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Improved effective potential in curved spacetime and quantum matter–higher-derivative gravity theory

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We develop a general formalism to study the renormalization-group- (RG-) improved effective potential for renormalizable gauge theories, including matter-R^2-gravity, in curved spacetime. The result is given up to quadratic terms in curvature, and one-loop effective potentials may be easily obtained from it. As an example, we consider scalar QED, where dimensional transmutation in curved space and the phase structure of the potential (in particular, curvature-induced phase transitions) are discussed. For scalar QED with higher-derivative quantum gravity (QG), we examine the influence of QG on dimensional transmutation and calculate QG corrections to the scalar-to-vector mass ratio. The phase structure of the RG-improved effective potential is also studied in this case, and the values of the induced Newton and cosmological coupling constants at the critical point are estimated. The stability of the running scalar coupling in the Yukawa theory with conformally invariant higher-derivative QG, and in the standard model with the same addition, is numerically analyzed. We show that, in these models, QG tends to make the scalar sector less unstable.

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I. INTRODUCTION

It is a common belief in modern cosmology that, during its evolution, the Universe went through one or more inflationary stages (for an introduction, see [1, 2] and references therein). Some of the models of an inflationary universe, in particular the so-called "new inflation" one [3], are based on the scalar field effective potential [4–6] calculated in flat space, and the corresponding phase transitions are very important in such models. The basic observation is, then, that curvature corrections are not too essential in the effective potential for applications in the inflationary epoch. However, because of the fact that we consider curved spacetimes, for consistency one obviously has to work with curved spacetime effective potentials.

Furthermore, it seems that inflationary universe models based on Coleman-Weinberg-type phase transitions [5] are not quite consistent (usually, the one-loop approach is used). In these circumstances, it may happen to be very useful to analyze the effective potential in curved spacetime (for a general review, see [7]) beyond the one-loop approximation, in order to find more reliable forms of it. It is well known that effective potentials in curved spacetimes can produce curvature-induced phase transitions [7] which may become relevant in different context of cosmology.

From another viewpoint, it would be of interest to include also in such an analysis quantum gravitational corrections. Even in the absence of a consistent quantum gravity (QG) theory, we may work with some effective model for QG, which can be the Einstein theory [8, 9], which is not renormalizable [9], or higher-derivative theory [7, 10, 11] at scales below $\mu_{\rm Pl} \simeq 10^{19}$ GeV. Despite its perturbative nonunitarity, higher-derivative QG may be considered as some effective theory, while the problem of unitarity should be addressed in a more complete and fundamental theory. Of course, the physics which can be adressed in that framework is between $\mu_{\rm GUT} \simeq 10^{15}$ GeV and $\mu_{\rm Pl}$.

In the present paper we develop a general formalism to study the renormalization-group- (RG-) improved effective potential (and also the one-loop one) for gauge theories in curved spacetime and also in gauge theories

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interacting with higher-derivative QG. The RG-improved effective potential, which gives the leading-logarithmic behavior on the whole perturbation theory and hence goes beyond the one-loop approximation, is quite well known in flat-space theories after the seminal paper by Coleman and Weinberg [5] (see also [12, 13]), and has numerous applications. The conception of the RG-improved effective potential may be extended to curved spacetime [14] and, as we shall show below, to the situation where we have an interacting renormalizable quantum matter-gravity theory. We discuss a few relevant phenomena caused by the RG-improved effective potential.

The paper is organized as follows. In the next section, our general formalism for obtaining the RG-improved effective potential in gauge theories on curved backgrounds, up to quadratic terms in curvature, is presented. One-loop effective potentials may be derived from it after expanding for a small RG parameter t. This formalism is easily applied when, in addition to matter, we have QG (any matter-gravity unified theory should, of course, be renormalizable in our context). In Sec. III, the RG-improved effective potential, in the linear curvature approximation, is explicitly given for scalar QED. Dimensional transmutation in curved space and curvature corrections to the scalar-to-vector mass ratio are discussed. The phase structure of the potential is numerically investigated for some choices of the parameters. Section IV is devoted to the study of the RG-improved (and one-loop) effective potentials in scalar QED with higher-derivative QG. Gravity corrections to the scalar-to-vector mass ratio are calculated for two versions of QG (one of which is conformally invariant higher-derivative QG). The influence of QG on the stability of the effective potential is numerically discussed. Spontaneous symmetry breaking and curvature-induced phase transitions are shown to exist for some choice of the theory parameters, and induced values of the Newton and cosmological constants are estimated. In Sec. V we consider the stability of the Yukawa theory with conformally invariant R^2 gravity. Numerical analysis shows that QG corrections change the running of the scalar coupling constant, making it less unstable. By way of some speculation, we repeat such a discussion in the standard model interacting with the same QG theory. An increase in the initial value of the QG coupling constant can overcome the instability of the scalar coupling, which, in turn, may change the bounds between the Higgs and top quark masses. Finally, we end by giving a summary and some outlook.

II. RENORMALIZATION-GROUP-IMPROVED EFFECTIVE POTENTIAL IN CURVED SPACETIME

We begin with the presentation of a general formalism for the RG-improved effective potential. Our starting point will be some multiplicatively renormalizable theory on a general curved background. In principle, such a theory may include quantum gravity (QG) as well; then, the curved background is simply the background part of the metric (we use the background field method here).

First of all, let us consider some matter theory in curved spacetime, where the Lagrangian corresponding to a multiplicatively renormalizable model reads

$$\mathcal{L} = \mathcal{L}_m + \mathcal{L}_{\text{ext}}, \tag{2.1}$$

with

$$\mathcal{L}_{\text{ext}} = \Lambda + \kappa R + a_1 R^2 + a_2 C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} + a_3 G + a_4 \Box R. \tag{2.2}$$

$C_{\mu\nu\alpha\beta}$ is the Weyl tensor and G is the Gauss-Bonnet invariant. The Lagrangian for matter contains gauge fields, some multiplets of scalars φ and spinors ψ, and kinds of interaction which are typical of any grand unified theory (GUT). Symbolically,

$$\mathcal{L}_m = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - h\varphi - M)\psi + \frac{1}{2}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi - \frac{1}{2}m^2\varphi^2 + \frac{1}{2}\xi R\varphi^2 - \frac{1}{4!}f\varphi^4, \tag{2.3}$$

where $\mathcal{D}_\mu = \nabla_\mu - igA_\mu$, ∇_μ is the covariant derivative, and all indices are suppressed. Note that the necessity of $\mathcal{L}_{\text{ext}}$ is dictated by the condition of multiplicative renormalizability in curved space [7]. Because of the fact that we are considering an external gravitational field, also total derivative terms have to be included in $\mathcal{L}_{\text{ext}}$.

We will be interested in the calculation of the effective potential [4–6] for the scalar field, i.e., the effective action on a constant background φ, R. Since the theory is multiplicatively renormalizable, its effective potential satisfies the standard RG equation

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_i\frac{\partial}{\partial\lambda_i} - \gamma\varphi\frac{\partial}{\partial\varphi}\right)V(\mu,\lambda_i,\varphi) = 0, \tag{2.4}$$

where $\lambda_i = (\xi, f, h^2, g^2, m^2, \mu, \Lambda, \kappa, a_1, a_2, a_3)$, β_i are the corresponding beta functions, and φ is the background scalar. The $\Box R$ term from (2.2) has disappeared owing to our choice of background space of the form $R = \text{const}$. For the gauge fields, a Landau-type gauge is supposed to be chosen. As a result, in the one-loop approach, which we apply for handling (2.4), the β function for the gauge parameter is zero and its associated term in (2.4) may be dropped. The RG equation for V has been discussed in [5, 12, 13] (and references therein) for flat space and in [7, 14] for the case of curved space.

