Class of supersymmetric solitons with naked singularities

Mirjam Cvetič and Donam Youm
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104-6396
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We study vacuum domain walls in a class of four-dimensional $N = 1$ supergravity theories where along with the matter field, forming the wall, there is more than one "dilaton," each respecting SU(1,1) symmetry in their subsector. We find *supersymmetric* (planar, static) walls, interpolating between a Minkowski vacuum and a new class of supersymmetric vacua which have a naked (planar) singularity. Although such walls correspond to idealized configurations, i.e., they correspond to planar configurations of infinite extent, they provide the first example of supersymmetric classical solitons with naked singularities.

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Some of the solutions of gravity theory correspond to configurations with naked space-time singularities, i.e., singularities which are not hidden behind horizons. This uncomfortable feature is remedied by Penrose's conjecture, which states that generic initial conditions do not evolve to form naked singularities [1]. Such a conjecture is difficult to prove, and the dynamical formation of naked singularities has been addressed only for specific cases [2].

On the other hand, it has been observed [3–5] that in supersymmetric theories the allowed black hole configurations are only those with mass M bounded from below by the Bogomol'nyi bound, e.g., $M \geq \sqrt{P^2 + Q^2}$, where P and Q correspond to the magnetic and electric charges of the black hole, respectively. Incidentally, such a bound coincides with the one of cosmic censorship. Namely, black holes in supersymmetric theories have singularities hidden behind (or at) the horizon. This observation prompted a conjecture [5] that supersymmetry acts as a cosmic censor; i.e., in supersymmetric theories, configurations do not have naked singularities. The conjecture applies [5] only to configurations which have asymptotically flat space-time. In a related context, it has been proven [4] that in ungauged extended supergravity theories there are no classical "solitons" without horizons. Such solitons were assumed to have a nontrivial structure in the interior and to tend at large distances toward a supersymmetric vacuum *without matter sources.* The notion that "supersymmetry does not like naked singularities" [5] is intriguing and thus warrants further investigation.

In this paper we present supersymmetric vacuum domain wall configurations in a class of four-dimensional (4D) $N = 1$ supergravity models, where along with the matter field, forming the wall, there are $n \geq 2$ "dilaton" fields, each respecting SU(1,1) symmetry in their subsector. Such walls are *supersymmetric* (planar, static) configurations, where on one side of the wall the space-time is Minkowski, while on the other side the space-time has a (planar) naked singularity [6]. Although such configurations correspond to idealized solutions—i.e., they correspond to planar configurations with infinite extent—they provide the first example of classical solitons with naked singularities in a supersymmetric theory. Such solitons provide another counterexample to the notion that in supersymmetric theories naked singularities are hard to realize [7]. One is able to trace the origin of these singularities to the fact that these walls interpolate between (supersymmetric) Minkowski vacuum and a *new class of supersymmetric vacua* associated with a nontrivial matter source due to dilatons. Namely, in such supersymmetric vacua, $n \geq 2$ dilatons render the vacuum energy positive and for $n > 3$ the stress-energy tensor violates the strong energy condition. Such vacua are in sharp contrast with the Minkowski and anti–de Sitter space times, i.e., unique supersymmetric vacua without matter sources [4]. Such walls should also be contrasted with ordinary ($n = 0$) [8–11] and dilatonic ($n = 1$) [12] supergravity walls.

We consider 4D $N = 1$ supergravity theory with $n \geq 2$ dilatons $S_i \equiv e^{-2\phi_i} + ia_i$ $(i = 1, \ldots, n)$, which we choose to describe as scalar components of the chiral supermultiplets [13]. Dilatons have a Kähler potential $K(S_i, \overline{S}_i) = -\sum_{i=1}^{n} \ln(S_i + \overline{S}_i)$ and no superpotential $[W(S_i) = 0]$, thus respecting SU(1,1) noncompact symmetry in each subsector [14]. The scalar component T of the matter multiplet has a Kähler potential $K_M(T, \overline{T})$ and superpotential $W_M(T)$, which allow for isolated minima of the matter potential and thus for T to form the wall. A crucial property of the effective action is that it can be written in terms of the separable Kähler potential $K = K_M(T, \overline{T}) + K(S_i, \overline{S}_i)$ and superpotential $W = W_M(T)$, which depends *only* on the matter field T.

