Vector dominance model and Gari-Krümpelmann formula for the nucleon electromagnetic form factor

Keiji Watanabe and Hiroyuki Takahashi*
Department of Physics, Meisei University, Hino, Tokyo 191, Japan
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The electromagnetic form factors of the nucleon are investigated by using the vector dominance model (VDM) and the formulas that are proposed by Gari and Krümpelmann (GK) to synthesize the VDM and perturbative QCD. We simplify and generalize the formulas given by GK. The existing data of nucleon electromagnetic form factors are realized remarkably well. Although the incorporation of a QCD term improves the fit, the present experimental data on the form factors are not accurate enough to exclude a simple naive vector dominance model.

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Recently, experimental data for the electromagnetic form factors have become accurate from low to high squared momentum transfer $t = Q^2$ [1–12]. For high t, the magnetic form factors of nucleons decrease more rapidly than the prediction of the dipole formula, $G_D = (1 + t/0.71)^{-2}$, where t is given in terms of GeV^2. The neutron electric form factor is very small up to $t = 4$ GeV^2. For small t we may explain the experimental data in terms of hadron dynamics, for instance, with recourse to the vector dominance model (VDM) [13] or dispersion theoretical calculations for the pion and nucleon system. In the vector dominance model it is necessary to take the ρ meson mass smaller than the experimental one by 20%. The following two possibilities were considered for the small ρ meson mass. One is an explanation based on hadron dynamics; the uncorrelated two-pion contribution works in reducing the effective mass of the ρ meson [14]. The other is the incorporation of an additional photon–ρ-meson form factor that effectively makes the ρ meson mass smaller [15]. For larger t we have the prediction of perturbative QCD (PQCD), according to which the charge and magnetic moment form factors F_1 and F_2 are given asymptotically as $F_1 \sim t^{-2} \ln(t/Q_0^2)^{-\gamma}$ and $F_2 \sim t^{-3} \ln(t/Q_0^2)^{-\gamma'}$ where Q_0, γ, and γ' are constants. The experimental results for large t seem to agree with the prediction of PQCD.

Considering that the experimental data have become very accurate and that in the near future ep collider experiments will provide data for very high t, it is a problem of importance to derive formulas that explain the form factors for the low and high t regions systematically. Furuichi and one of the authors assumed superconvergent dispersion relations [16,17] for F_i $(i = 1, 2)$ to synthesize the low and high t regions of form factors. They are able to explain the experimental data very well, including the recent SLAC data [9–12]. Gari and Krümpelmann (GK) [18,19] proposed simple formulas, which show the VDM for the low t region with ρ and ω meson poles and for the high t region PQCD behavior. Their formulas reproduce the magnetic form factor of the proton very well but for the other form factors the agreement with new experiments was not good.

It is the purpose of this paper to investigate the nucleon electromagnetic form factor based on simple phenomenological formulas. First, we examine the VDM to see to what extent the simple vector dominance model reproduces the experimental data throughout the low and high t regions. Second, we investigate the effect of PQCD by using the improved formulas of Gari and Krümpelmann with simplification of the QCD term and incorporation of the vector bosons appearing in the Particle Data Group book [20].

In the vector dominance model the charge and magnetic moment form factors of nucleons, F_1^I and F_2^I (I being the isospin), are given as

$$F_i^I(t) = P_i^I(t) = c_i^I - t\sum_{k=1}^{N_I} \frac{a_i(M_k^I)}{t + (M_k^I)^2} \qquad (I = 0, 1; i = 1, 2) \ . \quad (1)$$

The M_k^I's denote the mass of vector bosons with isospin I, $a_i(M_k^I)$ are constants, and the normalizations of P_i^I at $t = 0$ are taken as $c_1^I = 1$ and $c_2^I = 2\kappa_I$, where κ_I is the anomalous magnetic moment with isospin I; $\kappa_0 = (\kappa_p + \kappa_n)/2$ and $\kappa_1 = (\kappa_p - \kappa_n)/2$ with κ_p and κ_n being the anomalous magnetic moments of proton and neutron, respectively.

The following two constraints are considered on the asymptotic behavior of P_i^I for $t \to \infty$: (a) $P_1^I \to 0$ and $tP_2^I \to 0$; (b) $tP_1^I \to 0$ and $t^2P_2^I \to 0$.

The constraint (a) leads to

$$\sum_{i=1}^{N_I} a_1(M_i^I) = 1 \,, \quad \sum_{i=1}^{N_I} a_2(M_i^I) = 2\kappa_I \ ,$$

$$\sum_{i=1}^{N_I} (M_i^I)^2 a_2(M_i^I) = 0 \ .$$

*Present address: Department of Physics, College of Science and Technology, Nihon University, Chiyoda 101 Tokyo, Japan.

