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Fermion mass predictions in a generalized extended technicolor scenario

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One family extended technicolor (ETC) models with technifermion mass spectra compatible with the experimental data for the precision parameters S, T, U, V, W, and X, and ETC interactions compatible with the CERN LEP measurements of the $Z \to b\bar{b}$ vertex have recently been proposed. To investigate whether these scenarios are consistent with the third family fermion masses we develop a generalized ETC model in which ETC interactions are represented by four-Fermi interactions. We discuss the reliability of the gap equation approximation to the nonperturbative dynamics. Two generic scenarios of couplings fit the precision data and third family masses: one is an unpredictive existence proof, the other, which generates the large top mass by direct top quark condensation, has a minimal number of interactions that break the global symmetry of the light fermions in the observed manner. This latter scenario makes surprisingly good predictions of the charm, strange, and up quark masses.

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I. INTRODUCTION

The holy grail of the next generation of accelerator experiments is a renormalizable, predictive model of the gauge boson and fermion masses that break electroweak symmetry. Models in which electroweak symmetry is broken by a condensate of strongly interacting fermions [1,2] are an enticing possibility since they appeal to the successes of the BCS theory of superconductivity and chiral symmetry breaking in QCD. While strongly interacting models such as technicolor [1] provide a simple explanation of electroweak symmetry breaking (EWSB) and the W and Z gauge boson masses, the diverse light-fermion masses are much harder to understand. In the past theorists have tended to concentrate on building models that extended the basic technicolor scenario [3–5] to include the light-fermion masses [extended technicolor (ETC) models] as an existence proof that technicolor models can generate the diverse spectrum observed. Many of these models [4], by virtue of being existence proofs, have been very complicated having at least as many free parameters as there are elements in the light-fermion mass matrices. The hope is that experimental discoveries will shed light on a simpler model along these lines which predicts some or all of the light-fermion masses.

Recent precision tests of particle interactions below the Z mass from experiments [6] at the CERN e^+e^- collider LEP and low-energy atomic measurements [7] have tightly constrained the parameters in the low-energy effective theory of the electroweak symmetry-breaking sector. The effects of particles heavier than $M_Z/2$ (which are integrated from the effective theory probed at LEP) on low-energy observables have been neatly summarized in terms of the parameters S, T, U, V, W, and X [8,9] as well as the deviation from the tree-level prediction for the process $Z \to b\bar{b}$ [10–12]. These new data have been used to rule out many of the ETC models constructed prior to LEP. Recent work [12–16] has concentrated on finding technifermion mass spectra and extended technicolor interactions that are consistent with the new precision data. The conclusion has been that ETC scenarios with light technifermions and ETC interactions broken above 10 GeV still provide valid existence proofs of "realistic" strongly interacting models of EWSB.

In this paper we wish to investigate whether the conclusions reached in Refs. [12–16] are compatible with an ETC model that correctly generates the third-family masses and whether such a model sheds light on the form of a simple predictive ETC scenario. In Sec. II we review the analysis of the precision data and the conclusion as to the form a realistic technifermion mass spectrum must take. In Sec. III we introduce a generalized form of ETC model with ETC interactions represented by four-Fermi interactions. We imagine that these operators will be generated by heavy ETC gauge boson exchange in a renormalizable model. However, given the lack of experimental data about such a sector and hence the large number of possible models we prefer to use the more generic four-Fermi interactions. In order to simplify the initial treatment we set the weak mixing angles to zero and concentrate on the charged leptons and quarks. The Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and neutrino mass generation are extremely model dependent; we stress that we make this approximation in order to generate generic statements about ETC. We discuss how, in principle, such a model could be predictive. To perform a numerical search of the parameter space of this model we must make some approximation to the full nonperturbative strong dynamics. We shall use the familiar gap equation approximation [17]. In Sec. IV, we review the successes and failures of the gap equation with some numerical examples. In Sec. V, we present two general scenarios of ETC model with technifermion and third-family mass spectra compatible with all available experimental data. One of these scenarios is entirely unpredictive while the other, which generates the large top mass by top condensation makes, when extended to include the first and second families of fermions, surprisingly good predictions for the up, charm, and strange quark masses. We present these predictions in Sec. VI. Finally, in Sec. VII, we conclude by discussing the implications of our generalized model for ETC model building and the need to extend the analysis to the neutrino sector and the CKM matrix elements.

II. PRECISION CONSTRAINTS ON ETC

Recent precision LEP data [6] and low-energy atomic physics measurements [7] provide stringent constraints on the physics responsible for EWSB. In this section we review these constraints and the results of Refs. [12–15] which suggest one-family ETC models compatible with these constraints may exist. We divide our discussion of these constraints into two types: oblique corrections and nonoblique corrections.