Such a class of supergravity theories is motivated by the no-scale supergravity models [15] as well as by the effective theory of (4D) superstring vacua [16]. In the latter case, one field corresponds to the dilaton field of the string theory and the other $(n-1)$ fields are the compactification moduli. Note, however, that for superstring vacua the $n - 1$ moduli cannot be rewritten as scalar components of linear multiplets and matter fields in general do couple to the corresponding moduli fields in the Kähler potential. The above proposed class of supergravity models should thus be viewed primarily as a specific framework which illustrates the existence of classical supersymmetric solitons with naked singularities.

The scalar part of the tree-level $N = 1$ supergravity Lagrangian is of the form

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{2}R + K_{T\overline{T}}g^{\mu\nu}\partial_\mu T\,\partial_\nu\overline{T} + \sum_{i=1}^{n} g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i - 2^{-n}\exp\left(2\sum_{i=1}^{n}\phi_i\right)\tilde{V}\right] , \quad (1)$$

where

$$\tilde{V} = e^{K_M}(K^{T\overline{T}}|D_T W_M|^2 - (3-n)|W_M|^2 \quad (2)$$

is the part of the potential which depends only on the matter field T. Here $K^{T\overline{T}} \equiv (K_{T\overline{T}})^{-1} \equiv (\partial_T\partial_{\overline{T}}K)^{-1}$ and $D_T W_M \equiv e^{-K_M}\partial_T(e^{K_M}W_M)$. We use the convention $(+---)$ for the metric and the gravitational constant $\kappa = 8\pi G = 1$. In (1) we have already set $a_i = 0$ (which turns out to correspond to the solution of equation of motion) and turned off the gauge fields. We also assume that the models are free of mixed Kähler-Lorentz anomalies [17].

We assume that $\tilde{V}$ has isolated minima, thus allowing for T field to form a wall configuration. Note that $\tilde{V}$ is modified as a result of the presence of n real scalar fields ϕ_i, which yield an additional contribution $e^K K^{S_i\overline{S}_i}|D_{S_i}W|^2 = \frac{1}{2}e^{2\phi_i}e^{K_M}|W_M|^2$ to $\tilde{V}$ for each field ϕ_i. For supersymmetric minima, $D_T W_M = 0$ and $\tilde{V} = (n-3)e^{K_M}|W_M|^2$. Therefore, at such minima dilatons ϕ_i screen the matter potential by $2^{-n}\prod_{i=1}^{n} e^{2\phi_i}$ [see Eq. (1)] as well as changing an overall scale factor of the matter potential from -3 (for the ordinary supersymmetric vacuum) to $(-3+n)$, thus rendering the matter potential (2) less negative. For supersymmetric minima $\tilde{V}$ is nonpositive for $0 \le n \le 2$, it vanishes identically for $n = 3$, and it is always non-negative for $n \ge 4$. We have therefore constructed examples of supergravity models, where supersymmetric minima can have *positive* vacuum energy. This is counter to the prevailing lore that for the supersymmetric vacua the vacuum energy is nonpositive.

We start with the metric ansatz for planar [in (x,y) plane], static domain wall solutions:

$$ds^2 = A(z)(dt^2 - dz^2 - dx^2 - dy^2) , \quad (3)$$

and the scalar fields $T(z)$ and $\phi_i(z)$ depend only on z. Using a technique of the generalized Israel-Nester-Witten form developed in Ref. [8] for the study of supergravity walls, one obtains the following Bogomol'nyi bound for the energy density σ of the planar domain wall configuration:

$$\sigma - |C| = \int_{-\infty}^{\infty}\left[-\delta_\epsilon\psi_i^\dagger g^{ij}\delta_\epsilon\psi_j + K_{T\overline{T}}\delta_\epsilon\chi^\dagger\delta_\epsilon\chi + \sum_{j=1}^{n} K_{S_j\overline{S}_j}\delta_\epsilon\eta_j^\dagger\delta_\epsilon\eta_j\right]dz \ge 0 . \quad (4)$$

