

Gravitons and light cone fluctuations

L.H. Ford

Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155

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Gravitons in a squeezed vacuum state, the natural result of quantum creation in the early Universe or by black holes, will introduce metric fluctuations. These metric fluctuations will introduce fluctuations of the light cone. It is shown that when the various two-point functions of a quantized field are averaged over the metric fluctuations, the light cone singularity disappears for distinct points. The metric-averaged functions remain singular in the limit of coincident points. The metric-averaged retarded Green's function for a massless field becomes a Gaussian which is nonzero both inside and outside of the classical light cone. This implies some photons propagate faster than the classical light speed, whereas others propagate slower. The possible effects of metric fluctuations upon one-loop quantum processes are discussed and illustrated by the calculation of the one-loop electron self-energy.

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I. INTRODUCTION

It was conjectured several years ago by Pauli [1] that the ultraviolet divergences of quantum field theory might be removed in a theory in which gravity is quantized. The basis of Pauli's conjecture was the observation that these divergences arise from the light cone singularities of two-point functions, and that quantum fluctuations of the spacetime metric ought to smear out the light cone, possibly removing these singularities. This conjecture was discussed further by Deser [2], in the context of a path integral approach to the quantization of gravity, and by Isham, Salam, and Strathdee [3]. However, there seems to have been little progress on this question in the intervening years. Indeed, it is well known that perturbative quantum gravity, far from being a universal regulator, is afflicted with nonrenormalizable infinities of its own. In the present work, the issue of light cone fluctuations will be examined in a context where they are produced by gravitons propagating on a flat background. We assume that the gravitons are in a squeezed vacuum state, which is the appropriate state for relic gravitons created by quantum particle creation processes in the early Universe [4] or by black hole evaporation. More generally, a squeezed vacuum state is the quantum state which arises in any quantum particle creation process in which the state of the created particles is an in-vacuum state represented in an out-Fock space. It will be shown that averaging over the metric fluctuations associated with such gravitons has the effect of smearing out the light cone.

It should be noted that the metric fluctuations being considered in this paper are distinct from those due to fluctuations in the energy-momentum tensor of the source [5, 6]. It is possible for the energy density, for example, to exhibit large fluctuations. This arises in the Casimir effect and in quantum states in which the expectation value of the energy density is negative. This means that the gravitational field of such a system is not described by a fixed classical metric, but rather by a fluctuating metric. However, these metric fluctuations

are "passive" in the sense that they are driven by fluctuations in the degrees of freedom of the matter field. In contrast, the metric fluctuations due to gravitons in a squeezed state are "active" fluctuations produced by quantized degrees of freedom of the gravitational field itself.

In Sec. II, the retarded, Hadamard, and Feynman functions will be averaged over metric fluctuations. The resulting smearing of the light cone is also discussed. The average of the square of the Feynman propagator for a scalar field is also calculated. The results are given in terms of the mean square of the squared geodesic separation between points. In Sec. III, this quantity is calculated explicitly for various cases. In this section, gravitons in an expanding universe are also discussed, and some estimates for the present background of relic gravitons are given. The one-loop electron self-energy in the presence of metric fluctuations is calculated and discussed in Sec. IV. The results of the paper are summarized and discussed in Sec. V.

II. AVERAGING TWO-POINT FUNCTIONS OVER METRIC FLUCTUATIONS

A. The retarded Green's function

Let us consider a flat background spacetime with a linearized perturbation $h_{\mu\nu}$ propagating upon it. Thus the spacetime metric may be written as

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu \\ &= dt^2 - d\mathbf{x}^2 + h_{\mu\nu} dx^\mu dx^\nu. \end{aligned} \quad (1)$$

In the unperturbed spacetime, the square of the geodesic separation of points x and x' is $2\sigma_0 = (x - x')^2 = (t - t')^2 - (\mathbf{x} - \mathbf{x}')^2$. In the presence of the perturbation, let this squared separation be 2σ , and write

$$\sigma = \sigma_0 + \sigma_1 + O(h_{\mu\nu}^2), \quad (2)$$

so σ_1 is the shift in σ to first order in $h_{\mu\nu}$.

Let us consider the retarded Green's function for a massless scalar field. In flat spacetime, this function is

$$G_{\text{ret}}^{(0)}(x - x') = \frac{\theta(t - t')}{4\pi} \delta(\sigma_0), \quad (3)$$

which has a δ -function singularity on the future light cone and is zero elsewhere. In the presence of a classical metric perturbation, the retarded Green's function has its δ -function singularity on the perturbed light cone, where $\sigma = 0$. In general, it may also become nonzero on the interior of the light cone due to backscattering off the curvature. However, we are primarily interested in the behavior near the new light cone, and so let us replace $G_{\text{ret}}^{(0)}(x - x')$ by

$$G_{\text{ret}}(x, x') = \frac{\theta(t - t')}{4\pi} \delta(\sigma). \quad (4)$$

We are assuming that the curved space Green's functions have the Hadamard form, in which case their leading asymptotic behavior near the light cone is the same as in flat space [7]. One may regard this assumption as a restriction on the physically allowable quantum states. If we terminate the expansion of σ at first order (higher orders will be discussed below), then Eq. (4) may be expressed as

$$G_{\text{ret}}(x, x') = \frac{\theta(t - t')}{8\pi^2} \int_{-\infty}^{\infty} d\alpha e^{i\alpha\sigma_0} e^{i\alpha\sigma_1}. \quad (5)$$

We now replace the classical metric perturbations by gravitons in a squeezed vacuum state $|\psi\rangle$. Then σ_1 becomes a quantum operator which is linear in the graviton field operator, $h_{\mu\nu}$. A squeezed vacuum state is a state such that σ_1 may be decomposed into positive and negative frequency parts. Thus we may find σ_1^+ and σ_1^- so that

$$\sigma_1^+ |\psi\rangle = 0, \quad \langle\psi| \sigma_1^- = 0, \quad (6)$$

where $\sigma_1 = \sigma_1^+ + \sigma_1^-$. In terms of annihilation and creation operators, $\sigma_1^+ = \sum_j a_j f_j$ and $\sigma_1^- = \sum_j a_j^\dagger f_j^*$, where the f_j are mode functions. We now write

$$e^{i\alpha\sigma_1} = e^{i\alpha(\sigma_1^+ + \sigma_1^-)} = e^{i\alpha\sigma_1^-} e^{-\frac{1}{2}\alpha^2[\sigma_1^+, \sigma_1^-]} e^{i\alpha\sigma_1^+}. \quad (7)$$

