations can be defined as

$$G_c^{(2)}(|\mathbf{r}|) = \langle\phi(0)\cdot\phi(\mathbf{r})\rangle - |\langle\phi\rangle|^2 \ , \tag{13}$$

where ϕ is the order parameter of the system in question and may have, in principle, D different internal components (for example, in the Ising model, the order parameter has only one component; in a Heisenberg system, three; and in the He^3 superfluid transition as many as 18 [3]).

Away, but not far from the critical region, one can write

$$G_c^{(2)}(r) \sim \frac{\exp\{-r/\xi\}}{r^{d-2+\eta}} \ , \quad r \ \text{large} \ . \tag{14}$$

Here d is the (spatial) dimensionality of the system, η is a further critical exponent, and ξ is the correlation length. As one approaches the critical curve ξ diverges as

$$\xi \sim |T - T_c|^{-\nu} \ . \tag{15}$$

Here ν is another critical exponent.

Kadanoff [15] studied the scaling properties of the correlation functions and found a new scaling law relating the critical exponents:

$$(2-\eta)\nu = \gamma. \tag{16}$$

With an additional assumption about the scaling behavior of the correlation function [16] one obtains the hyperscaling law

$$\nu d = 2 - \alpha \ . \tag{17}$$

Note that only here the dependence with the dimensionality of the system appears. By use of the renormalization group equations, one can show [17] that hyperscaling holds for $d \leq 4$ but breaks down for $d > 4$.

Now, let us consider the black hole in equilibrium with a radiation bath. By use of the quantum field theory technics in the curved spacetime of the black hole one can obtain an approximate expression for the correlation function of the fluctuations of fields in this curved background. In equilibrium, the field will be in the Hartle-Hawking vacuum state. The correlation function of a scalar field in the Schwarzschild background, for large distances r, is given by [18] (ω being the frequency of the mode in the Hartle-Hawking state considered)

$$G_\omega(r) \sim \frac{\omega}{2\pi\left[\exp\left(\frac{2\pi\omega}{k_H}\right) - 1\right]} \ , \tag{18}$$

and thus, independent of the distance r. Here, we shall make the hypothesis that in equilibrium gravitational correlations from black hole fluctuations behave in a similar qualitative way to the scalar field fluctuations, even considering charged and rotating black holes. We will propose below that the black hole itself can be represented effectively, near criticality, by an order parameter ϕ having the same critical exponents.

From Eq. (14) we thus conclude that

$$d - 2 + \eta = 0 \ . \tag{19}$$

The correlation length can be formally defined from [3]

$$\xi^2 = -\frac{1}{2K_T}\left(\frac{\partial^2 G(\omega)}{\partial\omega^2}\right)_{\omega=0} \ . \tag{20}$$

By use of Eq. (18) and since $K_T^{-1} \sim |T - T_c|^{-\gamma}$ we find that

$$\xi^2 \sim |T - T_c|^{-\gamma} \quad \Rightarrow \quad \nu = \frac{1}{2} \ , \tag{21}$$

where we have used the definition of ν, Eq. (15), and that in our case $\gamma = 1$.

We see that our two new critical exponents [Eqs. (19) and (21)] take values that fulfill the scaling relation (16) only if the dimension of the system is $d = 2$. In this case, the hyperscaling relation (17) is also satisfied, as expected for $d < 4$, and we have

$$\eta = 0 \ . \tag{22}$$

This is, in fact, the first hint that our system behaves as an effective two-dimensional one. [Alternatively, from the results below, where we obtain $d = 2$, and by asking the system to satisfy the scaling and hyperscaling relations, we would find the results (21) and (22) for the static correlation function's critical exponents.]

Additional insight can be gained by comparison to the Gaussian model. This model can be described in its continuous version by the partition function [4]

$$\mathcal{Z}_G(J) = \mathcal{N} \int \mathcal{D}\phi \exp\left\{ - \int d^d x \left[\frac{1}{2} c^2 |\nabla \phi|^2 + \frac{1}{2} \mu \phi^2 - J\phi \right] \right\} . \tag{23}$$

The Hamiltonian appearing in the exponential can be seen as a truncation of orders ϕ^4 or higher in a Ginzburg - Landau model. The Gaussian model was originally studied [19] for a discrete spin variable. It has the advantage of being exactly soluble, and it presents a critical point with critical exponents (for a one-component field ϕ, i.e., $D = 1$) given by [17]

$$\alpha = 2 - d/2 \quad , \quad \beta = (d-2)/4 \quad , \quad \gamma = 1 \quad ,$$

$$\delta = \frac{d+2}{d-2} \quad , \quad \eta = 0 \quad , \quad \nu = \frac{1}{2} \ . \tag{24}$$

It is worth remarking here that all these critical exponents can be made to take exactly the same values as for the black hole case, i.e., Eqs. (9)–(11), for $d = 2$. Thus, $d = 2$ appears here as the effective spatial dimensionality of black holes near critical conditions.