To effect the constraint (b) the following additional conditions are required:

$$\sum_{i=1}^{N_I}(M_i^I)^2 a_1(M_i^I) = 0\ ,\quad \sum_{i=1}^{N_I}(M_i^I)^4 a_2(M_i^I) = 0\ .$$

When the condition (a) is imposed, P_i^I are given in terms of the independent parameters $\bar{A}_1(M_i^I)$ $(i = 1, \ldots, N_I - 1)$ and $\bar{A}_2(M_i^I)$ $(i = 1, \ldots, N_I - 2)$. We have

$$P_1^{I(\mathrm{a})}(t) = \left[1 - t\sum_{i=1}^{N_I-1}\frac{\bar{A}_1(M_i^I)}{t+(M_i^I)^2}\right]\frac{(M_{N_I}^I)^2}{t+(M_{N_I}^I)^2}\ , \tag{2}$$

$$P_2^{I(\mathrm{a})}(t) = \left[2\kappa_I - t\sum_{i=1}^{N_I-2}\frac{\bar{A}_2(M_i^I)}{t+(M_i^I)^2}\right] \times\frac{(M_{N_I-1}^I)^2(M_{N_I}^I)^2}{[t+(M_{N_I-1}^I)^2][t+(M_{N_I}^I)^2]}\ . \tag{3}$$

For the case (b) P_i^I are expressed as

$$P_1^{I(\mathrm{b})}(t) = \left[1 - t\sum_{i=1}^{N_I-2}\frac{\bar{A}_1(M_i^I)}{t+(M_i^I)^2}\right]\frac{(M_{N_I-1}^I)^2(M_{N_I}^I)^2}{[t+(M_{N_I-1}^I)^2][t+(M_{N_I}^I)^2]}\ , \tag{4}$$

$$P_2^{I(\mathrm{b})}(t) = \left[2\kappa_I - t\sum_{i=1}^{N_I-3}\frac{\bar{A}_2(M_i^I)}{t+(M_i^I)^2}\right]\frac{(M_{N_I-2}^I)^2(M_{N_I-1}^I)^2(M_{N_I}^I)^2}{[t+(M_{N_I-2}^I)^2][t+(M_{N_I-1}^I)^2][t+(M_{N_I}^I)^2]}\ . \tag{5}$$

For the vector dominance model we examine the cases with conditions (a) and (b); the charge and magnetic moment form factors are given by $F_i^I = P_i^{I(\mathrm{a})}$ or $F_i^I = P_i^{I(\mathrm{b})}$.

Let us now give the formulas which satisfy the prediction of PQCD for large t. We simplify the QCD factor and generalize the pole terms in the Gari-Krümpelmann formulas. The form factors are

$$F_i^I(t) = P_i^{I(K)}(t)F^{\mathrm{QCD}}(t)\ , \tag{6}$$

where $K = a, b$ for $i = 1, 2$, respectively, and

$$F^{\mathrm{QCD}}(t) = \frac{\Lambda^2}{\Lambda^2 + t\left[\ln\left(\frac{t+\Lambda^2}{Q_0^2}\right)\Big/\ln\left(\frac{\Lambda^2}{Q_0^2}\right)\right]^{\gamma}}\ . \tag{7}$$

Here Q_0 and $\gamma = 2 + 4/(3\beta)$ with β given as $\beta = 11 - 2n_f/3$. The number of flavors is $n_f = 6$. The proton and neutron form factors are $F_i^p = (F_i^0 + F_i^1)/2$ and $F_i^n = (F_i^0 - F_i^1)/2$ and the electric and magnetic form factors of nucleon N, G_E^N and G_M^N, are $G_E^N = F_1^N - tF_2^N/(4m^2)$ and $G_M^N = F_1^N + F_2^N$ with m the nucleon mass.

We analyze experimental data by using the VMD and our improved formula of Gari and Krümpelmann, to as the Watanabe-Takahashi (WT) formula referred hereafter. Totally, we use 172 data on the nucleon electromagnetic form factors, the compilation of world data summarized in Ref. [17]. In this calculation we take account of the following vector bosons: for the isovectors $\rho(770)$, $\rho'(1450)$, $\rho''(1700)$, and the unconfirmed one $\rho'''(2110)$ and for the isoscalars $\omega(783)$, $\omega'(1390)$, $\omega'(1600)$, $\phi(1020)$, and $\phi'(1680)$. The number given in parentheses denotes the mass of vector bosons in MeV. We enumerate the vector bosons in the order given above in applying the formulas (2)–(5). We examine the following cases for the number of vector bosons: $N_V = 3$ and 4, and $N_S = 3$ and 5. The case of $N_S = 3$ implies that the Okubo-Zweig-Iizuka (OZI) rule is strictly valid and the couplings of the ϕ and ϕ' mesons to the nucleon are neglected.