In the second step we used the Campbell-Baker-Hausdorff formula, that $e^{A+B} = e^A e^{\frac{1}{2}[A,B]} e^B$ for any pair of operators A and B that each commute with their commutator, $[A, B]$. We now take the expectation value of this expression and use the facts that $e^{i\alpha\sigma_1^+} |\psi\rangle = |\psi\rangle$ and $\langle\psi| e^{i\alpha\sigma_1^-} = \langle\psi|$, which follow immediately from Eq. (6) if the exponentials are expanded in a power series. Finally, we use $[\sigma_1^+, \sigma_1^-] = \sum_j f_j f_j^* = \langle\sigma_1^2\rangle$ to write

$$\langle e^{i\alpha\sigma_1} \rangle = e^{-\frac{1}{2}\alpha^2 \langle\sigma_1^2\rangle}. \quad (8)$$

Thus when we average over the metric fluctuations, the retarded Green's function is replaced by its quantum expectation value:

$$\langle G_{\text{ret}}(x, x') \rangle = \frac{\theta(t - t')}{8\pi^2} \int_{-\infty}^{\infty} d\alpha e^{i\alpha\sigma_0} e^{-\frac{1}{2}\alpha^2 \langle\sigma_1^2\rangle}. \quad (9)$$

The expectation value of σ_1^2 is formally divergent. However, in flat spacetime this divergence may be removed by subtraction of the expectation value in the Minkowski vacuum state. Henceforth, we will take $\langle\sigma_1^2\rangle$ to denote this renormalized expectation value.

The above integral converges only if $\langle\sigma_1^2\rangle > 0$, in which case it may be evaluated to yield

$$\langle G_{\text{ret}}(x, x') \rangle = \frac{\theta(t - t')}{8\pi^2} \sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}} \exp\left(-\frac{\sigma_0^2}{2\langle\sigma_1^2\rangle}\right). \quad (10)$$

Note that this averaged Green's function is indeed finite at $\sigma_0 = 0$ provided that $\langle\sigma_1^2\rangle \neq 0$. Thus the light cone singularity has been smeared out, as illustrated in Fig. 1. Note that the smearing occurs in both the timelike and spacelike directions.

This smearing may be interpreted as due to the fact that photons may be either slowed down or boosted by the metric fluctuations. Photon propagation now becomes a statistical phenomenon; some photons travel slower than light on the classical spacetime, whereas others travel faster. We have now the possibility of "faster than light" signals. This need not cause any causal paradoxes, however, because the system is no longer Lorentz invariant. The graviton state defines a preferred frame of reference. The usual argument linking superluminal signals with causality violation assumes Lorentz invariance [8].

The effects of light cone fluctuations upon photon propagation are in principle observable. Consider a source which emits evenly spaced pulses. An observer at a distance D from the source will detect pulses whose spacing varies by an amount of the order of Δt . For a pulse which is delayed by time Δt ,

$$\sigma = \frac{1}{2}[(D + \Delta t)^2 - D^2] \approx D\Delta t, \quad \Delta t \ll D. \quad (11)$$

Thus the typical time delay or advance is of the order of

$$\Delta t \approx \frac{\sqrt{\langle\sigma_1^2\rangle}}{D}. \quad (12)$$

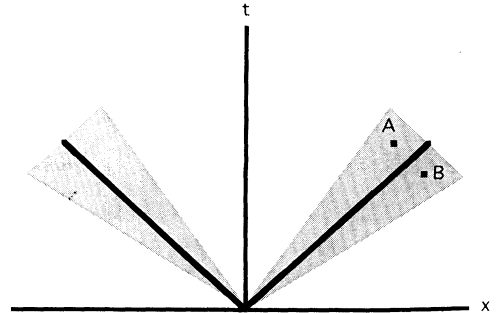


FIG. 1. The smearing of the light cone due to metric fluctuations. A photon which arrives at Point A from the origin has been slowed by the effect of metric fluctuations. A photon which arrives at Point B has been boosted by metric fluctuations, and appears to travel at a superluminal velocity in the background metric.

This effect leads to the broadening of spectral lines. The observer will detect a line which is broadened in wavelength by $\Delta\lambda = \Delta t$. Some observational aspects of this effect will be discussed in more detail in Sec. III.

Note that it is essential that the gravitons be in a non-classical state, such as a squeezed vacuum, in order to obtain light cone smearing. Gravitons in a coherent state will represent a classical gravity wave. In this case, the retarded Green's function will still have a δ -function singularity on the light cone of the perturbed spacetime.

In the above calculation of $\langle G_{\text{ret}}(x, x') \rangle$, the expansion of σ was truncated after the first order. However, it is of interest to consider the effect of second-order terms. This is particularly pertinent in view of the fact that the crucial corrections involve $\langle \sigma_1^2 \rangle$, which is itself second order in $h_{\mu\nu}$ [9]. We now write

$$\sigma = \sigma_0 + \sigma_1 + \sigma_2 + O(h_{\mu\nu}^3), \quad (13)$$

so that σ_2 is the second-order correction. We now wish to include this correction in the calculation of $\langle G_{\text{ret}}(x, x') \rangle$. Let us first write

$$\sigma_2 =: \sigma_2 : + \langle \sigma_2 \rangle, \quad (14)$$

where the colons denote normal ordering with respect to the state $|\psi\rangle$, and the expectation value is understood to be in this state. Equation (7) is now replaced by

$$e^{i\alpha(\sigma_1 + \sigma_2)} = e^{i\alpha\sigma_1^-} e^{-\frac{1}{2}\alpha^2[\sigma_1^+, \sigma_1^-]} e^{i\alpha\sigma_1^+} e^{i\alpha\langle\sigma_2\rangle} e^{i\alpha:\sigma_2:}. \quad (15)$$