The Gaussian model is not fully satisfactory because it has no "ordered" phase. The integral in Eq. (23) diverges for $T < T_c$ and thus one must include higher-order terms (e.g., ϕ^4) to stabilize this integral. It is interesting to note here that black holes themselves pass through the critical curve (at $T = T_c$) from a region of canonical instability to a region of canonical stability as one lowers their temperature (see Figs. 1 and 2 of Ref. [9]).

One might think that the resulting effective dimension of the system, $d = 2$, relies only on comparison to the Gaussian model and that other possibilities are still open. To explore this possibility we can recall some results from the renormalization group theory. Let us suppose that our effective Hamiltonian contains terms of order higher than in the Gaussian model. We then can write

$$H_{\text{eff}}(\phi) = \frac{1}{2} c^2 |\nabla \phi|^2 + \frac{1}{2} \mu \phi^2 + \frac{\lambda}{4!} \phi^4 + b\phi^2 |\nabla \phi|^2 + \cdots \ . \tag{25}$$

The scaling properties of the additional operators, with n powers of ϕ and p derivatives, can be studied in terms of the sign of

$$\Delta = n - p - \frac{1}{2} d(n-2) \ . \tag{26}$$

If Δ is positive (negative) the operator is relevant (irrelevant) [4].

Thus, if the dimensionality of the system were larger than or equal to four, the renormalization group analysis tells us [5,4] that the operators we have added to the Gaussian Hamiltonian are "irrelevant" in the sense that they do not contribute to modify the critical exponents, which will be those of the Gaussian model or the mean field (Landau) theory. Thus, no matching with the black hole results can be made for $d \geq 4$ models, since those values do not coincide with Eqs. (9)–(11). There is still the possibility of having $d = 3$, as is the case of the most realistic system, e.g., those studied in the laboratory. In $d = 3$, ϕ^4 becomes a relevant operator. One can make a perturbation theory based on the Gaussian part of the Hamiltonian and obtain a set of critical exponents [4] that fit very well with laboratory experiments but are not those of black holes. Thus, we are left with $d = 2$ (since for $d < 2$ no critical phenomena take place). The problem here is that *all* operators of the form ϕ^{2n} and $|\nabla\phi|^{2p}\phi^{2n}$ are relevant and thus will modify the critical exponents. Since all operators are relevant, we expect this theory to be renormalizable. In fact, we know that field theory (as well as gravity) in two dimensions is asymptotically free in the ultraviolet allowing us to build up a finite quantum field theory.

We can now conclude that the first-order approximation to quantum effects in black holes corresponds to the Gaussian approximation. Let us recall that the path integral formulation of the Hawking [20] radiation and black hole gravitational thermodynamics relies on the stationary phase approximation to obtain a convergent Gaussian integral. The next-order approximation should include back reaction and self-interaction effects as well as higher-order quantum corrections. In fact, whatever would be the final form of the quantum theory of gravity, we can assume that the Kerr metric should be a classical solution to the vacuum field equations. The critical exponents of this black hole solution will then be those given in Eqs. (9)–(11). By applying the universality hypothesis, these exponents will be the same (at first quantum order) for the full family of black hole solutions to that theory. We can thus conjecture that critical phenomena in black holes will survive to higher quantum order corrections. The scaling laws will continue to hold, but the critical exponents that will satisfy these laws, when quantum higher-order corrections are taken into account, will be different from those given by Eqs. (9)–(11).

III. THE BLACK HOLE HORIZON AS A QUANTUM CRITICAL SYSTEM

Now that we have established that the dimensionality of the system is $d = 2$, it remains to identify this two-dimensional surface. A natural choice is the horizon of the black hole[1] (or better, a slightly shifted outwards 2+1 hypersurface [21]). One observer far away from the black hole sees all the matter of the collapsing body that will form it to accumulate on the two-dimensional horizon forming some kind of "membrane" [22] or subtle "skin" [8].