Solving Eq. (2.4) by the method of the characteristics gives

$$V(\mu,\lambda_i,\varphi) = V(\mu e^t, \lambda_i(t), \varphi(t)), \tag{2.5}$$

where

$$\frac{d\lambda_i(t)}{dt} = \beta_i(\lambda_i(t)), \qquad \lambda_i(0) = \lambda_i,$$
$$\frac{d\varphi(t)}{dt} = -\gamma(t)\varphi(t), \qquad \varphi(0) = \varphi. \tag{2.6}$$

Physically, (2.5) means that the effective potential can be (locally) found provided that its functional at some certain t is known.

Using the classical Lagrangian (2.1) for constant background as the initial value of V at $t = 0$, and working only in one-loop aproximation, we can find [14]

$$V_{\rm RG} = \frac{1}{4!}f(t)\varphi^4(t) - \frac{1}{2}\xi(t)R\varphi^2(t) + \frac{1}{2}m^2(t)\varphi^2(t) + \Lambda(t) + \kappa(t)R + a_1(t)R^2 + a_2(t)C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} + a_3(t)G, \tag{2.7}$$

where

$$\begin{aligned} &\dot{f}(t) = \beta_f(t), f(0) = f, \dot{\kappa}(t) = \beta_\kappa(t), \kappa(0) = \kappa, \\ &\dot{\xi}(t) = \beta_\xi(t), \xi(0) = \xi, \dot{a}_i(t) = \beta_{a_i}(t), a_i(0) = a_i, i = 1,2,3, \\ &\dot{m}^2(t) = \beta_{m^2}(t), m^2(0) = m^2, \dot{\varphi}(t) = -\gamma(t)\varphi(t), \varphi(0) = \varphi, \\ &\dot{\Lambda}(t) = \beta_\Lambda(t), \Lambda(0) = \Lambda. \end{aligned} \tag{2.8}$$

The one-loop β functions which appear on the right-hand side (RHS) of (2.8) for any particular model are, as a rule, known or may be obtained without serious problems. It should be observed that the RG-improved effective potential (2.7) was written in the approximation up to curvature invariants of second order.

This RG-improved effective potential (EP) is given in leading-logarithmic approximation (summing all leading logarithms in perturbation theory) [5], which, in this sense, is much richer than the standard one-loop version. One-loop EP's can be obtained from RG-improved EP's in some limit (small t, weak couplings). However, notice that, contrary to what happens in the nonimproved case, Eq. (2.7) is actually valid at all t's for which $V_{\rm RG}$ does not diverge (this is an improvement).

Now, we ask ourselves this natural question: What is the choice of the RG parameter t which leads to the summation of all logarithms to all orders? In fact, for massive theories it is not easy to answer [13, 14]. One has to introduce a few massive scales (which make the discussion technically complicated), use the decoupling theorem [15] and the effective field theory [16] (and references therein) to construct the RG-improved EP's at all these scales. In the present paper we shall consider, for simplicity, either massless theories, where the choice is actually unique $t = \frac{1}{2}\ln\frac{\varphi^2}{\mu^2}$, or massive theories limited to the case of very high φ, such that $\varphi^2 \gg m^2_{\rm eff}$, $m^2_{\rm eff}$ being the largest effective mass of the theory. Then, we may drop all massive terms in (2.7) and make our choice of t, again, as above.

In massless theories the RG-improved potential (2.7) may be expanded for small t and weak coupling in a general form, thus obtaining a very general expression for the one-loop effective potential [17] (see also [7]):

$$V^{(1)} = \frac{1}{4!}f\varphi^4 + \frac{1}{48}(\beta_f - 4f\gamma)\varphi^4\left(\ln\frac{\varphi^2}{\mu^2} - \frac{25}{6}\right) - \frac{1}{2}\xi R\varphi^2 - \frac{1}{4}(\beta_\xi - 2\xi\gamma)R\varphi^2\left(\ln\frac{\varphi^2}{\mu^2} - 3\right), \tag{2.9}$$

where $\beta_f, \beta_\xi, \gamma$ are the one-loop beta functions, and μ^2 is a mass parameter in the range $\mu^2 \lesssim \mu^2_{\rm GUT} \simeq 10^{15}$ GeV. Note that Coleman-Weinberg-type normalization conditions have been used to derive Eq. (2.9) (see [17] for more details). This is a very useful expression which will be employed for explicit analysis in a few different theories below.

Next, we would like to point out that the above-developed formalism can be easily applied to multiplicatively renormalizable QG theories with matter. As is widely known, Einstein QG is a nonrenormalizable theory [9]. That is why we have chosen to work with higher-derivative QG, which is multiplicatively renormalizable [7, 10] and asymptotically free [7, 11, 18]. It should be noted that it may be asymptotically free for all couplings when such a theory interacts with some GUT model [7, 18]. Of course, R^2 gravity with matter cannot be considered as a reasonable candidate for a consistent QG theory, due to the open question about its perturbative nonunitarity, which is typical of any higher-derivative interaction. However, one can regard it as an effective model for some yet unknown, consistent, QG theory at scales below $\mu_{\rm Pl} \simeq 10^{19}$ GeV. That will be our viewpoint throughout this paper.

Working, for simplicity, with massless theories, our initial Lagrangian, using standard notations, is

$$\mathcal{L} = \mathcal{L}_m + \mathcal{L}_{\rm QG}, \quad \mathcal{L}_{\rm QG} = \frac{1}{\lambda}W - \frac{\omega}{3\lambda}R^2, \tag{2.10}$$

where $W = C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}$. Even in this case, starting from higher-derivative QG without Einstein and cosmological terms, we may recover them as a result of a curvature-induced phase transition at lower energies, as we will see below. When dealing with such a theory, we get again an RG-improved EP of the form (2.7), and a one-loop effective potential such as (2.9). The only difference, in comparison with the no-QG case, is that all β functions have now changed due to explicit QG corrections. Hence, our formalism is general enough for being applied to a QG theory also.

It should also be observed that, as the metric is now quantized, one has to introduce the term $-\gamma_g g_{\mu\nu}\frac{\delta}{\delta g_{\mu\nu}}$ in Eq. (2.4) [7]. However, in the one-loop approximation and using the background field method, $\gamma_g = 0$, and therefore this term vanishes. Having at hand the general formalism developed in this section, we may now set out to study explicit examples.

III. RENORMALIZATION-GROUP-IMPROVED EFFECTIVE POTENTIAL IN GAUGE THEORIES IN CURVED SPACETIME

Let us begin with the simplest model for an Abelian gauge theory: massless electrodynamics in curved spacetime. The classical Lagrangian for this theory is

$$\mathcal{L}_m = \frac{1}{2}\left(\partial_\mu\varphi_1 - eA_\mu\varphi_2\right)^2 + \frac{1}{2}\left(\partial_\mu\varphi_2 - eA_\mu\varphi_1\right)^2 + \frac{1}{2}\xi R\varphi^2 - \frac{1}{4!}f\varphi^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{3.1}$$

where $\varphi^2 = \varphi_1^2 + \varphi_2^2$. Using the Landau gauge for calculating the one-loop β functions, these turn out to be [5, 14]

$$\beta_f = \frac{1}{(4\pi)^2}\left(\frac{10}{3}f^2 - 12e^2 f + 36e^4\right),$$

$$\beta_{e^2} = \frac{2e^4}{3(4\pi)^2}, \gamma = -\frac{3e^2}{(4\pi)^2}, \beta_\xi = \frac{\left(\xi - \frac{1}{6}\right)}{(4\pi)^2}\left(\frac{4}{3}f - 6e^2\right). \tag{3.2}$$

From here on, limiting ourselves to the linear curvature approximation, the β_{a_i}'s will be no longer necessary in our discussion.