Here we have ignored all terms which are of third order or higher, including those which arise when $:\sigma_2:$ is commuted past σ_1^\pm . We use the fact that

$$e^{i\alpha:\sigma_2:}|\psi\rangle = |\psi\rangle, \quad (16)$$

to write the analogue of Eq. (8):

$$\langle e^{i\alpha(\sigma_1 + \sigma_2)} \rangle = e^{i\alpha\langle\sigma_2\rangle - \frac{1}{2}\alpha^2\langle\sigma_1^2\rangle}. \quad (17)$$

As in the case of $\langle\sigma_1^2\rangle$, we assume that $\langle\sigma_2\rangle$ is a renormalized expectation value. Now the metric averaged Green's function becomes

$$\langle G_{\text{ret}}(x, x') \rangle = \frac{\theta(t-t')}{8\pi^2} \sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}} \exp\left(-\frac{\sigma_0^2 + \langle\sigma_2\rangle}{2\langle\sigma_1^2\rangle}\right). \quad (18)$$

Comparison with Eq. (10) reveals that the effect of retaining the σ_2 term is simply to shift slightly the position of the peak of the Gaussian. Thus $\langle\sigma_2\rangle$ enters in a different way from $\langle\sigma_1^2\rangle$, due to the different powers of α in Eq. (17). The same phenomenon would occur for the other functions to be discussed below, so henceforth the σ_2 terms will be ignored.

It should be noted that although we are expanding σ in powers of the metric perturbation $h_{\mu\nu}$, the averaging procedure used to obtain $\langle G_{\text{ret}}(x, x') \rangle$ retains terms of all orders in $h_{\mu\nu}$. This is essential in order to obtain nontrivial results. We can think of this as an expansion of the argument of the exponential functions in Eqs. (5) or (15)

but not of the functions themselves. This seems to be self-consistent in that retaining successively higher terms in σ leads to small changes in the form of the results, as we saw in going from Eq. (10) to Eq. (18).

B. The Hadamard function

In addition to the retarded and advanced Green's functions discussed in the previous subsection, there are several other singular functions in quantum field theory which can be expressed as vacuum expectation values of products of field operators. In particular, the *Hadamard function* for a scalar field ϕ is defined as

$$G_1(x, x') \equiv \langle 0|\phi(x)\phi(x') + \phi(x')\phi(x)|0\rangle, \quad (19)$$

where $|0\rangle$ is the vacuum state. In the massless case in flat spacetime, it has the explicit form

$$G_1(x, x') = -\frac{1}{4\pi^2\sigma}. \quad (20)$$

Recall that σ is one-half of the square of the geodesic distance between x and x' , and in flat spacetime, $\sigma = \frac{1}{2}(x-x')^2$. Even in the massive case, and/or in curved spacetime, Eq. (20) gives the asymptotic behavior of $G_1(x, x')$ near the light cone. As in the case of the retarded Green's function, we now wish to replace σ by $\sigma_0 + \sigma_1$ and take the quantum expectation value of the result. Let us use the identities

$$\int_0^\infty d\alpha e^{i\alpha x} = \frac{i}{x} + \pi\delta(x), \quad (21)$$

and

$$\int_0^\infty d\alpha e^{-i\alpha x} = -\frac{i}{x} + \pi\delta(x), \quad (22)$$

to write

$$\frac{1}{\sigma_0 + \sigma_1} = -\frac{i}{2} \int_0^\infty d\alpha [e^{i(\sigma_0 + \sigma_1)\alpha} - e^{-i(\sigma_0 + \sigma_1)\alpha}]. \quad (23)$$

Now use Eq. (8) to take the expectation value of the above expression and write

$$\begin{aligned} \langle G_1(x, x') \rangle &= -\frac{1}{4\pi^2} \left\langle \frac{1}{(\sigma_0 + \sigma_1)} \right\rangle \\ &= -\frac{1}{4\pi^2} \int_0^\infty d\alpha \sin \sigma_0 \alpha e^{-\frac{1}{2}\alpha^2\langle\sigma_1^2\rangle}. \end{aligned} \quad (24)$$

This expression gives us the Hadamard function averaged over metric fluctuations for the case that $\langle\sigma_1^2\rangle > 0$.

Let us examine the asymptotic forms of this result. Near the classical light cone, $\sigma_0 \rightarrow 0$. If we expand the integrand of the above expression to lowest order in σ_0 , and perform the integration, we find that

$$\langle G_1(x, x') \rangle \sim -\frac{\sigma_0}{4\pi^2\langle\sigma_1^2\rangle}, \quad \sigma_0 \rightarrow 0. \quad (25)$$

Thus the light cone singularity is removed so long as $\langle\sigma_1^2\rangle \neq 0$, which will generally be the case for noncoincident points. Equation (24) may be rewritten as

$$\begin{aligned} \langle G_1(x, x') \rangle = & -\frac{1}{4\pi^2\sigma_0} \left[1 - \frac{\langle \sigma_1^2 \rangle}{\sigma_0^2} \int_0^\infty dt t \cos t \right. \\ & \left. \times \exp\left(-\frac{\langle \sigma_1^2 \rangle t^2}{2\sigma_0^2}\right) \right]. \end{aligned} \quad (26)$$

In the limit that $\sigma_0^2 \gg \langle \sigma_1^2 \rangle$, the second term above is negligible and we recover the classical form of G_1 :

$$\langle G_1(x, x') \rangle \sim -\frac{1}{4\pi^2\sigma_0}. \quad (27)$$

The above expression for $\langle G_1(x, x') \rangle$ is valid for $\langle \sigma_1^2 \rangle > 0$. We can, however, obtain an alternative form valid for the case that $\langle \sigma_1^2 \rangle < 0$. To do so, we use the representation

$$\frac{1}{\sigma_0 + \sigma_1} = \int_0^\infty d\alpha e^{-(\sigma_0 + \sigma_1)\alpha}. \quad (28)$$

Now we have

$$\begin{aligned} \langle G_1(x, x') \rangle = & -\frac{1}{4\pi^2} \left\langle \frac{1}{(\sigma_0 + \sigma_1)} \right\rangle \\ = & -\frac{1}{4\pi^2} \int_0^\infty d\alpha e^{-\sigma_0\alpha} e^{\frac{1}{2}\alpha^2\langle \sigma_1^2 \rangle}. \end{aligned} \quad (29)$$