A. Oblique corrections

The major contributions to low-energy observables from fermions and scalars with masses greater than $M_Z/2$ occur at one loop as oblique corrections to gauge boson propagators [18]. These corrections have been parametrized by Peskin and Takeuchi [8] and by Burgess *et al.* [9] in terms of the six parameters S, T, U, V, W, and X. LEP's precision measurements have been performed on the Z mass resonance and hence the parameters associated with charged current interactions, U and W, are the least well constrained experimentally. A global fit [19] to the experimental data in which all six parameters S, T, U, V, W, and X are allowed to vary simultaneously gives the one standard deviation bounds

$$\begin{aligned} &S \sim -0.93 \pm 1.7, \quad V \sim 0.47 \pm 1.0 \ , \\ &T \sim -0.67 \pm 0.92, \quad X \sim 0.1 \pm 0.58 \ . \end{aligned} \tag{2.1}$$

If V and X both fall to zero the global fit to data for S and T is much more restrictive; the one standard deviation bounds are [19]

$$S \sim -0.5 \pm 0.6, \quad T \sim -0.3 \pm 0.6 \ . \tag{2.2}$$

References [13,14] have proposed that a one-family technicolor model is compatible with the data if the technifermion spectrum is of the form

$$\begin{aligned} &m_Q \sim \text{degenerate} \ , \\ &m_E \sim 150 - 250 \text{ GeV} \ , \\ &m_N \sim 50 - 100 \text{ GeV} \ . \end{aligned} \tag{2.3}$$

Treating the technifermions as weakly interacting doublets and calculating the contributions perturbatively we obtain $S \sim 0.09 N_{\rm TC}$, $T \sim 0.3 N_{\rm TC}$, $X \sim 0$, and $V \sim -(0.15 - 0.02) N_{\rm TC}$, where $N_{\rm TC}$ is the number of technicolors. Including the nonpertubative technicolor interactions is not possible directly but a number of authors [13–15,20] have constructed effective theories of strong interactions that have been used to estimate the resulting deviations from the perturbative results. The estimates away from the isospin-preserving limit are extrapolations based on perturbative results and effective-field theory results that agree with a scaled up version of QCD (the custodial isospin-preserving limit). The estimates of [13,14,16] per doublet are

$$\begin{aligned} S_{\rm weak} &< S_{\rm strong} < S_{\rm weak} + 0.05 \ , \\ T_{\rm weak} &< T_{\rm strong} < 2 T_{\rm weak} \ , \\ V_{\rm strong} &\sim V_{\rm weak} \ . \end{aligned} \tag{2.4}$$

We conclude that the contributions to precision parameters are enhanced somewhat due to the strong interactions. Nevertheless the technifermion spectrum in Eq. (2.3) is plausibly consistent with the experimental data for $N_{\rm TC} < 6$. Although such a spectrum may be consistent with the experimental data it is unclear whether a technicolor model compatible with both this spectrum and the need to generate the third-family masses exists. In Secs. III–V we shall propose a generalized ETC model and search its parameter space for a model satisfying both these requirements.

B. Nonoblique corrections

The ETC gauge bosons responsible for the light-fermion masses give rise to nonoblique corrections to fermion-antifermion production rates at LEP [10]. If the ETC interactions are orthogonal to the standard model gauge group then these nonoblique effects serve to correct the left-handed fermion couplings by

$$\delta g_L^{\rm ETC} \sim -\frac{1}{2} \frac{g_{\rm ETC}^2}{M_{\rm ETC}^2} F_\pi^2 \frac{e}{s_{\theta_W} c_{\theta_W}} I_3 \ , \tag{2.5}$$

where $g_{\rm ETC}$ and $M_{\rm ETC}$ are the ETC gauge boson coupling and mass, respectively, I_3 is the external fermion's weak isospin, and F_π is the electroweak symmetry-breaking scale. Only the coupling of the ETC gauge boson, $g_{\rm ETC}^2/M_{\rm ETC}^2$, that is responsible for the top quark's mass is sufficiently large for the experimental data to constrain. These nonoblique effects are potentially visible in the $Z \to b\bar{b}$ vertex, measured by the ratio of Z boson decay widths to $b\bar{b}$ over that to all non-$b\bar{b}$ hadronic final states [10]:

$$\Delta_R = \frac{\delta(\Gamma_b/\Gamma_{h\neq b})}{\Gamma_b/\Gamma_{h\neq b}} \sim \frac{2\delta g_L g_L}{g_L^2 + g_R^2} \ , \tag{2.6}$$

where $g_L = (e/s_\theta c_\theta)(-\frac{1}{2} + \frac{1}{3}s_\theta^2)$, $g_R = (e/s_\theta c_\theta)(\frac{1}{3}s_\theta^2)$.

If the top quark mass ($m_t > 130$ GeV) is generated by a perturbative ETC gauge boson (i.e., $g_{\rm ETC}^2 \sim 1$), then the ETC breaking scale must be of order 1 TeV. The ETC contribution to $\Delta_R \sim 4\%$ [10,11] is approximately double the maximum experimentally consistent value [6]. However, if the ETC coupling is allowed to rise to 40–

80 % of its critical coupling ($g_C^2 = 8\pi^2$) at a breaking scale of 10 TeV then a physical top mass can be obtained for a realistic value of Δ_R [12]. We shall, therefore, take the lightest ETC gauge boson to have mass $M_{\rm ETC} \sim 10$ TeV.