This bound is saturated if and only if the supersymmetry variations $\delta_\epsilon\psi_\mu, \delta_\epsilon\chi$, and $\delta_\epsilon\eta_j$ of the fermionic partners of the fields $g_{\mu\nu}$, T, and S_j, respectively, vanish. For this case, one has *supersymmetric* bosonic backgrounds, and the metric and scalar fields satisfy coupled first-order differential equations (self-dual or Bogomol'nyi equations):

$$\begin{aligned} 0 &= \mathrm{Im}\left(\partial_z T\frac{D_T W_M}{W_M}\right) , \\ \partial_z T &= -\zeta\left[2^{-n}A\exp\left(2\sum_1^n\phi_i\right)\right]^{1/2} \times e^{K_M/2}|W_M|K_M^{T\overline{T}}\frac{D_{\overline{T}}\overline{W}_M}{\overline{W}_M} , \\ \partial_z \ln A &= 2\zeta\left[2^{-n}A\exp\left(2\sum_1^n\phi_i\right)\right]^{1/2} e^{K_M/2}|W_M| , \\ \partial_z\phi_i &= -\zeta\left[2^{-n}A\exp\left(2\sum_1^n\phi_i\right)\right]^{1/2} e^{K_M/2}|W_M| , \end{aligned} \quad (5)$$

where ζ is either $+1$ or -1 and can change sign when and only when W vanishes [8,9]. The above coupled first-order differential equations can be viewed as "square roots" of the corresponding Einstein and Euler-Lagrange equations; they provide special solutions of equations of motion which saturate the Bogmol'nyi bound (4).

The topological charge $|C|$ can be determined in the thin wall approximation. Then in the wall region ($z \sim z_0 = 0$, without loss of generality) the matter field T is a quickly varying function, resembling a step function centered at the wall, while the metric $A(z)$ and $\phi_i(z)$ fields vary slowly. With the choice $A(0) = 1$ and the boundary conditions $\phi_i(0) = (\phi_i)_0$ ($i = 1,\ldots,n$), one obtains [8,12]

$$\begin{aligned} \sigma = |C| &\equiv 2|(\zeta|We^{K/2}|)_{z=0^+} - (\zeta|We^{K/2}|)_{z=0^-}| \\ &= 2(\alpha_1 \pm \alpha_2) , \end{aligned} \quad (6)$$

where $\alpha_{1,2} \equiv 2^{-n/2}\exp[\sum_1^n(\phi_i)_0]e^{K_M/2}|W_M|_{1,2}$. Here the subscript 1 (or 2) refers to the side of the wall with the larger (or smaller) value of α. The plus and minus signs correspond to a solution with W_M crossing zero and $W_M \neq 0$ everywhere, respectively. Note that there are *no* walls corresponding to $\alpha_1 = \alpha_2 = 0$; i.e., the superpotential W_M has to have a nonzero value at least on one side of the wall.

The first two equations in (5) describe the evolution of the matter field $T = T(z)$ with z. The first equation is the "geodesic" equation [8] for the complex T field. It is the same as for the ordinary and dilatonic supergravity walls. The third and fourth equations in (5) for the conformal factor $A(z)$ and the real scalar fields $\phi_i(z)$ imply

$$A(z)e^{2\phi_i(z)} = e^{2(\phi_i)_0} , \quad i = 1,\ldots,n . \quad (7)$$

Here we have used the boundary conditions $A(0) = 1$ and $\phi_i(0) = (\phi_i)_0$. Note that this equation is true *everywhere* in the domain wall background. If one chooses to take one of the fields, say, ϕ_i, to be the dilaton field of the 4D string vacua, then Eq. (7) implies that the string frame metric $[A_s(z) \equiv A(z)e^{2\phi_1(z)}]$ is *flat* everywhere in the domain wall background. For $n \neq 1$ the second equation in (5) for the matter field T does not decouple from the conformal factor $A(z)$ and the scalar fields $\phi_i(z)$, and therefore the evolution of $T(z)$ will be affected by the presence of $\phi_i(z)$ [18]. For a thin wall, one sets $\tilde{V} = (n-3)e^{K_M}|W_M|^2 = \text{const}$ outside the wall and then the solutions can be found explicitly [19,20].

One can classify solutions into type I, II, and III walls in manner similar to those of ordinary supergravity walls ($n = 0$) [8,9] and dilatonic walls ($n = 1$) [12]. Here we concentrate [21] on the type I solutions corresponding to the case where $\alpha_1 \neq 0$ and $\alpha_2 = 0$; i.e., on one side of the wall, $W_M = 0$. The thin wall solution has the form

$$\sigma = 2\alpha_1 : \quad \begin{cases} A(z) = [1-(n-1)\alpha_1|z|]^{2/(n-1)} \ , \quad \phi_i = (\phi_i)_0 - \frac{1}{n-1}\ln[1-(n-1)\alpha_1|z|] \ , \quad z < 0 \ , \\ A(z) = 1 \ , \quad \phi_i = (\phi_i)_0 \ , \quad z > 0 \ . \end{cases} \tag{8}$$