By analogy to the models for spin systems able to suffer critical transitions, we can think of the event horizon as having only a finite (and eventually discrete) number of degrees of freedom at every (lattice) cell of Planckian dimensions. We know that if there is a continuous internal symmetry in the order parameter, no long range order, or broken symmetry, will occur in two space dimensions. If the symmetry is discrete it is possible (e.g., Ising model).

It is interesting to compare our approach to black hole quantization with that of 't Hooft [23,24] since several points in common can be drawn. In this approach to the problem of black hole quantization one postulates the existence of an S matrix to describe the evaporation process. This hypothesis seems to be supported by new evidence revealing that processes such as stimulated emission in the Hawking radiation might play an important role in helping to solve the loss of quantum information and/or coherence paradox [25]. The horizon shift produced by light particles going out or coming into the horizon is an essential ingredient in the construction of the S matrix. Its elements are given by

$$\langle p^{\rm out}(\Omega)|p^{\rm in}(\Omega)\rangle = \mathcal{N}\exp\left\{i\int\int p^{\rm out}(\Omega)f(\Omega-\Omega') \times p^{\rm in}(\Omega')d^2\Omega d^2\Omega'\right\}, \tag{27}$$

where $p^{\rm in}(\Omega)$ and $p^{\rm out}(\Omega)$ are the momentum distribution at angle $\Omega = (\vartheta,\varphi)$ of the in- and outgoing particles, respectively. The shift function f is the Green function defined on the event horizon [26–28] satisfying

$$\nabla_\perp^2 f - af = b\delta^2(\Omega-\Omega') , \tag{28}$$

$$a = d_\perp r_+\kappa(r_+) \quad , \quad b = 32\pi p r_+^2 g_{uv}(r_+) . \tag{29}$$

This expression can be integrated by using the properties of Legendre polynomials [29]:

$$f(\Omega-\Omega') = -\frac{b}{4}\frac{P_{-1/2+i\sqrt{a-1/4}}[-\cos(\Omega-\Omega')]}{\cosh\left(\pi\sqrt{a-1/4}\right)} ,$$
$$d_\perp = 2. \tag{30}$$

The Legendre functions $P_{-1/2+is}(z)$ are called conical functions and are defined positives for $z<1$.

The poles of the S matrix (27) can be evaluated as follows. First, we note that the structure of these poles depends essentially on the short distance behavior of the function $f(\Delta\Omega)$ (see Ref. [30]). For our function (30) this is given by

$$f(\Delta\Omega) \approx \frac{b\sin(\pi\lambda)}{4\pi\cosh\left(\pi\sqrt{a-1/4}\right)}\left\{2\ln\left(\frac{\Delta\Omega}{4}\right)+\gamma +\psi(\lambda+1)+\pi\cot(\pi\lambda)\right\} , \tag{31}$$

where

$$\lambda = -\frac{1}{2}+i\pi\sqrt{a-1/4} \ ;$$
$$\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)} \quad \text{and} \quad \gamma = 0.577215\ldots . \tag{32}$$

What is essential here is the logarithmic dependence of $f(\Delta\Omega)$. For this dependence the poles of the S matrix are found to lie in [31]

$$E_p^2 = -ilM_{\rm Pl}^2 \quad \text{with} \quad l \ \text{ a positive integer} . \tag{33}$$

These imaginary poles give the bound state spectrum and correspond to the ultrarelativistic hydrogenlike poles.

A functional integral representation can be given for the S matrix [23]:

$$\langle p^{\rm out}(\Omega)|p^{\rm in}(\Omega)\rangle = \mathcal{N}\int\mathcal{D}x(\Omega)\exp\left\{\int d^2\Omega\left[-\frac{i}{2\pi}\left[(\partial_\Omega x)^2+ax^2\right]+ixp^{\rm ext}\right]\right\} . \tag{34}$$

The similarity between this expression and the partition function of our model, Eq. (23), is apparent if we identify there the two-dimensional surface with the event horizon and the scalar order parameter with the "membrane coordinates," x.

