Multiparticle data in high energy hadronic interaction with nuclei and Koba-Nielsen-Olesen-Golokhvastov scaling

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In this Brief Report, multiplicity data of π-nucleus interactions are studied in the light of the newly proposed KNO-Golokhvastov scaling, the validity of which was previously reported only for e^+e^- and pp interactions. Furthermore, it is shown that the energy-independent scaling function can be described well by the log normal distribution. A comparison of the scaling function of the multiplicity distribution in π-nucleus interaction data with the e^+e^- and pp data is presented.

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A lot of what we know about the dynamics of multiparticle production comes from the study of multiplicity distributions [1]. Various methods have been suggested by different authors for describing the multiplicity distribution of the particles produced in high-energy interactions at ultrarelativistic energies. They are multiplicity scaling [2,3], the negative binomial distribution (NBD) [4], the two sources model [5], partially coherent laser distribution (PCLD) [6], the statistical boot strap model [7], the three fireball model [8], the modified two sources model [9], the sum of two NBD's [10], etc. In this connection, scaling properties of multiplicity distributions draw a great deal of attention because here the distributions of multiplicity data of various energies can be united in the same framework. The multiplicity data of different energies can be described by a single energy-independent scaling function and the parameters for such a function are fixed for all energies. Standard Koba-Nielsen-Olesen (KNO) scaling was derived from Feynman scaling for asymptotic energies [2]. Some experimental works reported [11,12] the validity of KNO scaling even at finite energies. In contrast, violation of KNO scaling to the UA5 data of $p\bar{p}$ collisions [13] at the CERN Collider has provoked a great deal of discussion. Moreover, KNO scaling is not self-consistent. This scaling has been redefined by Golokhvastov to remove the inconsistency in its mathematical formulation [3]. After Golokhvastov, a modified version of KNO scaling has been known as KNOG scaling. The e^+e^- and pp data have been reported to obey this KNOG scaling [14–16]. For universality of the scaling law, it is necessary at this stage to check the validity of the KNOG scaling for more complex processes such as hadron-nucleus and nucleus-nucleus interactions. Recently, a successful attempt was made to describe the energy-independent scaling function in the form of a log normal distribution [14,15,17,18], which is the most fundamental distribution encountered in nature. The KNOG scaling behavior and the log normal form of the scaling function can be explained by assuming a Polyakov scale-invariant branching process [19] as the mechanism of multiparticle production [17]. It is interesting to note that the intermittency phenomenon [20,21], which has created much controversy in recent years, can also be a manifestation of the same branching process. This probable correlation between the intermittency and log normal distribution prompts one to study this scaling behavior of the multiplicity distributions.

In this paper we try to verify KNOG scaling with a hadron probe. Our experimental data of the π^--Ag/Br interaction at 200 and 350 GeV/c have been used for the analysis. We have also tried to fit our data sets in the log normal form as in earlier ones. Details of the exposure of the plates, the identification of the tracks, and their angle measurement with respect to the beam axis have been described in our earlier papers [22,23]. To minimize the variation of the impact parameter, we selected those Ag/Br events for which the number of heavy tracks, N, lies within $10 \leq N \leq 14$. With this selection criteria, a sample of 484 and 457 events for 350 and 200 GeV/c π^- interactions was chosen for this present investigation.

According to KNO scaling, derived by Koba, Nielsen, and Olesen [2], the probability P of n secondaries to be produced in an interaction can be expressed in the form

$$P_n = \frac{1}{\langle n\rangle}\psi(n/\langle n\rangle) \;, \tag{1}$$

where $\langle n\rangle = \sum_{n=0}^{\infty} nP_n$ and $\psi(n/\langle n\rangle)$ is an energy-independent scaling function normalized by the equation

$$\int_0^\infty \psi(z)dz = \int_0^\infty z\psi(z)dz = 1, \quad z = n/\langle n\rangle \;.$$

This scaling is not self-consistent at any scale $\langle n\rangle$. Only for large $\langle n\rangle$ (i.e., energy) can the following conditions be written:

$$1 = \sum_{n=0}^{\infty} P_n = \sum_{n=0}^{\infty}\frac{1}{\langle n\rangle}\psi(n/\langle n\rangle) \simeq \int_0^\infty \psi(z)dz = 1 \;. \tag{2}$$