On one side of the wall ($z > 0$) the space-time is flat, and on the other side ($z < 0$) $A(z)$ vanishes at the finite coordinate distance $|z|_{\text{sing}} = 1/(n-1)\alpha_1$ from the wall. Both the scalar fields $\phi_i(z)$ and the curvature invariants blow up in this region [22]. For $n = 2$, $R = 0$, but $R_{\mu\nu}R^{\mu\nu} = \infty$ at the singularity. For $n > 2$, not only $R_{\mu\nu}R^{\mu\nu} = \infty$, but also $R = 3 \times 2(2-n)\alpha^2[1-(n-1)\alpha_1|z|]^{-2n/(n-1)}$ blows up at $|z|_{\text{sing}}$ and the space-time becomes more singular as the number n of dilatons $\phi_i(z)$ increases. Clearly, $|z|_{\text{sing}}$ is a finite proper distance $d = \int \sqrt{A(z)}dz = 1/n\alpha_1 < \infty$ as well as within a finite affine parameter from the wall [23]. Thus the singularity is naked [24].

One can trace the origin of the naked singularity to the nature of the stress-energy tensor on the side of the wall with varying dilaton fields. The stress energy is diagonal with nonzero components: $T_{tt} = -T_{xx} = -T_{yy} = (2n-3)\alpha_1^2[1-(n-1)\alpha_1|z|]^{-2}$ and $T_{zz} = 3\alpha_1^2[1-(n-1)\alpha_1|z|]^{-2}$. Thus it satisfies the weak energy condition $T_{tt} \geq 0$ for $n \geq 2$ and the dominant energy condition $T_{tt} \geq |T_{ii}|$ ($i = x, y, z$) for $n \geq 3$; however, it violates the strong energy condition $T_t^t - \frac{1}{2}\sum_{i=t,x,y,z} T_i^i \geq 0$ for any $n > 3$. Our supersymmetric solutions are thus in agreement with the theorem [25] that static planar solutions are singular when the stress energy satisfies the weak energy condition $T_{tt} \geq 0$. On the other hand, such new supersymmetric vacua violate the strong energy condition for $n > 3$. This is in sharp contrast with the Minkowski anti–de Sitter space-times, i.e., unique supersymmetric vacua without matter sources [4].

We found new *supersymmetric* vacuum domain walls within a class of 4D, $N = 1$ supergravity models, where along with the matter field forming the wall there are $n \geq 2$ dilatons, each of them respecting SU(1,1) symmetry in their subsector. Such walls interpolate between (supersymmetric) Minkowski vacuum and a new class of supersymmetric vacua with the naked singularity. The origin of the naked singularity is traced to the nontrivial matter source due to the dilaton fields. Although such walls correspond to idealized solutions—i.e., they are planar configurations of infinite extent—they correspond to the first example of classical supersymmetric solitons with naked singularities.

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[1] R. Penrose, Riv. Nuovo Cimento **1**, 252 (1969).
[2] E.g., see a recent work addressing the formation of naked singularities in the collision of black holes in the space-time with the positive cosmological constant: D. R. Brill, G. T. Horowitz, D. Kastor, and J. Traschen, Phys. Rev. D **49**, 840 (1994); J. Horne and G. T. Horowitz, *ibid.* **48**, 5457 (1994).
[3] G. Gibbons, Nucl. Phys. **B207**, 337 (1982).
[4] G. Gibbons, in *Supersymmetry, Supergravity and Related Topics*, edited by F. del Aguila *et al.* (World Scientific, Singapore, 1985), pp. 124–166.
[5] R. Kallosh, A. Linde, T. Ortín, A Peet, and A. Van Proeyen, Phys. Rev. D **46**, 5278 (1992).
[6] Note that the space-time is Minkowski only on one side of the wall. Thus the space-time of such walls is not asymptotically flat.
[7] Within black hole configurations, a counterexample with asymptotically flat space-time has been addressed by R. Kallosh and T. Ortin "Supersymmetry, Trace Anomaly and Naked Singularities," Report No. N194002, hep-th/9404006 (unpublished). This example corresponds to a Kerr-Newman black hole with $M = Q$, with the rotation parameter a passing its extremal limit. Here M and Q are the mass and the charge of the black hole, respectively. Such a black holes have naked singularities; however, they are supersymmetric *only* at the classical level. Another example of extreme charged black holes with naked singularities within $N = 8$ supersymmetric 5D Kaluza-Klein theories was found in Ref. [3]. Supersymmetric black holes with a cosmological constant and with naked singularities were also found by L. Romans, Nucl. Phys. **B383**, 395 (1992). In the latter case, however, the asymptotic space-time is not flat.
[8] M. Cvetič, S. Griffies, and S.-J. Rey, Nucl. Phys. **B381**, 301 (1992).
[9] M. Cvetič and S. Griffies, Phys. Lett B **285**, 27 (1992).
[10] M. Cvetič, R. L. Davis, S. Griffies, and H. H. Soleng, Phys. Rev. Lett. **70**, 1191 (1993).
[11] G. Gibbons, Nucl. Phys. **B394**, 3 (1993).
[12] M. Cvetič, Phys. Rev. Lett. **71**, 815 (1993).
[13] An alternative description can be in terms of a linear supermultiplet. Since our solutions have imaginary components a_i set to zero and gauge fields are turned off, both descriptions are equivalent [e.g., P. Adamietz, P. Binetruy, G. Girardi, and R. Grimm, Nucl. Phys. **B401**, 257 (1993), and references therein]. Note that a linear