III. A GENERALIZED ONE-FAMILY ETC MODEL

We wish to study a range of ETC models without restricting to any particular scenario to investigate whether the technifermion spectrum of Eq. (2.3) and heavy ETC gauge bosons are compatible with the generation of the third-family masses. Since we have argued that the experimental constraints restrict models to an ETC breaking scale of 10 TeV or greater it will be a good approximation to model the ETC interactions by simple four-Fermi operators (we expect higher-dimensional operators to be sufficiently suppressed). Thus our general model will consist of an SU(N) technicolor group and all $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(3)_C$ invariant four-Fermi operators acting on a full technifamily (N, E, U^c, D^c: c is a color index). In addition we consider the third family of fermions which are technicolor singlets but interact with the technifermions by ETC interactions again modeled by all gauge-invariant four-Fermi operators. The technicolor group becomes strongly interacting at the scale $\Lambda_{\rm TC} \sim 1$ TeV forming technifermion condensates and breaking electroweak symmetry. We shall allow the ETC charges to vary over all possible values and search for a general scenario(s) of couplings compatible with the experimental data discussed in Sec. II and the third-family fermion masses. These solutions will hopefully provide a general basis from which to build more specific (renormalizable) models.

The ETC interactions in our model can be split into two categories, sideways and horizontal. Sideways interactions feed the technifermion condensates down to the light three families of fermions. There are four such operators connecting the technifermion and third family:

$$\begin{aligned} &\frac{g_{\nu 3}^2}{M_{\rm ETC}^2}\bar{\Psi}_L N_R \bar{\nu}_{\tau R}\psi_L \ , \\ &\frac{g_{\tau}^2}{M_{\rm ETC}^2}\bar{\Psi}_L E_R \bar{\tau}_R\psi_L \ , \\ &\frac{g_{t}^2}{M_{\rm ETC}^2}\bar{Q}_L U_R \bar{t}_R q_L \ , \\ &\frac{g_{b}^2}{M_{\rm ETC}^2}\bar{Q}_L D_R \bar{b}_R q_L \ , \end{aligned} \tag{3.1}$$

where $\Psi = (N, E)$, $\psi = (\nu_\tau, \tau)$, $Q = (U, D)$, and $q = (t, b)$. For readers who wish to have a renormalizable ETC model in mind these correspond to operators generated by breaking $\mathrm{SU}(N+1)_{\rm ETC} \to \mathrm{SU}(N)_{\rm TC}$+third family at the scale $M_{\rm ETC} \sim 10$ TeV.

Horizontal interactions correspond to technifermion and light-fermion self-interactions of the form

$$\frac{g_f^2}{M_{\rm ETC}^2}\bar{F}_L f_R \bar{f}_R F_L \ , \tag{3.2}$$

where F is the left-handed doublet containing the general fermion f and where there may, in general, be such an interaction for each fermion in the model. We might expect the third-family fermions and their respective technifermion counterparts to share quantum numbers and hence horizontal interactions. Our models will respect this constraint except when direct top condensation is investigated. Again the reader may envision that these interactions are generated at the scale $M_{\rm ETC}$ perhaps most naively by the breaking of an additional U(1) gauge group (allowing for the different fermions within a family to have different horizontal charges). We also note that all the four-Fermi operators will have charges below their critical couplings hence we may skip any discussion of the strong properties of isolated U(1) gauge interactions.

To simplify the initial analysis of this paper we shall neglect the discussion of the neutrino masses in the model since their masses do not fit any obvious pattern in relation to the other light-fermion masses. We effectively assume that there are no right-handed neutrinos though we maintain right handed technineutrinos. The precise mechanism for suppressing neutrino masses is extremely model dependent.

A realistic ETC scenario must agree with experimental data for m_τ, m_t, m_b, V, T, and S. The general model has 8 independent four-Fermi charges (12 if we allow the third family horizontal interactions to differ from their technicounterparts) and, therefore, we might expect solutions to exist compatible with the data. In Sec. V, we will verify that such a solution exists, however, it is clearly unpredictive.

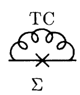
We might hope that a scenario of couplings that are more constrained and hence predictive might also be compatible with the experimental data. In addition to the search for models compatible with third-family masses we shall seek amongst such solutions those that can be further constrained. In this light it is interesting to propose the minimal model in principle capable of reproducing the fermion mass spectra. The EWSB scale must be set by the technicolor dynamics corresponding to the scale $\Lambda_{\rm TC}$ at which the technicolor group becomes strongly interacting. The third-family masses are suppressed relative to this scale by a factor of ~ 10. The minimal set of family symmetry-breaking interactions is a single sideways interaction for each of the third-family fermions (corresponding to $g_3 \equiv g_\tau = g_t = g_b$). The quarks are more massive than the leptons so we must break the symmetry between them by the addition of at least one extra interaction; we shall introduce a single horizontal interaction for the quarks ($g_Q \equiv g_D = g_U$). Finally the top quark is more massive than the bottom quark and hence there must be an additional interaction on the up-type quarks to break the symmetry between them; we introduce a single additional horizontal interaction for top-type quarks (g'_U). Thus, there must be a minimum of three ETC interactions in our model to break the global symmetries that would otherwise leave the light fermions degenerate. Such a model cannot therefore be predictive of the third-family fermion masses.

The first- and second-family ETC interactions may be trivially incorporated into the generalized model by the introduction of all gauge-invariant four-Fermi operators involving the fermions now in the model. The minimal symmetry-breaking constraint on these couplings would be to simply introduce a single additional sideways interaction connecting each family to the heavier families and for their horizontal interactions to be identical to their heavier partners. In Sec. VI, we shall make such a constrained addition of couplings to make light quark mass predictions.