A successful generalization was introduced by Golokhvastov to remove the mathematical inconsistency in KNO scaling [3]. After this introduction, P_n can be redefined as

$$P_n = \int_n^{n+1} P(\tilde{n})d\tilde{n} = \int_{n/\langle\tilde{n}\rangle}^{(n+1)/\langle\tilde{n}\rangle} \psi(z)dz \ . \tag{3}$$

Here $P(\tilde{n})$ is the probability distribution of a continuous parameter (multiplicity) $\tilde{n}$, which can be written in the form

$$P(\tilde{n}) = \frac{1}{\langle\tilde{n}\rangle}\psi(\tilde{n}/\langle\tilde{n}\rangle) \ . \tag{4}$$

$\psi(z)$ is the energy-independent scaling function normalized by the equation

$$\int_0^\infty \psi(z)dz = \int_0^\infty z\psi(z)dz = 1, \quad z = \tilde{n}/\langle\tilde{n}\rangle \ .$$

The continuous average multiplicity can be approximated [16] as $\langle\tilde{n}\rangle = \langle n\rangle + 0.5$, for $\langle n\rangle \gtrsim 1$.

Then the normalization condition (2) is true for any scale $\langle n\rangle$. It was shown in Refs. [14,17] that if KNOG scaling holds, the multiplicity data plotted as $S_n(\sum_{i=n}^\infty P_i)$ versus reduced multiplicity $n/\langle\tilde{n}\rangle$ should overlap as a single function. From the definition of S_n, it has been shown [14] that

$$S_n = \sum_{i=n}^\infty P_i = \int_z^\infty \psi(z)dz = -\phi(z) \ , \tag{5}$$

where $\phi(z)$ is a primitive function of $\psi(z)$.

Now the probability can be calculated as

$$P_n = \int_{n/\langle\tilde{n}\rangle}^{(n+1)/\langle\tilde{n}\rangle} \psi(z)dz = \phi[(n+1)/\langle\tilde{n}\rangle] - \phi(n/\langle\tilde{n}\rangle) \ . \tag{6}$$

The above integral can be approximated [15] by the value at the middle of the integral range as

$$\begin{aligned} P_n &= \int_{n/\langle\tilde{n}\rangle}^{(n+1)/\langle\tilde{n}\rangle} \psi(z)dz \\ &= 1/\langle\tilde{n}\rangle\psi[(n+0.5)/\langle\tilde{n}\rangle] \\ &= 1/(\langle n\rangle + 0.5)\psi[(n+0.5)/(\langle n\rangle + 0.5)] \ . \end{aligned} \tag{7}$$

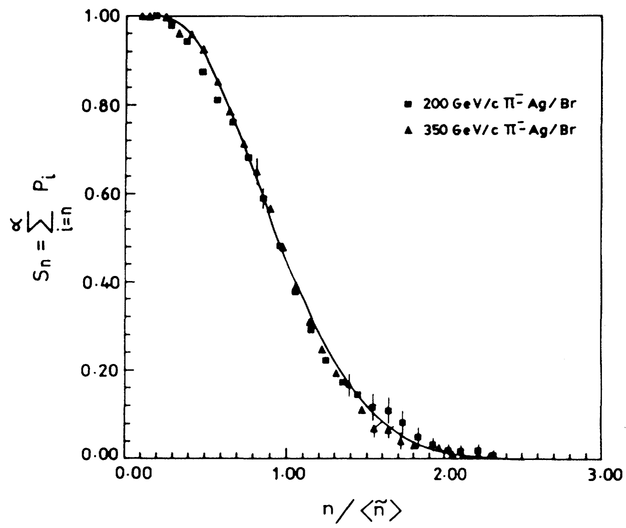

FIG. 1. Plot of $S_n = \sum_{i=n}^\infty P_i$ against $n/\langle\tilde{n}\rangle$ for data of π^--Ag/Br interactions at 200 and 350 GeV/c. The symbol represents the experimental points and the solid line indicates the theoretical curve.