The solutions of the RG equations for the coupling constants are

$$e^2(t) = e^2\left(1 - \frac{2e^2 t}{3(4\pi)^2}\right)^{-1}, \quad \varphi^2(t) = \varphi^2\left(1 - \frac{2e^2}{3(4\pi)^2}\right)^{-9},$$

$$f(t) = \frac{1}{10}e^2(t)\left[\sqrt{719}\tan\left(\frac{1}{2}\sqrt{719}\ln e^2(t) + C\right) + 19\right],$$

$$C = \arctan\left[\frac{1}{\sqrt{719}}\left(\frac{10f}{e^2} - 19\right)\right] - \frac{1}{2}\sqrt{719}\ln e^2,$$

$$\xi(t) = \frac{1}{6} + \left(\xi - \frac{1}{6}\right)\left[\frac{e^2(t)}{e^2}\right]^{-26/5}\frac{\cos^{2/5}\left(\frac{1}{2}\sqrt{719}\ln e^2 + C\right)}{\left(\frac{1}{2}\sqrt{719}\ln e^2(t) + C\right)}. \tag{3.3}$$

Using these effective coupling constants, the RG-improved potential is found to be [14]

$$V = \frac{1}{4!}f(t)\varphi^4(t) - \frac{1}{2}\xi(t)R\varphi^2(t). \tag{3.4}$$

This RG-improved EP in scalar electrodynamics has already been discussed in a more general case, namely, with a massive scalar field [14].

Before starting to work with (3.4), we write down the one loop EP (2.9) for this specific theory:

$$V^{(1)} = \frac{1}{4!}f\varphi^4 + \frac{1}{48(4\pi)^2}\left[\frac{10}{3}f^2 + 36e^4\right]\varphi^4\left(\ln\frac{\varphi^2}{\mu^2} - \frac{25}{6}\right) - \frac{1}{2}\xi R\varphi^2 + \frac{1}{12(4\pi)^2}\left[\left(\xi - \frac{1}{6}\right)\left(\frac{4}{3}f - 6e^2\right) + 6\xi e^2\right]R\varphi^2\left(\ln\frac{\varphi^2}{\mu^2} - 3\right). \tag{3.5}$$

This one-loop EP in linear approximation was also obtained in Ref. [19], where a background de Sitter space was considered for explicit calculations.

A few remarks concerning dimensional transmutation are in order. Restricting $V^{(1)}$ first to flat space ($R = 0$), choosing next $\mu = \varphi_m$ where $\varphi = \varphi_m$ is the vacuum state, and repeating the analysis by Coleman and Weinberg [5], one can easily get

$$V^{(1)} = \frac{3e^4}{64\pi^2}\varphi^4\left(\ln\frac{\varphi^2}{\varphi_m^2} - \frac{1}{2}\right),$$

$$\frac{f}{24} = \frac{11}{144(4\pi)^2}(36e^4). \tag{3.6}$$

When carrying these considerations over to curved spacetime, the transmutation mechanism acts in a different way [14]. Indeed, supposing again $f \sim e^4$ we arrive at

$$V^{(1)} = \frac{1}{4!}f\varphi^4 + \frac{3e^4}{64\pi^2}\varphi^4\left(\ln\frac{\varphi^2}{\mu^2} - \frac{25}{6}\right) - \frac{1}{2}\xi R\varphi^2 - \frac{1}{4(4\pi)^2}e^2 R\varphi^2\left(\ln\frac{\varphi^2}{\mu^2} - 3\right). \tag{3.7}$$

Choosing $\mu = \varphi_m$, where $\varphi = \varphi_m$ is now the vacuum state in curved spacetime (supposing that it exists), we do not find such a precise connection between f and e^4 as in flat space [5] but, instead, we obtain

$$\frac{V^{(1)\prime}}{4\varphi_m} = \left[\frac{f}{4!} - \frac{11e^4}{64\pi^2}\right]\varphi_m^2 + R\left[-\frac{1}{4}\xi + \frac{e^2}{4(4\pi)^2}\right] = 0. \tag{3.8}$$

Working in the linear curvature approximation (assuming that curvature corrections are not large) one can impose the second condition in (3.6) by hand. Then, and only then, is dimensional transmutation switched on in curved space, yielding a condition to fix ξ in terms of e^2:

$$\xi = \frac{e^2}{(4\pi)^2}. \tag{3.9}$$

In this case, the universal normalization-independent expression for the one-loop effective potential turns into

$$V^{(1)} = \frac{3e^4}{64\pi^2}\varphi^4\left(\ln\frac{\varphi^2}{\mu^2} - \frac{1}{2}\right) - \frac{e^2}{64\pi^2}R\varphi^2\left(\ln\frac{\varphi^2}{\mu^2} - 1\right). \tag{3.10}$$

Obviously, the above arguments do not hold for models in the presence of strong curvature, where other choices leaving ξ arbitrary seem to be more natural.

By use of (3.10), one can find the constant curvature corrections to the scalar-to-vector mass ratio in the form (see [5])

$$\frac{m^2(S)}{m^2(V)} = \frac{V^{(1)\prime\prime}(\varphi_m)}{e^2\varphi_m^2} = \frac{3e^2}{8\pi^2} - \frac{R}{16\pi^2\varphi_m^2}. \tag{3.11}$$

That gives the nonzero curved space generalization of the corresponding flat space Coleman-Weinberg result [5].

Our purpose now will be to discuss the phase structure, i.e., symmetry-breaking and curvature-induced phase transitions, in scalar QED, using the RG-improved effective potential. We shall be interested in first-order phase transitions, i.e., those in which the order parameter jumps sharply at some critical value of the curvature R_c. The conditions on such phase transitions are [19]

$$V(\varphi_c, R_c) = 0, \qquad \left.\frac{\partial V}{\partial\varphi}\right|_{\varphi_c,R_c} = 0, \qquad \left.\frac{\partial^2 V}{\partial\varphi^2}\right|_{\varphi_c,R_c} > 0. \tag{3.12}$$

The behavior of this RG-improved potential in one of the most interesting cases, associated with $e^2 = 0.1$, $\xi = 0$, is shown in Fig. 1. If R is smaller than a certain value R_1 around 1.9×10^{-6}, a local minimum for some $\varphi > 0$ exists. However, that state is just metastable until the value of R is lowered down to $R_c \simeq 1.44 \times 10^{-6}$. Below this figure, the new minimum is the global one; i.e., $\varphi = 0$ has become metastable while the $\varphi \neq 0$ associated with the global minimum is now the physical vacuum. Therefore, a symmetry-breaking phase transition, induced by curvature itself, takes place at $R = R_c$, even in a situation with $\xi(0) = 0$. The fact that scalar QED in curved spacetime undergoes a curvature-induced phase transition was already noted some time ago [19], using the one-loop effective potential.