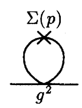
In addition we note that the CKM matrix elements only significantly vary from the identity for the first (lightest) family of fermions whose masses are generated by the weakest ETC operators. We conclude that quark mixings and CP violation are generated by those weak interactions and, therefore, in discussion of the heavier two generations of fermions we may neglect the CKM matrix elements. There is no clear understanding of the origin of the CKM matrix elements and hence we wish to neglect their generation in this discussion since we wish to make model-independent predictions. Making this approximation will clearly upset any predictions of the first-family masses which are associated with large mixings and indeed in Sec. VI we shall see this manifest.

IV. THE GAP EQUATION APPROXIMATION

Before we can discuss the success or failure of scenarios such as those discussed in Sec. III, we must have a reliable method of calculating physical quantities in strongly interacting theories. We make use of the familiar gap equation approximation [17,21]. The two gap equations [17] for the fermion self-energy from $\mathrm{SU}(N)$ gauge interactions (in the Landau gauge and with a running gauge coupling) and four-Fermi interactions, respectively, are

$$\Sigma(p) = $$


$$= \frac{3C(R)}{4\pi}\int_0^{\Lambda^2} \alpha(\mathrm{Max}(k^2,p^2))\frac{k^2dk^2}{\mathrm{Max}(k^2,p^2)}\frac{\Sigma(k)}{k^2+\Sigma^2(k)}\ , \tag{4.1}$$

$$\Sigma(p) = $$


$$= \frac{g^2}{8\pi^2\Lambda^2}\int_0^{\Lambda^2} k^2dk^2\frac{\Sigma(k)}{k^2+\Sigma^2(k)}\ , \tag{4.2}$$

where $C(R)$ is the Casimir operator of the fermion's representation of the gauge group, α the running gauge coupling, g the four-Fermi interaction strength, and Λ the UV cutoff.

Clearly, in the case of the gauge coupling, the precise value of the critical coupling and the form of the solution depend upon the form of the running of the coupling both in the high-momentum regime and in the nonperturbative regime. We shall make use of the parametrization

$$\begin{aligned} \alpha(q^2) &= \alpha_{\max},\quad q^2<\Lambda_{\mathrm{TC}}\ ,\\ \alpha(q^2) &= \frac{\alpha_{\mathrm{LAM}}}{1+\alpha_{\mathrm{LAM}}\beta\,\ln(q/\Lambda_{\mathrm{TC}})},\quad q^2>\Lambda_{\mathrm{TC}}\ , \end{aligned} \tag{4.3}$$

and allow $\alpha_{\max}$, α_{LAM}, and β to vary over a range of plausible values in order to check the gap equation results for numerical stability.

The scale Λ_{TC} is determined by requiring the correct Z mass which is given by the technipion decay constant, F_{π^3}, which we estimate by the dynamical perturbation theory (DPT) result [22,23]

$$F_{\pi^3} = \frac{N_{\mathrm{TC}}}{32\pi^2}\int_0^{\Lambda^2} dk^2k^2\left(\frac{\Sigma_U^2-k^2(\Sigma_U^2)'/4}{(k^2+\Sigma_U^2)^2}+U\leftrightarrow D\right)\ , \tag{4.4}$$

$$F_{\pi^\pm} = \frac{N_{\mathrm{TC}}}{32\pi^2}\int_0^{\Lambda^2} dk^2k^2\frac{F(k^2)}{(k^2+\Sigma_U^2)(k^2+\Sigma_D^2)}\ , \tag{4.5}$$

$$\begin{aligned} F(k^2) = {} & (\Sigma_U^2+\Sigma_D^2)-\tfrac{1}{4}k^2(\Sigma_U^2+\Sigma_D^2)'-\tfrac{1}{8}[(\Sigma_U-\Sigma_D)^2]'-\tfrac{1}{4}(\Sigma_U-\Sigma_D)(\Sigma_U+\Sigma_D)'(\Sigma_U'\Sigma_D-\Sigma_U\Sigma_D')\\ & +[\tfrac{1}{2}k^2(\Sigma_U^2-\Sigma_D^2)-\tfrac{1}{4}k^2(k^2-\Sigma_U\Sigma_D)(\Sigma_U-\Sigma_D)(\Sigma_U+\Sigma_D)']\left(\frac{1+(\Sigma_U^2)'}{k^2+\Sigma_U^2}-\frac{1+(\Sigma_D^2)'}{k^2+\Sigma_D^2}\right)\ , \end{aligned} \tag{4.6}$$

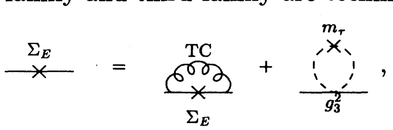
where Σ_U and Σ_D are the self-energies of the fermions and the prime indicates the derivative with respect to k^2. For simplicity we neglect the mass splitting within the lepton doublet in the calculation of the F_π's; since the Z mass is dominated by the techniquark contribution to F_{π^3} this will introduce only small errors and allows us to avoid the complication of specifying the neutrino sector. The three four-Fermi couplings are determined, for a given value of M_{ETC}, by requiring that the correct τ, top, and bottom masses are obtained as solutions. We tune to two significant figures in the fermion masses and use $m_t \sim 170$ GeV as a representative value. We cut the integrals off at $M_{\rm ETC}$.