The parameters are determined by minimizing χ^2. We summarize in Table I the χ^2 values for the above mentioned cases. As we have remarked before, the ρ meson mass is taken as an adjustable parameter and is changed in the interval 0.60–0.77 GeV. In the case of the WT formula, Λ appearing in (7) is taken as a parameter and is changed in the interval 0.5–5 GeV. The best fit is obtained for $\Lambda \sim 1$ GeV. In the WT formula the parameters are $\bar{A}(M_i^I)$, the ρ meson mass, and Λ. The QCD param-

TABLE I. χ^2 values vs the number of vector bosons N_V and N_S and the ρ meson mass m_ρ for the VDM and WT formulas. (a) Vector dominance model. (b) The improved GK Formula: the WT Formula. Λ is the parameter appearing in (7).

(a)

N_V	N_S	m_ρ	χ^2_{min}	$\chi^2_{\mathrm{min}}/N_{\mathrm{DF}}$
VDM(a): $F_1 \to 0$, $tF_2 \to 0$ for $t \to \infty$				
3	3	0.63	356.8	2.22
4	3	0.60	264.5	1.63
3	5	0.65	205.3	1.28
4	5	0.64	201.6	1.27
VDM(b): $tF_1 \to 0$, $t^2F_2 \to 0$ for $t \to \infty$				
3	3	0.60	492.3	2.91
4	3	0.60	457.1	2.74
4	5	0.64	214.5	1.32

(b)

N_V	N_S	m_ρ	Λ	χ^2_{min}	$\chi^2_{\mathrm{min}}/N_{\mathrm{DF}}$
3	3	0.60	1.5	465.0	2.78
4	3	0.60	1.0	200.7	1.24
3	5	0.60	1.0	177.9	1.13
4	5	0.63	0.8	174.5	1.10
4	5	0.7681	1.0	180.3	1.13

TABLE II. $\bar{A}_j(M_i^I)$ appearing in (2)–(5). (a) $\bar{A}_1(M_i^I)$ and $\bar{A}_2(M_i^I)$ for the VDM. (b) $\bar{A}_1(M_i^I)$ and $\bar{A}_2(M_i^I)$ for the WT formula. The dashes mean that there is no corresponding $\bar{A}_j(M_i^I)$.

(a)

M_i^I (GeV)	VDM(a) $m_\rho = 0.64$ GeV $\bar{A}_1(M_i^I)$	VDM(a) $\bar{A}_2(M_i^I)$	VDM(b) $m_\rho = 0.64$ GeV $\bar{A}_1(M_i^I)$	VDM(b) $\bar{A}_2(M_i^I)$
$\rho(m_\rho)$	0.8451	4.0564	0.4009	3.5223
$\rho(1.465)$	−0.0548	−0.1994	0.5092	---
$\rho(1.700)$	0.2250	---	---	---
$\rho(2.110)$	---	---	---	---
$\omega(0.783)$	3.8458	0.1118	1.8847	−0.2760
$\omega'(1.394)$	9.2941	1.3958	−4.8003	0.0879
$\omega''(1.549)$	−5.6236	−2.0204	3.8150	---
$\phi(1.020)$	−6.5428	---	---	---
$\phi'(1.680)$	---	---	---	---

(b)

M_i^I (GeV)	WT1 $m_\rho = 0.60$ GeV $\bar{A}_1(M_i^I)$	WT1 $\bar{A}_2(M_i^I)$	WT2 $m_\rho = 0.63$ GeV $\bar{A}_1(M_i^I)$	WT2 $\bar{A}_2(M_i^I)$	WT3 $m_\rho = 0.7681$ GeV $\bar{A}_1(M_i^I)$	WT3 $\bar{A}_2(M_i^I)$
$\rho(m_\rho)$	0.6866	2.4861	1.7345	1.6834	2.0173	3.5925
$\rho'(1.465)$	−2.1712	−1.0536	−13.0701	−2.5905	−11.1483	−2.1433
$\rho''(1.700)$	0.8580	---	9.0132	---	8.8659	---
$\rho'''(2.110)$	---	---	---	---	---	---
$\omega(0.783)$	1.3669	0.0249	3.1114	3.7354	6.0116	3.3699
$\omega'(1.394)$	−1.3466	---	41.6280	−22.0795	47.5315	−25.0373
$\omega''(1.594)$	---	---	−32.9222	17.9500	−34.3330	22.3358
$\phi(1.020)$	---	---	−13.5000	---	−19.8864	---
$\phi'(1.680)$	---	---	---	---	---	---

eter Q_0 is fixed at 0.2 GeV.