Near criticality the "mass term" μ in Eq. (23) vanishes like $\mu\sim|T-T_c|$. Thus, the model becomes conformally invariant. In this case we can write the functional integral formulation in a covariant "stringy" way:

$$\mathcal{Z}_G(J) = \mathcal{N}\int\mathcal{D}\phi^\rho(\sigma)\mathcal{D}g^{ab}(\sigma)\exp\left\{-\int d^2\sigma[-i\sqrt{g}g^{ab} \times\partial_a\phi^\rho\partial_b\phi^\rho + i\phi^\rho J^\rho]\right\} , \tag{35}$$

where σ stands for the two horizon coordinates (in Euclidean space), the order parameter has now ρ internal dimensions, and g^{ab} is the metric on the horizon surface (for a membrane interpretation see Ref. [32]).

Summarizing, we have started by showing the scaling of black holes near criticality. This property of critical systems can be embodied in the conformal invari-

[1]Finite size effects are not expected to affect the scaling properties derived in the thermodynamic limit [3].

ance theory [33,34]. Then, we are led to string theory, which is a realization of a conformal field theory on the two-dimensional world sheet [35]. Equation (35) corresponds to the bosonic string case. The fermionic degrees of freedom can be eventually incorporated in it, this corresponding to the addition of a fermionic order parameter in the effective Hamiltonian Eq. (25).

Actually, for a continuum model, the lower critical dimension is precisely two [4]. This means that to have critical phenomena we should consider $d = 2 + \epsilon$, where ϵ remains small whenever the hole is big compared to the Planck scale [36]. Otherwise, if we consider a discrete model, the lower critical dimension is one. We could thus keep $d = 2$ and deal with a discrete order parameter on the surface of a black hole transformed now in a lattice with a spacing of the order of the Planck length.

Of particular interest here is the result that the continuum limit of the two-dimensional tricritical Ising model near the critical point is supersymmetric [37] (in Ref. [9] we remarked the existence of tricritical points in extreme Kerr-Newman black holes).

The results presented in this paper lead us to consider the following effective model for dealing with a coarse-grained quantization of black holes: *A black hole appears to an external observer as if it had all its quantum degrees of freedom concentrated on a thin membrane tightly covering the horizon.*

This "phenomenological" model is of course observer-dependent, since in a reference system falling with the matter that will form the black hole, nothing special nor the membrane is seen when crossing the horizon. It is also clear that it is the "skin" on the horizon that we propose to consider as a system to quantize by using the standard rules of quantum field theory. In particular, we expect a unitary S matrix to exist, and to describe the process of formation and evaporation of a black hole without leaving room for loss of quantum coherence.

IV. DISCUSSION

From the viewpoint of field theory the nonrenormalizability of quantum gravity is seen as a particularly annoying problem. Especially since the establishment of the standard model of weak, electromagnetic, and strong interactions, renormalizability has become a natural requirement to any good theory. On the other hand, from the point of view of statistical physics [38] the nonrenormalizability of gravity appears natural, since its weakness suggests it is irrelevant (in critical phenomena language) and therefore nonrenormalizable. At low energies, far below $E_{\rm Pl}$, only the relevant operators (which lead to renormalizable theories) will survive. This explains why all current experimental observations can be accurately described in terms of an effective long distance gravitational theory. As β decay is the low-energy remnant of much richer physics above the electroweak scale, new physics should be expected at energies $E > E_{\rm Pl}$. Our effective model, introduced in the last section, can be thought of as the low-energy version of the physics above $E_{\rm Pl}$ obtained by eliminating the details of its structure in a similar way as is done, for example, with the details of copper and zinc atoms from the description of β brass to obtain the Ising model [4].

We have shown conclusive evidence that black holes undergo critical phenomena. Under this condition their characteristic behavior is as if they had an effective dimension equal to two (plus, eventually, ϵ). This dimensionality was first obtained by asking that the critical exponents η and ν, obtained from the correlation functions, satisfy the scaling (and hyperscaling) relations. Then it was shown that by comparison of the black holes other critical exponents with those of the Gaussian model in d dimensions, complete agreement can only be found for $d = 2$. Finally, by quite general arguments coming from the renormalization group theory, we have argued that the effective dimension *cannot* be $d \geq 3$.

The effective two dimensionality [we remark that here this is not imposed externally as in the case of two-dimensional (2D) black holes [39]] has several interesting consequences. Here we shall briefly discuss two of them.