We seek solutions that are compatible with the mass spectrum in Eq. (2.3). We measure the degeneracy of the techni-up and -down quarks by their contribution to the T parameter calculated in DPT:

$$T_Q = \frac{1.37}{F^2_{\pi^3_Q}}(F^2_{\pi^\pm_Q} - F^2_{\pi^3_Q}) \; . \tag{4.7}$$

A solution consistent with Eq. (2.3) must have a contribution T_Q of at most a few tenths. Since we are neglecting the mechanism that determines the neutrino sector masses we shall simply display the technielectron mass [given by the condition $M_E = \Sigma(M_E)$] for comparison with that in Eq. (2.3).

In order to investigate the consistency of solutions within the gap equation approximation let us consider the minimal predictive set of couplings in the ETC model proposed in Sec. III. The gap equations for the technifamily and third family are technielectron,



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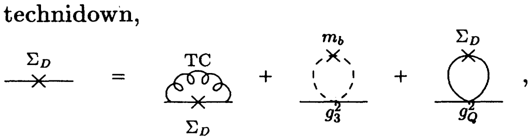

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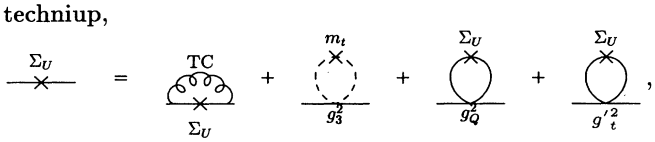

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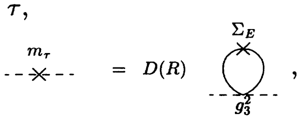

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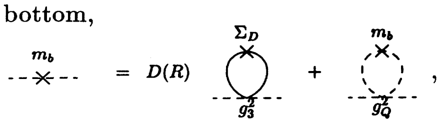

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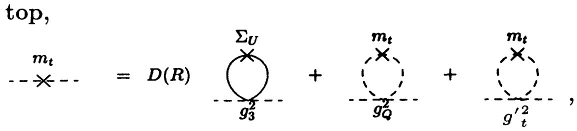


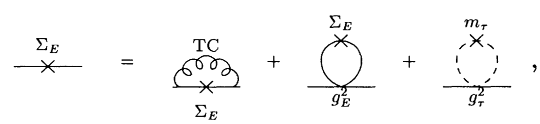
where $D(R)$ is the dimension of the technifermions' representation under the technicolor group. Numerical solutions to these equations are presented in Table I for a range of values of $\alpha_{\rm max}$, $\alpha_{\rm LAM}$, β, $M_{\rm ETC}$, and $N_{\rm TC}$. The form of the solutions is insensitive to the variation of parameters at least up to errors of at most order 1. The precision electroweak parameters are, however, sensitive to errors of this magnitude. The quark contribution to the T parameter is a measure of 1% differences between our calculated values of $F^Q_{\pi^3}$ and $F^Q_{\pi^\pm}$ and vary between $T_Q = 8.9$ and $T_Q = 24.2$. Similarly we have argued that to achieve a realistic value of S and V we require the technielectron mass [determined by the condition $\Sigma(M_E) = M_E$] to lie in the range 150–250 GeV. The calculated value of M_E given in Table I shows large variation, $M_E = 90$–260 GeV. We shall only be able to argue about the gross features of the technifermion spectrum and on where these are compatible with the realistic mass spectra in Eq. (2.10).

V. SUCCESSFUL SCENARIOS

We have searched the full parameter space of the generalized model for coupling scenarios compatible with the third-family masses and the technifermion spectrum in Eq. (2.3). In this section we present the two successful scenarios, one model is completely unpredictive the other is a variation on the minimal predictive model with direct top condensation.

A. An existence proof

In principle the ETC couplings in the generalized ETC model described in Sec. III need not be related and we obtain the gap equations technielectron,



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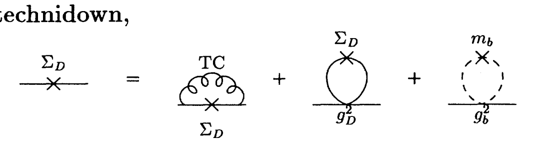

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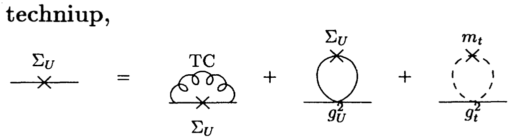

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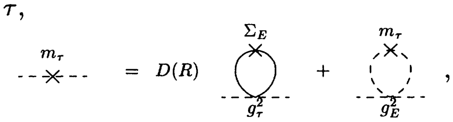

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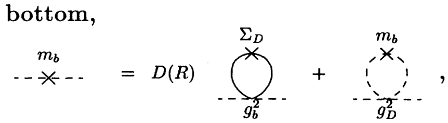

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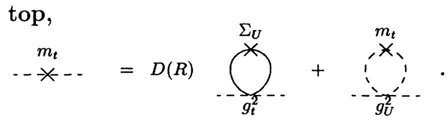


TABLE I. Numerical solutions of gap equations in Eq. (2.3) and values of the couplings and scales used. $\alpha_{\rm max}$ and $\alpha_{\rm LAM}$ are expressed in terms of the critical coupling in the fixed point theory $[\alpha_C = \pi/3C(R)]$. The four-Fermi couplings are given as percentages of the critical coupluing ($g_C^2 = 8\pi^2$). T_Q is the contribution to T from the techniquarks. Solutions are obtained by tuning parameters to give the correct Z mass, m_τ, m_b, and m_t.