We give in Table II the values of $\bar{A}_j(M_i^I)$ for the VDM and WT formulas for the following cases. For the VDM: VDM(a), VDM with the asymptotic condition (a) for $N_V = 4$, $N_S = 5$, and $m_\rho = 0.64$ GeV; VDM(b), VDM with the condition (b) for $N_V = 4$, $N_S = 5$, and $m_\rho = 0.64$ GeV. For the WT formulas: WT1, $N_V = 4$, $N_S = 3$ for $m_\rho = 0.6$ GeV and $\Lambda = 1.0$ GeV; WT2, $N_V = 4$, $N_S = 5$ for $m_\rho = 0.63$ GeV and $\Lambda = 0.8$ GeV; WT3, $N_V = 4$, $N_S = 5$ with m_ρ kept at the experimental value and $\Lambda = 1$ GeV. The residues a_i at the poles of $P_j^{(a)}$ are evaluated by using $\bar{A}_j(M_i^I)$ that are given in Table II.

We illustrate our numerical calculations in Figs. 1–4 for VDM(b) and WT3. We also enter in the figures the calculation by GK. The simple vector dominance model realizes the experimental data for cases (a) and (b) if we take $N_V = 4$ and $N_S = 5$ and the ρ meson mass $m_\rho \sim 0.6$ GeV. We note that in the VDM it is necessary to take $N_S = 5$, namely, the ϕ and ϕ' mesons are required to fit the data. Although the experimental data are reproduced fairly well via case (a), the asymptotic conditions may not be acceptable, because in the high t region the ratio of the calculated electric and magnetic

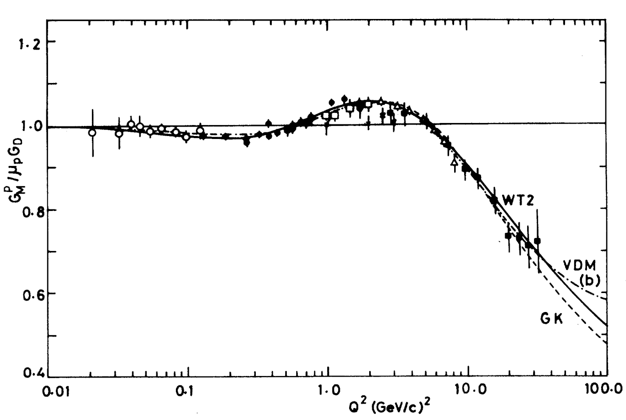

FIG. 1. $G_M^p/(\mu^p G_D)$. The solid curve is the calculation by WT2 with $N_V = 4$, $N_S = 5$, $m_\rho = 0.63$ GeV, and $\Lambda = 0.8$ GeV; the dashed one is the GK formula [18]; and the dash-dotted one VDM(b) with $N_V = 4$, $N_S = 5$, and $m_\rho = 0.64$ GeV. ○: Borkowski *et al.* [7]. •: Price *et al.* [2]. ▽: Ch. Berger *et al.* [3]. △: Bosted *et al.* [10]. □: Bartel *et al.* [4]. ■: Sill *et al.* [12].

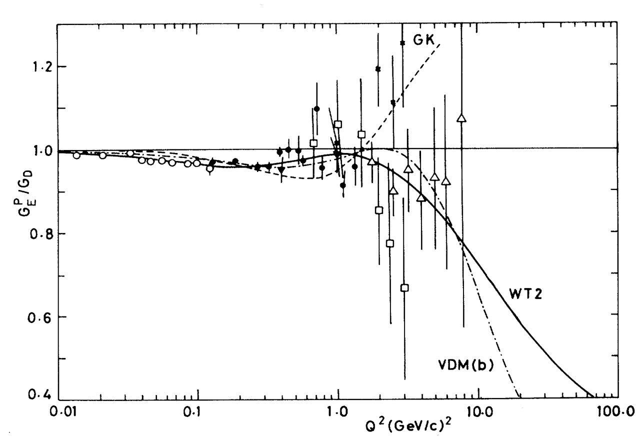

FIG. 2. G_E^p/G_D. × Walker *et al.* [9]. The dashed curve is the calculation by GK model 3 [19]. See the caption of Fig. 1.

