PHYSICAL REVIEW D VOLUME 51, NUMBER 3 1 FEBRUARY 1995

Test of CPT symmetry conservation in K^0 decays

G. B. Thomson and Y. Zou
Department of Physics and Astronomy, Rutgers–The State University of New Jersey, Piscataway, New Jersey 08855
(Received 7 July 1994)

Recent measurements have been made of several of the decay parameters of the K^0-$\bar{K}^0$ system: η_{+-0}, $\eta_{+-\gamma}$, Δm, τ_S, ϕ_{+-}, ϕ_{00}, and ϵ'/ϵ. These measurements make it possible to place new limits on the violation of CPT symmetry. To do this we use the Bell-Steinberger relation, which relates the time derivative of the square of the kaon wave function to the sum of the partial decay rates. We evaluate each partial decay rate and discuss the experimental uncertainties.

PACS number(s): 11.30.Er, 13.25.Es, 14.40.Aq

CPT symmetry conservation is satisfied in almost all local field theories and is considered a cornerstone of high energy physics. However, because of the nonlocal nature of some recent candidate theories, e.g., string theory, and because whether CPT is conserved or not is ultimately an experimental question, a considerable amount of theoretical and experimental effort has gone into tests of CPT invariance. Perhaps the most natural place to search for CPT -violation is in the K^0-$\overline{K^0}$ system. Because of the small mass difference between the K_L^0 and K_S^0 mesons, this system is extraordinarily sensitive to small effects such as CP or CPT symmetry violation. For example it is the only place to date where CP violation has been found.

To investigate possible CPT violation in kaon decays we choose to evaluate the Bell-Steinberger relation [1]. This equation is based on the conservation of probability. It relates the time derivative of the square of the kaon wave function to the sum of partial decay rates. Our choice is motivated by the availability of new measurements of many of the parameters describing decays of the K^0-$\overline{K^0}$ system, namely η_{+-0} (the CP violation parameter for $K_S^0 \to \pi^+\pi^-\pi^0$), $\eta_{+-\gamma}$ (the CP violation parameter for $K_L^0 \to \pi^+\pi^-\gamma$), Δm (the K_L-K_S mass difference), τ_S (the K_S lifetime), ϕ_{+-} and ϕ_{00} (the phases of η_{+-} and η_{00}), and ϵ'/ϵ (the parameter ratio describing direct CP violation). An exhaustive review [2] of CPT violation in kaon decays in 1984 found a "not quite satisfactory agreement" between data and CPT conservation due to the then current measurements of ϕ_{+-} and ϕ_{00}. We shall see that the new measurements have clarified the situation.

The derivation of the Bell-Steinberger relation starts from the parametrization of a state of amplitude a ($\overline{a}$) of the K^0 ($\overline{K}^0$). The Schrödinger equation for this state is

$$-\frac{d}{dt}\begin{pmatrix} a \\ \overline{a} \end{pmatrix} = \left(iM + \frac{1}{2}\Gamma\right)\begin{pmatrix} a \\ \overline{a} \end{pmatrix}, \qquad (1)$$

where M and Γ are 2×2 Hermitian matrices called the mass and decay matrices. The eigenstates, called K_S^0 and K_L^0, are given by

$$|K_S\rangle = \frac{1}{\sqrt{2(1+|\epsilon+\Delta|^2)}}[(1+\epsilon+\Delta)|K^0\rangle + (1-\epsilon-\Delta)|\overline{K}^0\rangle], \qquad (2a)$$

$$|K_L\rangle = \frac{1}{\sqrt{2(1+|\epsilon-\Delta|^2)}}[(1+\epsilon-\Delta)|K^0\rangle - (1-\epsilon+\Delta)|\overline{K}^0\rangle], \qquad (2b)$$

where, in terms of the elements of M and Γ,

$$\epsilon = \frac{\mathrm{Im}(\Gamma_{12}) + i\mathrm{Im}(M_{12})}{\Gamma_S - \Gamma_L - 2i\Delta m}, \qquad (3a)$$

$$\Delta = \frac{\Gamma_{11} - \Gamma_{22} + i(M_{11} - M_{22})}{\Gamma_S - \Gamma_L - 2i\Delta m}, \qquad (3b)$$

where Γ_S (Γ_L) is the K_S (K_L) decay rate. Δ (ϵ) describes CP violation with CPT violation (conservation). This fact can be seen from Eq. (3b) because Δ is nonzero if there is a difference in either masses or decay rates of the K^0 and $\overline{K^0}$.