$N_{\rm TC}$	$\frac{\alpha_{\rm max}}{\alpha_C}$	$\frac{\alpha_{\rm LAM}}{\alpha_C}$	β	$M_{\rm ETC}$/TeV	$\Lambda_{\rm TC}$/TeV	g_3/g_C (%)	g_Q/g_C (%)	g_t/g_C (%)	T_Q	M_E/GeV
3	2	2	1.00	10	0.60	40.0	51.8	52.66	16.4	170
6	2	2	1.00	10	0.20	13.4	33.2	35.1	20.4	90
3	2	2	0.75	10	0.50	41.6	49.5	50.1	17.5	160
3	2	2	0.50	10	0.35	45.0	44.5	46.1	19.5	140
3	2	2	0.22	10	0.10	52.9	29.3	36.0	24.2	90
3	2	2	1.00	5	0.52	30.4	50.64	60.8	19.0	150
3	2	2	1.00	50	0.49	70.7	17.8	14.5	15.8	140
3	3	1	1.00	10	0.60	36.0	55.1	57.7	8.9	260
3	1.5	1.5	1.00	10	1.10	48.4	38.3	47.8	22.9	100

The top and bottom quark masses within this general model are determined by their separate sideways interactions. Although the top and bottom masses feed back into the technifermions' self-energies tending to enhance the techni-up self-energy it is clear that the separate horizontal interactions on the top- and bottom-type quarks can be used to enhance the technibottom self-energy to oppose this custodial SU(2)-violating effect. We can tune a set of couplings to give $T_Q = 0$ and which correctly describe the Z, τ, top, and bottom masses, e.g., a scenario with $g_E = g_U = 0$:

$N_{\rm TC}$	$\frac{\alpha_{\rm max}}{\alpha_C}$	$\frac{\alpha_{\rm LAM}}{\alpha_C}$	β	$M_{\rm ETC}$	$\Lambda_{\rm TC}$	g_τ/g_C%	g_b/g_C%	g_t/g_C%	g_D/g_C%	T_Q
3	2.0	2.0	1.00	10	0.5	48.4	5.9	70.1	85.5	0.0

which give the technifermion masses

$$M_U \sim 400 \text{ GeV}, \quad M_D \sim 400 \text{ GeV}, \quad M_E \sim 140 \text{ GeV} . \tag{5.2}$$

Such a scenario is consistent with the technifermion mass spectrum in Eq. (2.3) and hence with all available experimental data. The renormalizable models of [4] can give rise to precisely this spectrum of ETC interactions, however, the degeneracy of the techniquarks (and hence the low-T parameter) arises from a conspiracy in the four-Fermi couplings which seems unnatural. Nevertheless this scenario does provide an existence proof for ETC models.

B. Direct top condensation

The minimal predictive model of Eq. (4.8) fails because the techniup self-energy must be enhanced by too much relative to the technidown in order to generate the top bottom mass splitting. Recently there has been much discussion in the literature of direct top condensation [2] giving rise to the large top mass. If the condensation occurs at a scale close to the EWSB scale (ν) then the top mass must be large ($\sim$ 400 GeV) to generate the entire Z mass on its own. The top mass may be lowered by raising the scale Λ of the condensating interaction but since the theory naturally produces a top mass of order Λ this can only be achieved at the expense of fine-tuning the associated coupling by ν/Λ. In models with a large scale Λ, below Λ the model may be approximated by an effective light scalar theory with coupling constant fixed points and, therefore, it is possible to make very precise predictions of the top and scalar mass. The top mass predictions are excluded by the T parameter measurements. When Λ is close to the EWSB scale the fine-tuning is reduced but the theory does not give rise to a light scalar and we must rely on gap equation approximations to calculate the details of the theory with the corresponding loss of precision. If the top condensate is not the sole source of EWS breaking then the vacuum expectation value of the condensate may be lowered and hence a lower top mass generated. We can construct such an ETC model with top condensation simply by removing the horizontal interaction on the techni-up quark in the minimal predictive model [we set $g'_U = 0$ in Eq. (4.8) and introduce a new self-interaction for the top only of the form $g_{tc}^2/M_{\rm ETC}^2 \bar{\Psi}_L t_R \bar{t}_R \Psi_L$]. Since the large top mass is no longer generated by the sideways ETC interactions there is less constraint upon the ETC breaking scale, $M_{\rm ETC}$, from the $Z \to b\bar{b}$ vertex measurements. We shall allow $M_{\rm ETC}$ to fall to 5 TeV. We discuss the degree of tuning induced in the conclusions.

In Table II, we show some solutions for this scenario and their predictions for the contribution to the T parameter from the techniquarks.

The solutions with a low ETC scale seem consistent with the technifermion mass spectrum proposed in Sec. II though the technielectron mass is somewhat high. Within the gap equation approximation it is certainly not possible to discount this scenario so we shall consider it

TABLE II. Numerical values of the couplings and scales of solutions for the top condensating scenario of couplings. The four-Fermi couplings are given as percentages of the critical coupling $(g_C^2/8\pi^2)$. T_Q is the contribution to T from the techniquarks. Solutions are obtained by tuning parameters to give the correct Z mass, m_τ, m_b, and m_t.