A simple way to show how the mass of a black hole should be quantized can be obtained by describing the black hole by a wave function corresponding to the order parameter in a critical system (collective variable) having one component (a single scalar field) depending only on the two angular coordinates that cover the horizon surface and the proper time τ. In this simplified model the only effect of the black hole's gravitational field is to provide the background geometry, i.e., the spherical surface representing the horizon. If we impose to this wave function the Klein-Gordon equation (which corresponds to the Gaussian approximation)

$$\left\{-\partial_\tau^2 + \frac{1}{r_H^2}\nabla_\Omega^2 + \mu^2\right\}\phi(\vartheta,\varphi,t) = 0 \; , \tag{36}$$

where $r_H = M + \sqrt{M^2 - a^2 - Q^2}$ is the horizon radius, we have the following set of eigenfunctions:

$$\phi_{lm} = \exp\{-iE_l\tau\}Y_{lm}(\vartheta,\varphi) \; , \tag{37}$$

where Y_{lm} are the spherical harmonics and the energy of the system is given by

$$E_l^2 = \mu^2 + \frac{\hbar^2 l(l+1)}{r_H^2} \quad , \quad l = 0,1,2,\ldots \; . \tag{38}$$

Since $r_H \simeq 2GM/c^2$ (for a Schwarzschild hole) and (if we consider that the whole black hole is represented by[2] ϕ) $E_l \simeq M$, Eq. (38) implies

$$M^2 \simeq \frac{\mu^2}{2} + \frac{1}{2}\sqrt{\mu^4 + M_{\rm Pl}^4 l(l+1)} \; .$$

We have that, for big l (where we expect this approach to be valid),

[2]For the alternative view of $M_{\rm BH} = M + E_l$, i.e., quantization of the fluctuations around the classical value, see Ref. [32].

$$M \simeq \frac{1}{\sqrt{2}} M_{\rm Pl}\sqrt{l} \quad , \quad \text{with } l \quad \text{a positive integer.} \qquad (39)$$

This represents a quantization of the black hole mass in units of the Planck mass. It is interesting to remark that the $\sqrt{l}$ dependence has also been found in Eq. (33), and by Bekenstein [40] using the quantization of adiabatic invariant action integrals (see Refs. [41–45] for still other independent derivations). We note that Eq. (39) consists of three factors. While the first $1/\sqrt{2}$ term is expected to be model dependent, the $M_{\rm Pl}$ factor could have been guessed on dimensional grounds. There seems to be some agreement in the cited literature as well as in our Eqs. (33) and (39) on the $\sqrt{l}$ dependence. We thus think that Eq. (39) represents a first approximation to the black hole mass quantization. The black hole radiation will now come out in the form of a line spectrum with most of the radiation at the frequency $\hbar\omega_l = \Delta M c^2 = \frac{M_{\rm Pl}^2 c^2}{4M}$ (also in multiples of this frequency), which corresponds to the maximum of the (continuum) Hawking spectrum, i.e., $\hbar\omega_{\rm max} \sim k_B T_{BH} \sim \frac{M_{\rm Pl}^2 c^2}{M}$.

Since our black hole system, as we have seen, has an associated effective dimension equal to two, the proportionality entropy area can be expected to appear in a natural way. In fact, since the mass of the black hole is quantized there must be a finite number of internal states. They can be counted by noting [42] that a black hole of mass M given by Eq. (39) can be formed in 2^{l-1} different (and equivalent) ways from units of $M_{\rm Pl}$. The entropy associated with the ignorance of the exact way in which the black hole formed, can be evaluated, in a first approximation, as $S_{\rm BH} \simeq k_B \ln[2^{l-1}]$. For large l we have

$$S_{\rm BH} \simeq k_B \ln 2 \left(\frac{M}{M_{\rm Pl}}\right)^2 \simeq \frac{k_B l_{\rm Pl}}{4\pi} \ln 2 \left(4\pi r_H^2\right) \quad , \qquad (40)$$

which gives the well-known proportionality between entropy and area of a black hole.

One should not be bewildered by these results, since they are founded on a crude approximation to the quantum black hole problem. The model is necessarily incomplete (a second quantized description should be considered, for example). Also 't Hooft suggests that the quantum states labeled by E_l in Eq. (38) are enormously degenerated [46]. It is also important to evaluate the width of each energy level (to account for the quantum instability of black holes) and compare it to the separation between energy levels [47]. However, what we wanted to rescue from the above crude model[3] is the relevance of the essentially two-dimensional nature of semiclassical black holes and the possibility of representing them, in a first approximation, by a single scalar field (playing the role of the order parameter in a critical system).

Thus, in conclusion, we can say that black holes may have "no hair" [48,49], but instead they seem to behave as if they had "skin."

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[3]A more detailed treatment is under study by the present author [32].

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