By our explicit numerical results, we have shown that the RG-improved effective potential behaves qualitatively like its one-loop counterpart. Using the one-loop

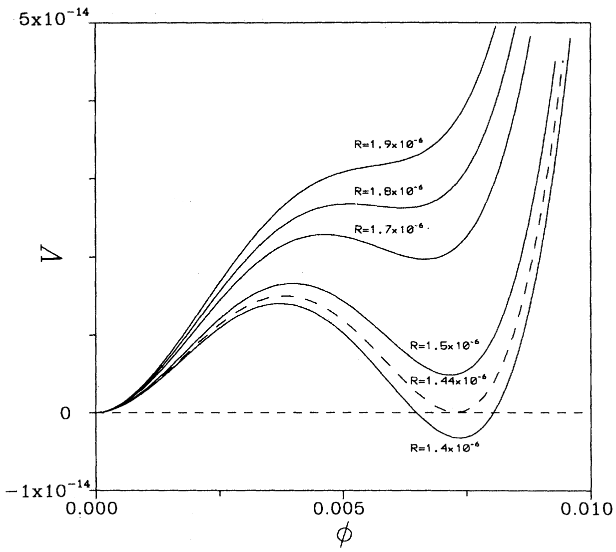

FIG. 1. RG-improved effective potential for scalar QED (SQED), with $e^2(0) = 0.1$, $f(0) = 0.01$, $\xi(0) = 0$ ($\mu^2 = 1$ is used throughout), for different values of the curvature R. Note the curvature-induced phase transition, which takes place when $\varphi = 0$ becomes metastable for $R < R_c \simeq 1.44 \times 10^{-6}$, while the local minimum at $\varphi > 0$ becomes the global one.

potential (3.5) (keeping, for simplicity, only logarithmic terms in the one-loop correction) we have found curves which differ just very slightly from the ones plotted in Fig. 1. Of course, it would not be difficult to repeat the same analysis for different choices of the initial values of the theory parameters.

IV. EFFECTIVE POTENTIAL IN SCALAR QED WITH HIGHER-DERIVATIVE GRAVITY

We will now look at the effective potential in massless scalar QED interacting with QG in the forms (2.10) and (3.1). QG corrections to QED β functions, which may be taken from Ref. [18] (see also [17]), have a universal form for any matter theory. Unfortunately, the complexity of the RG equations for the total system of β functions prevents us from finding the RG-improved EP (2.7) explicitly. It can be obtained only in an implicit form (applying linear curvature approximation):

$$V = \frac{1}{4!}f(t)\varphi^4(t) - \frac{1}{2}\xi(t)R\varphi^2(t), \tag{4.1}$$

where

$$\lambda(t)=\frac{\lambda}{1+\dfrac{\alpha^2\lambda t}{(4\pi)^2}},\quad \alpha^2=\frac{203}{15},\quad e^2(t)=\frac{e^2}{1-\frac{2e^2t}{3(4\pi)^2}},$$
$$\frac{d\omega}{dt}=\beta_\omega=-\frac{1}{(4\pi)^2}\lambda\left[\frac{10}{3}\omega^2+\left(5+\alpha^2\right)\omega+\frac{5}{12}+3\left(\xi-\frac{1}{6}\right)^2\right],$$
$$\frac{d\xi}{dt}=\beta_\xi+\Delta\beta_\xi,\quad \frac{df}{dt}=\beta_f+\Delta\beta_f,\quad -\frac{1}{\varphi}\frac{d\varphi}{dt}=\gamma+\Delta\gamma. \tag{4.2}$$

In expressions (4.2), β_ξ,β_f,γ are the ones given in (3.2), and the universal QG corrections [7, 18] are

$$\Delta\beta_\xi=\frac{1}{(4\pi)^2}\lambda\xi\left[-\frac{3}{2}\xi^2+4\xi+3+\frac{10}{3}\omega+\frac{1}{\omega}\left(-\frac{9}{4}\xi^2+5\xi+1\right)\right],$$
$$\Delta\beta_f=\frac{1}{(4\pi)^2}\left[\lambda^2\xi^2\left(15+\frac{3}{4\omega^2}-\frac{9\xi}{\omega^2}+\frac{27\xi^2}{\omega^2}\right)-\lambda f\left(5+3\xi^2+\frac{33}{2\omega}\xi^2-\frac{6}{\omega}\xi+\frac{1}{2\omega}\right)\right],$$
$$\Delta\gamma=\frac{1}{(4\pi)^2}\frac{\lambda}{4}\left(\frac{13}{3}-8\xi-3\xi^2-\frac{1}{6\omega}-\frac{2\xi}{\omega}+\frac{3\xi^2}{2\omega}\right). \tag{4.3}$$

As one can see from (4.2), there are no QG corrections to $e^2(t)$ (that is prohibited by local gauge invariance). The calculation of $\Delta\gamma$ took place in the harmonic gauge [18]. For simplicity, we have not explicitly written in (4.2), (4.3) the t dependence. Moreover, for these equations, the standard initial conditions $\omega(0)=\omega$, $f(0)=f$, $\xi(0)=\xi$, $\varphi(0)=\varphi$ were assumed (here, the ω, f, ξ, φ on the RHS denote *truly* t-independent quantities).

For the conformal version of higher-derivative gravity with scalar QED, one has

$$V=\frac{1}{4!}f(t)\varphi^4(t)-\frac{1}{12}R\varphi^2(t), \tag{4.4}$$

where

$$\lambda(t)=\frac{\lambda}{1+\frac{\alpha_1^2\lambda t}{(4\pi)^2}},\quad \alpha_1^2=\frac{27}{2},\qquad e^2(t)=\frac{e^2}{1-\frac{2e^2t}{3(4\pi)^2}},$$
$$\frac{df}{dt}=\frac{1}{(4\pi)^2}\left(\frac{10}{3}f^2-12e^2f+36e^4-\frac{41}{8}\lambda f+\frac{5}{12}\lambda^2\right),$$
$$-\frac{1}{\varphi}\frac{d\varphi}{dt}=\frac{1}{(4\pi)^2}\left(-3e^2+\frac{27}{32}\lambda\right), \tag{4.5}$$

and the QG corrections in this conformal model have been taken from [18]. Note that, in order to make such a theory multiplicatively renormalizable, one has to use the so-called special conformal regularization (see [7, 18], third Ref. of [11], and also references therein).

Using the above β functions, it is easy to find the one-loop effective potential (2.9) for higher-derivative QG with scalar QED. In the general version (2.10), taking into account (4.2), (4.3) one can obtain (see also [20])

$$V^{(1)}=\frac{1}{4!}f\varphi^4+\frac{1}{48(4\pi)^2}\left[\frac{10}{3}f^2+36e^4+\lambda^2\xi^2\left(15+\frac{3}{4\omega^2}-\frac{9\xi}{\omega^2}+\frac{27\xi^2}{\omega^2}\right)\right.$$
$$\left.-\lambda f\left(\frac{28}{3}+18\frac{\xi^2}{\omega}-\frac{8\xi}{\omega}-8\xi+\frac{1}{3\omega}\right)\right]\varphi^4\left(\ln\frac{\varphi^2}{\mu^2}-\frac{25}{6}\right)$$
$$-\frac{1}{2}\xi R\varphi^2-\frac{1}{4(4\pi)^2}\left\{\left(\xi-\frac{1}{6}\right)\left(\frac{4}{3}f-6e^2\right)+6\xi e^2\right.$$
$$\left.+\lambda\xi\left[8\xi+\frac{5}{6}+\frac{10}{3}\omega+\frac{1}{\omega}\left(-3\xi^2+6\xi+\frac{13}{12}\right)\right]\right\}R\varphi^2\left(\ln\frac{\varphi^2}{\mu^2}-3\right). \tag{4.6}$$

In this case, we suppose that $\mu^2_{\rm GUT}<\mu^2<\mu^2_{\rm Pl}$.