multiplet has no superpotential.

[14] A Kähler potential $K(S_i, \overline{S}_i) = -\sum_{i=1}^{n} \beta_i \ln(S_i + \overline{S}_i)$ with $\beta_i > 0$ yields a straightforward generalization. We motivate the choice $\beta_i = 1$ from examples in string theory and no-scale supergravity models.

[15] E.g., J. Ellis, C. Kounnas, and D. Nanopoulos, Nucl. Phys. **B247**, 373 (1984), and references therein.

[16] E. Witten, Phys. Lett. **155B**, 151 (1984).

[17] G. Lopes Cardoso and B. Ovrut, Nucl. Phys. **B418**, 535 (1994), and references therein.

[18] One can obtain explicit numerical solutions of Eq. (5) for a wall of any thickness with similar qualitative features as discussed for the thin walls in the text. Namely, on one side of the wall the matter field $T(z)$ reaches the supersymmetric minimum at a finite $z_{\rm sing}$. At $z_{\rm sing}$ the conformal factor $A(z)$ degenerates, yielding the space-time singular.

[19] Outside the thin wall ($\partial_z T = 0$), Einstein's equations for the conformal factor $A(z)$ and Euler-Lagrange equations for $\phi_i(z) = \phi(z) + (\phi_i)_0$ $(i = 1, \ldots, n)$, as derived from Lagrangian (1), are of the form

$$-H' - \tfrac{1}{4}H^2 = n(\phi')^2 + (n-3)\alpha_i^2 e^{2n\phi} A\ ,$$
$$\tfrac{3}{4}H^2 = n(\phi')^2 - (n-3)\alpha_i^2 e^{2n\phi} A\ ,$$
$$(A\phi')' = (n-3)\alpha_i^2 e^{2n\phi} A^2\ ,$$

where $H = A'/A$ and $A' \equiv \partial_z A$. Here α_i ($i = 1$ and 2 for $z > 0$ and $z < 0$, respectively) is defined after Eq. (6) in terms of the matter Kähler potential and superpotential on either side of the wall. Note that $\tilde{V}_i = (n-3)\alpha_i^2$ [see Eq. (2)] correspond to the value of the matter part of the potential on either side of the wall. [Some of the solutions of the above equations have been found by H. Soleng (unpublished).] The stress-energy tensor at the wall is of the form $T_{\mu\nu} = \sigma\delta(z)(1, -1, -1, 0)$. The matching conditions [W. Israel, Nuovo Cimento **B44**, 1 (1966); **48**, 463 (E) (1967)] for $A(z)$ and $\phi(z)$ at the wall are $A'(0^+) - A'(0^-) = -\sigma$ and $\phi'(0^+) - \phi'(0^-) = \frac{1}{2}\sigma$. They impose boundary conditions on solutions of the above Einstein's and Euler-Lagrange equations. Note that such matching conditions are automatically satisfied by the solutions of Eqs. (5).