If we form a state $|K(t)\rangle = a_S|K_S\rangle + a_L|K_L\rangle$ then the conservation of probability requires that the slope of the normalization of this state at time t=0 be equal to the sum of the decay rates:

$$-\frac{d}{dt}|\langle K(0)|K(0)\rangle|^2 = \sum_f |a_S A(K_S \to f) + a_L A(K_L \to f)|^2, \qquad (4)$$

where the sum runs over all decay channels f, and the A's are decay amplitudes. Since a_S and a_L are arbitrary parameters Eq. (4) is really three equations, for the factors multiplying $|a_S|^2$, $|a_L|^2$, and the cross terms, $a_S^* a_L$ and its complex conjugate. Using the equation for the cross terms, and Eq. (1) for the time derivatives, yields the Bell-Steinberger relation

$$(1 + i\tan\phi_{\mathrm{sw}})[\mathrm{Re}(\epsilon) - i\mathrm{Im}(\Delta)] = \sum_f \alpha_f, \qquad (5)$$

where ϕ_{sw} is the superweak phase, $\tan\phi_{\mathrm{sw}} = 2\Delta m/(\Gamma_S - \Gamma_L)$, the sum again runs over all decay channels f, and $\alpha_f = (1/\Gamma_S)A^*(K_S \to f)A(K_L \to f)$. Although derived from the principle of conservation of probability, Eq. (5) is a test of CPT symmetry conservation through the appearance of the CPT-violating parameter Δ. Table I shows all of the decays that contribute to the states, f, and the formulas for each α_f. In the table, δ_l is the

TABLE I. All of the α_f that contribute to the Bell-Steinberger relation.

Decay mode	α_f
$K_L \to \pi^+\pi^-$	$\alpha_{+-} = B_{+-}^{(S)}\eta_{+-}$
$K_L \to \pi^0\pi^0$	$\alpha_{00} = B_{00}^{(S)}\eta_{00}$
$K_L \to \pi^+\pi^-\gamma$	$\alpha_{+-\gamma} = B_{+-\gamma}^{(S)}\eta_{+-\gamma}$
$K_L \to \pi e\nu$ and $\pi\mu\nu$	$\alpha_{l3} = \frac{\tau_S}{\tau_L}[B_{\pi e\nu}^{(L)} + B_{\pi\mu\nu}^{(L)}][\delta_l(1+2\mathrm{Re}(x)) - 2i\mathrm{Im}(x)]$
$K_S \to \pi^+\pi^-\pi^0$	$\alpha_{+-0} = \frac{\tau_S}{\tau_L}B_{+-0}^{(L)}\eta_{+-0}$
$K_S \to \pi^0\pi^0\pi^0$	$\alpha_{000} = \frac{\tau_S}{\tau_L}B_{000}^{(L)}\eta_{000}$

charge asymmetry in semileptonic decays, and x is the $\Delta S = \Delta Q$ violation parameter.

Each of the CP-violation parameters, $\eta_{+-}, \eta_{00}, \eta_{+-\gamma}$, and δ_l is measured to be of order 10^{-3} [3]. The standard model predicts that η_{+-0} and η_{000} are of the same order and that $x = 0$. If these predictions are true, then the size of each α_f is clear. α_{+-} and α_{00} dominate the sum. $\alpha_{+-\gamma}$ is smaller because of its smaller branching ratio, and $\alpha_{l3}, \alpha_{+-0}$, and α_{000} are reduced by the factors of $\tau_S/\tau_L(= 1/580)$ shown in Table I.

Table II shows the experimental values of each α_f from the world's data on kaon decays. In addition, Table II shows the quantities $\alpha_{+-} + \alpha_{00}$ and $(1 + i\tan\phi_{\mathrm{sw}})\mathrm{Re}(\epsilon)$ and the difference between them. If CPT is conserved and the theoretical prejudice about x, η_{+-0}, and η_{000} is true then this difference should be consistent with zero within uncertainties.

The values are calculated in two ways. The first, in the center column of Table II , is from the 1992 Particle Data Group (PDG) compendium of Ref. [3]. The entry for $\alpha_{+-\gamma}$ is blank because $\eta_{+-\gamma}$ had not been measured in 1992. If we form a sum of all the α_f's the small experimental uncertainties on α_{+-} and α_{00} would be overcome by the large uncertainties on $\alpha_{l3}, \alpha_{+-0}$, and α_{000}. To avoid this, we can for the moment assume that CPT is conserved and that the standard model predictions for x, η_{+-0}, and η_{000} are correct, and compare $\alpha_{+-} + \alpha_{00}$ with $(1 + i\tan\phi_{\mathrm{sw}})\mathrm{Re}(\epsilon)$ to see if the data are consistent with these assumptions. They are consistent at the $\sim 1\sigma$ level.