As for the conformal version, the one-loop EP turns out to be

$$V^{(1)}=\frac{1}{4!}f\varphi^4+\frac{1}{48(4\pi)^2}\left[\frac{10}{3}f^2+36e^4+\frac{5}{12}\lambda^2-\frac{17}{2}\lambda f\right]\varphi^4\left(\ln\frac{\varphi^2}{\mu^2}-\frac{25}{6}\right)$$
$$-\frac{1}{12}R\varphi^2+\frac{1}{12(4\pi)^2}\left[-3e^2+\frac{27}{32}\lambda\right]R\varphi^2\left(\ln\frac{\varphi^2}{\mu^2}-3\right). \tag{4.7}$$

First, we discuss the dimensional transmutation in the presence of QG (see also [20]) on a flat background. Taking

Eq. (4.6), supposing either λ^2 and e^4 of the same order or λ^2 dominant, choosing $\mu^2 = \varphi_m^2$ (where φ_m is the minimum), and applying $\frac{\partial V^{(1)}}{\partial\varphi}\Big|_{\varphi=\varphi_m} = 0$ as in Sec. III, one finds that, due to dimensional transmutation, $f \sim \lambda^2 + e^4$, and therefore

$$V^{(1)} = \frac{1}{48(4\pi)^2}\left[36e^4 + \lambda^2\xi^2\left(15 + \frac{3}{4\omega^2} - \frac{9\xi}{\omega^2} + \frac{27\xi^2}{\omega^2}\right)\right]\varphi^4\left(\ln\frac{\varphi^2}{\varphi_m^2} - \frac{1}{2}\right),$$
$$2(4\pi)^2 f = \frac{11}{3}\left[\lambda^2\xi^2\left(15 + \frac{3}{4\omega^2} - \frac{9\xi}{\omega^2} + \frac{27\xi^2}{\omega^2}\right) + 36e^4\right]. \tag{4.8}$$

If e^4 is leading as compared with the QG term, we are in the case already studied by Coleman and Weinberg [5]: QG corrections are negligible. However, in the opposite situation, where $\lambda^2 \gg e^4$, dimensional transmutation is a purely quantum gravitational effect.

From Eq. (4.8) one can obtain the scalar-to-vector mass ratio in the presence of QG:

$$\frac{m^2(S)}{m^2(V)} = \frac{1}{6(4\pi)^2}\left[36e^2 + \frac{\lambda^2\xi^2}{e^2}\left(15 + \frac{3}{4\omega^2} - \frac{9\xi}{\omega^2} + \frac{27\xi^2}{\omega^2}\right)\right]. \tag{4.9}$$

This is to be compared with the original Coleman-Weinberg [5] result ($\lambda^2 = 0$ case); see (3.11). As one can notice, the QG corrections in (4.9) may become the dominant ones.

We also find the analogous ratio for the conformal version, which reads

$$\frac{m^2(S)}{m^2(V)} = \frac{1}{6(4\pi)^2}\left[36e^2 + \frac{5}{12}\frac{\lambda^2}{e^2}\right]. \tag{4.10}$$

Again, QG corrections may turn out to be the leading ones.

It is interesting to realize that, in principle, curvature corrections to (4.9) or (4.10) may be calculated as we did in Sec. III. Next, our aim is to define ξ in terms of e^2, λ in the general version. To this end, we take (4.6) assuming $f \sim \lambda^2 + e^4$, and get

$$V^{(1)} = \frac{1}{4!}f\varphi^4 + \frac{1}{48(4\pi)^2}\left[36e^4 + \lambda^2\xi^2\left(15 + \frac{3}{4\omega^2} - \frac{9\xi}{\omega^2} + \frac{27\xi^2}{\omega^2}\right)\right]\varphi^4\left(\ln\frac{\varphi^2}{\mu^2} - \frac{25}{6}\right)$$
$$-\frac{1}{2}\xi R\varphi^2 - \frac{1}{4(4\pi)^2}\left\{e^2 + \lambda\xi\left[\frac{5}{6} + 8\xi + \frac{10}{3}\omega + \frac{1}{\omega}\left(-3\xi^2 + 6\xi + \frac{13}{12}\right)\right]\right\}R\varphi^2\left(\ln\frac{\varphi^2}{\mu^2} - 3\right). \tag{4.11}$$

Choosing $\mu = \varphi_m$ and requiring $\frac{\partial V^{(1)}}{\partial\varphi}\Big|_{\varphi=\varphi_m} = 0$ on (4.11), we do not obtain the same connection among f, λ^2, and e^4 as on a flat background (the situation is the same as in Sec. II). If we impose such a condition, following from flat space considerations, we get expression (3.9), as the connection between ξ and e^2, even in the presence of QG. This fact is caused by the particular form of the QG corrections to the $R\varphi^2$ term in (4.11) (it is always proportional to ξ). It is also very interesting to note that, if we started from the theory without the electrodynamic sector, i.e., $e^2 = 0$, we would get, from the condition $\frac{\partial V^{(1)}(\mu=\varphi_m)}{\partial\varphi}\Big|_{\varphi_m} = 0$, the equation

$$\varphi_m^2\left\{\frac{f}{6} - \frac{11}{3}\frac{1}{12(4\pi)^2}\lambda^2\xi^2\left(15 + \frac{3}{4\omega^2} - \frac{9\xi}{\omega^2} + \frac{27\xi^2}{\omega^2}\right)\right\}$$
$$-R\left\{\xi - \frac{1}{(4\pi)^2}\lambda\xi\left[\frac{5}{6} + 8\xi + \frac{10}{3}\omega + \frac{1}{\omega}\left(-3\xi^2 + 6\xi + \frac{13}{12}\right)\right]\right\} = 0. \tag{4.12}$$

Generally speaking, this is an equation to determine the minimum (assuming that it exists) in terms of the curvature and the theory parameters. However, if, as before, we put by hand the flat space condition for f, we are led to

$$1 - \frac{\lambda}{(4\pi)^2}\left[\frac{5}{6} + 8\xi + \frac{10}{3}\omega + \frac{1}{\omega}\left(-3\xi^2 + 6\xi + \frac{13}{12}\right)\right] = 0. \tag{4.13}$$

This condition is inconsistent, as it causes one of the QG coupling constants to be larger than unity, which contradicts the implicit assumptions in our perturbative treatment of the theory. Hence, without the electromagnetic coupling constant e, dimensional transmutation does not work order by order in curvature.