Near the light cone, this quantity is finite:

$$\langle G_1(x, x') \rangle \rightarrow -\frac{1}{4\pi^2} \sqrt{\frac{\pi}{2|\langle \sigma_1^2 \rangle|}}, \quad \sigma_0 \rightarrow 0. \quad (30)$$

We may rewrite Eq. (29) as

$$\begin{aligned} \langle G_1(x, x') \rangle = & -\frac{1}{4\pi^2\sigma_0} \left[1 + \frac{\langle \sigma_1^2 \rangle}{\sigma_0^2} \int_0^\infty dt t e^{-t} \right. \\ & \left. \times \exp\left(\frac{\langle \sigma_1^2 \rangle t^2}{2\sigma_0^2}\right) \right]. \end{aligned} \quad (31)$$

From this form, we again obtain Eq. (27) when $\sigma_0^2 \gg |\langle \sigma_1^2 \rangle|$. Alternatively, Eq. (31) may be derived by expanding $(\sigma_0 + \sigma_1)^{-1}$ in a power series in σ_1 , using Wick's theorem to replace $\langle \sigma_1^{2n} \rangle$ by $(2n-1)!! \langle \sigma_1^2 \rangle^n$, and finally resumming the result by Borel summation.

C. The Feynman propagator

The average of the Feynman propagator G_F over the metric fluctuations can readily be obtained by combining the results of the previous two subsections. We use the identity

$$G_F(x, x') = -\frac{1}{2} [G_{\text{ret}}(x, x') + G_{\text{adv}}(x, x')] - \frac{i}{2} G_1(x, x'), \quad (32)$$

and the fact that the advanced Green's function is related to the retarded Green's function by

$$G_{\text{adv}}(x, x') = G_{\text{ret}}(x', x). \quad (33)$$

We restrict our attention to the case that $\langle \sigma_1^2 \rangle > 0$, both because it is only here that we have a formula for $\langle G_{\text{ret}} \rangle$,

and it is the case of greater physical interest. Combining Eqs. (10) and (24), we obtain

$$\begin{aligned} \langle G_F(x, x') \rangle = & -\frac{1}{16\pi^2} \sqrt{\frac{\pi}{2\langle \sigma_1^2 \rangle}} \exp\left(-\frac{\sigma_0^2}{2\langle \sigma_1^2 \rangle}\right) \\ & + \frac{i}{8\pi^2} \int_0^\infty d\alpha \sin \sigma_0 \alpha e^{-\frac{1}{2}\alpha^2\langle \sigma_1^2 \rangle}. \end{aligned} \quad (34)$$

Again, this quantity is finite except in the coincidence limit, $x' \rightarrow x$.

Alternately, we can write

$$G_F(x, x') = \frac{1}{8\pi^2} \left[\frac{i}{\sigma} - \pi \delta(\sigma) \right] = -\frac{1}{8\pi^2} \int_0^\infty d\alpha e^{-i\alpha\sigma}. \quad (35)$$

Averaging this integral form for G_F over metric fluctuations yields

$$\langle G_F(x, x') \rangle = -\frac{1}{8\pi^2} \int_0^\infty d\alpha e^{-i\sigma_0\alpha} e^{-\frac{1}{2}\alpha^2\langle \sigma_1^2 \rangle}. \quad (36)$$

This form is equivalent to Eq. (34). Note that whereas the real part of the above integral may be expressed in terms of elementary functions, the imaginary part may not.

D. The square of the Feynman propagator

Earlier in this section, we obtained expressions for the various singular functions averaged over metric fluctuations. However, the Feynman diagrams for one-loop processes often involve products of at least two Feynman propagators. Thus, if we wish to study the effect of metric fluctuations upon these processes, we need an expression for quantities such as $\langle G_F^2 \rangle$, the average of the square of the Feynman propagator. We will again assume that $\langle \sigma_1^2 \rangle > 0$. We may use Eq. (35) to write

$$G_F^2 = \frac{1}{(8\pi^2)^2} \int_0^\infty d\alpha d\beta e^{i(\alpha+\beta)\sigma}. \quad (37)$$

If we set $\sigma = \sigma_0 + \sigma_1$, and average over the metric fluctuations, the result is

$$\langle G_F^2 \rangle = \frac{1}{(8\pi^2)^2} \int_0^\infty d\alpha d\beta e^{-i(\alpha+\beta)\sigma_0} e^{-\frac{1}{2}(\alpha+\beta)^2\langle \sigma_1^2 \rangle}. \quad (38)$$

We next change the integration variables, first to polar coordinates defined by $\alpha = \rho \cos \theta$ and $\beta = \rho \sin \theta$, and then to a rescaled radial coordinate defined by $t = (\cos \theta + \sin \theta)\rho$:

$$\begin{aligned} \langle G_F^2 \rangle = & \frac{1}{(8\pi^2)^2} \int_0^{\frac{\pi}{2}} d\theta \int_0^\infty d\rho \rho e^{-i(\cos\theta + \sin\theta)\sigma_0\rho} \\ & \times \exp\left[-\frac{1}{2}(\cos\theta + \sin\theta)^2\langle \sigma_1^2 \rangle\rho^2\right] \\ = & \frac{1}{(8\pi^2)^2} \int_0^{\frac{\pi}{2}} \frac{d\theta}{(\cos\theta + \sin\theta)^2} \\ & \times \int_0^\infty dt t e^{-i\sigma_0 t} e^{-\frac{1}{2}\langle \sigma_1^2 \rangle t^2}. \end{aligned} \quad (39)$$

We now use the identity

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{(\cos\theta + \sin\theta)^2} = 1, \quad (40)$$

to write our result (for later use, it is convenient to relabel the integration variable):

$$\langle G_F^2 \rangle = \frac{1}{(8\pi^2)^2} \int_0^\infty d\alpha \alpha e^{-i\sigma_0 \alpha} e^{-\frac{1}{2} \langle \sigma_1^2 \rangle \alpha^2}. \quad (41)$$

In this case, the imaginary part of the integral can be expressed in terms of elementary functions to write

$$\langle G_F^2 \rangle = \frac{1}{64\pi^4} \int_0^\infty d\alpha \alpha \cos \sigma_0 \alpha e^{-\frac{1}{2} \langle \sigma_1^2 \rangle \alpha^2} + i \frac{\sqrt{2\pi} \sigma_0}{128\pi^4 \langle \sigma_1^2 \rangle^{\frac{3}{2}}} \exp\left(-\frac{\sigma_0^2}{2 \langle \sigma_1^2 \rangle}\right). \quad (42)$$

Again, these forms hold for the case that $\langle \sigma_1^2 \rangle > 0$. As in the case of the averaged Green's functions, this quantity is finite on the classical light cone, $\sigma_0 = 0$, so long as the points are not actually coincident, so $\langle \sigma_1^2 \rangle \neq 0$.