The second method of calculating the α_f's starts with the PDG numbers, but supplements them with more recently published results from Fermilab experiments E621 [4] and E731 [5], and from CERN experiment NA31 [6]. The recent result on η_{+-0} by the E621 Collaboration tests the standard model prediction that α_{+-0} should not contribute appreciably to the sum, and upholds that prediction. The uncertainties on α_{+-0} are now much smaller than those of α_{+-} and α_{00}. The measurement of $\eta_{+-\gamma}$ by the E731 Collaboration fills the blank in Table II. Because the branching ratio of $K_S \to \pi^+\pi^-$ quoted in PDG includes the $\pi^+\pi^-\gamma$ radiative decay and since, within uncertainties, $\eta_{+-\gamma} = \eta_{+-}$, $\alpha_{+-\gamma}$ is already included in α_{+-} to sufficient accuracy. We need not make any correction here.

Both the E731 and NA31 Collaborations have published accurate results on ϵ'/ϵ and on $\Delta\phi$. We have used these results to calculate η_{00} from the more precisely known η_{+-}. We used weighted averages of the E731 and NA31 values.

In addition, the E731 group used their data to measure $\Delta m, \tau_S, \Delta\phi$, and ϕ_{+-}. The determination of all these parameters by one experiment minimizes systematic errors. When fitting data on K_L-K_S interference in the $\pi^+\pi^-$ channel to determine Δm and ϕ_{+-} the variables are correlated. In the E731 publication [5] they recompute other groups' values of ϕ_{+-} using their own value of Δm, and find excellent agreement among the experiments. Our second method used the E731 values of Δm and τ_S. The agreement between $\alpha_{+-} + \alpha_{00}$ and $(1 + i\tan\phi_{\mathrm{sw}})\mathrm{Re}(\epsilon)$ is somewhat better here [7].

It is clear from our analysis that two experiments are needed to improve the uncertainties on the Bell-Steinberger relation. The $\Delta S = \Delta Q$ violation parameter x and CP-violation parameter η_{000} (in $K_S^0 \to \pi^0\pi^0\pi^0$) must be measured with smaller uncertainties. The value (and uncertainty) of $\mathrm{Re}(\alpha_{l3})$ is small because it is proportional to δ_l. But $\mathrm{Im}(\alpha_{l3}) \propto \mathrm{Im}(x)$ and its uncertainty

TABLE II. Values for each α_f, calculated from the 1992 Particle Data Group (PDG) compendium (center column), and also using more recent publications (right-hand column). Note that the values in the table have been scaled by a factor of 10^3.

α_f	10^3 PDG (1992) value	10^3 current value
α_{+-}	$(1.069 \pm 0.027) + i(1.131 \pm 0.027)$	$(1.152 \pm 0.030) + i(1.045 \pm 0.031)$
α_{00}	$(0.486 \pm 0.019) + i(0.514 \pm 0.018)$	$(0.536 \pm 0.017) + i(0.464 \pm 0.018)$
$\alpha_{+-\gamma}$		$(0.003 \pm 0.005) + i(0.010 \pm 0.002)$
α_{l3}	$(0.004 \pm 0.001) + i(0.007 \pm 0.059)$	$(0.004 \pm 0.001) + i(0.007 \pm 0.059)$
α_{+-0}	$(0.028 \pm 0.040) + i(-.036 \pm 0.057)$	$(0.001 \pm 0.001) + i(0.003 \pm 0.006)$
α_{000}	$(-.030 \pm 0.067) + i(0.019 \pm 0.101)$	$(-.030 \pm 0.067) + i(0.019 \pm 0.101)$
$\alpha_{+-} + \alpha_{00}$	$(1.555 \pm 0.033) + i(1.644 \pm 0.032)$	$(1.689 \pm 0.034) + i(1.510 \pm 0.036)$
$(1 + i\tan\phi_{\mathrm{sw}})\mathrm{Re}(\epsilon)$	$(1.635 \pm 0.060) + i(1.564 \pm 0.057)$	$(1.635 \pm 0.060) + i(1.546 \pm 0.057)$
Difference	$(-.080 \pm 0.069) + i(0.081 \pm 0.066)$	$(0.054 \pm 0.069) + i(-.037 \pm 0.068)$

is large compared to that of α_{+-} and α_{00}. The uncertainty in α_{000} is due to that of η_{000}, and is the largest of any of Table II. After these measurements are made, the largest uncertainty will be that of δ_l (known to 3.7%), which dominates the uncertainty in $(1 + i \tan \phi_{\rm sw})\mathrm{Re}(\epsilon)$ [because $\delta_l = 2\mathrm{Re}(\epsilon)$].