Some properties of the model under consideration are illustrated by means of Figs. 2–5. The scalar coupling $f(t)$ is represented in Fig. 2, for the conformal version, and in Fig. 3, for a case of the general model where this function is seen to go negative in a certain region, thus turning the effective potential unstable in that range. While conformal invariance seems to prevent instabilities of this type, in a general situation the positiveness

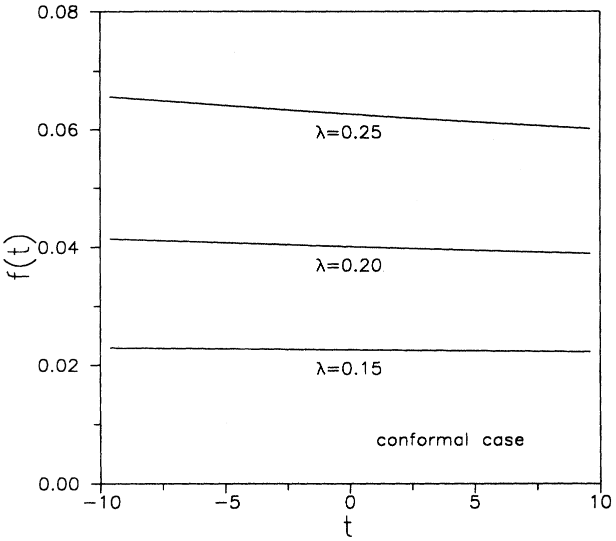

FIG. 2. Running scalar coupling $f(t)$, in the conformal version of SQED with R^2 gravity, for different initial values of $\lambda(0)$. In this and the next figures, the initial values of $f(0)$ are taken equal to $e^4(0)+\lambda^2(0)$, for reasons already explained in the text. Since we are concerned with the gravity-dominated regime, the value of $\lambda(0)$ has been chosen to be one order of magnitude larger than $e^2(0)$. If instead of the conformal version we consider the general theory with $\xi(0)=1/6$ [and $\omega(0)=1$], the resulting picture practically overlaps the one here displayed.

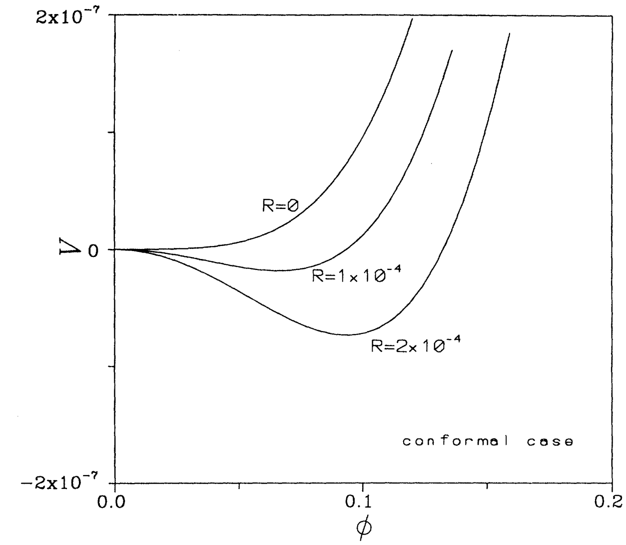

FIG. 4. Effective potentials for the conformal version of SQED with higher-derivative quantum gravity. In this range, the one-loop and RG-improved approximations coincide to the extent that their associated curves completely overlap one another. However, closer numerical examination around $\varphi=0$, at smaller scales than those visible in the figure, reveals differences between both. A $\xi(0)=1/6$ choice in the general model would give rise to practically the same figure as in the conformal case.

of this coupling cannot be taken for granted.

As for the chances of symmetry breaking, the appearance of a global minimum for $\varphi \neq 0$ is depicted in Figs. 4 and 5. Figure 4 indicates symmetry breaking for all positive values of R in the conformal version. The particular settings in Fig. 5, which depict a given situation

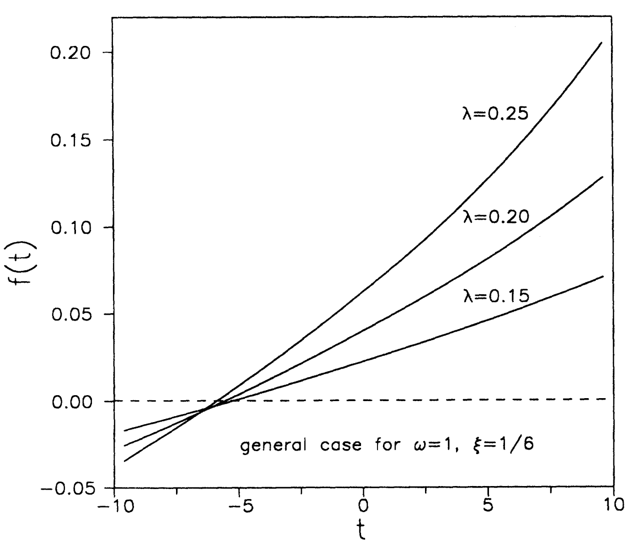

FIG. 3. $f(t)$ for the general version of the same theory, in the case $\omega(0)=1$, $\xi(0)=1$. For $t<t_i \simeq -6$ these couplings become negative.

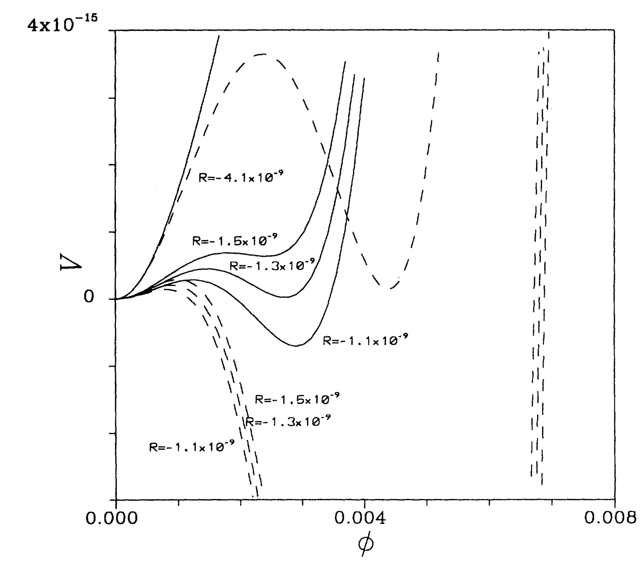

FIG. 5. One-loop (dashed line) and RG-improved (solid line) potentials, in the general version of our QG-corrected SQED model, with $\omega(0)=1$, $\xi(0)=1$, $e^2(0)=10^{-2}$, $\lambda(0)=0.15$, $f(0)=e^4(0)+\lambda^2(0)$, for three different values of the curvature R. At the scale shown, there is already quantitative disagreement between both approximations in the precise value of R_c, which would be of $\simeq -4.1\times10^{-9}$ at one loop and of $\simeq -1.3\times10^{-9}$ for the RG improvement, but the nature of the overall picture still coincides. The existence of a curvature-induced phase transition may be inferred from this image.

in the general model, make the passage from symmetric to symmetry-breaking phase take the specific form of a first-order transition, induced, of course, by the change in curvature. The "broken" phase exists for every positive R below a critical value R_c. Thus, we have found the existence of a curvature-induced phase transition in scalar QED with QG. Examining the RG-improved EP at the critical point, we obtain the following estimates for the induced Newton ($G_{\rm ind}$) and cosmological ($\Lambda_{\rm ind}$) couplings:

$$\frac{1}{16\pi G_{\rm ind}} \simeq 4.2\times 10^{-15}\mu^2, \quad \frac{2\Lambda_{\rm ind}}{16\pi G_{\rm ind}} \simeq -3.3\times 10^{-6}\mu^4, \tag{4.14}$$

where, as we have already mentioned, $\mu^2 < \mu_{\rm Pl}^2$. As one can see, Einstein gravity is induced, with the large cosmological constant. In principle, it is not difficult to generalize the above results for more realistic gauge theories. For example, let us consider the minimal SU(5) GUT (without fermions) with a 24-plet of gauge bosons A_μ and a 24-plet of Higgs bosons φ transforming under the adjoint representation of the group SU(5). The Higgs sector of this theory has the form

$$\mathcal{L}_H = -\frac{1}{2}{\rm Tr}(\partial_\mu\phi - ig[A_\mu,\phi])^2 - \frac{1}{4}f_1({\rm Tr}\phi^2)^2 - \frac{1}{2}f_2{\rm Tr}\phi^4 + \frac{1}{2}\xi R{\rm Tr}\phi^2. \tag{4.15}$$