$N_{\rm TC}$	$\frac{\alpha_{\rm max}}{\alpha_C}$	$\frac{\alpha_{\rm LAM}}{\alpha_C}$	β	$M_{\rm ETC}$/TeV	$\Lambda_{\rm TC}$/TeV	g_3/g_C (%)	g_Q/g_C (%)	g_{tc}/g_C (%)	T_Q	M_E/GeV
3	2	2	1.00	10	1.10	20.3	51.2	83.8	0.71	320
3	1.5	1.5	1.00	10	2.65	21.4	39.4	89.1	2.92	225
3	3	1	0.95	10	0.95	20.3	58.0	79.9	0.20	410
3	2	2	0.50	10	0.78	19.7	47.3	86.1	1.00	300
6	2	2	1.00	10	0.45	24.2	48.4	82.4	5.08	185
3	2	2	1.00	5.	1.07	13.5	47.3	87.1	0.09	300
4	2	2	1.00	5.	0.85	12.4	47.3	87.2	0.11	270
5	2	2	1.00	5.	0.65	13.5	46.1	87.8	0.24	230

a successful ETC model. This maximally constrained set of couplings is a subset of solutions to the full generalized model of this form. Similar solutions exist with, for example, different sideways couplings for each fermion but where T_Q is kept small because the large top mass is the result of a large value for g_{tc}. We present results for the maximally constrained scenario of couplings since we consider it of interest that such a scenario of couplings is compatible with the experimental data and because it naturally gives fermion mass predictions when extended to the lighter two families.

VI. QUARK MASS POSTDICTIONS

We have argued in Sec. III that a model of EWSB and the third-family masses (excluding neutrinos) must have at least four couplings and hence cannot be "postdictive" of the third-family masses. However, it is conceivable that only one additional parameter need be added to generate the second-family masses (a parameter that suppresses the second-family masses relative to the third) since quark lepton and custodial isospin symmetry breaking already exist in the model. Similarly one additional parameter might suffice to suppress the first-family masses below the second but of course our neglection of the CKM matrix elements which are substantial for the first family makes this seem less likely to be successful. In this section we investigate the possibility of such postdiction in the scenarios we have discussed above.

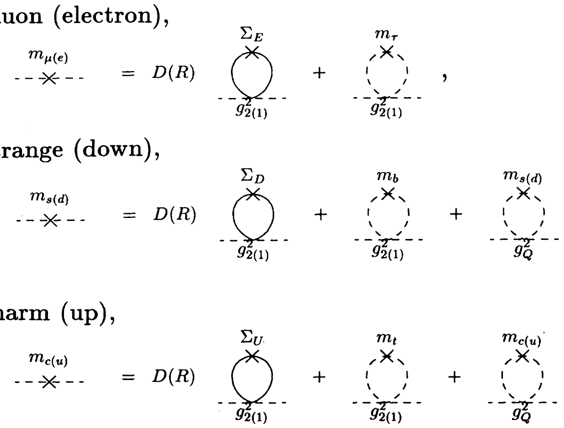
The "existence proof" scenario does not lend itself to postdiction since to follow the pattern of the model of the third-family masses we could simply introduce additional sideways interactions for each new light fermion sufficient to generate their mass. There are no constraints on the couplings so they are unpredictive. The first- and second-family masses are at least 2 orders of magnitude smaller than the technifermion masses and hence any feedback of the light two families masses into the technifermion self-energies are negligible and do not upset our calculations of S and T. Although unpredictive the scenario still provides an existence proof of a realistic ETC model.

The top condensation scenario however is potentially predictive as described above. We introduce the additional sideways interactions

muon (electron),

strange (down),

charm (up),



(6.1)

which we would expect to be generated if there was a single breaking scale associated with each of the first and second families in the breaking of $SU(N+3)_{\rm ETC} \to SU(N)_{\rm TC}$+three families. Again the feedback of the first- and second-family masses to the technifermions and third family are negligible. We set the coupling strength of the new sideways interaction by requiring that we generate the correct muon and electron masses. The up, down, charm, and strange quark masses are now predictions of the model. Explicitly,

$$\begin{aligned} &\Lambda_{\rm TC} \text{ determined by } M_Z, \quad g_t \text{ determined by } m_t, \\ &g_3 \text{ determined by } m_\tau, \quad g_2 \text{ determined by } m_\mu\ , \\ &g_Q \text{ determined by } m_b, \quad g_1 \text{ determined by } m_e\ . \end{aligned} \tag{6.2}$$

Although the predictions of the model are clearcut, our ability to calculate is limited as discussed in Sec. IV. The gap equation solutions are, however, moderately well bounded since the integrals over the technifermion's self-energies are fixed to a good degree by the imposed requirements that they correctly give the Z, τ, bottom, and top masses. We shall quote the range of predictions from all the coupling values in Table II as an estimate of our theoretical errors. We obtain

$$\begin{aligned} m_c &= 1.5 \pm 0.8 \text{ GeV}, \quad m_s = 0.32 \pm 0.02 \text{ GeV}\ , \\ m_u &= 6.6 \pm 3.7 \text{ MeV}, \quad m_d = 1.5 \pm 0.2 \text{ MeV}\ . \end{aligned} \tag{6.3}$$

We immediately notice that these predictions are in surprisingly good agreement with the observed mass spectra except for the down quark. The failure to predict the down quark mass however is to be expected since we have neglected the generation of the CKM matrix which has large elements for the first family. Conservatively we can conclude that ETC models with the minimal number of ETC interactions that are sufficient to break the global symmetry of the light fermions in the observed pattern seem capable of reproducing the pattern of the observed light-fermion mass spectrum.