[20] The results obtained from Lagrangian (1) should be contrasted with those for the Jordan-Brans-Dicke (JBD) Lagrangian which in the Einstein frame is of the form

$$\mathcal{L}_{\rm JBD} = \sqrt{-g}\left[-\tfrac{1}{2}R + \tfrac{1}{2}g^{\mu\nu}\partial_\mu\tilde{\phi}\,\partial_\nu\tilde{\phi} + e^{-\beta\tilde{\phi}}\mathcal{K} - e^{-2\beta\tilde{\phi}}V\right]\ ,$$

where $\beta \equiv (\omega + \frac{3}{2})^{-1/2}$, while $\mathcal{K}$ and V correspond to the kinetic and potential energy of the matter field forming the wall, respectively. ω is the coefficient in front of the kinetic energy of the JBD field $\Phi \equiv e^{\beta\tilde{\phi}}$ in the JBD frame defined by $g_{{\rm JBD}\mu\nu} = g_{\mu\nu}/\Phi$ [P. Jordan, Z. Phys. **157**, 112 (1959); C. Brans and C. Dicke, Phys. Rev. **124**, 925 (1961)]. Outside the thin wall, $\mathcal{K} = 0$. In this case the Lagrangian (1) can be written in the form of the JBD Lagrangian provided $\phi_i(z) = \tilde{\phi}(z)/\sqrt{2n} + (\phi_i)_0$ $(i = 1, \ldots, n)$, $V = 2^{-n}\exp(2\sum_{i=1}^{n}(\phi_i)_0)\tilde{V}$, and $\omega = 2/n - \frac{3}{2}$. However, inside the wall region, $\mathcal{K} \neq 0$, and the JBD Lagrangian *is not* of the form (1). Thus, for the JBD Lagrangian, the matching conditions at the wall are different: $A'(0^+) - A'(0^-) = -\sigma$ and $\tilde{\phi}'(0^+) - \tilde{\phi}'(0^-) = -\frac{3}{2}\beta\sigma$ and are obviously not satisfied by the solutions of Eq. (5).

[21] Type II and type III walls correspond to the walls with $\alpha_1 \neq 0$ and $\alpha_2 \neq 0$; i.e., on both sides of the wall, $W_M \neq 0$. Type II walls have W_M traversing zero, a plus sign in Eq. (6), and conformal factors $A(z)_{1,2} = [1 - (n-1)\alpha_{1,2}|z|]^{2/(n-1)}$. That is, they have naked singularities on both sides of the wall. Type III walls have $W_M \neq 0$ everywhere, a minus sign in Eq. (6), and conformal factors $A(z)_1 = [1 - (n-1)\alpha_1|z|]^{2/(n-1)}$ for $z < 0$ and $A_E(z)_2 = [1 + (n-1)\alpha_2|z|]^{2/(n-1)}$ for $z > 0$; i.e., they have a naked singularity on one side of the wall ($z < 0$) and are geodesically complete on the other side of the wall ($z > 0$).

[22] Note that ordinary supergravity walls ($n = 0$) have the thin wall solution $A(z) = [1 - \zeta\alpha_i z]A^{-2}$ $(i = 1, 2)$, while dilatonic walls ($n = 1$) have the thin wall solution $A(z) = \exp[2\zeta\alpha_i z]$ $(i = 1, 2)$. On the side with larger α_i, the space-time of the ordinary walls [10,11] has no singularities with $|z| = \infty$ corresponding to the Cauchy horizon, while for dilatonic walls the singularity at $|z| = \infty$ coincides with the horizon [12]. Note that on that side of the ordinary and dilatonic supergravity walls the weak and the dominant energy conditions are violated, while the strong energy condition is satisfied.

[23] For the null geodesics the affine parameter $\lambda_0(z) = E^{-1}\int^z A(z')dz'$ is finite as $|z| \to [1/(n-1)]\alpha_1$. Here E is a conserved energy parameter. The proper time of the massive test particle is $\tau(z) = \int^z mA(z')dz'/\sqrt{E^2 - m^2A(z')}$. With $A(z) = [1 - (n-1)\alpha_1|z|]^{2/(n-1)}$ $(n \geq 2)$, $\tau(z)$ is finite as $|z| \to [1/(n-1)]\alpha_1$.

[24] It can be shown, by applying Killing spinor identities [R. Kallosh and T. Ortín, "Killing Spinor Identities," Report No. SU-ITP-93-16, hep-th/9306085 (unpublished)] that our bosonic solutions do not acquire quantum corrections which respect $N = 1$ supersymmetry.

[25] A. D. Dolgov and I. B. Khriplovich, Gen. Relativ. Gravit. **21**, 13 (1989).