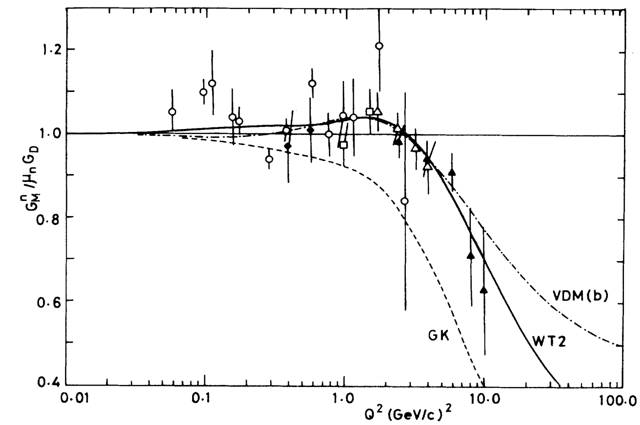

FIG. 3. $G_M^n/(\mu^n G_D)$. The dashed curve is the calculation by the GK formula [18]. See the caption of Fig. 1. △: Rock *et al.* [8]. △: Lung *et al.* [11]. □: Bartel *et al.* [4]. ○: Noncoincidence data cited in Bartel *et al.* [4]. ◇: Combined analysis; Stein *et al.* [1] and Bartel *et al.* [4].

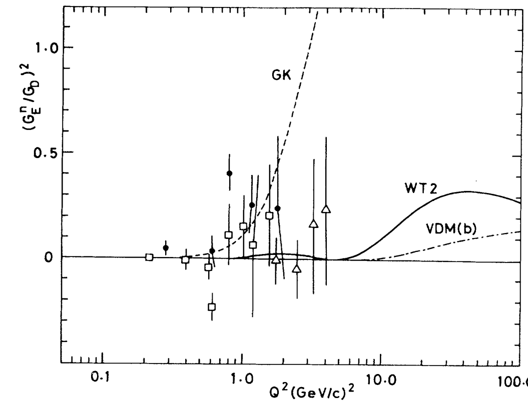

FIG. 4. $(G_E^n/G_D)^2$. The dashed curve is the calculation by GK model 3 [19]. ○: Bartel *et al.* [4]. •: Hanson *et al.* [5]. △: Lung *et al.* [11].

form factors to the dipole formula increases. By using the factorized formulas (6), where the PQCD constraints are satisfied asymptotically, we may improve the fit remarkably as illustrated in Figs. 1–4. In this case, it is possible to reproduce the experimental data by leaving out the couplings of ϕ and ϕ' mesons to the nucleon, but the results are improved by incorporation of the couplings of ϕ mesons to the nucleon. The χ^2 value becomes considerably smaller; $\chi^2_{\min}/N_{\rm DF} = 1.10$ for $N_V = 4$, $N_S = 5$, while $\chi^2_{\min}/N_{\rm DF} = 1.24$ for $N_V = 4$, $N_S = 3$. The cases VDM and WT1 reproduce the experimental data very well except for G_E^p in the small t region less than 0.1 GeV^2; the calculated result becomes a little larger than the experimental data of G_E^p, and consequently χ^2 becomes large. It must be noticed that in the WT formulas we are able to reproduce the experimental data quite well by keeping the ρ meson mass at the experimental value as is shown in Table I(b). In this case the factor $F^{\rm QCD}$ in (6) works as an additional incorporation of the photon–ρ-meson form factor as in Ref. [15]. Although the assumption of PQCD behavior for the form factors greatly improves the results, information at higher momentum transfer is required to draw the conclusion that the contribution from vector boson poles vanishes asymptotically so that the effect of PQCD actually dominates in the large t region for the nucleon electromagnetic form factor. In this paper we have simplified the QCD factor as given by (7) to reduce the number of parameters; however, we may perform similar calculations by using the same function for $F_{\rm QCD}$ that was proposed by Gari and Krümpelmann. By taking account of observed vector bosons in the original GK formula, the experimental data are parametrized very well throughout the low and high t regions; we have $\chi^2_{\min}/N_{\rm DF} \sim 1.0$. When experimental data of form factors in the region t above 100 GeV^2 become available, it will be necessary to elaborate the QCD factor by considering higher order effects, etc.

The factorized formula of the VDM and $F^{\rm QCD}$ given by (6) is analogous to the form factor version of the Veneziano amplitude, given by Di Vecchia and Drago [21], which represents the dual nature of Regge pole and vector bosons. The formula is an interpolation of the VDM and PQCD, and it implies the existence of duality in the vector boson resonances and the quark-gluon system.

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