Considering the breaking SU(5)$\to$SU(3)$\times$SU(2)$\times$U(1), with $\phi = \varphi\left(1,1,1,-\frac{3}{2},-\frac{3}{2}\right)$ and working, for simplicity, on a flat background, we obtain the following one-loop effective potential with QG corrections in the SU(5) GUT (for a discussion in curved space with no QG, see [17]):

$$\begin{aligned} V = {}& \frac{15}{16}(15f_1+7f_2)\varphi^4 + \frac{15}{32(4\pi)^2}\Bigg[15\times 64 f_1^2 + 1296 f_1 f_2 + \frac{32\times 91}{5}f_2^2 \\ & + \frac{375}{2}g^4 + \lambda^2\xi^2\left(15 + \frac{3}{4\omega^2} - \frac{9\xi}{\omega^2} + \frac{27\xi^2}{\omega^2}\right) \\ & - \lambda(15f_1+7f_2)\left(\frac{28}{3} + 18\frac{\xi^2}{\omega} - 8\frac{\xi}{\omega} - 8\xi + \frac{1}{3\omega}\right)\Bigg]\varphi^4\left(\ln\frac{\varphi^2}{\mu^2} - \frac{25}{6}\right), \end{aligned} \tag{4.16}$$

where the Landau gauge has been used. This result shows explicitly the universality of QG corrections and the possibility of applying them to different theories.

V. STABILITY IN THE YUKAWA MODEL WITH CONFORMALLY INVARIANT HIGHER-DERIVATIVE GRAVITY

The purpose of this section is to discuss the issue of stability in a Yukawa theory with conformal higher-derivative QG. The Lagrangian of the Yukawa theory that we consider reads

$$\mathcal{L}_m = \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{12}R\varphi^2 - \frac{1}{4!}f\varphi^4 + \bar\psi(i\gamma^\mu\nabla_\mu - h\varphi)\psi, \tag{5.1}$$

where ψ is a massless Dirac spinor. It is known that, because of Yukawa interactions, the scalar coupling constant becomes negative at large t (high energies), thus rendering the scalar effective potential unstable. Here, using the simplest, i.e., conformal, version of our QG theory (2.10), we will try to understand whether it is possible to change the stability properties of the EP to the better side by virtue of QG corrections.

The RG-improved EP looks like (4.4), but the following substitutions must be done for the RG equations (see [7, 18] for QG corrections to Yukawa couplings)

$$\lambda(t) = \frac{\lambda}{1+\frac{\alpha_1^2\lambda t}{(4\pi)^2}}, \qquad \alpha_1^2 = \frac{803}{60},$$

$$\begin{aligned} \frac{dh^2}{dt} &= \frac{1}{(4\pi)^2}\left(10h^4 - \frac{61}{16}h^2\lambda\right), \\ \frac{df}{dt} &= \frac{1}{(4\pi)^2}\left(3f^2 + 8fh^2 - 48h^4 - \frac{41}{8}\lambda f + \frac{5}{12}\lambda^2\right), \\ -\frac{1}{\varphi}\frac{d\varphi}{dt} &= \frac{1}{(4\pi)^2}\left(2h^2 + \frac{27}{32}\lambda\right). \end{aligned} \tag{5.2}$$

In the one-loop approach, the EP is given by

$$V^{(1)} = \frac{1}{4!}f\varphi^4 + \frac{1}{48(4\pi)^2}\left[\frac{5}{12}\lambda^2 - 48h^4\right]\varphi^4\left(\ln\frac{\varphi^2}{\mu^2} - \frac{25}{6}\right) - \frac{1}{12}R\varphi^2 + \frac{1}{12(4\pi)^2}\left[2h^2 + \frac{27}{32}\lambda\right]R\varphi^2\left(\ln\frac{\varphi^2}{\mu^2} - 3\right), \tag{5.3}$$

where we have supposed that $f\sim\lambda^2 - h^4$ and, therefore, higher-order terms such as f^2, fh^2, $f\lambda$ have been dropped out. We shall also consider the situation in which λ and h^2 are of the same order, but $\lambda > 10h^2$ (otherwise the potential is necessarily unstable). From expression (5.3), it is possible to get the QG corrections to the scalar-to-fermion mass ratio (the fermion becomes massive after spontaneous symmetry breaking):

$$\frac{m^2(S)}{m^2(F)} = \frac{V^{(1)\prime\prime}(\varphi_m)^2}{h^2\varphi_m^2} = \frac{1}{6(4\pi)^2}\left[\frac{5}{12}\frac{\lambda^2}{h^2} - 48h^2\right]. \tag{5.4}$$

In this model, spontaneous symmetry breaking can take place only as a result of QG corrections (if $\lambda > 10h^2$). On the other hand, Fig. 6 illustrates the occurrence of instabilities in this theory, both with and without gravity.

Analogously, we shall briefly outline how to discuss a variant of the standard model (SM) coupled to conformal higher-derivative gravity. The required one-loop RG functions for such a theory, in the absence of gravity, were given in [13] (first reference)—see also [21]. After adding to them the corresponding QG corrections (using the notation of Ref. [13], but for the scalar coupling, which now is called f), we are posed with the set of equations

$$g^2(t) = \frac{g^2}{1 + \frac{19}{3(4\pi)^2}g^2 t}, \quad g'^2(t) = \frac{g'^2}{1 - \frac{41}{3(4\pi)^2}g'^2 t}, \quad g_3^2(t) = \frac{g_3^2}{1 + \frac{14}{(4\pi)^2}g_3^2 t},$$

$$\lambda(t) = \frac{\lambda}{1 + \frac{\alpha_2^2\lambda t}{(4\pi)^2}}, \qquad \alpha_2^2 = \frac{481}{30},$$

$$\frac{df}{dt} = \frac{1}{(4\pi)^2}\left(4f^2 + 12fh^2 - 36h^4 - 9fg^2 - 3fg'^2 + \frac{9}{4}g'^4 + \frac{9}{2}g^2g'^2 + \frac{27}{4}g^4 - \frac{41}{8}\lambda f + \frac{5}{12}\lambda^2\right),$$

$$\frac{dh}{dt} = \frac{1}{(4\pi)^2}\left(\frac{9}{2}h^3 - 8g_3^2h - \frac{9}{4}g^2h - \frac{17}{12}g'^2h - \frac{61}{16}\lambda h^2\right). \tag{5.5}$$

As shown in Fig. 7, at $\lambda = 0$, i.e., no gravity, the scalar coupling can sometimes be negative, but by virtue of λ corrections, the sign can be reversed thus restoring the stability of the model. Although the general appearance of this figure is quite similar to one included in [13], the effect exhibited is different: There, no gravity was present and the negativeness could be corrected by raising the value of $f(0)$; in our case, $f(0)$ is held at a fixed value while we change the strength of QG perturbations. In these circumstances, such a stability restoration can be regarded a purely quantum gravitational effect.