It is interesting to note that the scaling function $\psi(z)$ for the e^+e^- and pp interactions was found to have a log normal form [14,15,17,18]. In this form the function $\psi(z)$ can be written as

$$\begin{aligned} \psi(z) &= (N/\sqrt{2\pi}\sigma)[1/(z+c)] \\ &\times \exp[-(\ln(z+c)-\mu)^2/2\sigma^2] \ . \end{aligned} \tag{8}$$

The parameters N, μ, c, and σ are constrained by the following two normalization conditions [18]:

$$\text{(I)} \quad \sum_{n=0}^\infty P_n = \int_0^\infty P(\tilde{n})d\tilde{n} = \int_0^\infty \psi(z)dz = \frac{N}{2}\text{erfc}[(\ln c - \mu)/\sqrt{2}\sigma] = 1 \ ,$$

where erfc stands for the complementary error function [24] and

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-t^2)dt \ .$$

$$\text{(II)} \quad \langle z\rangle = \int_0^\infty z\psi(z)dz = R\exp(\mu + \sigma^2/2) - c = 1 \ ,$$

TABLE I. Parameters c and D obtained by fitting the data of π^--Ag/Br interactions at 200 and 350 GeV/c in log normal form.

Data	Momentum (GeV/c)	c	D	χ^2	NDF	χ^2/NDF
π^-	200	0.87±0.80	0.47±0.01	14.66	19	0.77
π^-	350	0.75±0.62	0.40±0.03	16.0	20	0.80
All		0.87±0.15	0.465±0.005	32.00	41	0.78

TABLE II. Comparative study of KNOG and KNO scaling behaviors.

Scaling	NDF	χ^2	χ^2/NDF	Confidence level
KNOG	41	32.00	0.78	≈85%
KNO	38	36.5	0.96	≈60%

where $R = \text{erfc}(\nu - \sigma/\sqrt{2})/\text{erfc}(\nu)$ and $\nu = (\ln c - \mu)/\sqrt{2}\sigma$.

For small values of σ and c, the following approximations can be used [15]:

$$N \simeq 1 \text{ and } \exp(\mu + \sigma^2/2) - c = 1 \; . \tag{9}$$

The dispersion D of the scaling function $\psi(z)$ is equal to

$$D = \sqrt{\langle z^2\rangle - \langle z\rangle^2} = \exp[\mu + \sigma^2/2]\sqrt{\exp(\sigma^2) - 1} \; . \tag{10}$$

Thus, from the normalization conditions (I) and (II), it is seen that the log normal function $\psi(z)$ has only two free parameters. Since μ and σ are strongly correlated, it is therefore better to take the shift c and dispersion D of the scaling function as free parameters to be fitted by χ^2 minimization and then μ and σ may be calculated from Eqs. (9) and (10). Hence

$$\sigma = \sqrt{\ln\{[D/(1+c)]^2 + 1\}} \text{ and } \mu = \ln(c+1) - \sigma^2/2 \; . \tag{11}$$

Then values of c, μ, and σ can be substituted in Eq. (8) to obtain $\psi(z)$, and hence P_n can be calculated using Eq. (7).

In the way described above, we have analyzed our own data of the π^--Ag/Br interaction at 200 and 350 GeV/c. We have calculated the experimental value of $S_n = \sum_{i=n}^{\infty} P_i$ for the full phase space data at each energy separately. Figure 1 shows the plot of experimental values of S_n with reduced multiplicity $n/\langle\tilde{n}\rangle$ for the data of each energy. The closed square and closed triangle represent the experimental points of 200 and 350 GeV/c π^- initiated interactions, respectively. Typical statistical errors are shown at a few points. From Fig. 1 it is observed that the plot of S_n versus $n/\langle\tilde{n}\rangle$ for different energies, in general, speaks in favor of KNOG scaling. We have fitted each of the data sets separately and also