III. GRAVITONS AND THE FORM OF $\langle \sigma_1^2 \rangle$

A. Gravitons in flat spacetime

So far, we have only assumed that the quantum state $|\psi\rangle$ of the gravitons is such that σ_1 can be decomposed into positive and negative parts which satisfy Eq. (6). However, we must have more information about the state before we can determine the explicit form of $\langle \sigma_1^2 \rangle$. Even the calculation of σ_1 for a given classical metric perturbation can be a difficult task, involving the integration of the square root of Eq. (1) along a geodesic. However, as we are interested in gravitational wave perturbations, we may simplify the analysis by the adoption of the transverse-tracefree gauge, which is specified by the conditions

$$h_j^j = \partial_j h^{ij} = h^{0\nu} = 0. \quad (43)$$

In particular, $h_{\mu\nu}$ has purely spatial components h_{ij} in a chosen coordinate system. Thus, in this gauge, a null geodesic is specified by

$$dt^2 = d\mathbf{x}^2 - h_{ij} dx^i dx^j, \quad (44)$$

and along a future-directed null geodesic, one has

$$dt = \sqrt{1 - h_{ij} n^i n^j} dr \approx \left(1 - \frac{1}{2} h_{ij} n^i n^j\right) dr. \quad (45)$$

Here $dr = |d\mathbf{x}|$, and $n^i = dx^i/dr$ is the unit three-vector defining the spatial direction of the geodesic. Thus the time interval Δt and spatial interval $\Delta r = r_1 - r_0$ traversed by a null ray are related by

$$\Delta t = \Delta r - \frac{1}{2} \int_{r_0}^{r_1} h_{ij} n^i n^j dr. \quad (46)$$

Denote the right-hand side of the above expression by $\Delta \ell$, the proper spatial distance interval between the end points. Now consider an arbitrary pair of points (not necessarily null separated). The square of the geodesic

separation between these points is

$$2\sigma = (\Delta t)^2 - (\Delta \ell)^2 \approx (\Delta t)^2 - (\Delta r)^2 + \Delta r \int_{r_0}^{r_1} h_{ij} n^i n^j dr, \quad (47)$$

so

$$\sigma_1 = \frac{1}{2} \Delta r \int_{r_0}^{r_1} h_{ij} n^i n^j dr. \quad (48)$$

If we now treat h_{ij} as a quantized metric perturbation, we obtain a formula for $\langle \sigma_1^2 \rangle$:

$$\langle \sigma_1^2 \rangle = \frac{1}{4} (\Delta r)^2 \int_{r_0}^{r_1} dr \int_{r_0}^{r_1} dr' n^i n^j n^k n^m \times \langle h_{ij}(x) h_{km}(x') \rangle. \quad (49)$$

Here the graviton two-point function $\langle h_{ij}(x) h_{km}(x') \rangle$ is understood to be renormalized, so that it is finite when $x = x'$ and vanishes when the quantum state of the gravitons is the vacuum state.

Of particular interest is the case where only modes with wavelengths long compared to Δr are excited, so the two-point function is approximately constant in both variables. Then

$$\langle \sigma_1^2 \rangle \approx \frac{1}{4} \langle h_{ij} h_{km} \rangle \Delta x^i \Delta x^j \Delta x^k \Delta x^m, \quad (50)$$

where $\Delta x^i = (dx^i/dr) \Delta r$ is the spatial coordinate separation of the end points. In this frame of reference, $\langle \sigma_1^2 \rangle$ will depend only upon Δx^i .

We may illustrate the calculation of $\langle \sigma_1^2 \rangle$ more explicitly. The field operator $h_{\mu\nu}$ may be expanded in terms of plane waves as

$$h_{\mu\nu} = \sum_{\mathbf{k}, \lambda} [a_{\mathbf{k}, \lambda} e_{\mu\nu}(\mathbf{k}, \lambda) f_{\mathbf{k}} + \text{H.c.}], \quad (51)$$

where H.c. denotes the Hermitian conjugate, λ labels the polarization states, $f_{\mathbf{k}} = (2\omega V)^{-\frac{1}{2}} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$ is a box normalized mode function, and the $e_{\mu\nu}(\mathbf{k}, \lambda)$ are polarization tensors. (Here units in which $32\pi G = 1$, where G is Newton's constant are used.) Let us consider the particular case of gravitons in a squeezed vacuum state of a single linearly polarized plane wave mode. Let the mode have frequency ω and be propagating in the $+z$ direction. Take the polarization tensor to have the nonzero components $e_{xx} = -e_{yy} = 1/\sqrt{2}$. This is the "+" polarization in the notation of Ref. [10]. Then we have that

$$\langle \sigma_1^2 \rangle = \frac{[(\Delta x)^2 - (\Delta y)^2]^2}{16\omega V} \text{Re}[\langle a^\dagger a \rangle + \langle a^2 \rangle e^{2i\omega(z-t)}]. \quad (52)$$

A squeezed vacuum state for a single mode can be defined by [11]

$$|\zeta\rangle = S(\zeta) |0\rangle, \quad (53)$$

where $S(\zeta)$ is the squeeze operator defined by

$$S(\zeta) = \exp \left[\frac{1}{2} \zeta^* a^2 - \frac{1}{2} \zeta (a^\dagger)^2 \right]. \quad (54)$$

Here

$$\zeta = r e^{i\delta} \quad (55)$$

is an arbitrary complex number. The squeeze operator has the properties that

$$S^\dagger(\zeta) a S(\zeta) = a \cosh r - a^\dagger e^{i\delta} \sinh r, \quad (56)$$

and

$$S^\dagger(\zeta) a^\dagger S(\zeta) = a^\dagger \cosh r - a e^{-i\delta} \sinh r. \quad (57)$$

From these properties, we may show that

$$\langle a^\dagger a \rangle = \sinh^2 r, \quad (58)$$

and

$$\langle a^2 \rangle = -e^{i\delta} \sinh r \cosh r. \quad (59)$$

Hence in this example

$$\langle \sigma_1^2 \rangle = \frac{[(\Delta x)^2 - (\Delta y)^2]^2}{16\omega V} \sinh r \times \left\{ \sinh r - \cosh r \cos [2\omega(z-t) + \delta] \right\}. \quad (60)$$

Here $\langle \sigma_1^2 \rangle$ will be positive in some regions and negative in others.