It was pointed out in Ref. [22] and the first reference of

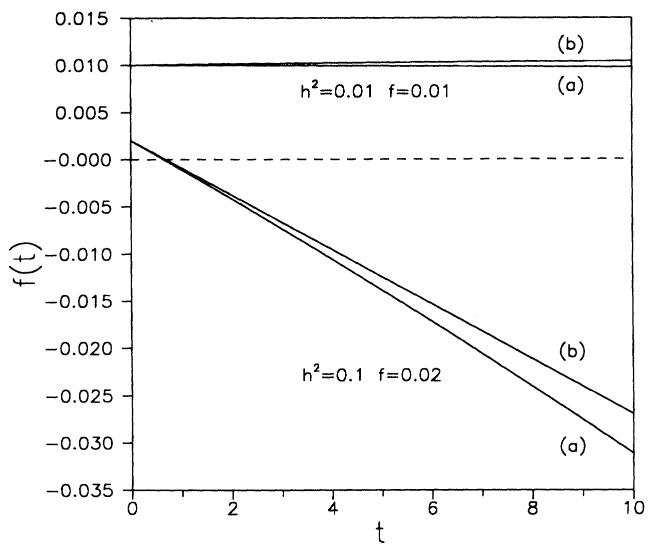

FIG. 6. Running scalar coupling $f(t)$ for the conformal Yukawa model. The picture represents the cases $h^2(0) = 0.01$, $f(0) = 0.01$ and $h^2(0) = 0.1$, $f(0) = 0.002$. For each of them, we have considered a pair of curves: (a) in the absence of QG, (b) with the corrections corresponding to $\lambda(0) = 0.25$. The instability is already noticeable for the second set of initial values. In the QG-corrected version, $f(t)$ tends to be marginally less negative than in the absence of gravity.

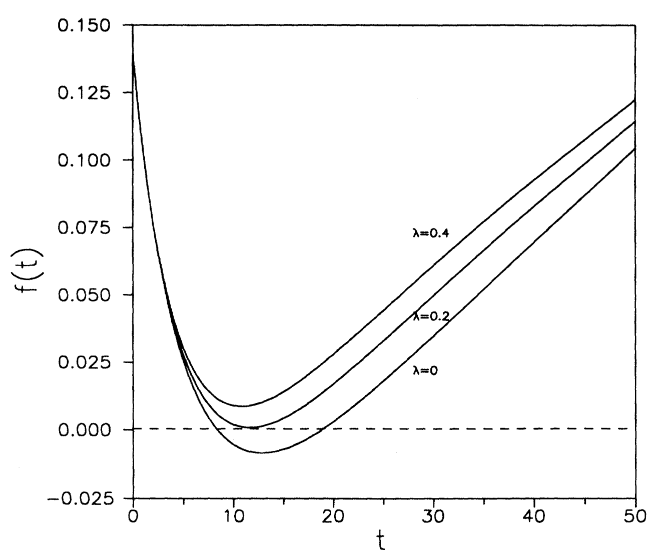

FIG. 7. Behavior of the running scalar coupling $f(t)$ for the SM with QG corrections. The present curves correspond to the initial values $g(0) = 0.65$, $g'(0) = 0.36$, $g_3(0) = \sqrt{4\pi\alpha_3}$, $\alpha_3 = 0.11$, $f(0) = 0.14$, $h(0) = 0.7$, and to the different values of $\lambda(0)$ quoted. An increase in this constant may correct the instability brought about by the negativeness of $f(t)$.

[13] that demanding absolute stability of the electroweak coupling implies that [13]

$$m_H \geq 1.95 m_t - 189 \text{ GeV}, \tag{5.6}$$

where m_H and m_t are the Higgs boson mass and the top mass, respectively. Thus, the stability of the scalar coupling constant puts some bounds on the relation between m_H and m_t. As we can infer from the above study, QG corrections may change or completely destroy these bounds. Of course, the mechanism of the appearance of large QG corrections in the SM β functions is not clear at all; neither do we have reliable estimations for the QG coupling initial values. Hence, the above results should be considered rather as a speculation which, however, may open quite an interesting field of QG applications to SM phenomenology.

VI. DISCUSSION

In this work we have studied the RG-improved (and one-loop) effective potential on a curved background. A general formalism has been applied to scalar QED with and without R^2 gravity on a curved background for both cases. We have discussed a few phenomena caused by QG, particularly dimensional transmutation in the presence of classical and quantum gravitational fields, influence of QG on the stability of the effective potential (in the examples of the Yukawa model and the standard model). In particular, we have shown that, as a result of QG effects in the conformal version of R^2 gravity, the running scalar coupling may become less unstable.

A numerical study of the phase structure in scalar QED on a curved background and also in the presence of quantum R^2 gravity has been done. We have shown the possibility of spontaneous symmetry-breaking and curvature-induced phase transitions. Is is interesting that, after the phase transition, one can get the Einstein theory in the low-energy limit even in situations where the Einstein sector was not present in the original R^2 quantum gravity.

QG corrections to the scalar-to-vector mass ratio are calculated in scalar QED. Because of their universal structure, it is not difficult to repeat the same analysis for any reasonable GUT theory with higher-derivative gravity.

The general formalism developed in this paper may be easily applied to any multiplicatively renormalizable theory of matter with QG. There are many questions pending in such more realistic GUT theories, such as the study of stability in the scalar sector, development of a clearer understanding of the connections between an effective theory for QG and GUT phenomenology, the inducing of Einstein gravity with realistic values of cosmological and Newton coupling constants at the critical point of a curvature-induced phase transition, and so on.

We would like to remark that in string theory, at the low-energy limit there also appear higher-derivative terms in the string effective action. However, the string effective action includes the dimensionless dilaton field and, hence, it is rather different from the starting action (2.1) for higher-derivative quantum gravity. Moreover, as a rule string quantum corrections are included into the string effective action, which represents the expansion on α' and derivative terms, and should therefore be treated classically. Hence, the results that we have obtained here are in fact restricted to the class of QG theories of the form (2.1) and cannot be applied to string-inspired models of QG. Moreover, in effective actions corresponding to string theories the Weyl tensor term squared that appears in (2.1) is prohibited, as a rule (apart from the fact that the coefficients of the curvature terms in such theories become dilatonic functions).

It is worthwhile to note also that higher-derivative QG with matter of the form (2.1) has been considered recently in Ref. [23] where the RG flow of the gravitational coupling constant has been studied. It was suggested there to use the dependence of the gravitational constant on the induced distance in actual cosmological applications, as for a possible solution of the dark matter problem [23]. Of course such a possibility may exist only within R^2-gravity models of type (2.1). In our paper we have been dealing with another circle of questions in matter–R^2-gravity models, i.e., the study of QG corrections in the matter sector. Certainly, however, in the model studied of QED with R^2-gravity the qualitative behavior of the gravitational coupling constant in (2.7) is of the same type as in Ref. [23]. Hence, the conclusions of that paper about the possibility of solving the dark matter problem remain true for the matter–R^2-gravity models studied in the present work.

Another interesting possibility is connected with an inflationary universe based on a Coleman-Weinberg-type effective potential. One may hope that taking into account QG corrections to this potential in the above discussed form may improve the situation and make such an inflationary universe more realistic. Viewed from another side, it would be of great interest to include QG corrections in the back-reaction problem analysis [24] (albeit which is not so easy). We plan to return to some of these questions elsewhere.

ACKNOWLEDGMENTS

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