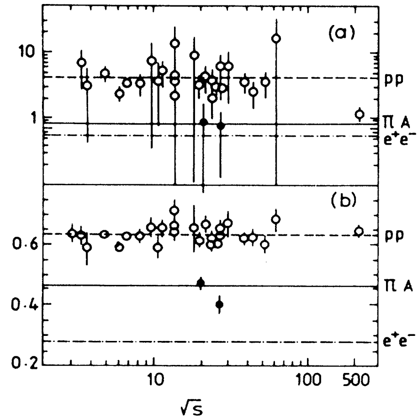

FIG. 2. Parameters c and D of the log normal distribution fitted to each of the data sets. The solid dots represent the values at each energy of our π-nucleus data and the horizontal line represents the corresponding values for simultaneous fitting of all data sets.

all data sets together in the form of a log normal distribution using Eq. (7). The MINUIT program of CERN is used for curve fitting. The resulting values of the parameters c, D, and also χ^2 are given in Table I. To get the theoretical curve, we have used the parameters obtained by simultaneous fitting of all the data sets. The solid line in Fig. 1 represents the theoretical curve. We have also tried to fit the multiplicity data in the KNO form using the usual $\psi(n/\langle n\rangle)$ function:

$$\psi(z) = (a_1 z + a_2 z^3 + a_3 z^5 + a_4 z^7)e^{-bz}, \quad z = n/\langle n\rangle \; .$$

To have a comparative study between the KNO and KNOG scaling behavior, we have presented the relevant parameters in Table II. The table shows that the level of confidence of fitting is about 85% for KNOG scaling whereas the confidence level falls to only about 60% in the case of KNO scaling. So the KNOG formulation provides us with a much better energy scaling law than KNO scaling.

The values of the parameters c and D from our π-nucleus data are plotted against $\sqrt{s}$ along with the parameter values of e^+e^- and pp data [18] in Fig. 2. In

TABLE III. Value of the parameters c and D for data sets of π-nucleus, lepton-lepton (e^+e^-) [15], and hadron-hadron (pp) [18] interactions.

Type of interaction	Data	Energy/momentum	c	D	χ^2	NDF	χ^2/NDF
π-Nucleus	π^--Ag/Br	200 GeV/c	0.87±0.15	0.465±0.005	32.00	41	0.78
	π^--Ag/Br	350 GeV/c					
Lepton-Lepton	e^+e^-	$\sqrt{s}$ = 7–91 GeV	0.56±0.03	0.277±0.001	208.55	282	0.74
Hadron-Hadron	pp	$\sqrt{s}$ = 3–62 GeV	4.25±0.20	0.629±0.003	458.49	316	1.45

this figure the solid dots represent the values at each energy of our πA data whereas the horizontal line is drawn through the value for all data sets. The figure suggests that the data of different energies can be described by a single log normal distribution with two fixed parameters.

The parameters obtained by simultaneous fitting of our π-nucleus full phase space data of all energies as well as the parameters of the lepton-lepton data (e^+e^-) [15] and hadron-hadron data (pp) [18] are presented in Table III. The shape of the scaling function for e^+e^-, pp, and our πA collisions are also compared in Fig. 3.

We end with the following conclusions.

(i) KNOG scaling is better supported by our multiplicity data of π-nucleus interactions at 200 and 350 GeV/c than KNO scaling.

(ii) The energy-independent scaling function has the log normal form, similar to the form obtained with e^+e^- and pp data. However, the values of the parameters are different in different types of interactions.

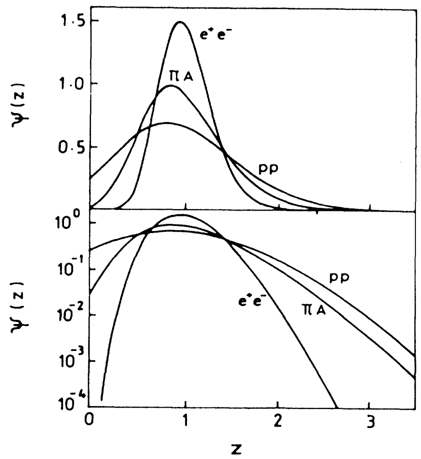

FIG. 3. Comparison of the shapes of the scaling function $\psi(z)$ for π-nucleus, lepton-lepton, and hadron-hadron interactions.

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