Of particular interest to us will be the case of an isotropic bath of gravitons. Here rotational symmetry and the tracelessness condition imply that

$$\langle h_{ij} h_{kl} \rangle = A \left(\delta_{ij} \delta_{kl} - \frac{3}{2} \delta_{ik} \delta_{jl} - \frac{3}{2} \delta_{il} \delta_{jk} \right), \quad (61)$$

where $A = -\frac{1}{15} \langle h_{ij} h^{ij} \rangle$. In this case, we have that

$$\langle \sigma_1^2 \rangle = h^2 r^4, \quad (62)$$

where $r = |\Delta \mathbf{x}|$ is the magnitude of the spatial separation, and

$$h^2 = -\frac{1}{2} A = \frac{1}{30} \langle h_{ij} h^{ij} \rangle \quad (63)$$

is a measure of the mean squared metric fluctuations.

In some cases, the gravitons may be regarded as being in a thermal state. Although a thermal state is a mixed state rather than a pure quantum state, quantum particle creation processes often give rise to a thermal spectrum of particles. In the case of gravitons created by the Hawking effect, this correspondence is exact. In the case of cosmological particle production, it is possible to obtain an approximately thermal spectrum in some cases [12]. We may find h^2 for a thermal bath of gravitons by noting that here, due to the two polarization states for gravitons, $\langle h_{ij} h^{ij} \rangle = 2 \langle \varphi^2 \rangle$, where φ is a massless scalar field. In a thermal state at temperature T , it is well known that $\langle \varphi^2 \rangle = \frac{T^2}{12}$. Thus, for a thermal bath of gravitons at temperature T ,

$$\langle \sigma_1^2 \rangle = \frac{1}{180} T^2 r^4. \quad (64)$$

Note that in this case, $\langle \sigma_1^2 \rangle > 0$, whereas more generally it may have either sign. Recall that the forms of the averaged Green's functions obtained in Sec. II depend upon the sign of $\langle \sigma_1^2 \rangle$, and it is only for the case $\langle \sigma_1^2 \rangle > 0$ that expressions were found for $\langle G_{\text{ret}} \rangle$ and $\langle G_F \rangle$.

B. Gravitons in an expanding universe

For the most part in this paper, we are concerned with gravitons and light cone fluctuations on a background of flat spacetime. However, relic gravitons from the early Universe are one of the more likely sources of metric fluctuations. Thus we need to discuss gravitons on a cosmological background, which we will take to be a spatially flat Robertson-Walker universe. The metric can be written as

$$ds^2 = dt^2 - a^2(t) d\mathbf{x}^2, \quad (65)$$

where $a(t)$ is the scale factor. Linearized perturbations of this metric were investigated by Lifshitz [13], who showed that it is still possible to impose the transverse-trace-free gauge conditions, Eq. (43). The nonzero components of the perturbation satisfy

$$a^{-3} \frac{\partial}{\partial t} \left(a^3 \frac{\partial h_j^i}{\partial t} \right) - a^{-2} \nabla^2 h_j^i = 0. \quad (66)$$

However, this is just the equation satisfied by a minimally coupled scalar field in this background,

$$\square \varphi = 0. \quad (67)$$

Thus the graviton field may be treated as a pair (one for each polarization) of massless, minimally coupled scalar fields. The quantization of cosmological metric perturbations in this framework was discussed in Ref. [14].

Consider a power law expansion, for which

$$a(t) = ct^\alpha. \quad (68)$$

In this case, the solutions of Eq. (66) are of the form $\psi_k e^{i\mathbf{k} \cdot \mathbf{x}}$, where

$$\psi_k = \eta^{\frac{1}{2b}} [c_1 H_\nu^{(1)}(k\eta) + c_2 H_\nu^{(2)}(k\eta)]. \quad (69)$$

Here $b = (\alpha - 1)(3\alpha - 1)^{-1}$ and $\nu = (2|b|)^{-1}$. Furthermore, c_1 and c_2 are arbitrary constants, and η is the conformal time given by

$$\eta = \int a^{-1} dt = [c(1 - \alpha)]^{-1} t^{1-\alpha}. \quad (70)$$

We are interested in the late time behavior of these solutions, which will indicate how quantities such as $\langle \sigma_1^2 \rangle$ or h^2 scale with the expansion of the universe. As $t \rightarrow \infty$, $\eta \rightarrow \infty$ if $\alpha < 1$, and $\eta \rightarrow 0$ if $\alpha > 1$. In the former case, we use the large argument limit of the Hankel functions:

$$|H_\nu^{(1)}(k\eta)| \sim |H_\nu^{(2)}(k\eta)| \sim \sqrt{\frac{2}{\pi|k\eta|}}, \quad (71)$$

as $|\eta| \rightarrow \infty$ for fixed k . In the latter case, we use the small argument limit:

$$|H_\nu^{(1)}(k\eta)| \sim |H_\nu^{(2)}(k\eta)| \sim \frac{\Gamma(\nu)}{\pi} \left(\frac{1}{2}|k\eta|\right)^{-\nu}, \quad |k\eta| \rightarrow 0. \tag{72}$$