VII. CONCLUSIONS

The precision data from LEP [6] have provided tight constraints on the form of models of EWSB. It has been argued [13,14] that technicolor models with a single technifamily with a light technineutrino and degenerate techniquarks give contributions to the S, T, and V parameters that lie within the experimentally allowed bands. If the top mass is generated by strong ETC interactions broken above 10 TeV then the model will lie within the experimental limits on nonoblique corrections to the $Zb\bar{b}$ vertex as well [12]. As a first step towards a fully renormalizable, predictive model of EWSB we have investigated whether an ETC model can be compatible both with the precision data and the light-fermion masses. To make this investigation we have used a generalized one-family ETC model in which the ETC interactions are represented by four-Fermi interactions.

To calculate within this generalized model we have used the gap equation approximation to the Schwinger-Dyson equations. Unfortunately, even within the gap equation approximation the solutions for the technifermions self-energies $\Sigma(k^2)$ are dependent on the precise form of the running of the technicolor coupling. The technicolor dynamics are fixed to some degree by the requirement that the model gives rise to the correct Z boson mass (given by an integral equation over the self-energies). Calculation of the light-fermion masses (also given by integral equations over the self-energies) are, therefore, moderately stable. However, the precision electroweak variables are very sensitive to shifts in, for example, $\Sigma(0)$ and are hence less well determined. Nevertheless we have argued that couplings exist in the generalized ETC model that very plausibly fit the experimental constraints.

Two scenarios in the generalized ETC model have been found consistent with the precision data and the third-family fermion masses. The first is an unpredictive existence proof in which sufficient ETC couplings are included that the fermion mass spectra may be tuned to match the data. The second scenario contains what we have argued is the minimum number of different strength ETC interactions required to break the global symmetry on the third family in the observed pattern. This model achieves a sufficiently large top mass by direct top condensation.

In order to obtain a large top mass in these models the ETC interactions must be tuned close to their critical values. The "fine-tuning" is at worst of order 10%, corresponding in our results to our need to quote ETC couplings to three significant figures in order to tune to two significant figures in the light-fermion masses. In fact the tuning is only this severe for the ETC couplings that generate the top mass. This degree of tuning may not be unnatural since gauge couplings naturally run between their critical value g_C and $\sim 0.1 g_C$ over many orders of magnitude of momentum. Clearly any greater degree of fine tuning which, for example, would be associated with significantly increasing $\Lambda_{\rm ETC}$, would be unsatisfactory. The authors of [24] have argued that when ETC interactions grow close to their critical values there will be light (relative to $M_{\rm ETC}$), scalar, ETC bound states of the light fermions (corresponding to the light scalar found in the usual top condensate scenario). These bound-state masses will fall to $\sim 2m_f$, where m_f is the mass of the consistent fermion, as the ETC interactions grow to their critical values. The strongest ETC interactions in our models above $\sim$ 80–90 % of g_C, which will presumably give rise to the lightest scalar spectrum, are associated with the top mass generation. We, therefore, expect the lightest such scalar to have a mass $>$ 100 GeV.

The top condensing scenario may be minimally extended to the first and second families. The model then makes predictions for the up, down, charm, and strange quark masses. Our calculation of these masses shows that the charm, strange, and up quark mass predictions are consistent (up to errors due to uncertainty in the gap equation approximation) with the experimental values. The model does no reproduce the up-down mass inversion observed in nature but we have argued that this is the result of our neglection of the mechanism for the generation of the CKM matrix which has large elements for the first-family quarks. In addition we have neglected a discussion of the neutrino sector since their masses do not fit any obvious pattern in the fermion mass spectra. In this paper we have concentrated on predictions which are potentially generic to ETC models. Clearly it would be of interest to continue the analysis to models of neutrino masses and the CKM matrix but such analysis would only serve to confuse the cleaner model of quarks and charged leptons.

Hopefully the successes of the generalized ETC model here will be translatable to a renormalizable ETC model. In this respect the proposal in [17] that the quark lepton mass splittings may result from QCD interactions, corresponding to $g_Q^2 \to \alpha_{\rm QCD}$ in the top condensate scenario, is appealing. At the EWSB scale $\alpha_{\rm QCD}(M_Z^2)/\alpha_{\rm QCD}^{\rm crit} \sim$ 15%. Our analysis suggests (see Table II) that g_Q/g_C needs to be of order 50% however. The value of g_Q/g_C can be reduced (see Table I) by increasing the maximum value the technicolor coupling reaches in the nonperturbative regime, or by increasing $N_{\rm TC}$ or $\Lambda_{\rm ETC}$ or finally by decreasing the technicolor β function towards a walking value. Unfortunately each of these changes tends to increase the T parameter contribution from the techniquarks. The uncertaintites in the gap equation analysis though does not preclude the possibility.

We conclude that our unpredictive model provides an existence proof that ETC models exist which satisfy the stringent precision measurement bounds. The scenario with direct top condensation provides the tantalizing possibility that ETC models can be constructed that are predictive.

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