In summary, we have calculated the limits that can be put on CPT violation in decays of the neutral kaon using the Bell-Steinberger relation. The recent measurement of η_{+-0} by the E621 Collaboration reduces its contribution to the uncertainty in the Bell-Steinberger relation to an insignificant level. If reasonable assumptions are made about the size of x and η_{000} then the world's data are completely consistent with CPT conservation. A much better test could be made if x, η_{000}, and δ_l were measured with better accuracy.

We wish to thank A. Beretvas, T. Devlin, N. Grossman, K. Heller, M. J. Longo, and S. V. Somalwar for many interesting discussions. This work was supported by the National Science Foundation.

---

[1] J. S. Bell and J. Steinberger, in *Proceedings of the Oxford International Conference on Elementary Particles*, 1965, edited by R. G. Moorhouse, A. E. Taylor, and T. R. Walsh (Rutherford Laboratory, Chilton, England, 1966).

[2] V. V. Barmin *et al.*, Nucl. Phys. **B24**, 293 (1984).

[3] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992). The value of η_{+-0} used in preparing the center column of Table II is that published in M. Metcalf *et al.*, Phys. Lett. **40B**, 703 (1972). The η_{000} value is that of V. V. Barmin *et al.*, *ibid.* **128B**, 129 (1983). For both columns of Table II we calculated Re(ϵ) from the relation $\delta_l = 2\mathrm{Re}(\epsilon)$.

[4] The E621 Collaboration published a new limit on η_{+-0} in Y. Zou *et al.*, Phys. Lett. B **329**, 519 (1994). For the right-hand column of Table II, we used the fit they performed using the constraint, $\mathrm{Re}(\eta_{+-0}) = \mathrm{Re}(\epsilon)$. In Hikasa *et al.* [3], p. VII.89 there is an interesting note written by T. Nakada and L. Wolfenstein. They describe this constraint as being valid if CPT is conserved. If CPT were not conserved, then there are two ways $\mathrm{Re}(\eta_{+-0})$ could change. One is through what the authors of Ref. [2] called "superstrong" CPT violation. This effect would also make the K^+ lifetime be different from that of the K^-. The current data on the K^+-K^- lifetime difference require that the change in $\mathrm{Re}(\eta_{+-0})$ be less than $\sim 0.2\epsilon$. The other way is through Re(Δ). In *The Physics of Time Reversal* (University of Chicago, Chicago, 1987), p. 227, R. G. Sachs shows that $\Delta \leq 0.3\epsilon$. These limits are much smaller than the statistical uncertainty reported by Zou *et al.* so use of the constraint is justified.

[5] The E731 Collaboration published new measurements of $\Delta m, \tau_S, \phi_{+-}$, and $\Delta\phi = \phi_{00} - \phi_{+-}$ (and hence ϕ_{00}) in L. K. Gibbons *et al.*, Phys. Rev. Lett. **70**, 1199 (1993), a new measurement of ϵ'/ϵ in L. K. Gibbons *et al.*, *ibid.*, **70**, 1203 (1993), the first measurement of $\eta_{+-\gamma}$ in E. J. Ramberg *et al.*, *ibid.*, **70**, 2529 (1993), and $\mathrm{B}^{(S)}_{+-\gamma}$ in E. J. Ramberg *et al.*, Phys. Rev. Lett. **70**, 2525 (1993).

[6] The NA31 Collaboration published a new measurement of $\Delta\phi$ in R. Carosi *et al.*, Phys. Lett. B **237**, 303 (1990), and one of ϵ'/ϵ in G. D. Barr *et al.*, *ibid.* B **317**, 233 (1990).

[7] The NA31 group calculated a limit on the mass difference between the K^0 and the $\overline{K^0}$ in their paper by Carosi *et al.* [6]. This limit was $(m_{K^0} - m_{\overline{K^0}})/m_{K^0} < 5 \times 10^{-18}$ at 95% confidence level. With the complete set of E731 and NA31 results used here that limit now becomes $(m_{K^0} - m_{\overline{K^0}})/m_{K^0} < 3 \times 10^{-18}$ at 95% confidence level.