From these forms, we find that $|\psi_k| \sim a^{-1}$ if $\alpha < 1$, and $|\psi_k| \sim \text{const}$ if $\alpha > 1$. Thus as $t \rightarrow \infty$,

$$h^2 \sim \frac{1}{a^2}, \quad \alpha < 1, \tag{73}$$

$$h^2 \rightarrow \text{const}, \quad \alpha > 1. \tag{74}$$

Now let us make some estimates of the magnitude of h^2 due to a background of relic cosmological gravitons. The creation of gravitons in an expanding universe is a topic upon which there is a vast literature [15]. Let us consider a model in which gravitons are created at the end of an inflationary epoch. This type of model was discussed in Ref. [16], where it was argued that the typical energy density of gravitons present just after inflation will be of the order of the energy density associated with the Gibbons-Hawking temperature of the de Sitter phase. Let ρ_V be the vacuum energy density during inflation and ρ_P be the Planck density. Then the energy density of the created gravitons at the end of inflation will be of the order of

$$\rho_i \approx \frac{\rho_V^2}{\rho_P}. \tag{75}$$

This energy density will subsequently be redshifted by the expansion of the universe to an energy density at the present time of the order of

$$\rho \approx \rho_i \left(\frac{3K}{T_R}\right)^4 \approx \frac{\rho_V^2}{\rho_P} \left(\frac{3K}{T_R}\right)^4, \tag{76}$$

where T_R is the temperature of reheating after inflation. Here we are assuming that the subsequent expansion rate of the universe corresponds to $\alpha < 1$, so that the gravitons redshift as ordinary massless particles. The typical wavelength of the gravitons at the time of creation is

$$\lambda_i \approx (\rho_i)^{\frac{1}{4}}, \tag{77}$$

and will be redshifted at the present time to a wavelength of the order of

$$\lambda \approx \lambda_i \left(\frac{3K}{T_R}\right). \tag{78}$$

The corresponding mean squared metric fluctuation will be of the order of

$$h^2 \approx \rho \lambda^2. \tag{79}$$

If, for example, inflation were to occur at an energy scale of 10^{15} GeV, and the reheating occurs to the same energy scale, this model would predict a present-day mean graviton wavelength of the order of $\lambda \approx 10^4$ cm and $h \approx 10^{-36}$. For most purposes, the effects of these gravitons will be completely negligible. For example, the light cone fluctuations will produce a spread in arrival times of pulses, from Eq. (12), of the order of $\Delta t \approx 10^{-36}D$, where $D \leq 10^4$ cm. This is a time spread of no more than one

Planck time and is hence unobservably small. The best hope for observing the effects of the light cone fluctuations seems to be through their indirect influence upon virtual processes, which will be the topic of the next section.

IV. ONE-LOOP PROCESSES: THE ELECTRON SELF-ENERGY

In this section, we wish to explore the extent to which quantum metric fluctuations can act as a regulator of the ultraviolet divergences of quantum field theory. These divergences typically appear in one-loop processes, which represent the lowest-order quantum corrections to the classical theory. We will focus our attention upon the one-loop electron self-energy illustrated in Fig. 2. The self-energy function $\Sigma(p)$ is formally given by the divergent momentum space integral:

$$\Sigma(p) = ie^2 \int \frac{d^4k}{(2\pi)^4} D_F^{\mu\nu}(k) \gamma_\mu S_F(p-k) \gamma_\nu. \tag{80}$$

Here $D_F^{\mu\nu}(k)$ and $S_F(p-k)$ are the momentum space photon and electron propagators, respectively, and the γ_μ are Dirac matrices. This integral is logarithmically divergent for large k . In the conventional approach to field theory, this divergence is absorbed by mass renormalization. Here we wish to investigate the effects of introducing metric fluctuations. First, let us rewrite the expression for Σ as a coordinate space integral by use of the following relations between momentum space and coordinate space propagators:

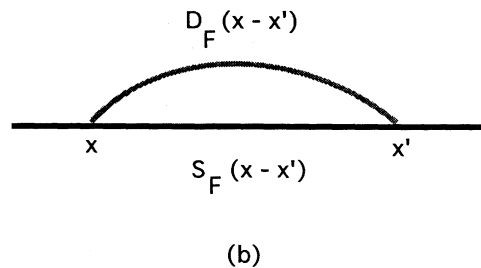
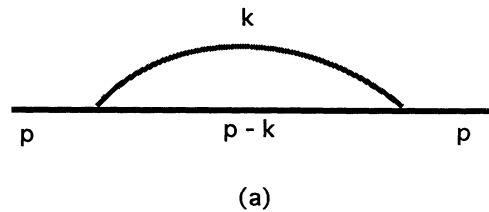


FIG. 2. The Feynman diagram for the one-loop electron self-energy in (a) momentum space and in (b) coordinate space. Here D_F is the photon propagator, and S_F is the electron propagator.

$$D_F^{\mu\nu}(k) = - \int d^4x e^{ikx} D_F^{\mu\nu}(x), \quad (81)$$

and

$$S_F(k) = \int d^4x e^{ikx} S_F(x). \quad (82)$$

The electron propagator $S_F(x)$ is expressible in terms of the scalar propagator by the relation

$$S_F(x) = -(i\gamma^\mu \nabla_\mu + m_0) G_F(x). \quad (83)$$

Here m_0 might be interpreted as a bare mass. If we adopt the Feynman gauge, the photon propagator becomes

$$D_F^{\mu\nu}(x) = -g^{\mu\nu} G_F(x). \quad (84)$$

Note that the scalar propagator, $G_F(x)$, in Eq. (83) is that for a massive field, whereas Eq. (84) is that for a massless field. However, we are interested in the behavior near the classical light cone, and so ignore the mass dependence of the former. Recall that Σ is a 4×4 matrix. The mass shift can be expressed as

$$\delta m = \frac{1}{4} \text{Re}[\text{Tr}\Sigma(0)]. \quad (85)$$

If we combine the above relations and use the fact that $\text{Tr}(\gamma^\mu) = 0$, this may be written as

$$\delta m = m_0 e^2 \text{Im} \int d^4x G_F^2(x). \quad (86)$$

This relation has been obtained assuming a fixed, flat background metric. However, we will assume that it also holds to leading order when we introduce small metric perturbations. Now we wish to average over metric fluctuations and write

$$\Delta m = \langle \delta m \rangle = m_0 e^2 \text{Im} \int d^4x \langle G_F^2(x) \rangle. \quad (87)$$

Use Eqs. (41) and (62) to write

$$\int d^4x \langle G_F^2(x) \rangle = \frac{1}{(8\pi^2)^2} \int_0^\infty d\alpha \alpha \times \int d^4x e^{-\frac{1}{2}i(t^2-r^2)\alpha} e^{-\frac{1}{2}h^2 r^4 \alpha^2}. \quad (88)$$

If we ignore any space or time dependence in h , then this integral may be explicitly evaluated. This should be an excellent approximation, as h is expected to vary on a cosmological time scale, whereas the dominant contributions to Δm should come from scales of the order of or less than the electron Compton wavelength. If we deform the contour for the α integration into the lower half plane, then the t integration becomes absolutely convergent, and we can write

$$\int_{-\infty}^\infty dt e^{-\frac{1}{2}it^2\alpha} = \sqrt{\frac{\pi}{\alpha}} e^{-\frac{1}{4}i\pi}. \quad (89)$$

If we perform the t and angular integrations in Eq. (88), and then replace α by the variable $u = \alpha r^2$, we find

$$\int d^4x \langle G_F^2(x) \rangle = \frac{\sqrt{\pi}}{16\pi^3} e^{-\frac{1}{4}i\pi} \int_0^\infty \frac{dr}{r} \int_0^\infty du \sqrt{u} e^{\frac{1}{2}u} \times e^{-\frac{1}{2}h^2 u^2}. \quad (90)$$

The r integration is logarithmically divergent at both limits. The infrared divergence at large r is an artifact of our having neglected the electron mass in the electron propagator. The ultraviolet divergence at small r is more serious, and reflects the failure of metric fluctuations to render quantum field theory fully finite. The basic problem is that although the light cone singularity has been removed, quantities such as $\langle G_F^2 \rangle$ are still singular at coincident points. Nonetheless, it is still of some interest to determine the h dependence of our expressions. The u integration may be performed explicitly [17] to yield

$$\int d^4x \langle G_F^2(x) \rangle = \frac{e^{-\frac{1}{4}i\pi}}{32\pi^2} h^{-\frac{3}{2}} e^{-(16h^2)^{-1}} D_{-\frac{3}{2}}\left(-\frac{i}{2h}\right) \times \int_0^\infty \frac{dr}{r}, \quad (91)$$

where $D_p(z)$ is the parabolic cylinder function.

If $h \ll 1$, we may use the large argument expansion [18] of $D_p(z)$:

$$D_p(z) \sim e^{-\frac{1}{4}z^2} z^p \left[1 - \frac{p(p-1)}{z^2} + \dots \right], \quad |\arg(z)| < \frac{3}{4}, \quad (92)$$

to write

$$D_{-\frac{3}{2}}\left(-\frac{i}{2h}\right) \sim e^{\frac{3}{4}i\pi} h^{\frac{3}{2}} e^{16h^2} (1 + 15h^2 + \dots), \quad h \ll 1. \quad (93)$$

If we now combine this result with Eqs. (87) and (91), we finally obtain the formal expression for the mass shift to be

$$\Delta m = \frac{m_0 e^2}{8\pi^2} (1 + 15h^2 + \dots) \int_0^\infty \frac{dr}{r}. \quad (94)$$

This expression is divergent, and hence still needs to be carefully regularized and renormalized. Here we will simply observe that the dependence of Δm upon h seems to be rather weak. If one were to absorb the divergent integral into a redefinition of m_0 , then the self-energy would seem to be time dependent if h decreases as the Universe expands. However, this time dependence would be extremely small at the present time. Even if one were to identify the renormalized one-loop self-energy with the observed mass of the electron (there could be a piece of nonelectromagnetic origin), one would have a time-dependent electron mass with $\dot{m}/m = 30h\dot{h}$. If $|\dot{h}/h| \approx 10^{-10}/\text{yr}$, and h is of the order of the estimate given in the last paragraph of Sec. III B, then $|\dot{m}/m| \approx 10^{-80}/\text{yr}$. This is well within the observational limits on the time variation of the electron mass, which are of the order of [19]

$$\left| \frac{\dot{m}}{m} \right| \leq 10^{-13}/\text{yr}. \quad (95)$$

V. SUMMARY AND DISCUSSION

We have seen that the introduction of metric fluctuations, such as those due to gravitons in a squeezed vac-

uum state, can modify the behavior of Green's functions near the light cone. For distinct but lightlike separated points, the usual singularity is removed. However, the singularity for coincident points remains. The smearing of the light cone leads to the possibility of "faster-than-light light," in the sense that some photons will traverse the interval between a source and a detector in less than the classical propagation time.

The smearing of the light cone is expected to modify virtual processes. This was explored through the calculation of the one-loop electron self-energy in the presence of metric fluctuations. The results were somewhat ambiguous, due to the presence of the remaining ultraviolet divergences. They can, however, be interpreted as supporting a very small time-dependent contribution to the mass of the electron in an expanding universe. Of course, the dominant source of metric fluctuations need not be relic gravitons. Any stochastic bath of gravitons will also contribute to h . It is possible that the major-

ity of gravitons at the present time are those due to local sources (thermal processes, etc.) rather than those of cosmological origin. It is also possible that passive metric fluctuations due to quantum fluctuations of the energy-momentum tensor of matter produce the dominant effect in smearing the light cone. It would be of particular interest to find a one-loop process which is rendered finite by the effects of the metric fluctuations. Such a process would presumably lead to observable quantities whose values depend upon the graviton background. Thus theories in which gravitons regulate ultraviolet divergences can have the property that local observable quantities may be determined by the large scale structure or history of the Universe.

ACKNOWLEDGMENTS

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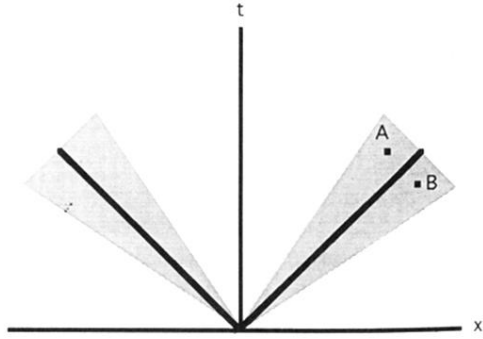


FIG. 1. The smearing of the light cone due to metric fluctuations. A photon which arrives at Point A from the origin has been slowed by the effect of metric fluctuations. A photon which arrives at Point B has been boosted by metric fluctuations, and appears to travel at a superluminal velocity in the background metric.